SLAC-PUB-3195
September 1983
(T)

A Triplet Code Model of Leptons and Quarks*

LIAO FU LUO† AND GUO CHEN YANG‡
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

A triplet code model of leptons and quarks is proposed. Assuming the internal A-spin, B-spin and C-spin of particles, from fundamental symmetry $SO(4)$ the color $SU(3)$, horizontal $SU(3)$, electroweak $SU(2) \times U(1)$ and other higher composite symmetries are derived. Using composite symmetry operators, the mass gap between third generation and the first two can be derived. The modification to the standard electroweak gauge theory is also discussed in some detail.

Submitted to Nuovo Cimento

---

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

† Permanent address: Inner Mongolia University, Huhehote, China.

‡ Permanent address: Hebei Institute of Technology, Tianjin, China.

1. A Model of Leptons and Quarks

Recently a series of composite models have been proposed in which leptons and quarks are regarded as composites of a smaller set of particles called preons or others[1]. The existence of generations of particles, the universality of weak interactions for leptons and quarks, etc. seem in favour of the compositeness of elementary fermions. One also expects that the many difficult problems left in the present form of particle theories may be solved or partly solved at the level of sub-particles. But in connection with the extreme smallness of radius $(\frac{1}{\Lambda})$ of leptons and quarks there are a lot of unsolved problems and difficulties common to these models. Such as: What are the properties of binding forces between preons? Why the lepton or quark possesses a mass much smaller that Λ? Especially it seems difficult to design a convincing and economical model in which only few unobserved exotic particles are predicted. (For example, how the spin $\frac{3}{2}$ particles could be excluded?) It is reasonable to doubt of the Nature is so tautologized that the situation of hadrons composed of quarks will recur once again. In 1981 we have proposed a new alternative subconstituent model of leptons and quarks.[2] We suppose that particles (leptons and quarks) are structureless in space-time but are composite in internal degrees of freedom (called A, B, and C) each is described by group $SO(4)$. One can decipher the "triplet code" of particles in terms of $G = SO^A(4) \times SO^B(4) \times SO^C(4)$. Set the A-spin wave functions denoted as $(A_2\ A_1\ \ -\bar{A}_1\ \bar{A}_2)$ which belongs to the representations (2,2) of $SO^A(4)$. $(A_2\ A_1)$ and $(-\bar{A}_1\ \bar{A}_2)$ are two spinor representations of $SU(2)$ in $SO^A(4)$ and $(A_2\ \ -\bar{A}_1)$ and $(A_1\ \bar{A}_2)$ are that of $SU(2)'$ in $SO^A(4)$. The charge is assumed to be $(\frac{2}{3}, \frac{1}{3}, -\frac{1}{3}, -\frac{2}{3})$ for $(A_2\ A_1\ \ -\bar{A}_1\ \bar{A})$ respectively the same is for B-spin and C-spin. The wave functions ("triplet code") of particles are listed in Table 1.

For the purpose of discussing gauge interactions it is necessary to divide the particle into chiral states. We suppose that the representation of $SU(2)$ part in $SO(4)$ is of

Vector-type, i.e., the left-handed and the right-handed states of one particle obey the same representation but the representation of $SU(2)'$ part is chiral, namely, the left and right-handed states transform according to the different representations $SU(2)'_L$ and $SU(2)'_R$. Denote the generators of G as $\vec{\tau}_\alpha$ and $\vec{\tau}'_\alpha$ ($\alpha = A, B, C$). Set $I_3 = \frac{1}{2}(\tau_{A3} + \tau_{B3} + \tau_{C3})$ and $I'_3 = \frac{1}{2}(\tau'_{A3} + \tau'_{B3} + \tau'_{C3})$ then

$$Q = \frac{1}{3} I_3 + I'_3$$
$$2I_3 = 3(B - L) \tag{1}$$

(B, L denote baryon and lepton number respectively). Particles may be classified according to the quantum numbers (I_3, I'_3). From Table 1 one finds $I_3 = \pm\frac{1}{2}$ for quarks (color triplet), $I_3 = \pm\frac{3}{2}$ for leptons (color singlet), $I'_3 = \pm\frac{1}{2}$ for known fermions (generation triplet) and $I'_3 = \pm\frac{3}{2}$ for exotic particles (generation singlet). An important feature is that the generations have been incorporated in the scheme. But no spin $\frac{3}{2}$ particle is necessary.

2. Composite Symmetries

The wave functions of particles listed in Table 1. are classified according to $SO^A(4) \times SO^B(4) \times SO^C(4)$ symmetry. However, the physical states are defined in terms of color, generation etc. in addition to charge, baryon and lepton number. How to distinguish the different states in a color triplet or a generation triplet? To answer the question, one should investigate these symmetries in the model.

We define the composite symmetry of particles as follows:

(a) begin Which is a group generated by the direct products of $\vec{\tau}_\alpha$ and $\vec{\tau}'_\alpha$ such as $\tau_{Ai}\ \tau_{Bj}\ \tau_{Ck}, \tau'_{Ai}\ \tau'_{Bj}\ \tau'_{Ck}$ etc; and

(b) Which classifies the leptons and quarks and defines the physical states of them.

(Though it is not a subgroup of G in general.)

By calculating the commutation relations one can prove that the following 15 operators

$$\begin{pmatrix} \tau_{A3} & \tau_{A+}\tau_{B-}\tau_{C3} & \tau_{A+}\tau_{B3}\tau_{C-} & -\tau_{A3}\tau_{B-}\tau_{C-} \\ \tau_{A-}\tau_{B+}\tau_{C3} & \tau_{B3} & \tau_{A3}\tau_{B+}\tau_{C-} & -\tau_{A-}\tau_{B3}\tau_{C-} \\ \tau_{A-}\tau_{B3}\tau_{C+} & \tau_{A3}\tau_{B-}\tau_{C+} & \tau_{C3} & -\tau_{A-}\tau_{B-}\tau_{C3} \\ -\tau_{A3}\tau_{B+}\tau_{C+} & -\tau_{A+}\tau_{B3}\tau_{C+} & -\tau_{A+}\tau_{B+}\tau_{C3} & \tau_{A3}+\tau_{B3}+\tau_{C3} \end{pmatrix} \quad (2)$$

generate the $SU(4)$ algebra. $U(d)$ quarks of three colors and $\nu(e)$ lepton belong to the representation $\underline{4}$ of the composite symmetry group $SU(4)$. Further one can prove that the first 3×3 elements of eq.(2) generate color $SU(3)$ algebra, namely

$$\begin{aligned} &T_+ = \tau_{A3}\tau_{B+}\tau_{C-} \qquad T_- = \tau_{A3}\tau_{B-}\tau_{C+} \qquad 2T_3 = [T_+, T_-] = \frac{1}{2}(\tau_{B3} - \tau_{C3}) \\ &V_+ = \tau_{A+}\tau_{B-}\tau_{C3} \qquad V_- = \tau_{A-}\tau_{B+}\tau_{C3} \qquad 2V_3 = [V_+, V_-] = \frac{1}{2}(\tau_{A3} - \tau_{B3}) \qquad (3) \\ &U_+ = \tau_{A-}\tau_{B3}\tau_{C+} \qquad U_- = \tau_{A+}\tau_{B3}\tau_{C-} \qquad 2U_3 = [U_-, U_-] = \frac{1}{2}(\tau_{C3} - \tau_{A3}) \\ &\qquad (T_3 + V_3 + U_3 = 0) \end{aligned}$$

Instead of $\vec{\tau}$ with $\vec{\tau}'$ in eq. (2) we obtain $SU(4)'$ algebra and $\nu_e\ \nu_\mu\ \nu_\tau\ \bar{E}^{(2)}$; $u_\alpha\ c_\alpha\ t_\alpha\ \bar{E}_\alpha(\frac{4}{3})$ ... belong to the representations $\underline{4}$ and $e^-\mu^-\tau^-E(1)$; $d_\alpha s_\alpha b_\alpha E_\alpha(\frac{5}{3})$ ... belong to the $\underline{4}^*$ respectively. Likewise, the horizontal $SU(3)'$ algebra can be deduced and the generators are those in eq. (3) by replacing $\vec{\tau}$ with $\vec{\tau}'$. The 48 known fermions are generation triplets and the exotic particles are generation singlets. The weak $SU(2)'_W$ symmetry is a kind of composite symmetries, too. One can show that

$$\begin{aligned} I_W^+ &= \tau'_{A+}\tau'_{B+}\tau'_{C-} + \tau'_{A+}\tau'_{B-}\tau'_{C+} + \tau'_{A-}\tau'_{B+}\tau'_{C+} \\ I_W^- &= \tau'_{A-}\tau'_{B-}\tau'_{C-} + \tau'_{A-}\tau'_{B+}\tau'_{C-} + \tau'_{A+}\tau'_{B-}\tau'_{C-} \qquad (4) \\ 2I_{W3} &= [I_W^+, I_W^-] = \frac{1}{4}(\tau'_{A3} + \tau'_{B3} + \tau'_{C3} - 3\tau'_{A3}\tau'_{B3}\tau'_{C3}) \end{aligned}$$

generate the $SU(2)'_W$ algebra. This interaction exists only for generation triplets. Another weak $SU(8)'_X$ symmetry described by

$$
\begin{aligned}
I_x^+ &= \tau'_{A+}\tau'_{B+}\tau'_{C+} \\
I_x^- &= \tau'_{A-}\tau'_{B-}\tau'_{C-} \\
2I_{X3} = [I_x^+, I_x^-] &= \frac{1}{4}(\tau'_{A3} + \tau'_{B3} + \tau'_{C3} + \tau'_{A3}\tau'_{B3}\tau'_{C3})
\end{aligned}
\tag{5}
$$

exists for generation singlets. Equation (1) can be rewritten in the form

$$
Q = \frac{1}{3} I_3 + I_{W3} + 3I_{X3} \tag{6}
$$

Therefore the various internal symmetries occured in the particle level can all be deduced as the composite symmetries. Of course, these symmetries are broken in general. The breaking mechanism is a difficult problem which we shall not discuss in this article. However, because the wave functions of particles and the composite symmetry operators in terms of internal coordinates have been found one may have some discussions about the mass spectrum of particles. In fact, the various states of particles can be defined in terms of (I_3, I'_3) and the color and generation quantum numbers. If we assume color a rigorous gauge symmetry which is not broken by the interaction of particles then the mass of leptons and quarks for given (I_3, I'_3) can be expressed as

$$
M = M(T'_{\pm}, V'_{\pm}, U'_{\pm}, T'_3, V'_3, U'_3) \tag{7}
$$

Under the assumption of S_3 symmetry between three degrees of freedom A, B, and C, the mass matrix of particles of three generations may be parametrized as

$$
M = \begin{pmatrix} c & a & a \\ a & c & a \\ a & a & c \end{pmatrix} \tag{8}
$$

The matrix is diagonalized by an unitary transformation

$$S^{-1}MS = \begin{pmatrix} c-a & 0 & 0 \\ 0 & c-a & 0 \\ 0 & 0 & c+2a \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{9}$$

We see that there exists a mass gap of $3a$ between third generation and the other two. Furthermore, if the S_3 symmetry is only an approximate one, then the degeneracy between the first two generations can be removed. For example, set

$$\begin{aligned} M = M_0 \ + \ 2a(T'_+ \ + \ T'_+) \ + \ 2b(U'_+ \ + \ U'_-) \ +2c(V'_+ \ + \ V'_-) \\ + (b+c)\tau'_{A3} \ + \ (c+a)\tau'_{B3} \ + \ (a+b)\tau'_{C3} \end{aligned} \tag{10}$$

Under the unitary transformation one obtains

$$M' = S^{-1}MS = M_0 + \begin{pmatrix} b+c-2a & \sqrt{3}(c-b) & 0 \\ \sqrt{3}(c-b & 2a-b-c & 0 \\ 0 & 0 & 2(a+b+c) \end{pmatrix} \tag{11}$$

If $M_0 = 2a - b - c$ is taken, then it is the mass matrix derived in[3] and the Cabbibo angle can be obtained. Of course, to obtain the coupling between third and first two generations and find the complete $K-M$ matrix, one should modify eq. (10) further and replace it by a more accurate one.

3. Gauge Symmetries

In order to investigate the interactions between particles, we shall discuss the gauge symmetries in this section. For simplicity, we consider $SO(4) = SU(2) \times SU(2)'_L$ only. That is, the $SU(2)'$ part in $SO(4)$ is assumed to be left-handed. However the following discussion can easily be extended to the $L-R$ symmetric theory.[4] Denote the $SU(2)\times SU(2)'_L$ generators as T_{Ai}, T_{Bi} and T_{Ci} ($i = 1...6$). For example,

$$\begin{aligned} T_{A1}, T_{A2}, T_{A3} &= \vec{\tau}_A \\ T_{A4}, T_{A5}, T_{A6} &= \frac{1}{2}\vec{\tau}'_A(A1+\gamma_5) \end{aligned} \tag{12}$$

Corresponding to a composite gauge transformation

$$\psi \rightarrow \ exp - i\theta_{ijk} T_{Ai} T_{Bj} T_{Ck} \psi \tag{13}$$

one may introduce a gauge field $(A_{ijk})_\mu$ through the covariant derivative

$$\partial_\mu + igT_{Ai}T_{Bj}T_{Cl}(A_{ijk})_\mu \tag{14}$$

For $(i, j, k) \leq 3$, the corresponding gauge field is of vector-type. For the remaining case, the field is left-handed.

The color $SU(3)$ as a composite gauge interaction can be introduced as usual. However the $SU(2) \times U(1)$ electroweak interaction should be generalized to include a new component. In fact, eq. (6) can be rewritten in the form

$$Q = I_{W3} \ + \ 3I_{X3} \ + \ \frac{1}{2}Y \tag{15}$$

Set the covariant derivative

$$D_\mu = \partial_\mu - ig_0 \vec{I}_W \cdot \vec{W}_\mu - \frac{i}{2} g_i Y B_\mu - ig_2 \vec{I}_x \cdot \vec{X}_\mu \tag{16}$$

here W_μ X_μ and B_μ describing $SU(2)'_W$, $SU(2)'_X$ and $U(1)$ gauge fields respectively. As in usual approach introduce Higgs scalar[5]

$$\begin{aligned} \Phi &= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad (I_w = \frac{1}{2}, I_x = 0, Y = 1) \\ \Phi &= \begin{pmatrix} \psi^{+++} \\ \psi^0 \end{pmatrix} \qquad (I_x = \frac{1}{2}, I_w = 0, Y = 3) \end{aligned} \tag{17}$$

Their vacuum expectations are $< \phi^0 >= v$, $< \phi^0 >= u$. Then one deduces the masses of charged gauge particles

$$\begin{aligned} m_W^2 &= \frac{1}{2} g_0^2 v^2 \\ m_X^2 &= \frac{1}{2} g_2^2 U^2 \end{aligned} \tag{18}$$

$$M_N^2 = \frac{1}{2}\begin{pmatrix} v^2 g_0^2 & -v^2 g_0 g_1 & 0 \\ -v^2 g_0 g_1 & (v^2 + 9u^2) g_1^2 & -3 g_1 g_2 u^2 \\ 0 & -3 g_1 g_2 u^2 & u^2 g_2^2 \end{pmatrix} \tag{19}$$

The photon corresponding to the eigenvalue zero of M_N^2 is

$$A_\mu = \left(\frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{9}{g_2^2}\right)^{1/2} (g^{-1} W_{3\mu} + g_1^{-1} B_\mu + 3 g_2^{-1} X_{3\mu}) \tag{20}$$

Under the approximation $u^2 >> v^2$ the masses of other two neutral gauge particles (denoting as Z and Y) are

$$\begin{aligned} m_z^2 &= \frac{1}{4} v^2 (g_0^2 + g_1^2) + \frac{1}{4} v^2 [(g_0^2 + g_1^2) g_2^2 + 9 g_1^2 (g_0^2 - g_1^2)](g_2^2 + 9 g_1^2)^{-1} \\ m_y^2 &= \frac{1}{2} u^2 (g_2^2 + 9 g_1^2) + \frac{1}{4} v^2 [(g_0^2 + g_1^2) g_2^2 + 9 g_1^2 (g_0^2 - g_1^2)](g_2^2 + 9 g_1^2)^{-1} \end{aligned} \tag{21}$$

The field Z_μ is expressed by

$$Z_\mu = \frac{1}{\sqrt{N}} (-g_0 (g^{-2} + 9 g_2^{-2}) W_{3\mu} + g_1^{-1} B_\mu + 3 g_2^{-1} X_{3\mu}) \tag{22}$$

One can show that the weak current coupled to Z_μ takes the same form of $T_3 - sin^2\theta'_W Q$ as in the standard theory. But $sin^2\theta'_W$ is defined by

$$sin^2\theta'_W = (1 + cot^2\theta_W + 9 cot^2\theta_x)^{-1} \tag{23}$$

Here

$$tan\theta_W = \frac{g_1}{g_0} \qquad tan\theta_x = \frac{g_2}{g_0} \tag{24}$$

On the other hand, from

$$\begin{aligned} \frac{G_F}{\sqrt{2}} &= \frac{1}{4v^2} \\ e^{-2} &= g_0^{-2} + g_1^{-2} + 9 g_2^{-2} \end{aligned} \tag{25}$$

and eq. (18) one obtains

$$m_W^2 = \frac{\alpha\pi}{\sqrt{2}G_F} \Big/ sin^2\theta'_W \tag{26}$$

From (21) and (18) one obtains

$$m_z^2/m_W^2 = \frac{1}{cos^2\theta'_W} \tag{27}$$

and

$$m_y^2/m_X^2 = 1 + 9\frac{tan^2\theta_W}{tan^2\theta_X} \tag{28}$$

Equations (26) and (27) are just the same as in the standard theory. Therefore to the v^2/u^2 we have reproduced all of the Weinberg-Salam's results.

For more accurate calculation, eq. (22) should be replaced by a rigorous expression

$$Z_\mu = \frac{1}{\sqrt{n'}}\left\{-g_0\left(\frac{1}{g_1^2} + \frac{9}{g_2^2 - 2m_z^2u^{-2}}\right)W_{3\mu} + \frac{1}{g_1}B_\mu + \frac{3g_2}{g_2^2 - 2m_z^2u^{-2}}X_{3\mu}\right\} \tag{29}$$

The weak current coupled to Z_μ still takes a form of $T_3 - sin\theta''Q$ and

$$\frac{1}{sin^2\theta''_W} = \frac{1}{sin^2\theta_W} + 9cot^2\theta_x\left(1 - \frac{2m_X^2}{g_2^2u^2}\right)^{-1} \tag{30}$$

Thus, by comparing (3) with (23) one may find the modification of the order m_W^2/m_X^2 to the standard $W - S$ theory. Another modification occurs in the mass of neutral gauge particle Z_μ, namely eq. (27) should be replaced by

$$\frac{m_z^2}{m_W^2} = \frac{1}{cos^2\theta'_W} - 9\frac{v^2}{u^2}\frac{g_1^4(g_1^2g_2^2 + g_0^2g_2^2 + 9g_0^2g_1^2)}{g_0^2(g_2^2 + 9g_1^2)^3} \tag{31}$$

or

$$\frac{1}{\rho} \equiv \frac{m_z^2cos^2\theta'_W}{m_W^2} = 1 - \frac{m_W^2}{m_X^2}\frac{9g_2^2g_1^4(g_1^2g_2^2 + g_0^2g_2^2 + 9g_0^2g_1^2)}{g_0^4(g_2^2 + 9g_1^2)^3} \tag{32}$$

Using $UA2$ data, if $\rho = 1.02$ is taken, then $m_X \sim 2m_W$ from eq. (32) for $g_0 \sim g_1 \sim g_2$.

Acknowledgements

The authors would like to thank Helen Quinn and Yi-Shi Duan for their valuable help and discussions.

References

1. A partial list of references about composite models can be found in Y. Tomozawa, Phys. Lett. 104B (1981) 136; M. Bander et.al., Phys. Rev. Lett. 47 (1981) 549.
2. L. F. Luo and G. C. Yang, Phys. Lett. 113B (1982) 162.
3. A. Ebrahim, Phys. Lett. 73B (1978) 181.
4. For example, R. Mohapatra et. al., Phys. Rev. D16 (1977) 2543; L. F. Luo, T. Lu and G. C. Yang, Phys. Lett. 95B (1980) 257.
5. V. Barger et. al., Phys. Rev. Lett. 46 (1981) 1501.

Table I

$\nu_e=(A_1B_1\bar{C}_2)$	$u_\alpha=(A_2B_2\bar{C}_2)$	$u_\beta=(-A_1B_2\bar{C}_1)$	$u_\gamma=(-A_2B_1\bar{C}_1)$
$e^+=(-A_2B_2\bar{C}_1)$	$\bar{d}_\alpha=(-A_1,B_1\bar{C}_1)$	$\bar{d}_\beta=(A_2B_1\bar{C}_2)$	$\bar{d}_\gamma=(A_1B_2\bar{C}_2)$
$e^-=(\bar{A}_2\bar{B}_2C_1)$	$d_\alpha=(\bar{A}_1\bar{B}_1C_1)$	$d_\beta=(-\bar{A}_2\bar{B}_1C_2)$	$d_\gamma=(-\bar{A}_1\bar{B}_2C_2)$
$\bar{\nu}_e=(\bar{A}_1\bar{B}_1C_2)$	$\bar{u}_\alpha=(\bar{A}_2\bar{B}_2C_2)$	$\bar{u}_\beta=(-\bar{A}_1\bar{B}_2C_1)$	$\bar{u}_\gamma=(-\bar{A}_2\bar{B}_1C_1)$
$\nu_\mu=(A_1\bar{B}_2C_1)$	$c_\alpha=(-A_2\bar{B}_1C_1)$	$c_\beta=(-A_1\bar{B}_1C_2)$	$c_\gamma=(A_2\bar{B}_2C_2)$
$\mu^+=(A_1\bar{B}_1C_2)$	$\bar{s}_\alpha=(A_1\bar{B}_2C_2)$	$\bar{s}_\beta=(A_2\bar{B}_2C_2)$	$\bar{s}_\gamma=(-A_1\bar{B}_1C_1)$
$\mu^-=(\bar{A}_2B_1\bar{C}_2)$	$s_\alpha=(-\bar{A}_1B_2\bar{C}_2)$	$s_\beta=(-\bar{A}_2B_2\bar{C}_1)$	$s_\gamma=(\bar{A}_1B_1\bar{C}_1)$
$\bar{\nu}_\mu=(\bar{A}_1B_2\bar{C}_1)$	$\bar{c}_\alpha=(-\bar{A}_2B_1\bar{C}_1)$	$\bar{c}_\beta=(-\bar{A}_1B_1\bar{C}_2)$	$\bar{c}_\gamma=(\bar{A}_2B_2\bar{C}_2)$
$\nu_\tau=(\bar{A}_2B_1C_1)$	$t_\alpha=(-\bar{A}_1B_2C_1)$	$t_\beta=(-\bar{A}_2B_2C_2)$	$t_\gamma=(-\bar{A}_1B_1C_2)$
$\tau^+=(-\bar{A}_1B_2C_2)$	$\bar{b}_\alpha=(\bar{A}_2B_1C_2)$	$\bar{b}_\beta=(-\bar{A}_1B_1C_1)$	$\bar{b}_\gamma=(\bar{A}_2B_2C_1)$
$\tau^-=(A_1\bar{B}_2\bar{C}_2)$	$b_\alpha=(-A_2\bar{B}_1\bar{C}_2)$	$b_\beta=(A_1\bar{B}_1\bar{C}_1)$	$b_\gamma=(-A_2\bar{B}_2\bar{C}_1)$
$\bar{\nu}_\tau=(A_2\bar{B}_1\bar{C}_1)$	$\bar{t}_\alpha=(-A_1\bar{B}_2\bar{C}_1)$	$\bar{t}_\beta=(A_2\bar{B}_2\bar{C}_2)$	$\bar{t}_\gamma=(-A_1\bar{B}_1\bar{C}_2)$
$\bar{E}(2)=(\bar{A}_2\bar{B}_2\bar{C}_2)$	$\bar{E}_\alpha(\frac{4}{3})=(\bar{A}_1\bar{B}_1\bar{C}_2)$	$\bar{E}_\beta(\frac{4}{3})=(\bar{A}_2\bar{B}_1\bar{C}_1)$	$\bar{E}_\gamma(\frac{4}{3})=(\bar{A}_1\bar{B}_2\bar{C}_1)$
$\bar{E}(1)=(-\bar{A}_1\bar{B}_1\bar{C}_1)$	$\bar{E}_\alpha(\frac{5}{3})=(-\bar{A}_2\bar{B}_2\bar{C}_1)$	$\bar{E}_\beta(\frac{5}{3})=(-\bar{A}_1\bar{B}_2\bar{C}_2)$	$\bar{E}_\gamma(\frac{5}{3})=(-\bar{A}_2\bar{B}_1\bar{C}_2)$
$E(1)=(A_1B_2C_1)$	$E_\alpha=(A_2B_2C_1)$	$E_\beta(\frac{5}{3})=(A_1B_2C_2)$	$E_\gamma(\frac{5}{3})=(A_2B_1C_2)$
$E(2)=(A_2B_2C_2)$	$E_\alpha(\frac{4}{3})=(A_1B_1C_2)$	$E_\beta(\frac{4}{3})=(A_2B_1C_1)$	$E_\gamma(\frac{4}{3})=(A_1B_2C_1)$