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FINITENESS CONDITIONS AND GAUGE SYMMETRY BREAKDOWN IN $N = 2$ SUPERSYMMETRIC MODELS*

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ABSTRACT

In the framework of $N = 2$ supersymmetric models, with matter representations chosen to ensure a vanishing β function, we work out the relations between soft $N = 1$ supersymmetric and/or supersymmetry breaking terms which preserve finiteness. We show explicitly in an $SU(2)$ example that these very simple conditions are supple enough to allow for a spontaneous symmetry breaking of the gauge group, with a potential bounded from below.

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1. Introduction

Recently the class of finite, globally supersymmetric theories has been enlarged by coupling the $N = 2$ supersymmetric gauge multiplet corresponding to the adjoint representation of a gauge group G to a certain number of $N = 2$ "matter" hypermultiplets so as to obtain a vanishing one loop β–function.[1] Finiteness is then insured to all orders by the $N = 2$ nonrenormalization theorems,[2] which rely on the $N = 2$ unconstrained superfield formulation of gauge[3] and matter[4] supermultiplets interactions.

In this paper we will analyze certain features related to the possibility of embedding realistic theories in such finite models.

For such a task one must first break the $N = 2$ supersymmetry softly, i.e., so that finiteness is preserved.[5] Subsequently, one must spontaneously break the gauge group (ideally in such a way that only one massless vector boson remains). In the process one must be able to lift the degeneracy of the particles mass spectra. Again, in a realistic theory, we must demand that the free parameters and symmetry breaking mechanism should allow us to give large masses to the unwanted particles so as to decouple them from the low energy sectors. In this way, the heavy mass particles become the natural physical cutoff of the present day low energy physics. This is a strong motivation for having a finite field theory.

Given the gauge group and the representation content corresponding to a finite model, the hard part of the Lagrangian is uniquely fixed. This is due to the absence of the $N = 2$ invariant couplings between the matter hypermultiplets.[6] They interact only with the gauge supermultiplet. One then has to turn to the soft part of the Lagrangian to find the free parameters which allow an explicit breaking of $N = 2$ supersymmetry and a spontaneous breaking of the gauge group.

In this paper we will restrict ourselves to a simple model invariant under $N = 2$

supersymmetry with an SU(2) gauge group.

We will work out in this model the most general soft breakings of $N = 2$ supersymmetry by looking at the finiteness condition of the effective potential computed at the one loop level.

This is a component computation which is not manifestly supersymmetric. However, it has the advantage of preserving at all stages the covariance under the global SU(2) group associated with the $N = 2$ supersymmetry algebra. By working in this nonsupersymmetric gauge we will encounter only a wave function renormalization, the finiteness condition being the equality of bare and renormalized physical parameters.

Subsequently we will analyze the breaking of the gauge group for this particular model of the interaction of $N = 2$ supersymmetric gauge multiplet with four matter hypermultiplets (flavors).

As we shall see the straightforward procedure of breaking the gauge group does not seem to work. Adding negative mass terms in certain scalar fields which conventionally forces the spontaneous symmetry breaking may lead to a potential which is unbounded fom below, because the hard part of the Higgs potential has null directions in the field space. Along these directions the negative mass terms make the potential decrease indefinitely.

In Section 2 we present the most general renormalizable $N = 2$ supersymmetric Lagrangian. In Section 3 we analyze the soft breakings of supersymmetry. In Section 4 we discuss the gauge breaking pattern and the particle spectra. The last section is devoted to conclusions.

2. N=2 Supersymmetric Lagrangian

The $N = 2$ supersymemtry algebra

$$\begin{aligned} \left\{Q^i_\alpha, Q^j_\beta\right\} &= \epsilon_{\alpha\beta}\, g^{ij}\, Z \\ \left\{Q^i_\alpha, \bar{Q}_{\dot{\beta}j}\right\} &= 2(\sigma_\mu)_{\alpha\dot{\beta}}\, P^\mu \delta^i_j \end{aligned} \tag{2.1}$$

has the following representations of interest with spin ≤ 1:

(a) hypermultiplets—with the spin content 2(1/2), 4(0) which can be massive if Z—the central charge—is nontrivially realized;

(b) $N = 2$ gauge multiplet which contains massless particles with the spin content 1(1), 2(1/2), 2(0).

If one enlarges the algebra, Eq. (2.1), with generators which rotate spinor charges, and form an SU(2) algebra, then one can have the following SU(2) assignments for case (a)$(1/2)_i$, $(\vec{0})$, (0) or 2(1/2), $2(0)_i$ (the isospinor index i = 1,2) and for case (b) 1(1), $(1/2)_i$, 2(0).

It turns out that if the $N = 1$ superfield Lagrangian corresponding to the multiplets (a) and (b) and invariant under $N = 2$ supersymmetry algebra is expressed in component fields, it acquires an additional global SU(2) invariance corresponding to the above assignment of fields in SU(2) representations.[7]

One can describe the $N = 2$ multiplets in terms of N=1 superfields. For describing the matter hypermultiplets one must consider two chiral superfields which transform in an opposite way with respect to the gauge group

$$\phi_1 \rightarrow e^{2ig\Lambda}\phi_1 \;\text{ and }\; \phi_2 \rightarrow e^{-2ig\Lambda}\phi_2 \tag{2.2}$$

where Λ is a Lie algebra valued chiral superfield.

The $N = 2$ gauge multiplet can be expressed in terms of a scalar superfield $V(x, \theta, \bar{\theta})$ and a chiral superfield $\chi(x, \theta)$in the adjoint representation of G, with transformation laws:

$$e^{2gV} \to e^{2ig\Lambda^{+}} e^{2gV} e^{-2ig\Lambda} \tag{2.3}$$

and

$$\chi \to e^{2ig\Lambda} \chi e^{-2ig\Lambda} \quad . \tag{2.4}$$

In terms of N=1 superfields, the most general renormalizable, N=2 invariant Lagrangian which describes the interaction of the N=2 gauge supermultiplet with an arbitrary number of flavors ϕ_1^m, ϕ_2^m is (m–flavor index which also denotes representations under the gauge group):

$$\begin{aligned} L =&\phi_1^{+m} e^{2gV} \phi_1^m + \phi_2^m e^{-2gV} \phi_2^{+m} + \frac{1}{k} \text{ Tr } \bar{\chi} e^{2gV} \chi e^{-2gV} \\ &+ \left[\frac{1}{4k(2g)^2} \text{ Tr } W^\alpha W_\alpha - m_m \phi_2^m \phi_1^m + ig\sqrt{2}\, \phi_2^m \chi \phi_1^m + \text{h.c.}\right] \quad , \end{aligned} \tag{2.5}$$

where Tr $t_a t_b = k\delta_{ab}$, $V = t_a V_a$, $\chi = t^a \chi_a$, $W_\alpha = -\frac{1}{4} \bar{D} \bar{D} e^{-2gV} D_\alpha e^{2gV}$. This

Lagrangian can be expressed in component fields

$$
\begin{aligned}
L = & \frac{1}{2} D^2 - \frac{1}{4} V^a_{\mu\nu} V^{a\mu\nu} - i \lambda_a \not{D} \bar{\lambda}_a \\
& + F_1^{m+} F_1^m + F_2^m F_2^{m+} + F_a^+ F_a \\
& + A_1^m D D A_1^m + A_2^m D D A_2^{m+} + M_a^+ D D M_a \\
& - i \bar{\psi}_1^m \tilde{\not{D}} \psi_1^m - i \psi_2 \not{D} \bar{\psi}_2^m - i \bar{\lambda}_2^a \tilde{\not{D}} \lambda_2^a \\
& - (m [A_2^m F_1^m + F_2^m A_1^m - \psi_2^m \psi_1^m] + \text{h.c.}) \\
& - ig \sqrt{2} \left[\bar{\psi}_1^m \bar{\lambda} A_1^m - A_1^{m+} \lambda \psi_1^m - A_2^m \bar{\lambda} \bar{\psi}_2^m + \psi_2^m \lambda A_2^{m+} \right] \\
& - ig \sqrt{2} [\bar{\lambda}_2 \bar{\lambda} M - M^+ \lambda \lambda_2] \\
& + g D^a \left[A_1^{m+} t_a A_1^m - A_2^m t_a A_2^{m+} \right] + g D^a M^+ t^a M \\
& + \Big[ig \sqrt{2} (A_2^m F_M A_1^m + F_2^m M A_1^m + A_2^m M F_1^m) \\
& - ig \sqrt{2} (\psi_2^m M \psi_1^m + A_2^m \lambda_2 \psi_1^m + \psi_2^m \lambda_2 A_1^m) + \text{ h.c.} \Big] \quad .
\end{aligned}
\tag{2.6}
$$

$$
\not{D} = D^\mu \sigma_\mu \qquad \tilde{\not{D}} = D^\mu \tilde{\sigma}_\mu
$$

Here $(V^{a\mu\nu}, \lambda^a, D^a)$ and (M_a, λ_{2a}, F_a) describe the gauge multiplet (V, χ) while $(A_1, \psi_1, F_1)^m$ and $(A_2, \psi_2, F_2)^m$ describe the m-th hypermultiplet (ϕ_1^m, ϕ_2^m).

The global SU(2) symmetry appears after use is made in the above Lagrangian of the equations of motion for the auxiliary fields and after we make the following identifications:

$$
\begin{aligned}
& A_i^m = i \begin{pmatrix} A_1^m \\ A_2^{m+} \end{pmatrix} \qquad \lambda^i = -i \begin{pmatrix} \lambda_2 \\ \lambda \end{pmatrix} \\
& \phi^m = \psi_2^m , \quad \psi^m = \psi_1^m ; \quad A^{+mi} = (A_i^m)^+ ; \quad \bar{\lambda}_i = (\lambda^i)^+ \quad .
\end{aligned}
\tag{2.7}
$$

To raise and lower the SU(2) indices, one uses the antisymmetric tensor g^{ij} (with $g^{12} = -g_{12} = 1$) The N=2 supersymmetric Lagrangian written in SU(2) covariant form is

$$
\begin{aligned}
L = & -\frac{1}{4} \text{ Tr } V_{\mu\nu} V^{\mu\nu} + \frac{1}{4} \text{ Tr } D_i^j D_j^i \\
& -\frac{1}{4} \text{ Tr } \bar{\lambda} \not{D} \lambda \\
& + A_m^{+i} D D A_{im} - \frac{i}{2} \phi_m \not{D} \bar{\phi}_m \\
& - \frac{i}{2} \bar{\psi}_m \tilde{\not{D}} \psi_m + F_m^{+i} F_{i\,m} \\
& \left(-i \frac{g}{2\sqrt{2}} \text{ Tr } \lambda^i [\lambda_i, M] + \text{ h.c.} \right) \\
& + \left(i\, g \sqrt{2}\, \bar{\psi}_m\, M^+\, \bar{\phi}_m + \text{ h.c.} \right) \\
& + i\, g \sqrt{2} [A_m^{+i}\, \bar{\lambda}_i\, \bar{\phi}_m + A_m^{+i} \lambda_i \psi_m \\
& - \bar{\psi}_m\, \bar{\lambda}_i\, A_m^i - \phi_m \lambda^i A_i^m] - \frac{g^2}{4} \text{ Tr } [M^+, M]^2 \\
& + \sqrt{2}\, g A_m^{+i} F_i^j A_{j\,m} - g^2 A_m^{+i} \{M, M^+\} A_{i_m} \\
& + i m_m (A_m^{+i} F_{i_m} - F_m^{+i} A_{i_m}) \\
& + i\, g \sqrt{2}\, m_m A_m^{+i} (M - M^+) A_{i_m} + m_m \phi_m \psi_m + m_m \bar{\psi}_m \bar{\phi}_m \quad .
\end{aligned}
\tag{2.8}
$$

This Lagrangian is invariant under the following supersymmetry transformations[8]:

$$
\begin{aligned}
\delta M &= i \sqrt{2}\, \epsilon_i^\alpha \lambda_\alpha^i \\
\delta V_\mu &= \frac{1}{\sqrt{2}} (\epsilon^i \sigma_\mu \bar{\lambda}_i + \bar{\epsilon}_i \tilde{\sigma}_\mu \lambda^i) \\
\delta \lambda_\alpha^i &= \sqrt{2} (\sigma_\mu D^\mu \bar{\epsilon})_\alpha^i M + \sqrt{2}\, D_j^i\, \epsilon_\alpha^j - i \sqrt{2}\, (\sigma_{\mu\nu} V^{\mu\nu} \epsilon)_\alpha^i + g[M, M^+] \epsilon_\alpha^i \\
\delta D_i j &= -\frac{i}{\sqrt{2}} \epsilon_{(i} \not{D} \bar{\lambda}_{j)} + \frac{i}{\sqrt{2}} \bar{\epsilon}_{(i} \tilde{\not{D}} \lambda_{j)} + i\, g\, \epsilon_{(i} [\lambda_{j)}, M^+] + i\, g\, \bar{\epsilon}_{(i} [\bar{\lambda}_{j)}, M]
\end{aligned}
\tag{2.9}
$$

for the gauge multiplet and

$$
\begin{aligned}
\delta A_i^m &= \sqrt{2}\,(i\,\epsilon\,\psi^m + i\,\bar{\epsilon}_i\,\bar{\phi}^m) \\
\delta\psi_\alpha^m &= -\sqrt{2}\,\epsilon_{\alpha i}\,F^{im} + \sqrt{2}\,(\not{D}\,\bar{\epsilon})_\alpha^i\,A_i^m - 2g\,M^+\,A_i\,\epsilon_\alpha^i \\
\delta\,\bar{\phi}_{\dot{\alpha}} &= -\sqrt{2}\,(\epsilon\;\not{D})_{\dot{\alpha}i}A_m^i - \sqrt{2}\,\bar{\epsilon}_{\dot{\alpha}}^i\,F_i - 2g\,M\,A_i\,\bar{\epsilon}_{\dot{\alpha}}^i \\
\delta F_i &= -\sqrt{2}\,\epsilon_i\;\not{D}\,\bar{\phi} - i\sqrt{2}\,\bar{\epsilon}_i\;\tilde{\not{D}}\,\psi \quad .
\end{aligned}
\tag{2.10}
$$

for the hypermultiplets.

The supersymmetric N=2 potential can be written in the form

$$
V_{N=2} = \frac{1}{2}\,d_i^{aj}\,d_j^{ai} + \frac{g^2}{4}\;\mathrm{Tr}\;[M^+, M]^2 + F_{1_m}^{+\;i}\,F_{1_m^i} + F_{2_m}^{+\;i}\,F_{2_m^i}
\tag{2.11}
$$

where

$$
\begin{aligned}
d_i^{aj} &= -\frac{1}{\sqrt{2}}\,g(A_{\underset{m}{i}}^{+}\,t^a A_m^j + A_m^{+j}\,t^a\,A_{\underset{m}{i}}) \\
F_{1_m^i} &= \frac{m_m}{\sqrt{2}}\,A_{\underset{m}{i}} + i\,M^+\,A_{\underset{m}{i}} \\
F_{2_m^i} &= \frac{m_m}{\sqrt{2}}\,A_{\underset{m}{i}} - i\,M\,A_{\underset{m}{i}} \quad .
\end{aligned}
\tag{2.12}
$$

The number of scalar auxiliary fields which make the potential positive definite is consistent with that for the N=2 gauge multiplet while for hypermultiplets this is the minimal set which is also SU(2) covariant.

The usual form of the potential for a supersymmetric theory

$$
V = F_{\mathcal{A}}^{+}\,F_{\mathcal{A}} + \frac{1}{2}\,D^2
\tag{2.13}
$$

apparently cannot be made covariant under the SU(2), with the corresponding number of degrees of freedom, $\mathcal{A}$ is a set of indices.

Before proceeding to the study of soft breaking, we will review the computations of the divergences in the one loop effective potential, using however (instead of usual form for the mass matrix) the SU(2) covariant version.

As is well known, the renormalized effective potential in the Landau gauge has the form

$$V = V_{c\ell} + u\, Str\, 1 + h\, Str\, M^2 - k\, Str\, M^4 + Str\, M^4\, lg\, M^2 \tag{2.14}$$

where

$$u \sim \int d^4k\,, \qquad h \sim \int \frac{d^4k}{k^2}\,, \qquad k \approx \int \frac{d^4k}{k^4} \tag{2.15}$$

and M^2 is the quadratic mass matrix for particles of spin 0, 1/2 and 1 defined as

$$m^{\circ}_{AB} = \begin{pmatrix} \frac{\delta \mathcal{L}}{\delta\psi_A\,\delta\psi_B^+} & \frac{\delta \mathcal{L}}{\delta\psi_A\,\delta\psi_B} \\ \frac{\delta \mathcal{L}}{\delta\psi_A^+\,\delta\psi_B^+} & \frac{\delta \mathcal{L}}{\delta\psi_A^+\,\delta\psi_B} \end{pmatrix} \tag{2.16}$$

where ψ_A are the scalars in the theory

$$m^{1/2}_{\substack{AB\\ \alpha\beta}} = \frac{\delta \mathcal{L}}{\delta\phi_{A\alpha}\,\delta\phi^+_{B\beta}} \tag{2.17}$$

where $\phi_{A\alpha}$ are the two component spinors. One gets (see also Ref. 9)

$$\begin{aligned} S\ \mathrm{Tr}\ M^2 = 0\ , \quad S\ \mathrm{Tr}\ M^4 = &-2n[(\vec{M}\,\vec{M}^+)^2 - \vec{M}^2\,\vec{M}^{+2}] \\ &- 3m_m^2\, A^{+i}_m A_{\substack{i\\m}} + 7i\sqrt{2}\, m_m A^{+i}_m (M - M^+) A_{\substack{i\\m}} - \frac{11}{2}\,\vec{M}\,\vec{M}^+\ A^{+i}_m A_{\substack{i\\m}} - 3d^{ja}_i\, d^{ai}_j \end{aligned} \tag{2.18}$$

where n is the number of hypermultiplets.

Correspondingly, we obtain for the renormalized quantities

$$\begin{aligned}
\hat{g} &= g[1 + 2kg^2(4-n)] \\
\hat{M} &= (1 - 4kg^2 + 2nkg^2)\, M \\
\hat{A} &= (1 + \frac{3k}{2}\, g^2)\, A \\
\hat{m} &= m \quad .
\end{aligned} \tag{2.19}$$

Thus, if we have n=4 hypermultiplets, the gauge coupling constant does not get renormalized and the model is finite.

This approach will be applied in the next section to the study of the soft breakings of the finite N=2 supersymmetric model.

3. N=2 Soft Breakings

To analyze the soft breakings of the finite N=2 supersymmetric and finite model considered above, we will add to the Lagrangian [Eqs. (2),(8)] the most general soft breaking terms, i.e., bilinears or trilinear terms with coefficients of dimensionality of $(\text{mass})^1$ or $(\text{mass})^2$, which are compatible with gauge invariance. In view of the pseudoreal nature of the spinor representation of $SU(2)$ this includes terms of the form $A^i_m U A^j_n$ where $U_{\alpha\beta}$ is the antisymmetric matrix $\epsilon_{\alpha\beta}$ acting on the gauge indices. We shall distinguish between two types of soft breakings: those which originate in a (N=1) superfield soft breaking for which one knows that they preserve the finiteness of the model and those which explicitly break the supersymmetry. For the flavor type particles any soft breaking can be rearranged in the following way: any fermionic mass term is completed to a superfield by adding the necessary bosonic terms so that one gets explicit soft supersymmetry breakings only in the bosonic sector. The mass terms for the fermions of the N=2 gauge multiplet cannot always be written in terms of superfields because a superfield which provides a mass for the gaugino does not exist. For the purpose of maintaining global SU(2) covariance, such terms will be left as free parameters in the Lagrangian.

The most general soft breaking is then of the form

$$\delta\mathcal{L} = \delta\mathcal{L}_{SF} + \delta\mathcal{L}_{EB} \quad , \tag{3.1}$$

where

$$\begin{aligned}
-\delta\mathcal{L}_{SF} =& \underset{mn}{m^2_{ij}}\, A^{+i}_m A^j_n + \underset{mn}{m'^2_{ij}}\, A^i_m U A^j_n + \underset{mn}{m'^{2*}_{ij}}\, \underset{m}{A^+_i}\, U \underset{n}{A^+_j} \\
&+ \Big[\frac{i}{2} \underset{mn}{P_{ij}}\, A^{+i}_m M A^j_n - \frac{i}{2} \underset{mn}{P^1_{ij}}\, A^i_m U M A^j_n + \frac{i}{2} \underset{mn}{P^2_{ij}}\, A^i_m U M^+ A^j_n \\
&- \frac{1}{2} m^I_{mn} \psi_m U \psi_n - m^{II}_{mn} \phi_m \psi_n - \frac{1}{2} m^{III}_{mn} \phi_m U \phi_n + \text{h.c.}\Big]
\end{aligned} \tag{3.2}$$

where $\underset{mn}{m^2_{ij}}$, $\underset{mn}{m'^2_{ij}}$, $\underset{mn}{P_{ij}}$, $\underset{mn}{P^{(1,2)}_{ij}}$ are parametrized in terms of the superfield mass terms:

$$m^I_{mn} \underset{m}{\phi_1} U \underset{n}{\phi_1} , \qquad m^{II}_{mn} \underset{m}{\phi_2} U \underset{n}{\phi_1} , \qquad m^{III}_{mn} \underset{m}{\phi_2} U \underset{n}{\phi_2} \tag{3.3}$$

and N=2 supersymmetric mass term $m_m \phi_2^m \phi_1^m$ in the following way:

$$\begin{aligned}
\underset{mn}{m^2_{12}} &= -(m^\circ m^{\circ +} + m^I m^{I+})_{nm} \\
\underset{mn}{m^2_{21}} &= (m^{\circ +} m^\circ + m^{III} m^{III+})_{nm} \\
\underset{mn}{m'^2_{12}} &= \frac{1}{2}[-(m^{I*} m^\circ)_{nm} + (m^{III} m^{\circ +})_{mn}] = -\underset{nm}{m'^2_{21}} \\
\underset{mn}{P_{12}} &= 2g\sqrt{2}\, m'_{\circ nm} \, , \qquad \underset{mn}{P_{21}} = -2g\sqrt{2}\, (m^+_\circ)_{nm} \\
\underset{mn}{P^1_{12}} = \underset{nm}{P^1_{21}} &= g\sqrt{2}\, m^{III}_{mn} , \quad \underset{mn}{P^2_{21}} = \underset{nm}{P^2_{12}} = g\sqrt{2}\, m^I_{mn} \\
m^\circ_{mn} &= m_m \delta_{mn} + m^{II}_{nm} , \quad m_0 = (m'_0)^+ \, .
\end{aligned} \tag{3.4}$$

The explicit breaking will be of the form:

$$
\begin{aligned}
-\delta \mathcal{L}_{EB} =& \delta m^2_{\substack{ij\\mn}}\, A^{+i}_m A^j_n + \delta m'^2_{\substack{ij\\mn}}\, A^i_m\, U\, A^j_n \\
&+\left[\frac{i}{2}\, \delta P_{\substack{ij\\mn}}\, A^{+i}_m\, M\, A^j_n - \frac{i}{2}\, \delta P^1_{\substack{ij\\mn}}\, A^i_m\, U\, M\, A^j_n \right. \\
&\left. +\frac{i}{2}\, \delta P^2_{\substack{ij\\mn}}\, A^i_m\, U\, M^+\, A^j_n + \text{ h.c.}\right] \\
&+ \delta m^2_M\, M_a\, M^+_a + (m'^2_M\, M^2_a - \frac{1}{2}\, m_{ij}\, \lambda^i_a \lambda^j_a + \text{h.c.}) \qquad .
\end{aligned}
\tag{3.5}
$$

Because the wave function renormalizations are already fixed by the N=2 supersymmetric counter–terms, the finiteness condition for Eq. (3.5) is given by imposing the requirement that $S\,\mathrm{Tr}\, M^4$ must be renormalized only by the wave functions of the fields $A_{\substack{i\\m}}$ and M.

Looking at the finiteness conditions for the trilinear coupling we get

$$
\begin{aligned}
\delta P_{\substack{ij\\mn}} &= -2\sqrt{2}\, m_{ij}\, \delta_{mn} \\
\delta P^1_{\substack{ij\\mn}} &= \delta P^2_{\substack{ij\\mn}} = 0 \qquad .
\end{aligned}
\tag{3.6}
$$

Analyzing the bilinear terms, one gets that arbitrary masses $\delta\, m'^2_{\substack{ij\\mn}}$ and m'^2_M are allowed while the other mass must fulfill the conditions

$$
\begin{aligned}
\delta m^2_M &= m_{ij}\, \bar{m}^{ij} + \frac{1}{4}\, (\delta m^2)_{\substack{e\\m}}{}^{e}_{m} \\
(\delta m^2)_{\substack{e\\pp'}}{}^{e} &= \frac{1}{4} (\delta m^2)_{\substack{e\\mm}}{}^{e}\, \delta_{pp'} \qquad .
\end{aligned}
\tag{3.7}
$$

We remark that in the computation of S Tr M^4 profuse cancellations occur. Once the condition (3.6) is fulfilled, all possible interference terms between $\delta\mathcal{L}_{SF}$ and $\delta\mathcal{L}_{EB}$ cancel, a feature which is actually easy to understand at the diagrammatic level. We note that the conditions (3.6) and (3.7) are also sufficient to insure the finiteness of the fermionic terms.

In the next section these soft breakings will be shown to be sufficient for breaking the gauge group.

4. Spontaneous Symmetry Breaking

The classical effective potential we want to study is

$$V_{c\ell} = V_{N=2} - \delta\, \mathcal{L}_{SF} - \delta\, \mathcal{L}_{EB} \ . \tag{4.1}$$

(In $\delta\mathcal{L}$'s, the fermion terms have been deleted.) One observes that there are certain directions along which the potential does not have any four boson interaction. These can be shown to correspond to the following cases

$$\text{(a)} \qquad A^i_m = 0 \quad \text{and} \quad \vec{M} \parallel \vec{M}^+ \tag{4.2}$$

$$\text{(b)} \qquad M_a = 0 \quad \text{and} \quad d^{a\ j}_{\ i} = 0 \tag{4.3}$$

Note that in either case, the trilinear interactions vanish as well. The situation obviously becomes dangerous if the bilinear terms have negative masses along these directions, because then the potential will be unbounded from below.

A systematic approach to avoid this catastrophe is to parameterize the fields to satisfy Eqs. (4.2) or (4.3), substitute them into Eq. (4.1), and then impose positive semi-definiteness conditions on the parameters of the bilinear terms.

For case (a), the solution of the constraint is as given. Here, it is possible to tune the breaking parameters in such a way that spontaneous symmetry breaking may occur, i.e., $\langle M_a \rangle \neq 0$. However, this solution is degenerate with the symmetry preserving solution $\langle M_a \rangle = 0$ and we discard it as being uninteresting.

For case (b), the general parameterization of A^i_m to satisfy $d^{a\ j}_{\ i} = 0$ is quite tedious to work out.

As it is our present intent to demonstrate that there exists some choice of parameters which lead to a potential that is bounded from below and which give a stable symmetry breaking solution, we shall just exhibit one example which does not rely on the procedure above.

For this purpose we make use of the fact that when both M^a and some of the A_{im} are non-vanishing, the quartic terms of Eq. (4.1) do not possess any zero (flat) direction. The trilinear couplings can then be used safely to shape a non-trivial minimum in field space. Since we are only concerned here with the existence of such a minimum, and since V vanishes at the origin, all we have to do is to exhibit some region in field space where V is negative.

Consider the choice

$$m_{ij} = \frac{1}{\sqrt{2}} m_0 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{4.4}$$

and

$$\delta m^2_{\substack{ij \\ mn}} = 2\mu^2 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \delta_{mn} \tag{4.5}$$

in which m_0 is real and $\mu^2 > 0$. The other parameters are set at zero values.

It follows from Eq. (3.7) that

$$\delta m^2_M = m_0^2 - 2\mu^2 \geq 0 \tag{4.6}$$

in which the non-negative requirement has been imposed to ensure that the potential stays bounded from below.

The effective potential in this example is

$$\begin{aligned} V_{c\ell} = & V_{N=2} + (m_0^2 - 2\mu^2) M_a^+ M_a \\ & + \delta m^2_{\substack{ij \\ mn}} A_m^{+i} A_n^j \\ & + \left(\frac{i}{2} \delta P_{\substack{ij \\ mn}} A_m^{+i}\, M\, A_n^j + h.c. \right) \end{aligned} \tag{4.7}$$

with (according to (3.6))

$$\delta P_{\substack{ij\\mn}} = -2\sqrt{2}\, m_{ij}\delta_{mn} \tag{4.8}$$

To show that the gauge symmetry is broken at the minimum of this potential, all we need to show is that there is a region in the field space in which $V_{c\ell} < 0$. For this purpose, we propose to look into the region where

$$i\, d^a_{ij} = -a\, m_{ij} M^a \tag{4.9}$$

where a is a complex coefficient to be adjusted later. Because of our choice of m_{ij}, we must require that $d^a_{11} = d^a_{22} = 0$. A simple way to satisfy this is to assume

$$A_{2m} = 0 \tag{4.10}$$

and

$$A_{1m} = \frac{m_0}{2\sqrt{2}} \begin{pmatrix} b_m \\ 0 \end{pmatrix} \tag{4.11}$$

Let us now define

$$x \equiv \sum_m |b_m|^2 \tag{4.12}$$

and

$$y \equiv \frac{\mu^2}{m_0^2}, \qquad 0 \le y \le 1/2 \tag{4.13}$$

We can write the potential in Eq. (4.7) as

$$V_{c\ell} = \frac{m_0^4 x^2}{256|a|^2} \left\{ 1 - 2Re\, a + \frac{|a|^2}{2} - 2y + \frac{1}{16}x \right\} \quad . \tag{4.14}$$

Clearly, there exist various choices of a and $0 \leq y \leq 1/2$ in which the potential is negative for some $x > 0$. This verifies our assertion that a stable symmetry breaking solution exists.

A few remarks are in order

1. In this model, the non-degenerate symmetry breaking solution reduces $SU(2)$ gauge symmetry to no symmetry at all, because some fields in the fundamental representation acquire vacuum expectation values.
2. Because of our choice of $\delta\, m^2_{\substack{ij \\ mn}}$, some scalars are massless. They can be made massive by modifying Eq. (4.5) by small quantities without changing the conclusions.

5. Concluding Remarks

We have used a simple $SU(2)$ model to illustrate systematically the procedure to obtain finiteness conditions in the soft breaking parameters and to bring out the inherent peculiarity of the effective potential, whose parameters must be chosen to avoid unboundedness from below. We have also shown that spontaneously broken symmetry can be accommodated in such theories.

It is quite clear that the methodology developed and some general features exposed here either are common or can be extended to cover other groups. A search for a realistic model along those lines is currently under way.

We have also looked into the alternative possibility of using radiative corrections to bend over classical potentials which are unbounded from below,[10] when other choices of parameters are made. The results have so far been negative. Radiative corrections either are unable to turn over in all directions negative bilinear terms, or they tend to drive the stable vacuum to the symmetric solution $< M >=< A >= 0$.

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