SLAC-PUB-3181
August 1983
T/E

ON THE FACTORISATION OF "LONG DISTANCE" CONTRIBUTIONS TO THE DRELL-YAN CROSS SECTION*

C. T. SACHRAJDA
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305
and
Physics Department, The University†
Southampton SO9 5NH, ENGLAND

ABSTRACT

We review the status of the hypothesis that all "long distance" contributions to the Drell-Yan cross section can be absorbed into parton distribution functions.

1. Introduction

The Drell-Yan process,[1] i.e. the production of massive lepton pairs in hadronic collisions, has proved to be an extremely valuable one in the development of high energy physics over the past decade. Apart from being a useful testing ground of parton model ideas and more recently of QCD,[2] it is in this process of massive lepton pair production that the W and Z bosons have been discovered[3] as well as new quark flavours.[4] In addition, if one assumes that the mechanism responsible for massive lepton pair production is quark-antiquark annihilation (with Quantum Chromodynamics (QCD) corrections), then this process provides us with information about the structure functions of hadrons which cannot be used as targets in deep inelastic lepton hadron scattering, such as π or K mesons and antiprotons. The results obtained for these structure functions are very reasonable.[5]

Can one make reliable predictions for the Drell-Yan process, using perturbation theory in QCD? The main problem here is that when we calculate higher order terms in perturbation theory we find that we obtain important contributions from regions of phase space in which the quark and gluon momenta

*Work supported by the Department of Energy, contract DE-AC03-76SF00515.
†Permanent address

Invited Talk presented at the XIVth International Symposium on Multiparticle Dynamics, Lake Tahoe, June 22-27, 1983.

are small. The evaluation of these "long distance" contributions is clearly outside the scope of perturbation theory, however it is conjectured that they can be absorbed into the quark and gluon distribution functions (as measured in deep inelastic scattering experiments e.g.). This "Factorization" conjecture was based on the observation that up to one loop order the relation between the Drell-Yan cross section and deep inelastic structure functions involved only calculable "short distance" contributions. These one loop studies were partially generalised to all orders of perturbation theory, however explicit two loop calculations have recently demonstrated the existence of new features, ones which had not been considered in the earlier all order "proofs". In this talk we will review these extra long distance contributions, in particular the Glauber multiple scattering contribution first discussed in this context by Bodwin, Brodsky and Lepage.[6]

We start by considering a simple one loop Feynman diagrams in order to demonstrate some of the singular low momentum regions. We will distinguish between three different regions, a distinction which we hope will be useful in the subsequent sections. The three regions however merge into each other and in general the contributions from the regions of momentum space where they overlap are the most difficult to evaluate. The diagram which we consider is that of Fig. 1, whose calculation requires the evaluation of the integral.

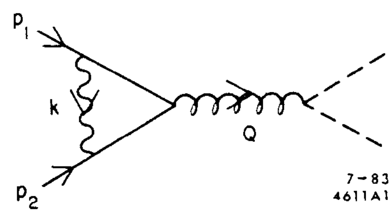

Fig. 1. Feynman Diagram (for the process quark-antiquark → lepton pair) used to illustrate the different types of mass singularity.

$$I \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2+i\epsilon]\,[(p_1-k)^2+i\epsilon]\,[(p_2+k)^2+i\epsilon]}\ , \tag{1}$$

where $p_1^2 = p_2^2 = 0$ and we have neglected all masses. The three singular regions we wish to demonstrate are:

(i) The region $k^\mu \to 0$ (all components of k vanish uniformly). From this region the contribution to I is

$$\int_{\substack{\text{small} \\ k_\mu}} \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2+i\epsilon]\,[-2p_1\cdot k+i\epsilon]\,[2p_2\cdot k+i\epsilon]}\ , \tag{2}$$

which is clearly singular. We call such singularities "infrared divergences".

(ii) The region $k^0, k^3 \sim k_\perp^2/\sqrt{s} \to 0$, where we work in the centre of mass frame of p_1 and p_2, whose components are in the 0 and 3 direction. In this region each of the three propagators vanishes like $k_\perp^2$ and the k_0 and k_3 phase space are each of magnitude $\sim k_\perp^2$ so that the contribution to I from this region is

$$\sim \int_{\substack{\text{small} \\ k_\perp}} \frac{d^2k_\perp (k_\perp^2)^2}{(k_\perp^2)^3} \tag{3}$$

which is also singular. We will call this region the Glauber Scattering region. The reason for this is that if we consider two body elastic scattering at large energy but fixed t, then we find that k_0 and k_3 are indeed $0(k_\perp^2/\sqrt{s})$, (see Fig. 2). Thus we can envisage this region as corresponding to a correction due to an elastic, small angle, almost on-shell scattering of the initial fermions.

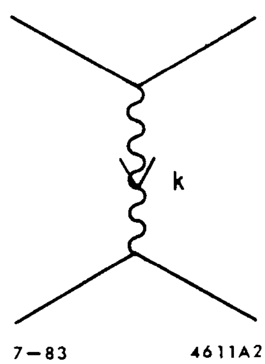

Fig. 2. Contribution to the amplitude for two body elastic quark quark scattering.

(iii) The region where k is parallel to p_1 or p_2. To demonstrate that the contribution to I is singular we start by introducing Sudakov Variables. These are defined by

$$k \equiv \alpha p_1 + \beta p_2 + k_\perp \tag{4}$$

and in terms of these variables

$$d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2k_\perp \tag{5}$$

$$k^2 = \alpha\beta s - k_\perp^2 \tag{6}$$

and

$$I = \frac{\pi s}{2} \int \frac{d\alpha\, d\beta\, dk_\perp^2}{(2\pi)^4} \frac{1}{[\alpha\beta s - k_\perp^2 + i\epsilon][-(1-\alpha)\beta s - k_\perp^2 + i\epsilon]} \frac{1}{[\alpha(1+\beta)s - k_\perp^2 + i\epsilon]} \tag{7}$$

By inspection we see that the region $\alpha \sim 0(1)$, $\beta s \sim k_\perp^2 \to 0$ is singular (as is the region $\beta \sim 0(1)$, $\alpha s \sim k_\perp^2 \to 0$), We call such singularities, collinear singularities.

As has been mentioned above, it is conjectured that the Drell-Yan cross section is related to Deep Inelastic Structure Functions in a calculable way, the relation depending only on short distance contributions. This clearly requires a cancellation of the various low momentum singularities. Such a cancellation is relatively straightforward to demonstrate for the collinear singularities, however the treatment of the infrared divergences and Glauber singularities is much more complicated, and we shall discuss these in some detail in the following sections.

If the factorisation hypothesis is correct, then the QCD prediction for the Drell-Yan cross section is that asymptotically (see Fig. 3)

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{9M^4} \sum_i Q_i^2 \int_0^1 dx_i dx_2 x_1 x_2\, \delta(x_1 x_2 - \tau) \left[q^{(1)}(x_1, M^2)\, \bar{q}^{(2)}(x_2, M^2) + \bar{q}^{(1)}(x_1, M^2) q^{(2)}(x_2, M^2)\right] \tag{8}$$

where $q(\bar{q})$ is the quark (antiquark) distribution function as measured in deep inelastic scattering at $|q^2| \sim O(M^2)$. There are logarithmic corrections to (8), specifically

$$\int dx_1 dx_2 \delta(x_1 x_2 - \tau) \to \int dx_1 dx_2 dz\ \delta(x_1 x_2 z - \tau)\left[\delta(z-1) + \frac{\alpha_s(M^2)}{\pi} f(z) + \cdots\right] + \text{ gluon-quark (antiquark) contributions.} \tag{9}$$

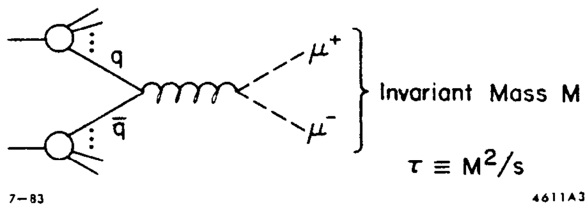

Fig. 3. The Drell-Yan model for massive lepton pairs production.

$f(z)$ is calculated from Feynman graphs involving only quarks and gluons and comes entirely from short distance regions of phase space.[7] The higher order corrections are large, their dominant effect is to modify the right hand side of Eq. (8) by a factor of 2-3 (the so-called K-factor). It is reassuring that experimental measurements seem to require such a factor. We will not discuss here the uncertainties in the calculation of the K-factor.

The plan for the rest of the lecture is as follows. In Section 2 we review the status of infrared divergences and their cancellations in Non-Abelian Gauge Theories. In Section 3 we present an introduction to Glauber Singularities and demonstrate how they can potentially spoil the factorisation conjecture. Section 4 contains the results of a full two loop calculation. Section 5 contains our conclusions.

2. Infrared Divergences

The Block-Nordsieck Theorem in Quantum Electrodynamics states that infrared divergences cancel for "physical" processes, i.e. for processes in which we allow for an arbitrary number of undetectable soft photons. There is no analogous theorem for Non-Abelian Gauge Theories, although with the demonstration that infrared divergences cancel for processes with none[8] or one[9] coloured parton in the initial state it was nevertheless hoped that a similar result would be true for all processes. It has since been shown, by means of an explicit example, that such a cancellation does not occur for processes with two coloured partons in the initial state.[10] For example, the process $qq \to \ell^+\ell^- X$ (see Fig. 4), where we average over the colours of the initial quarks, is infrared divergent in QCD, even though it is an inclusive quantity. These divergences first arise at the two loop order and are of the form

$$\sim \sigma_0 \, g^4 \frac{1}{\epsilon} \, \frac{1-\beta}{\beta} \left(\frac{1}{2\beta} \log \frac{1+\beta}{1-\beta} - 1 \right) \tag{10}$$

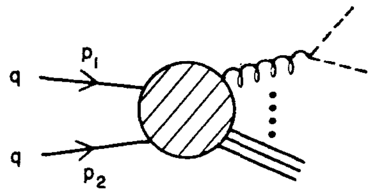

Fig. 4. Amplitude for the process $qq \to e^+e^-X$.

where we use dimensional regularisation to regulate the infrared divergences and $\epsilon \equiv 4$ — the number of dimensions, σ_0 is the lowest order cross section and $\beta = |\underline{p_1}|/E_1$ in the rest frame of p_2.

Of course in the physical Drell-Yan process the initial states are colour singlet hadrons and not coloured quarks and hence we do not expect any infrared divergences for this process. However, we do expect a residual "large logarithm", i.e. we expect the infrared divergence to be regulated as log "m^2" where "m^2" is a mass-scale characteristic of the hadronic wave function. This expectation is borne out by model calculations.[11] The reason that this infrared divergence is not direct evidence for the breakdown of factorisation is that it is accompanied by a factor $1 - \beta \sim m^4/S^2$, and hence is suppressed by two powers of s, (the authors of Ref. 12 obtain a different coefficient of $1/\epsilon$, one which is not suppressed by powers of s). A question which immediately arises is whether such a suppression occurs for all infrared divergences. The answer is "yes" for the leading infrared divergences in any order of perturbation theory,[13] i.e. the coefficient of g^{2n} $(1/\epsilon^{n-1})$ always contains a factor of $1 - \beta$. Recently Frenkel, Gatheral and Taylor[14] have presented an argument that all infrared divergences are suppressed by at least one power of s, although these authors caution that their work does not constitute a rigourous proof. This argument is based on an detailed study of Feynman graphs using in particular their unitarity and analyticity properties. Although it is somewhat disappointing that the arguments are so complicated, it is nevertheless reassuring that it now seems very likely that there is no breakdown of factorisation due to infrared divergences.

3. An Introduction to Glauber Multiple Scattering Singularities or Initial State Interactions*

We have already seen that the problem of verifying the factorisation conjecture in perturbation theory is a complicated one. A potential source of the

*This was the name given by Bodwin *et al.*[6] to this problem. We are reluctant to use it here since we shall not use light-cone time ordered perturbation theory, but only Feynman diagrams.

violation of this conjecture was pointed out by Bodwin, Brodsky and Lepage.[6] Consider Fig. 5 which includes some higher order corrections to the Drell-Yan model (Fig. 3), in which the partons, both "active" and "spectator", can scatter elastically off each other, staying close to their mass shells and exchanging only a small amount of transverse momentum. Below we will define more precisely what we mean by this. In QED we would find that the effect of these scatterings would be to modify the lowest order amplitude M_0 by a unitary Eikonal phase, so that (in impact parameter space)

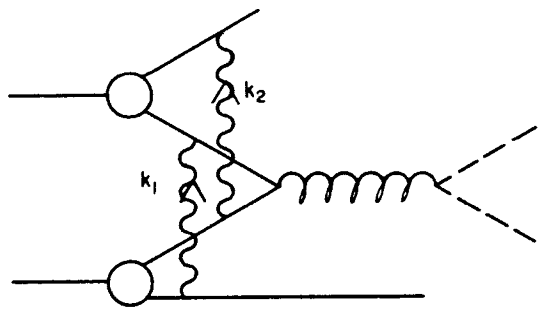

Fig. 5. Multiple scattering correction to the Drell-Yan model.

$$M_0 \to M_0 U \equiv M \tag{11}$$

and since $|M|^2 = |M_0|^2$, the integrated cross section is unaltered and the effect of all these Glauber Scatterings cancels. In a non-Abelian gauge theory there is no reason to expect such a cancellation since, because of the non-commuting nature of the interactions we have to distinguish between "initial state interactions" and "final state interactions" so that now

$$M_0 \to V M_0 U \equiv M \tag{12}$$

where U and V are unitary phases. Now $|M|^2 \neq |M_0|^2$ and hence there is no reason to expect factorisation to hold, and indeed Bodwin *et al.*[6] claimed that in two loop order there is a breakdown of the factorisation conjecture.

To get some idea of the nature of the problem we start with a simple example.[15] For simplicity we consider quark-hadron scattering (we return to the hadron-hadron scattering case later), for which, up to the two loop order in which we shall be interested, we can take the $q^{(1)}(x_1, M^2)$ of (8) to be a δ-function,

$$q^{(1)}(x_1, M^2) = \delta(x_1 - 1) \, , \tag{13}$$

so that if the factorisation conjecture is correct, equations (8) and (9) tells us that

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{9M^4}\, e_q^2\, \tau\, F_2(\tau, M^2) \tag{14}$$

where F_2 is the deep inelastic structure function. The fact that we do not have to consider higher order corrections on the right hand side of Eq. (13) follows from known crossing properties of the relevant low order graphs contributing to $d\sigma/dM^2$ and F_2.

In published calculations of the $f(z)$ of Eqs. (9)[7] one relates the cross section for $q\bar{q} \to e^+e^-X$ to the deep inelastic structure function of a quark. Neither the contribution of Fig. 6(a) to the Drell-Yan process not that from the graph of Fig. 6(b) to Deep Inelastic Scattering are considered. This is correct providing these two graphs are related by Eq. (14) and we now check whether this is so. For our purposes it can be shown that the magnetic moment interactions can be neglected and therefore we can consider scalar quarks. For simplicity we will consider the hadronic wave function to be given by a triple scalar coupling $\lambda(\phi\psi^+\psi)$ (where ϕ represents the hadron and ψ the quark field). Our results will depend only on the soft nature of this wavefunction, and on general analyticity and unitarity properties, in particular on the fact that there are no cuts below threshold.

We now compare the graph of Fig. 6(a) with that of 6(b). In terms of Sudakov variables we write

$$p = (0, 1, 0) \tag{15}$$

$$p_1 = (1, 0, 0) \tag{16}$$

and

$$q = (-x, 1, 0) \tag{17}$$

where we have neglected all masses. Using the mass-shell conditions we find that for the Drell-Yan process

$$p_2 = \left(1 - \tau\,,\ \frac{p_\perp^2}{(1-\tau)s}\,,\ \underset{\sim}{p}_\perp\right) \tag{18a}$$

and for Deep Inelastic Scattering

$$p_2 = \left(1 - x\,,\ \frac{p_\perp^2}{(1-x)s}\,,\ \underset{\sim}{p}_\perp\right) \tag{18b}$$

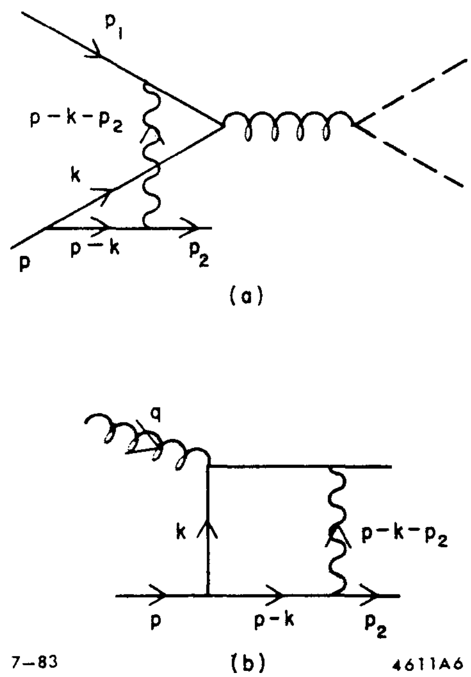

Fig. 6. One loop corrections to (a) the Drell-Yan Model and (b) Deep Inelastic Structure Function.

where $\underset{\sim}{p}_\perp$ is the transverse momentum of p_2 and in the deep inelastic case $s = 2p \cdot q$. Since we are interested in checking Eq. (14) we set $x = \tau$ in Eq. (18b) and notice that p_2 is now the same in both processes. Moreover by inspection three of the four propagators are now seen to be identical. Writing $k = (\alpha, \beta, \underset{\sim}{k}_\perp)$, we notice by simple power counting that the softness of the wavefunction implies that only the region where $\beta \simeq 0(k_\perp^2/s)$ contributes to the leading twist behaviour for $k_\perp^2 \ll s$. Restricting ourselves to this region we calculate the fourth propagator, the one which is different in the two cases, and find

$$\left[(p_1 + p - k - p_2)^2 + i\epsilon\right]^{-1} \simeq \left[(\tau - \alpha)s + i\epsilon\right]^{-1} \tag{19}$$

in the Drell-Yan case and

$$-\left[(q + k)^2 + i\epsilon\right]^{-1} \simeq \left[(\tau - \alpha)s - i\epsilon\right]^{-1} \tag{20}$$

where we have observed the terms depending on the transverse momenta into a new α, which is a shift of integration variables which has a negligible effect on the other propagators since β is small. The minus sign on the left hand side of (20) comes from the Feynman rules and the definition of the structure function. Thus, in the Feynman gauge, apart from the overall multiplication factor required from Eq. (14), the only difference between the contribution to the amplitude for the Drell-Yan process from the diagram of Fig. 6(a) and that form the diagram of Fig. 6(b) to deep inelastic scattering is the sign of the $i\epsilon$ on the right hand side of Eqs. (19) and (20). Taking the difference of that two integrals we find we can do the α integration trivially (since $(1/(\tau-\alpha)s+i\epsilon)-(1/(\tau-\alpha)s-i\epsilon) = -(2\pi i/s)\delta(\alpha-\tau))$ and the β integration by using Cauchy's Theorem to find that is equal to

$$4\pi^2 i\int\frac{d^2k_\perp}{k_\perp^2(p_\perp+k_\perp)^2}\ . \tag{21}$$

Thus the contributions to the amplitude for the two processes coming from finite (i.e. not too small) transverse momenta, differ only by an imaginary quantity. Hence to one loop order there is no violation of the factorisation conjecture (since to this order we convolute this amplitude with a real tree level diagram and add the complex conjugate).

The integral in Eq. (21) is divergent and although we know that these divergences will cancel, it is still possible that there is a non-factorising finite contribution, from the small transverse momentum region. To determine this, one has first to regulate these infrared divergences, and it has been verified to one loop order,[16] that there is no such non-factorising contribution. This calculation is interesting since individual Feynman diagrams give contributions which depend on the method of regulating the divergences, however they always sum to zero. For example, if one consider the graph of Fig. 1, and regulates the infrared divergence by dimensional regularisation or by giving the gluon a mass, then this divergence cancels against the corresponding diagrams with a real gluon leaving a finite contribution which contains a term

$$\frac{\alpha_s}{2\pi}\frac{4}{3}\pi^2\ . \tag{22}$$

This comes entirely from the short distance region of phase space and is perhaps the most important contribution to the K-factor. If, on the other hand, one regulates the infrared divergence by taking the initial quarks to be slightly off-shell then the corresponding contribution is

$$\frac{\alpha_s}{2\pi}\frac{4}{3}\pi^2\cdot 2\ . \tag{23}$$

Half of this contribution comes form the short distance region and the other half from the low momentum region. There are similar low momentum contributions from graphs involving interactions of spectator quarks, such as that of Fig. 6 and these contributions precisely cancel half of (23) leaving us with the short distance contribution (22). Thus there is no breakdown of the factorisation conjecture to one loop order.

Before going on to present the results of the analogous two loop calculations, we wish to mention that in a light like axial gauge* $n \cdot A = 0$, where $n = p_1$, the contribution to the Drell-Yan and Deep Inelastic amplitudes from the diagrams of Fig. 6(a) and 6(b) are each separately purely imaginary.[17]

4. Results of the Two Loop Calculation

The calculations of the effect of Glauber Multiple Scattering singularities have been extended to two loops.[17],[18] Indeed from the arguments at the beginning of Section 3 and noting that the only difference to one loop order between the Abelian and non-Abelian theories is a factor of $C_F = 4/3$, we would expect any problems with factorisation to start in this order. We start by presenting the results for diagrams with virtual gluons, evaluated in the Feynman gauge. For each diagram we subtract the contribution to the deep inelastic structure function (weighted by the factor required from Eq. (19)) from the Drell-Yan cross section. For most diagrams we obtain zero, but there are four exceptions each being proportional to B where

$$B = \frac{\alpha_s^2}{(2\pi)^5}\lambda^2\tau(1-\tau)\int\frac{d^2k_{\perp_1}d^2k_{\perp_2}d^2p_\perp}{p_\perp^2 k_{\perp_1}^2 k_{\perp_2}^2(p_\perp + k_{\perp_2})^2} \ . \tag{24}$$

The four non-vanishing contributions come from the diagrams of Fig. 7 and are:

$$\text{Fig.7a}: \quad \frac{C_F C_A}{4}\,B \tag{25a}$$

$$\text{Fig.7b}: \quad \frac{C_F C_A}{4}\,B \tag{25b}$$

$$\text{Fig.7c}: \quad -C_F^2\,B \tag{25c}$$

$$\text{Fig.7d}: \quad \left(C_F^2 - \frac{C_F C_A}{2}\right)B \tag{25d}$$

*Bodwin *et al.*[6] work predominantly in this gauge.

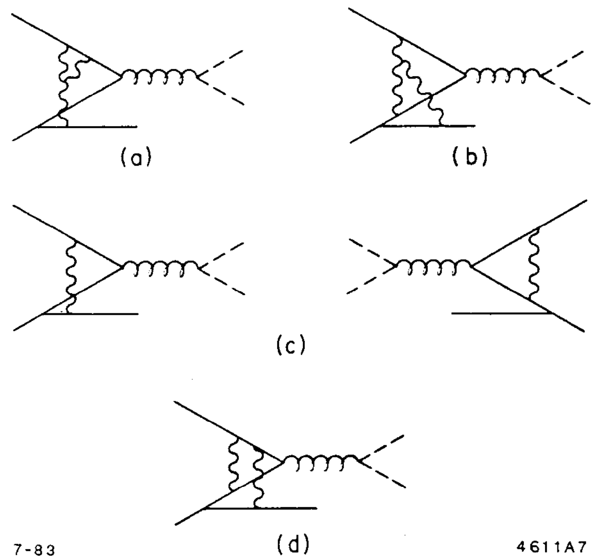

Fig. 7. Two loop corrections which individually give non factorising contributions in the Feynman Gauge. Figures (a), (b) and (d) are to be convoluted with the lowest order diagram, whereas figure (c) is to be understood as the convolution of the one loop graphs shown.

$C_F = 4/3$ and $C_A = 3$ are the eigenvalues of the quadratic Casimir operator in the fundamental and adjoint representations. The four contributions sum up to zero, so that these contributions also do not violate factorisation.

This result is in disagreement with Bodwin *et al.* who found a non-zero total and hence a counterexample of the factorisation conjecture. These authors, however, now agree with our result.[19] We point out that not all the contributions come from the Glauber region as defined by the gluons' Sudakov variables α and β being $0(k_\perp^2/s)$. For example integrals such as

$$\int_{-1}^{1} \frac{d\alpha\, d\beta}{(\alpha\beta + \delta)_p} = \pi^2 \,\mathrm{sign}(\delta) \tag{26}$$

appear frequently for $\delta = 0(k_\perp^2/s)$ and p stands for principle value. This integral has its main contribution from the region where $\alpha\beta = 0(k_\perp^2/s)$ and not $0(k_\perp^4/s^2)$. Moreover we were unable to define the Glauber Region satisfactorily, since, in the Feynman Gauge, the contribution from this region depends on the end points of the region. We prefer to integrate over all α's and β's and the results in Eq. (25) come from doing this.

The cancellation only occurs after integrating over the transverse momentum of the lepton pair, i.e. even though we express our results in terms of the quantity B of Eq. (24), $p_\perp$ corresponds to the transverse momentum of the lepton pair in (25a-c) whereas in (25d) the transverse momentum of the lepton pair is a linear combination of the integration variables. Hence we would expect "initial state interactions" to be relevant for the transverse momentum distribution at low transverse momentum (this is outside the scope of perturbation theory anyway). This effect is true in Abelian theories, indeed it is only the terms proportional to C_F^2 which need such a shift of integration variables in order to cancel.

We confirmed and extended our results in the light cone axial gauge. One nice feature of this gauge is that, up to two loop order, the Glauber region is now well defined, by this we man that we can define this region as being that where the Sudakov Variables α and β of the gluons are $0(k_\perp^2/s)$ and the relevant integrals do not depend on the precise choice of end points. The reason for this is that the $1/n \cdot k$ terms, which are present in the gluon propagators in this gauge, enhance the small α region so that there is sufficient convergence for the integrals not to depend on the end point. The relative contributions from the four non-vanishing diagrams (see Fig. 8) in which the incoming quark scatters twice of the spectator quark are as follows:

$$\text{Fig.(8a)} - \int \frac{C_F(C_F - (C_A/2))[k_{2\perp}^2 + 2k_{1\perp} \cdot k_{2\perp}]}{k_{1\perp}^2 k_{2\perp}^2 (k_1 + k_2)_\perp^2 (p + k_1 + k_2)_\perp^2 p_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \qquad (27a)$$

$$\text{Fig.(8b)} - \int \frac{C_F[k_{2\perp}^2 + 2k_{1\perp} \cdot k_{2\perp}]}{k_{1\perp}^2 k_{2\perp}^2 (k_1 + k_2)_\perp^2 (p + k_1 + k_2)_\perp^2 p_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \qquad (27b)$$

$$\text{Fig.(8c)} \int \frac{[C_F^2 + C_F(C_F - (C_A/2))]k_{1\perp} \cdot k_{2\perp}}{k_{1\perp}^2 k_{2\perp}^2 (k_1 + k_2)_\perp^2 (p + k_1 + k_2)_\perp^2 p_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \qquad (27c)$$

$$\text{Fig.(8d)} - \int \frac{C_F^2}{k_{1\perp}^2 k_{2\perp}^2 (p + k_1)_\perp^2 (p + k_2)_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \,. \qquad (27d)$$

Thus one sees that the terms proportional to C_F^2 cancel, which is a manifestation of the exponentiation of the Glauber singularities known to occur in an Abelian Theory. The terms proportional to $C_F C_A$ do not cancel in agreement with the expectations of Bodwin *et al.*[6] of a structure of the form VM_0U of Eq. (12). The contribution from Eq. (27) is cancelled by those of Fig. 9 in which the incoming quark scatters once off the spectator quark and once off the active antiquark.

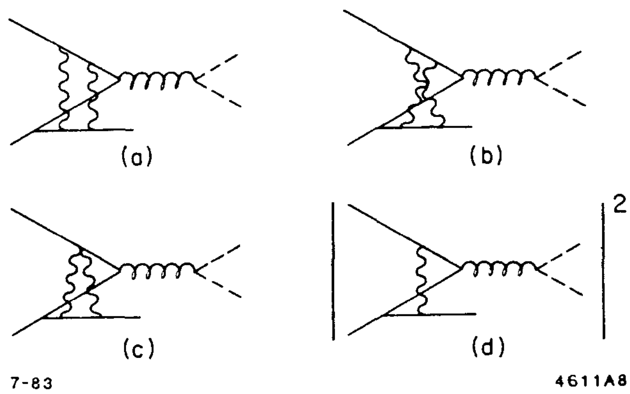

Fig. 8. Four diagrams which give non-vanishing contributions from the Glauber region in the light cone axial gauge. Figures (a)-(c) are to be convoluted with the lowest order graph.

These contribution are

$$\text{Fig.(9a)} \int \frac{C_F^2[k_{2\perp}^2 + 2k_{1\perp}\cdot k_{2\perp}]}{k_{1\perp}^2 k_{2\perp}^2 (p+k_1)_\perp^2 (k_1+k_2)_\perp^2 p_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \tag{28a}$$

$$\text{Fig.(9b)} \int \frac{\left(C_F^2 - \frac{C_F C_A}{2}\right)\left[k_{1\perp}^2 + 2k_{1\perp}\cdot k_{2\perp}\right]}{k_{1\perp}^2 k_{2\perp}^2 (p+k_1)_\perp^2 (k_1+k_2)_\perp^2 p_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \tag{28b}$$

$$\text{Fig.(9c)} - \int \frac{\left(C_F^2 + C_F\left(C_F - \frac{C_A}{2}\right)\right) k_{1\perp}\cdot k_{2\perp}}{k_{1\perp}^2 k_{2\perp}^2 (p+k_1)_\perp^2 (k_1+k_2)_\perp^2 p_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \tag{28c}$$

$$\text{Fig.(9d)} - \int \frac{C_F^2}{k_{1\perp}^2 k_{2\perp}^2 (p+k_2)_\perp^2 p_\perp^2} d^2k_{1\perp} d^2k_{2\perp} d^2p_\perp \,. \tag{28d}$$

Although this cancellation is interesting it is not very significant since it is straightforward to show that the Glauber region contribution of each of the diagrams of Fig. 8 and 9 are related to those of the corresponding diagrams in deep inelastic scattering by Eq. (14). This is also true for the diagrams in which the incoming quark scatters twice off the active anti-quark.

The Glauber region contributions are not the only relevant ones in the light cone axial gauge. For example one gets contributions from regions of phase space where one of the Sudakov variables β is $O(1/\delta)$ where δ is the cut-off in the Principle Value prescription defining $1/(n\cdot k)_p$. All these contributions cancel

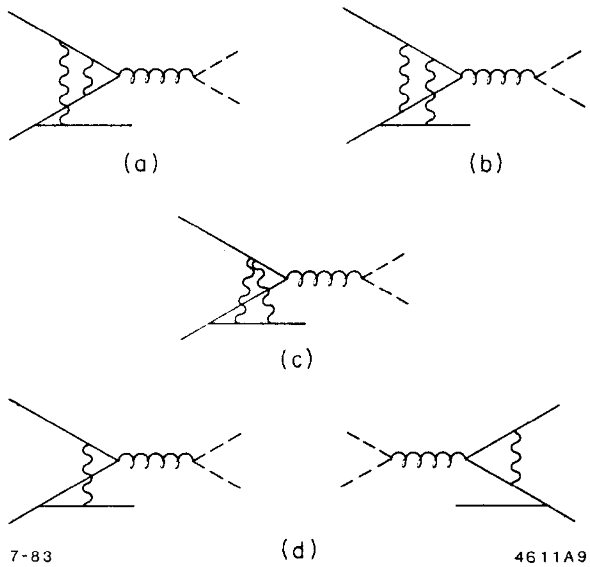

Fig. 9. Four more diagrams which give non-vanishing contributions from the Glauber region in the light cone axial gauge. Figures (a)-(c) are to be convoluted with the lowest order graphs whereas figure (d) is to be understood as the convolution of the two one loop graphs shown.

separately for the Drell-Yan process and for deep inelastic scattering. Diagram by diagram they are related not by Eq. (14), but by Eq. (14) modified by a factor (-1) on the right hand side, so this time the cancellation for each process separately is very significant. Thus we confirm the results of the Feynman Gauge calculation.

Before presenting our conclusions we would like to add the following results:

(i) We have not drawn any diagrams which include real gluons. In the light cone axial gauge it has been shown that, even individually, these do not have any non-factorising contributions of the kind discussed above.

(ii) A similar cancellation occurs for quark baryon scattering.

(iii) A similar cancellation occurs for meson-meson scattering.[20]

5. Conclusion

After several non-trivial calculations, there is no evidence that the factorisation of long distance contributions into parton distribution functions breaks down for the Drell-Yan process. This is in spite of the fact that the relevant theorems of QED do not carry over to Non-Abelian Gauge theories, and hence we have, a priori, no right to expect such a factorisation to hold. It is clear that our understanding of factorisation, even within the context of perturbation theory is far from complete.

One may have expected absorptive effects to have spoilt factorisation, however the calculations presented in this lecture indicate that, no matter how large the target, at sufficiently high energy there are no such effects (at least in perturbation theory). One possible interpretation is that the nature of the interaction is such that at high energies there is no time for multiple scattering interactions (which affect the colour quantum numbers of the partons) to alter the cross-section of the hard scattering process of lepton pair production. This is in analogy with the absence of bremsstrahlung from within the nucleus in high energy electron nucleus scattering.

Can one generalise the results presented above to all order of perturbation theory? This question is presently being studied[19],[21] by analytically continuing the Feynman integrals from the Glauber to the Collinear regions and then using technology already developed for the studies of collinear singularities (collinear Ward Identities). The problem, as always when trying to construct a proof to all orders of perturbation theory, is that it is hard to be sure one has identified all potentially non-factorising contributions. This is underlined by the surprises which have been discovered in the two loop calculations discussed above.

An important question which has not been studied so far is whether a similar cancellation of non-factorising contributions will occur for other hard scattering processes such as the inclusive production of jets (or single particles) at large transverse momentum.

Acknowledgments

It is a pleasure to thank G. T. Bodwin, S. J. Brodsky, S. D. Drell, R. D. Feynman, G. P. Lepage, H. D. Politzer, D. E. Soper, W. J. Stirling and in particular my collaborators W. W. Lindsay and D. A. Ross for many interesting discussions.

References

1. S. D. Drell and T. M. Yan, Phys. Rev. Lett. 25 (1970) 316; Ann. Phys. 66 (1971) 578.

2. G. Altarelli, Phys. Rep. 81 (1982) 1. Yu. L. Dokshitzer, D. I. Dyakonov and S. I. Troyan, Phys. Rep. 58C (1980) 269. C. T. Sachrajda in Gauge Theories in High Energy Physics (Proceedings of the 1981 Les Houches Summer School), edited by M. K. Gaillard and R. Stora (published by North Holland Publishing Company 1983).

3. G. Arnison *et al.*, Phys. Lett. 122B (1983) 103. M. Banner *et al.*, Phys. Lett. 122B (1983) 476. G. Arnison *et al.*, CERN preprint CERN-EP/83-73 (1983).

4. J. J. Aubert *et al.*, Phys. Rev. Lett. 33 (1974) 1404. S. W. Herb *et al.*, Phys. Rev. Lett. 39 (1977) 252.

5. G. Matthiae, La Rivista del Nuovo Cimento 4 (1981) 1. I. R. Kenyon, Rept. Math. Phys. 45 (1982) 1261.

6. G. T. Bodwin, S. J. Brodsky and G. P. Lepage, Phys. Rev. Lett. 47 (1981) 1799; SLAC-PUB-2860 (1981); SLAC-PUB-2927 (1982); SLAC-PUB-2966 (1982).

7. G. Altarelli, R. K. Ellis and G. Martinelli, Nucl. Phys. B143 (1978) 521 (Erratum B146 (1978) 544); B157 (1979) 461; J. Kubar-André and F. E. Paige, Phys. Rev. D19 (1979) 221; B. Humpert and W. L. Van Neerven, Phys. Lett. 84B (1979) 327; 85B (1979) 293.

8. T. Kinoshita, J. Math. Phys. 3 (1962) 1950; T. D. Lee and M. Nauenberg, Phys. Rev. 133 (1964) 1549; T. Appelquist, J. Carazzone, H. Kluberg-Stern and M. Roth, Phys. Rev. Lett. 36 (1976) 761.

9. S. V. Libby and G. Sterman, Phys. Rev. D19 (1979) 2468.

10. R. Doria, J. Frenkel and J. C. Taylor, Nucl. Phys. B168 (1980) 93; C. Di'Lieto, S. Gendron, I. G. Halliday and C. T. Sachrajda, Nucl. Phys. B183 (1981) 223.

11. A. Andrasi, M. Day, R. Doria, J. Frenkel and J. C. Taylor, Nucl. Phys. B182 (1981) 104.

12. H. Banerjee, M. Sengupta and A. Chatterjee, CERN preprint CERN-TH-3544 (1983); Phys. Lett. 124B (1983) 105.

13. C. E. Carneiro, M. Day, J. Frenkel, J. C. Taylor and M. T. Thomaz, Nucl. Phys. B183 (1981) 445.

14. J. Frenkel, J. G. M. Gatheral and J. C. Taylor, Sao Paulo preprint IFUSP/P-405 (1983).

15. W. W. Lindsay, D. A. Ross and C. T. Sachrajda, Phys. Lett. 117B (1982) 105.

16. P. V. Landshoff and W. J. Stirling, Z. Phys. C14 (1982) 251.

17. W. W. Lindsay, D. A. Ross and C. T. Sachrajda, Southampton University preprint SHEP 82/83-4, Nucl. Phys. B222 (1983) 189.

18. W. W. Lindsay, D. A. Ross and C. T. Sachrajda, Nucl. Phys. B214 (1983) 61.

19. G. T. Bodwin, S. J. Brodsky and G. P. Lepage (private communication).

20. W. W. Lindsay, Southampton University preprint SHEP 82/83-7 (1983).

21. J. C. Collins, D. E. Soper and G. Sterman, Phys. Lett. 126B (1983) 275.