SLAC-PUB-3166
July 1983
(T/E)

MONTE CARLO SENSITIVITY IN JET STUDIES – WHAT IS THE PHYSICS?*

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Abstract

Tests of perturbative QCD in hard processes involving jets have been found to depend on the jet fragmentation model used. We emphasize the need for testing the factorization of hard and soft processes, i.e., the independent fragmentation of jets. A method of analysis is suggested, which allows a model-independent determination of the gluon fragmentation function from e^+e^- data. We also comment on some simple features expected in $\bar{p}p \to$ (2 or 3 jets) $+X$ events.

1. QCD and Fragmentation Models

Many of our most interesting tests of perturbative QCD involve measuring quarks and gluons produced in hard processes. Particularly lower energy jets (such as the "third" jet produced in e^+e^- annihilations at present accelerators) are broad, however, and the hadrons associated with such jets cannot always be uniquely identified. It has therefore been common practice to compare data with QCD at the *hadron* level rather than at the parton (jet) level, by "fragmenting" the partons using Monte Carlo models.

There is an obvious drawback to this procedure. Predictions for hadron distributions depend on the fragmentation model used. Thus experimental tests of QCD get mixed up with fragmentation phenomenology. Since the strength of any chain of arguments is equal to its weakest link, this test of QCD is not as fundamental as we would like.

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*Work supported by the Department of Energy, contract DE-AC03-76SF00515.

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Review talk at the
XIV International Symposium on Multiparticle Dynamics
Lake Tahoe, June 1983

While the above difficulty has always been recognized, it was for some time generally felt not to be very important in practice, particularly for "clean" reactions like e^+e^- annihilations. By fitting "infrared insensitive" features of the hadron distributions, e.g., energy correlations, one expected to minimize the influence of parton fragmentation. It was furthermore shown that the parameters of the fragmentation model could be adjusted to give detailed agreement between "theory" and experiment.

2. Determinations of α_s from e^+e^- Annihilations.

The first comparisons[1)] between 3-jet events in e^+e^- annihilations and $O(\alpha_s)$ QCD predictions relied on the Field-Feynman[2)] fragmentation model applied independently[3)] to each of the jets. The comparisons were done at the hadron level. (The PLUTO Collaboration, however, used a cluster algorithm to reconstruct the jet axes.) Good agreement was found in each case, and the strong coupling α_s was determined to be $0.19 \pm 0.02 \pm 0.03$.

It soon became apparent, however, that even such "safe" quantities as energy correlations[4)] are strongly affected by fragmentation effects at present energies.[5)] This made it imperative to study the dependence of the α_s-determination on the fragmentation model assumed. CELLO[6)] undertook a critical comparison between the independent fragmentation ("IF") models[3)] used previously and the "string" fragmentation scheme developed in Lund.[7)]

In the string model, the gluon is split into a $q\bar{q}$ pair (plus a hadron). The quark and antiquark are (each) joined with the $\bar{q}$, q produced in the primary collision by a string, which fragments into hadrons in its own rest frame. Due to the Lorentz boost only ultra-relativistic hadrons follow the original q, $\bar{q}$, G directions in the e^+e^- CM system. At finite energies there are important correlations between the fragmentation of the three partons. This shows up, e.g., in an excess of slow particles emitted between the quark and gluon jets. Experimental evidence for such an excess has been reported by JADE.[8)]

The CELLO investigation[6)] confirmed the JADE observation[8)] that a good overall fit to the hadron distribution can be obtained using either the IF or string scheme. When they determined the value of α_s from various quantities such as sphericity, oblateness, etc., they found, however, that the result depended on the fragmentation scheme. The biggest difference was actually found when fitting energy correlations: $\alpha_s = 0.15 \pm 0.02$ in the IF scheme versus $\alpha_s = 0.25 \pm 0.04$ in the string model. Preliminary results from TASSO[9)] confirm this model sensitivity, although they find that the differences can be reduced somewhat by fitting all the parameters of the two models to the same data.

According to a recent result from MARK-J,[10)] the fragmentation model dependence seen when fitting energy correlations to $O(\alpha_s)$ is much smaller at $O(\alpha_s^2)$.

However, since the model dependence is basically a higher twist $(1/Q^2)$ effect, while the $O(\alpha_s^2)$ correction behaves like $1/\log Q^2$, the insensitivity seen by MARK-J is probably fortuitous, and presumably can only work for certain quantities at a given Q^2. This question needs further study.

The fact that the value of α_s, which characterizes the "hard" physics, comes out dependent on the "soft" physics described by the fragmentation model, is possible because the two physics domains are mixed in the analysis. Following this approach one has then two basic alternatives:

1. Pick one fragmentation model, on the basis of theoretical prejudice or fits to the data.
2. Postpone any determination of α_s to energies where the model dependence is unimportant.

Neither of these alternatives is very satisfactory, particularly considering that the IF and string schemes are the only two models for non-perturbative fragmentation that have been thoroughly studied by the experimental groups. Both models have arbitrary features, and one would prefer not to have one's choice of α_s depend on whether a model is, say, somewhat better than its competitors at describing the distributions of soft hadrons.

3. Jet Reconstruction Using Cluster Algorithms

In the present situation I believe emphasis should be put on *model-independent* analyses of data. Two very important questions are

A. *Are the jets clear enough to allow their reconstruction from the hadron data in an objective way?*

B. *Do the jets fragment independently?*

If (and only if!) an affirmative answer can be given to *both* questions, we do have a bona fide test of perturbative QCD. Independent jet fragmentation* ensures an effective factorization of soft and hard physics, as expected theoretically at sufficiently high energies.[11] The jet distributions obtained in A can be directly compared with perturbative QCD predictions, giving the value of α_s.

Once jets are shown to fragment independently we can go on to obtain reliable information also about the soft physics. Very important issues like the properties of gluon fragmention are difficult to settle in terms of the present models, which anyway treat gluons in a somewhat ad hoc way.

Beginning with the PLUTO study,[1] several groups have successfully applied

* Only sufficiently fast particles in a jet are expected to be produced independently of the hard vertex. There will be conventional short range correlations between soft particles in different jets.

cluster algorithms to their data. It is clearly important to do systematic studies of the dependence of the results on the algorithm employed, and to see how well jets produced by various Monte Carlo programs can be reconstructed. JADE has reported[12)] encouraging results in this regard. They used both the IF and string fragmentation models to estimate the "acceptance" corrections to their cluster algorithm. Within errors, the corrections given by both models turned out to be the same. If confirmed, this would mean that the answer to question A is *yes* – we can reconstruct the parton momenta in a model-independent way.

At first sight, the JADE result[12)] is puzzling, given the model dependence seen by CELLO[6)] and TASSO.[9)] However, the two sets of results are not obviously contradictory. Given a clear 3-jet event, such as there are in the data and in both models (since they fit the data), it is not surprising that the jet reconstruction is unambiguous. Yet such events will be produced at markedly different rates in the two models, for a fixed α_s.

More generally, the use of models only to estimate corrections to a cluster algorithm should result in less model dependence than when generating the hadron distributions directly. We may compare this with the use of Monte Carlos for calculating detector acceptances to hadrons – there as well it suffices for the model to be in only rough agreement with the data. In fact, as one reaches higher energies it may well become commonplace to talk about detector acceptances for *jets*, without even referring to hadrons, just as we today discuss electron detection without reference to its electromagnetic shower.

4. Model-independent Study of Jet Fragmentation

Assuming that the answer to question A above is positive – which appears to be a necessary condition for avoiding model-dependence – we have to tackle question B. Are there in fact correlations between the hadrons in different jets? Two types of correlations are expected: the conventional short-range correlations between soft particles, and long-range correlations introduced by the sum over different jet types (quark flavour/gluon).

I shall present a method for analyzing the long-range correlations,[13)] which has the advantage of giving model-independent results on the differences between light and heavy quark fragmentation. Furthermore, the gluon fragmentation function can (in principle at least) be determined from data on $e^+e^- \to 3$ jets. The analysis assumes independent fragmentation for jets – violations of this assumption will show up in various cross-checks of the method.

a) $e^+e^- \to 2$ Jets

Consider first 2-jet events in e^+e^- annihilations (with the jet reconstruction done using some algorithm satisfying A above). The fact that several quark flavors are produced introduces positive correlations between hadrons in the two jets –

if one jet looks like a light-quark jet then so will the other. This qualitative statement can be made precise as follows.

Consider ***any*** measurement related to the hadron (or lepton!) distributions in the jets. To be specific, let it be $D(z)$, the inclusive one-particle distribution at a given value of $z = E_h/E_{jet}$. When averaged over all events we have

$$D(z) = \sum_f \sigma_f \, D_f(z) \tag{1}$$

where $D_f(z)$ is the inclusive distribution for flavor f, and $\sigma_f = e_f^2 / \sum_f e_f^2$ is the relative production rate of this flavor. Now do the ***same*** measurement simultaneously for the two jets. Corresponding to (1) we thus get * the two-particle distribution $D(z_1, z_2)$, with the two particles belonging to different jets and having equal momenta:

$$D(z,z) = \sum_f \sigma_f \, D_f^2(z) \tag{2}$$

Now it is a matter of simple algebra to show that

$$D(z,z) - D^2(z) = \sum_f \sigma_f \left[D_f(z) - D(z)\right]^2 \tag{3}$$

Hence the correlation is indeed always positive, and can vanish only if $D_f(z) = D(z)$ for all f; i.e., if all flavors have identical fragmentation functions.

Comments:

– The result (3) relies only on a factorization between the hard and soft pysics, and remains valid after higher order corrections to the hard vertex.

– As evident from the trivial algebra, the result holds equally for features like the fraction of neutral energy, the number of leptons from weak decays, etc.

– Measurements of the correlation (3) can be used to estimate the differences between light ($\approx$ u) and heavy ($\approx$ c) quark fragmentation.

– If $D_f \neq D_{\bar{f}}$ (e.g., the π^+ inclusive distribution), the particles measured in opposite jets should be charge conjugate (π^+ and π^-). Equation (2) then generalizes to

$$D(z,z) = \frac{1}{2} \sum_f \sigma_f \left[D_f^2(z) + D_{\bar{f}}^2(z)\right] \tag{4}$$

* We are assuming quark and antiquark fragmentation for a given flavor to give the same $D_f(z) = D_{\bar{f}}(z)$. For the generalization, see below.

and Eq. (3) holds provided f and $\bar{f}$ are treated effectively as different flavors. A measurement of the *same* particle (π^+) in both jets,

$$\bar{D}(z,z) = \sum_f \sigma_f \, D_f(z) \, D_{\bar{f}}(z) \tag{5}$$

can be used to estimate the difference between quark and antiquark fragmentation:

$$D(z,z) - \bar{D}(z,z) = \frac{1}{2} \sum_f \sigma_f \Big[D_f(z) - D_{\bar{f}}(z) \Big]^2 \tag{6}$$

b) $e^+e^- \to 3$ jets

The above analysis can be repeated with minor modifications. * Averaging over all jets,

$$D(z) = \frac{2}{3} D_q(z) + \frac{1}{3} D_G(z) \tag{7}$$

where $D_q(z)$ is the average distribution for quark jets given by (1). Note that z is always defined using the energy of the jet in which the particle is found. Similarly,

$$D(z,z) = \frac{1}{3} \sum_f \sigma_f \, D_f^2(z) + \frac{2}{3} D_q(z) D_G(z) \tag{8}$$

$$D(z,z,z) = \sum_f \sigma_f \, D_f^2(z) \, D_G(z) \tag{9}$$

are the two- and three-particle distributions, all particles being in separate jets and carrying the same fraction of the jet momentum.

The correlations can now be expressed as

$$D(z,z) - D^2(z) = \frac{1}{3} \Big[\sum_f \sigma_f \, D_f^2(z) - D_q^2(z) \Big] - \frac{1}{9} \Big[D_q(z) - D_G(z) \Big]^2 \tag{10}$$

$$\begin{aligned} D(z,z,z) - D^3(z) = & \Big[\sum_f \sigma_f \, D_f^2(z) - D_q^2(z) \Big] D_G(z) \\ & - \frac{1}{27} \Big[8D_q(z) + D_G(z) \Big] \left[D_q(z) - D_G(z) \right]^2 \end{aligned} \tag{11}$$

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* From an experimental point of view the 3-jet sample may, of course, be more difficult to define than the 2-jet one.

The first terms on the r.h.s. of (10) and (11) are proportional to the 2-jet correlation (3), and are thus experimentally known. Hence a measurement of (10) and (11) can be directly used for determining the gluon fragmentation function $D_G(z)$.

The comments made above concerning 2-jet correlations apply equally to (10) and (11). In particular we note that the correlations should be independent of the jet production angles. On the other hand it is clear that new dynamical correlations will arise when the angle between two jets becomes small and the jets begin to coalesce. Hence a study of the angular (in)dependence of the 3-jet correlations could be particularly interesting.

5. $\bar{p}p \to 2$ jets

The new data[14] on jets at large p_T in $\bar{p}p$ collisions at $\sqrt{s} = 540$ GeV, as well as the recent ISR data[15] at large x_T will allow for model-independent tests of QCD in hadron collisions. The jets with high E_T are very clean, which will minimize the dependence on any cluster algorithm. I would like to point out[16] certain simple but non-trivial features of the QCD predictions that should be straightforward to test experimentally.

The exact $O(\alpha_s^2)$ QCD expression for 2-jet * production in hadron collisions contains a sum over many subprocesses,

$$\frac{d\sigma^{2\to 2}}{dx_a dx_b d\cos\theta^*} = \sum_{i,j} f_i^a(x_a, Q^2) f_j^b(x_b, Q^2) \frac{d\sigma_{ij}}{d\cos\theta^*} \tag{12}$$

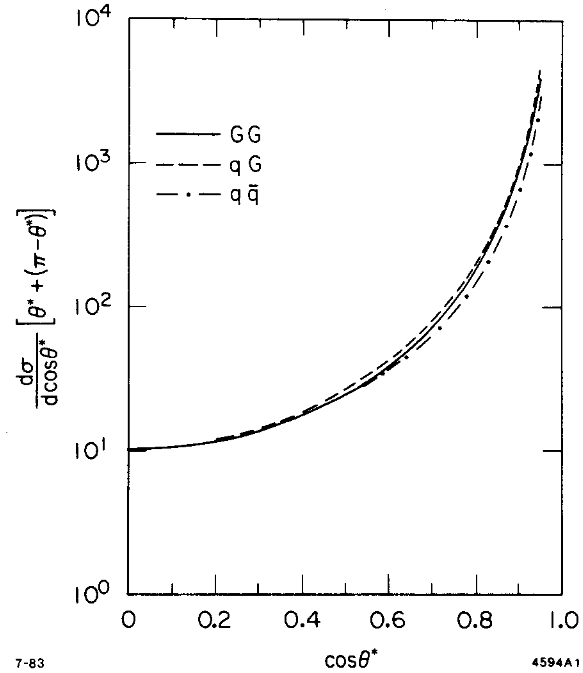
where partons i, j have structure functions f_i, f_j and cross-sections σ_{ij} at scattering angle θ^* in their own CM. How much of the rich structure of (12) can one hope to elucidate experimentally? Conversely, will experimental tests of (12) always be subject to assumptions about poorly known structure functions?

Actually the situation appears to be much simpler than (12) might suggest. There are a number general features of the 2-jet cross-section that are insensitive to the structure functions and which should be easy to verify in the data. On the other hand, separating the various subprocesses and deducing the structure functions from the data will require high accuracy measurements and identification of jet type (quark/gluon).

A primary reason for the simplification is the similar shape of all subprocess cross-sections $\sigma_{ij}(\theta^*)$. The most relevant ones for $\bar{p}p$ collisions are shown in Fig. 1, summed over final states and folded around $\cos\theta^* = 0$ (since we assume no identification of jet type). The relative difference between the curves in the range $|\cos\theta^*| \leq 0.8$ is $\leq 30\%$. From (12) we thus expect the 2-jet cross-section to have a nearly universal angular distribution, independent of the structure functions and

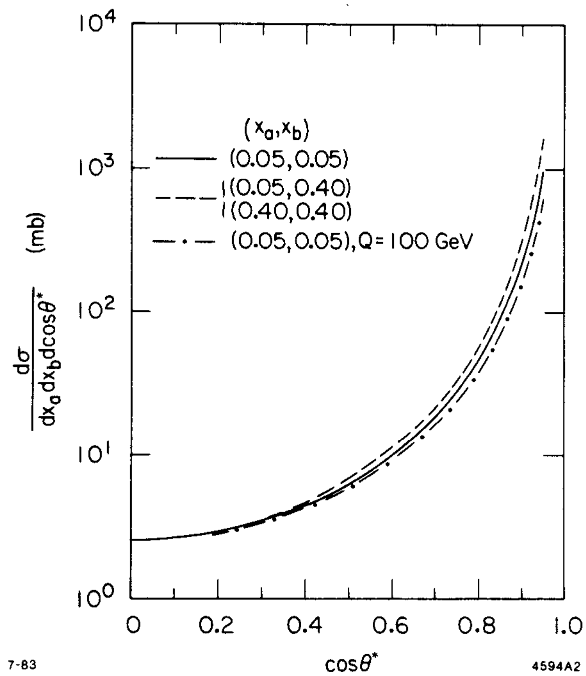
* We are not counting the spectator jets!

Fig. 1. Angular dependence of the QCD cross-sections for GG, qG and $q\bar{q}$ (equally flavored quarks) scattering assuming five quark flavors can be produced. The distributions have been folded around $\cos\theta^* = 0$ and are arbitrarily normalized to a common value at $\cos\theta^* = 0$, where their actual relative magnitude is 1:0.20:0.15.



of the parton energies x_a, x_b. This is shown in Fig. 2, where the influence of scaling violations is also displayed. Failure of the data to agree with the θ^*-distributions of Fig. 2 (within, say 30%) would essentially disprove the standard QCD ansatz (12).

Fig. 2. The parton CM angular distribution in $\bar{p}p \to 2$ jets for various values of the parton energy fractions x_a, x_b. The structure functions are from Ref. 17, with $Q^2 = 2\hat{s}\hat{t}\hat{u}/(\hat{s}^2 + \hat{t}^2 + \hat{u}^2)$ except for the dash-dotted curve, where a constant $Q = 100$ GeV is assumed.

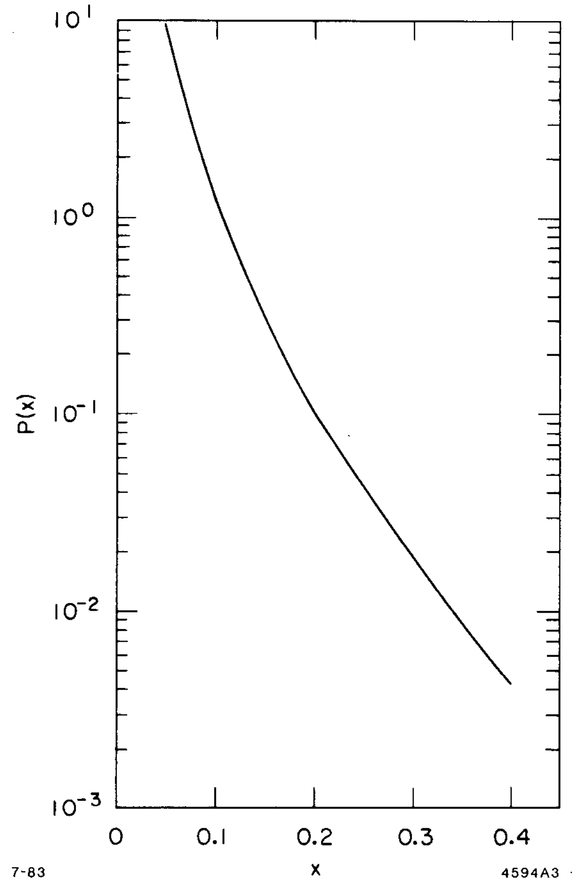


There is a further "effective" simplification occurring in (12). To a remarkably good approximation the QCD prediction *factorizes* in its dependence on x_a, x_b and θ^*. Thus (12) can be replaced by

$$\frac{d\sigma_{fact}^{2\to 2}}{dx_a dx_b d\cos\theta^*} = P(x_a)P(x_b)\sigma(\theta^*) \tag{13}$$

Here $P(x)$ is an effective structure function (independent of Q^2), which for the structure functions we used[17)] is shown in Fig. 3. The factorized expression (13) agrees with (12) at the 10...20% level in the entire kinematic range $|\cos\theta^*| \leq 0.8$; $0.05 \leq x_a, x_b \leq 0.40$.

Fig. 3. The effective structure function $P(x)$ used in Eq. (13) to describe the exact QCD expression (12).



It thus seems natural to test the factorization property (13) in the data by observing how closely the ratios of cross-sections at various values of x_a and x_b,

$$F(x_a, x_b) = \frac{\sigma(x_a, x_a)\sigma(x_b, x_b)}{[\sigma(x_a, x_b)]^2} \tag{14}$$

equal unity (at any θ^*). This is a non-trivial test of QCD since (14) should be within about 20% of unity in the experimentally accessible kinematic range, where cross-sections vary by six or more orders of magnitude.

To a rather good approximation we thus expect that it will be possible to summarize the data by Eq. (13), with $\sigma(\theta^*)$ given by the curves in Fig. 2 and $P(x)$ being an effective structure function as in Fig. 3. To go further than (13) probably requires identification of the final state jets (e.g., a quark + gluon final state singles out the qG subprocess).

The differences between quark and gluon fragmentation can be studied in much the same way as in e^+e^- annihilation (section 4). In $\bar{p}p \to 2$ jets we have effectively one quark flavor (neglecting differences between u and d fragmentation and the production of heavy quarks). The correlation analogous to (3) is

$$D(z,z) - D^2(z) = \left[\sigma_{qq}\sigma_{GG} - \frac{1}{4}\sigma_{qG}^2\right][D_q(z) - D_G(z)]^2 \tag{15}$$

Here σ_{qq}, σ_{qG} and σ_{GG} are the relative cross-sections for $q\,\bar{q}$, qG and GG final states ($\sigma_{qq} + \sigma_{qG} + \sigma_{GG} = 1$), and thus depend on the production variables x_a, x_b, θ^*. The final bracket in (15), on the other hand, is independent of the production mechanism and depends only on the differences between (light) quark and gluon fragmentation.

6. $\bar{p}p \to 3$ jets

The study of multi-jet configurations in hadron collisions is complicated by the ever-present spectator "jets". The best separation between parton and spectator jets is achieved when the production plane of the three jets is perpendicular to the beams (in the parton CM). In the laboratory, jets lying in this transverse plane are characterized by equal rapidities: $y_1 = y_2 = y_3$. There are in fact further reasons why this configuration should be favorable one:[16]

- By symmetry, only two variables are needed to describe the 3-jet configuration (in addition to the parton CM energy). Just as in e^+e^-, these can be taken as the scaled energies x_i ($x_1 + x_2 + x_3 = 2$), determined either from the jet energy fractions or their relative angles.

- The experimental acceptance is usually the same for jets with equal rapidity, and peaks around $y = 0$.

The $2 \to 3$ QCD subprocesses were first calculated three years ago.[18] Subsequently the expressions were dramatically simplified.[19] Requiring the jets to lie in the transverse plane leads to further simplifications. Writing the 3-jet cross-section as

$$\left.\frac{d\sigma^{2\to3}}{dx_a dx_b dx_1 dx_2 d\cos\theta_n}\right|_{\cos\theta_n = \pm1} = \sum_{ij} f_i^a(x_a)\, f_j^b(x_b)\, \frac{\alpha_s^3}{8} |M_{2\to3}|^2 \tag{16}$$

where θ_n is the polar angle of the normal to the production plane, the amplitudes M are relatively simple functions[19] of the jet energy fractions x_1, x_2, x_3. For example,[16]

$$|M(q\,\bar{q} \to GGG)|^2 = \frac{(N^2-1)}{9N^4}\,\frac{x_1^4+x_2^4+x_3^4}{x_1^2x_2^2x_3^2} \tag{17}$$

$$\times\left\{1+N^2\left(1-\frac{1}{2}\,\frac{x_1x_2}{1-x_3}-\cdots\right)+\frac{N^4}{8}\left[\frac{x_1x_2x_3^2}{(1-x_1)(1-x_2)}+\cdots\right]\right\}$$

where $N = 3$ for QCD and $\cdots$ stands for the two cyclically permuted terms: $x_1 \to x_2 \to x_3 \to x_1$. Spin, color and statistics (1/3!) factors are included in (17).

The 3-jet distributions in the transverse plane can be analyzed using the methods of e^+e^-. In Fig. 4 are shown the thrust distributions ($T = \max\{x_1, x_2, x_3\}$) for the most relevant subprocesses. Apart from the normalization, they are similar to each other and to $e^+e^- \to q\,\bar{q}\,G$.

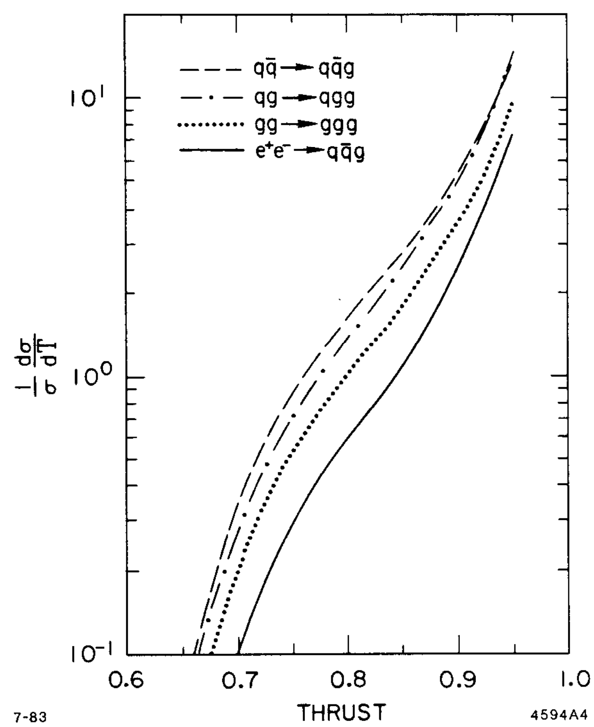
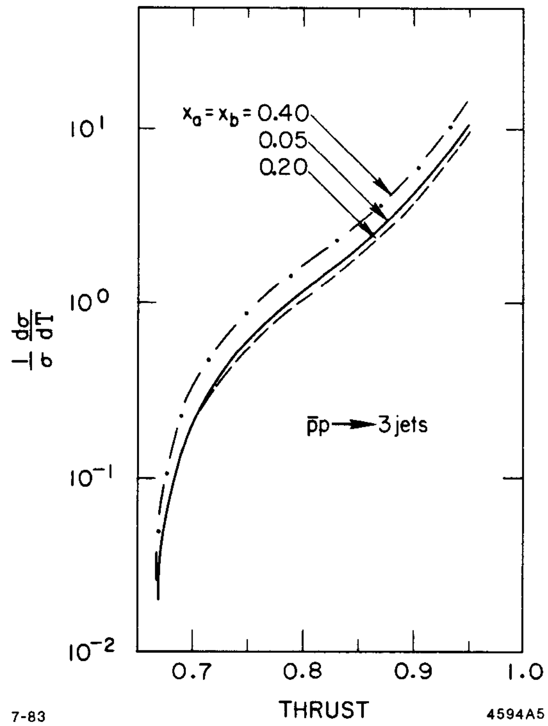


Fig. 4. Thrust distributions for some QCD processes. The production plane is orthogonal to the incoming particles, and the distribution is normalized to the corresponding $2 \to 2$ processes at $\theta^* = 90^o$.

Combining them with the structure functions[17)] through Eq. (16) one finds the corresponding thrust distributions for $\bar{p}\,p \to 3$ jets, which are only weakly dependent on x_a, x_b (Fig. 5).

Fig. 5. Thrust distributions for $\bar{p}p \to$ 3 jets in the transverse plane, normalized to the $\bar{p}p \to 2$ jet cross-section at $\theta^* = 90^o$.



7. Conclusions

The fact[6,9] that the value of the strong coupling α_s depends on the fragmentation model assumed in analyzing e^+e^- data shows that present methods do not separately test the hard and soft physics, nor has a factorization between these been demonstrated experimentally. The encouraging indication[12] that jets can be reconstructed in a model-independent way should be further studied. If this is confirmed, it will be essential to look for correlations between hadrons belonging to different jets. Only when the theoretically expected[11] factorization between hard parton production and soft fragmentation has been established can one conclusively test perturbative QCD.

A method for studying the correlations between jets introduced by the production of several quark flavors and gluons was discussed in sections 4 and 5. In principle this method allows a model-independent study of the differences between light and heavy quark fragmentation, as well as a determination of the gluon fragmentation.

The data on $\bar{p}p \to 2$ jets should to a good approximation factorize in its dependence o;n the parton momenta x_a, x_b and their scattering angle θ^* according to Eq. (13). The nearly universal angular distribution $\sigma(\theta^*)$ (Fig. 2) should be confirmed by experiment, which also can measure the effective structure function $P(x)$. To separate the various subprocesses probably requires identification of jet types.

Three-jet final states can be analyzed analogously to e^+e^--annihilations when the produced jets lie in the transverse plane. The thrust distribution should be weakly dependent on x_a, x_b and about a factor two higher than in e^+e^-.

ACKNOWLEDGEMENTS

Many of the results presented here grew out of a collaboration with Francis Halzen. I also wish to thank Carsten Peterson for a discussion on correlations. I am grateful to the organizers of this conference for their invitation, and to the SLAC Theory Group for their hospitality.

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