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THE DETERMINATION OF NEUTRINO MASS FROM CLUSTERING PHENOMENA IN THE EARLY UNIVERSE*

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Abstract

The clustering of neutrinos in the early universe is discussed. From the deduced connection between the mass of galaxies and neutrinos, the neutrino mass can be determined.

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The neutrino clustering phenomena have been investigated by several authors.[1-3] The phenomena are related to the nonvanishing mass of neutrinos. We shall discuss the problem in three models.

Model A: The three kinds of Neutrinos have nearly the same mass, $m(\nu_e) \sim m(\nu_\mu) \sim m(\nu_\tau)$ (hereafter denoted as m_1, m_2 and m_3, respectively).

Model B: The third generation neutrino is much heavier than the other two, $m_3 \gg m_1 \sim m_2$, but there is no oscillation between them.

Model C: $m_3 \gg m_1 \sim m_2$ and the neutrino oscillation is assumed.

According to the distribution after decoupling

$$dn_i \equiv dn(\nu_i) = \frac{4\pi}{(2\pi \hbar)^3} \; g_{\nu_i} P^2 \, dP \left[exp \, \frac{PC}{(\frac{4}{11})^{1/3} kT} + 1 \right]^{-1} \tag{1}$$

$$dn_\nu = dn_1 + dn_2 + dn_3$$

one can deduce the partial pressure P_i of the neutrino of i-th kind.[4] The velocity of disturbance propagation in a nonrelativistic neutrino gas is

$$V_i^2 \equiv V(\nu_i)^2 = \frac{\partial \rho_i}{\partial \rho_i} = 6.9 \times 1^{-10} C^2 \left(\frac{10}{m_i(eV)} \right)^2 \left(\frac{T}{2.7} \right)^2 \tag{2}$$

for the i-th kind of neutrino (ρ_i—its mass density). If there exists the oscillation between them (Model C) then

$$\begin{aligned} V_\nu^2 &= \frac{\partial(P_1 + P_2 + P_3)}{\partial(\rho_1 + \rho_2 + \rho_3)} = \frac{5}{3} \frac{P_1 + P_2 + P_3}{\rho_1 + \rho_2 + \rho_3} \\ &= 1.4 \times 10^{-9} C^2 \, \frac{100}{m_1 \, m_3 \, (ev)} \left(\frac{T}{2.7} \right)^2 \end{aligned} \tag{3}$$

By use of the hydrodynamic equations of neutrinos with self-gravity[5] and solving the secular equation for $\omega^2 = 0$, [3]

$$\begin{pmatrix} k^2 V_1^2 - 4\pi G \rho_1 & -4\pi G \rho_1 & -4\pi G \rho_1 \\ -4\pi G \rho_2 & k^2 V_2^2 - 4\pi G \rho_2 & -4\pi G \rho_2 \\ -4\pi G \rho_3 & -4\pi G \rho_3 & k^2 V_3^2 - 4\pi G \rho_3 \end{pmatrix} \begin{pmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \end{pmatrix} = \omega^2 \begin{pmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \end{pmatrix} \tag{4}$$

(here k denotes the wave vector of disturbance), one obtains the Jeans wave length $\lambda_{J\nu}$

$$\left(\frac{2\pi}{\lambda_{J\nu}}\right)^2 = k^2 = 4\pi G\left(\frac{\rho_1}{V_1^2}+\frac{\rho_2}{V_2^2}+\frac{\rho_3}{V_3^2}\right) \tag{5}$$

So

$$\lambda_{J\nu}^{-2} = \lambda_{J1}^{-2}+\lambda_{J2}^{-2}+\lambda_{J3}^{-2} \tag{6}$$

$$\lambda_{Ji} = \left(\frac{\pi V_i^2}{G\rho_i}\right)^{1/2} \tag{7}$$

Inserting Eq. (2) into Eq. (7), it follows

$$\lambda_{Ji} \propto m_i^{-3/2}\,T^{-1/2} \tag{8}$$

Therefore, for a given temperature the Jeans wave length λ_J is dominated by the contribution of the heaviest neutrino. If the oscillation between neutrinos is considered (Model C), then from Eq. (3), it follows

$$\lambda_{J\nu} = \left(\frac{\pi V_\nu^2}{G\rho_\nu}\right)^{1/2} \propto m_1^{-1/2}\,m_3^{-1}\,T^{-1/2} \quad . \tag{9}$$

So far we have discussed the gravitational instability of pure neutrino system. Generally speaking, the neutrino and the ordinary matter interact through gravitation and are mixed with each other. Using the same method as in the derivation of Eq. (6), one obtains[6]

$$\lambda_J^{-2} = \lambda_{J\nu}^{-2}+\lambda_{Jm}^{-2} \quad . \tag{10}$$

Here

$$\lambda_{Jm} = \left(\frac{\pi V_m^2}{G\rho_m}\right)^{1/2} \tag{11}$$

is the Jeans wave length of ordinary matter. As pointed out by Fang et al.,[6] $\lambda_{Jm} \gg \lambda_{J\nu}$ and $\lambda_J \simeq \lambda_{J\nu}$ before recombination. That is, the gravitational instability in the early universe is controlled by the neutrino clustering.

The formation of galaxies is due to the gravitational instability. As is well known, all the masses of galaxies (M_G) are around 10^{11} $M_\odot$. How to explain it is an unsolved problem. Of course, the contraction and clustering of ordinary matter take place after the recombination principally. However, at the temperature of recombination, near 4000° K, the clustering of neutrinos may seriously influence the contraction processes of ordinary matter.[3,4] As shown by Fang et al., and Lu et al.,[4,6] $\lambda_J \sim \lambda_{J\nu}$ before recombination. Therefore, one may assume a preferential clustering mass of ordinary matter in the region of radius $\lambda_{J\nu}$. Let us define the Jeans mass of a system of pure neutrinos

$$M_J^{(\nu)} = \frac{4}{3}\pi \rho_\nu \lambda_{J\nu}^3 \quad ; \tag{12}$$

we obtain

$$\begin{aligned} M_J^{(\nu)} &= 1.3 \times 10^{47}\, m_1^{-7/2}(eV)\, T^{3/2}\,\mathrm{gr} \qquad \text{(Model A)} \\ M_J^{(\nu)} &= 2.2 \times 10^{47}\, m_3^{-7/2}(eV)\, T^{3/2}\,\mathrm{gr} \qquad \text{(Model B)} \\ M_J^{(\nu)} &= 6.2 \times 10^{47}\, m_1^{-3/2}(eV)\, m_3^{-2}(eV)\, T^{3/2}\,\mathrm{gr} \qquad \text{(Model C)} \quad . \end{aligned} \tag{13}$$

The mass of galaxy M_G is determined by the quantity of ordinary matter contained in the radius $\lambda_{J\nu}$ of gravitational instability at recombination temperature. Thus

$$\begin{aligned} M_G &= \frac{n_N m_N}{3 n_1 m_1} M_J^{(\nu)} \qquad \text{(Model A)} \\ M_G &= \frac{n_N m_N}{n_3 m_3} M_J^{(\nu)} \qquad \text{(Model B \& C)} \quad . \end{aligned} \tag{14}$$

Here n_N and m_N refer to the number density and mass of nucleons, respectively. Inserting Eq. (13) into Eq. (14), taking $T = 4200°$ K, we obtain

$$m_1^{9/2}(eV) = \frac{n_N}{n_1}\ 0.54 \times 10^{11} \Big/ \left(\frac{M_G}{10^{11}\,M_\odot}\right) \qquad \text{Model A}$$

$$m_3^{9/2}(eV) = \frac{n_N}{n_3}\ 2.8 \times 10^{11} \Big/ \left(\frac{M_G}{10^{11}\,M_\odot}\right) \qquad \text{Model B} \qquad (15)$$

$$m_3^3(eV)\, m_1^{3/2}(eV) = \frac{n_N}{n_1}\ 8 \times 10^{11} \Big/ \left(\frac{M_G}{10^{11}\,M_\odot}\right) \qquad \text{Model C} \quad .$$

If $n_N/n_{1,3} \sim 2 \times 10^{-9}$, $M_G \sim 10^{11}\,M_\odot$, then

$$m_1 = 2.8\,eV \qquad \text{(Model A)}$$

$$m_3 = 4.1\,eV \qquad \text{(Model B)} \qquad (16)$$

$$m_3\, m_1^{1/2} = 12\,eV \qquad \text{(Model C)} \quad .$$

The mass derived is consistent with the upper bound for the matter density in the universe, $\sum N_\nu m_\nu < \rho_{max}$.[7] If, instead of the galaxy mass, the mass of cluster of galaxies $M_G \sim 10^{14}\,M_\odot$ is used, then

$$m_1 = 0.61\,eV \qquad \text{(Model A)}$$

$$m_3 = 0.88\,eV \qquad \text{(Model B)} \qquad (17)$$

$$m_3\, m_1^{1/2} = 1.2\,(eV)^{3/2} \qquad \text{(Model C)} \quad .$$

Eqs. (16) and (17) give the neutrino masses estimated from the mass of galaxy and galaxy cluster, respectively. It is interesting to note that these formulae relate the masses of some of the lightest particles and the most giant objects in the universe.

Another approach to determine the neutrino mass from the clustering phenomena is by observation of a new type of astronomical objects—Neutrino Astronomical Objects (NAOs). Resulting from the gravitational instability and clustering, the number density of neutrinos in the clustering region should be greater than that given by the equilibrium distribution [Eq. (1)]. One may estimate the mass of NAO by assuming a degenerate state equation for neutrinos. By use of the Oppenheimer–Volkoff's method one deduces the mass of NAO[3]

$$M_{NAO} = 2.3 \times 10^{15} C_M \left(\frac{10}{m_1 (eV)}\right)^2 M_{\odot} \qquad \text{(Model A)}$$
$$M_{NAO} = 4 \times 10^{15} C_M \left(\frac{10}{m_3 (eV)}\right)^2 M_{\odot} \qquad \text{(Model B \& C)} \tag{18}$$

where $C_M \leq 1$. When the mass of NAO reaches its maximum, $C_M \sim 1$. Equating Eq. (18) with Eq. (13) one obtains the Jeans temperature of pure neutrino system:[3]

$$T_J = 0.24 \times 10^4 \, C_M^{2/3} \left(\frac{m_1 (eV)}{10}\right) {}^\circ K \qquad \text{(Model A)}$$
$$T_J = 0.24 \times 10^4 \, C_M^{2/3} \left(\frac{m_3 (eV)}{10}\right) {}^\circ K \qquad \text{(Model B)} \tag{19}$$
$$T_J = 0.12 \times 10^4 \, C_M^{2/3} \left(\frac{m_1 (eV)}{10}\right) {}^\circ K \qquad \text{(Model C)} \quad .$$

Only below these temperatures can the NAO be formed. Therefore, if an NAO is observed and the time scale of its formation is known, then, by use of Eq. (19), one may deduce some ranges of neutrino masses. In this way, Ma et al., [8] obtained the mass of neutrino greater than 1 eV by the statistical analyses of quasars.

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