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SEARCH FOR NEUTRAL GAUGE FERMIONS IN e^+e^- ANNIHILATION*

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Abstract

Many broken supersymmetric theories contain two light neutral fermions χ, χ' which are mixtures of supersymmetric partners of the gauge and Higgs bosons. The heavier of these (χ') can be produced in e^+e^- annihilation by $e^+e^- \rightarrow \chi\chi'$ or $\chi'\chi'$ with cross-sections that may be comparable to that for $e^+e^- \rightarrow \bar{\nu}_\mu \nu_\mu$: $\sigma = (0.1$ to $10)$ pb at $\sqrt{s} = 30$ GeV. The χ' likes to decay into an e^+e^- or $\mu^+\mu^-$ pair and a χ, which is likely for cosmological reasons to be almost a pure photino $\tilde{\gamma}$. Associated production $e^+e^- \rightarrow \chi\chi'$ can lead to a one-sided "zen" event structure with visible decay products in one hemisphere only, while $e^+e^- \rightarrow \chi'\chi'$ can give 4-lepton final states with missing energy.

There is great interest at the moment in theories with broken supersymmetry (SUSY) and in their experimental signatures.[1] In particular, much attention has recently been paid [2,3,4,5] to the phenomenology of gauge fermions, which are in general mixtures of the SUSY partners of weak gauge bosons ($\tilde{W}^{\pm}$, $\tilde{W}^{3}$, $\tilde{B}$) and of Higgs bosons ($\tilde{H}_i$). In the minimal SUSY extension of the standard $SU(3) \times SU(2) \times U(1)$ model [6] there are two charged gauge fermions and four Majorana neutrals. Both model-building arguments and cosmological constraints [7] suggest that the lightest SUSY particle may well be a neutral gauge fermion χ which can be much lighter than the Z^0 or $W^{\pm}$ bosons. In many models an additional neutral gauge fermion χ',[3,4] and a charged gauge fermion $\chi^{\pm}$,[8] can also be lighter than the $W^{\pm}$ and Z^0. Hence there is some chance of seeing gauge fermions at present e^+e^- colliding ring energies [2,3] as well as in Z^0 and $W^{\pm}$ decays.[3,4,8] Estimates have already been made of $e^+e^- \rightarrow \chi\chi$,[2] $\chi\chi'$,[3] and $\chi'\chi'$,[3,4] of $Z^0 \rightarrow \chi\chi$, $\chi\chi'$ and $\chi'\chi'$,[3,4] and of $W^{\pm} \rightarrow \chi^{\pm}\chi$, $\chi^{\pm}\chi'$.[3,4]. Possible signatures for $e^+e^- \rightarrow \chi\chi'$, $Z^0 \rightarrow \chi\chi'$ and $W^{\pm} \rightarrow \chi^{\pm}\chi$ include one-sided "zen" events [3,4] in which one event hemisphere contains visible decay products of the χ' or $\chi^{\pm}$, while there are no visible particles in the recoil hemisphere. Other possible sources of one-sided events include sneutrino $\tilde{\nu}$ pair production [9] and $W^{\pm} \rightarrow \nu +$ heavy lepton.[4,10]

In this paper we study the reactions $e^+e^- \to \chi\chi'$ and $\chi'\chi'$ in detail. We present exact cross-sections including crossed channel selectron $\tilde{e}$ exchange and direct channel Z^0 exchange contributions. We also give approximate forms which are convenient for scaling cross-sections as functions of particle masses and of the center-of-mass energy $\sqrt{s}$. We then impose cosmological constraints [11,4,7] on the minimal SUSY extension of the Standard Model and present numerical calculations of the $e^+e^- \to \chi\chi'$, $\chi'\chi'$ cross-sections in the cosmologically allowed domain.[7] The results we display are for $\sqrt{s} = 30$ GeV with $m_{\tilde{e}} = 20$, 40 GeV,* and can easily be scaled to other values of $\sqrt{s}$ and $m_{\tilde{e}}$. We find that even within the cosmological constraints the cross-sections for $e^+e^- \to \chi\chi'$ (figs. 2 and 3) and $\chi'\chi'$ (figs. 4 and 5) can both be comparable to the cross-section for $e^+e^- \to \bar{\nu}_\mu \nu_\mu$, ranging from 0.1 to 10 pb at $\sqrt{s} = 30$ GeV. It is likely that the lightest neutral gauge fermion χ is almost a pure photino $\tilde{\gamma}$, while χ' is mainly a Higgsino $\tilde{H}$. However, in much of the cosmological domain the χ' has a substantial gaugino component, so that $e^+e^- \to \chi\chi'$ has an observable cross-section. In this case the χ' is likely also to decay via its gaugino component: $\chi' \to \chi + (e^+e^-, \mu^+\mu^-$ or $\bar{q}q)$. We argue that since squarks $\tilde{q}$ are likely [12,13] to be at least as heavy as sleptons $\tilde{\ell}$, probably e^+e^-, $\mu^+\mu^-$ and perhaps $\tau^+\tau^-$ dominate the χ' decay products. To aid experimental searches for the signatures $e^+e^- \to (\ell^+\ell^-) +$ nothing or $e^+e^- \to (\ell^+\ell^-) + (\ell^+\ell^-) +$ missing energy-momentum we present formulae for the energy distributions of the decay leptons $\ell^\pm$ and for the invariant mass of $\ell^+\ell^-$ pairs.

There are two important types of diagrams for neutral gauge fermion production in e^+e^- annihilation, namely crossed channel $\tilde{e}_{L,R}$ exchange and direct channel Z^0 exchange. Because of the small electron mass, the $\tilde{H}$ components of the mass eigenstates χ and χ' have negligible couplings to the selectrons, while the gaugino components do not couple to the Z^0. Thus for an arbitrary neutral Majorana fermion

$$\chi_i \equiv \alpha_i \tilde{W}^3 + \beta_i \tilde{B} + \gamma_i \tilde{H}_1^0 + \delta_i \tilde{H}_2^0 \qquad (i = 1, \ldots, 4) \tag{1}$$

*We have chosen to present our numerical results for the case $m_{\tilde{e}_L} = m_{\tilde{e}_R} \equiv m_{\tilde{e}}$ to simplify the discussion.

where α_i, β_i, γ_i, δ_i are elements of an orthogonal mixing matrix [4], we have

$$\sigma(e^+e^- \to \chi_i\chi_j) = \sigma^{ij}_{\tilde{e}\tilde{e}} + \sigma^{ij}_{\tilde{e}Z} + \sigma^{ij}_{ZZ} \tag{2a}$$

where *

$$\sigma^{ij}_{\tilde{e}\tilde{e}} = \frac{G_F^2}{2\pi}\,\frac{k}{\sqrt{s}}\left\{O_L^2\,\frac{m_Z^4}{m_{\tilde{e}_L}^4}\left[\frac{E_iE_j + k^2 - \sqrt{s}\,k\frac{a_L}{b_L}}{a_L^2 - b_L^2} + 2\,\frac{k^2}{b_L^2}\right.\right.$$
$$\left.\left. + \frac{1}{2}\left(\frac{\sqrt{s}\,k}{b_L^2} - 2\,\frac{k^2}{b_L^2}\,\frac{a_L}{b_L} - \eta_i\eta_j\,\frac{m_im_j}{a_Lb_L}\right)\ell n\,\frac{a_L+b_L}{a_L-b_L}\right] + (L\to R)\right\} \tag{2b}$$

$$\sigma^{ij}_{\tilde{e}Z} = \frac{G_F^2}{2\pi}\,\frac{k}{\sqrt{s}}\left\{(-R_eP_Z)O_LO_Z\,\frac{g_L}{b_L}\,\frac{m_Z^2}{m_{\tilde{e}_L}^2}\left[2\eta_L\sqrt{s}\,k - 2k^2\,\frac{a_L}{b_L}\right.\right.$$
$$\left.\left. + \left(E_iE_j + k^2\,\frac{a_L^2}{b_L^2} - \eta_i\eta_jm_im_j - \eta_L\,\sqrt{s}\,k\,\frac{a_L}{b_L}\right)\ell n\,\frac{a_L+b_L}{a_L-b_L}\right] - (L\to R)\right\} \tag{2c}$$

$$\sigma^{ij}_{ZZ} = \frac{G_F^2}{2\pi}\,\frac{k}{\sqrt{s}}\left[O_Z^2\,|P_Z|^2\,(g_L^2+g_R^2)\left(E_iE_j + \frac{1}{3}\,k^2 - \eta_i\eta_jm_im_j\right)\right] \tag{2d}$$

Here $E_{i(j)}$, k and $m_{i(j)}$ are the energy, the momentum and the mass of $\chi_{i(j)}$, $a_{L(R)} = 1 + (s/m^2_{\tilde{e}_{L(R)}})(1-(m_j^2+m_i^2)/s)$, $b_{L(R)} = k\sqrt{s}/m^2_{\tilde{e}_{L(R)}}$, $g_L = -1+2\sin^2\theta_W$ and $g_R = 2\sin^2\theta_W$ are the electron neutral current couplings, $P_z = (1-(s/M_Z^2) - i(\Gamma_Z/M_Z))^{-1}$, $\eta_{L(R)} = \overset{+}{(-)}1$, $\eta_{i(j)} = (+1)$ or (-1) is the sign in the Majorana condition, $\chi_{i(j)} = \eta_{i(j)}C\,\bar{\chi}_{i(j)}$ ($\eta_i\eta_j$ is equal to the relative sign of the corresponding eigenvalues obtained by diagonalizing the neutral fermion mass matrix), and $O_L = \sin^2\theta_W\,\beta_i\beta_j + \cos^2\theta_W\,\alpha_i\alpha_j + (\alpha_i\beta_j+\alpha_j\beta_i)\sin\theta_W\cos\theta_W$, $O_R = 4\sin^2\theta_W\,\beta_i\beta_j$ and $O_Z = \delta_i\delta_j - \gamma_i\gamma_j$.

It should be noted that the threshold behavior of $\sigma(e^+e^- \to \chi_i\chi_j)$ exhibits a strong dependence on the factor $\eta_i\eta_j$. If $\eta_i\eta_j = +1$, the threshold suppression is P-wave and remains substantial well above threshold. If $\eta_i\eta_j = -1$ it is an S-wave suppression and the cross-section is considerably larger in the threshold region. Both values of $\eta_i\eta_j$ are possible in the class of models we shall consider.

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* The expressions on the right-hand side of eqs. (2b – 2d) should be divided by two in the case $i = j$; χ and χ' correspond to χ_1 and χ_2 in this notation.

The exact cross-section formula (2) used in our numerical computations is quite complicated, but several simplifications are possible. First, at center-of-mass energies of 30 to 40 GeV the Z^0-exchange contribution to $e^+e^- \to \chi\chi'$ is unobservably small [$\ll 0.1 \times \sigma(e^+e^- \to \bar{\nu}_\mu \nu_\mu)$]. Hence any observable contribution to $e^+e^- \to \chi\chi'$ is due to $\tilde{e}$ exchange (2b). Secondly, there is a relatively simple analytic approximation for the $\tilde{e}$-exchange contribution $\sigma^{ij}_{\tilde{e}\tilde{e}}$ which factorizes as a product of a $\tilde{e}$ propagation factor and a threshold phase-space suppression factor:

$$\sigma^{ij}_{\tilde{e}\tilde{e}} \propto \left[\frac{1}{s}\left(1 - \frac{2m^2_{\tilde{e}_L}}{s}\ell n\left(1+\frac{s}{m^2_{\tilde{e}_L}}\right) + \frac{m^2_{\tilde{e}_L}}{s+m^2_{\tilde{e}_L}}\right) + (m_{\tilde{e}_L} \to m_{\tilde{e}_R})\right] \times \left[\frac{k}{s^{3/2}}\left(E_iE_j + \frac{k^2}{3} - \eta_i\eta_j m_i m_j\right)\right] . \tag{3}$$

The approximate scaling formula (3) is very convenient for scaling cross-sections as functions of $m_{\tilde{e}}$, $m_{i,j}$, and $\sqrt{s}$ when $\sqrt{s} \le 40$ GeV. Near threshold it is accurate to within about 30%, and becomes more precise where the phase space effects are smaller.

To proceed further we need to know the mixing coefficients $(\alpha_i, \beta_i, \gamma_i, \delta_i)$ in the decomposition (1). To determine these we specify a minimal SUSY extension of the Standard Model whose relevant Lagrangian terms are [4]

$$\mathcal{L} \ni \epsilon\epsilon_{\alpha\beta}\tilde{H}^\alpha_1\tilde{H}^\beta_2 - M_2\tilde{W}^a\tilde{W}^a - M_1\tilde{B}\tilde{B} \tag{4}$$

where the $\tilde{W}^a$ and $\tilde{B}$ are $SU(2)$ and $U(1)$ gauginos, and $\alpha, \beta(a)$ are doublet (triplet) $SU(2)$ indices. The quantities ϵ, M_2 and M_1 are mass parameters that are generally expected to be $O(M_W)$. We shall assume

$$M_1 = \frac{5}{3}\,\frac{\alpha_1}{\alpha_2}\,M_2 \tag{5}$$

where $\alpha_{1,2} \equiv g^2_{1,2}/4\pi$ are the $SU(2)$ and $U(1)$ gauge couplings, which holds to leading order in the renormalization group equations if weak $SU(2) \times U(1)$ is eventually embedded in a unifying non-Abelian group. With this simplifying assumption the Majorana mass matrix for $\tilde{W}_3$, $\tilde{B}$, $\tilde{H}^0_1$, and $\tilde{H}^0_2$ is

$$\left(\tilde{W}^3,\tilde{B},\tilde{H}_1^0,\tilde{H}_2^0\right)=\begin{pmatrix} M_2 & 0 & \frac{-g_2v_1}{\sqrt{2}} & \frac{g_2v_2}{\sqrt{2}} \\ 0 & \frac{5}{3}\frac{\alpha_1}{\alpha_2}M_2 & \frac{g_1v_1}{\sqrt{2}} & \frac{-g_1v_2}{\sqrt{2}} \\ \frac{-g_2v_1}{\sqrt{2}} & \frac{g_1v_1}{\sqrt{2}} & 0 & \epsilon \\ \frac{g_2v_2}{\sqrt{2}} & \frac{-g_1v_2}{\sqrt{2}} & \epsilon & 0 \end{pmatrix}\begin{pmatrix}\tilde{W}^3\\ \tilde{B}\\ \tilde{H}_1^0\\ \tilde{H}_2^0\end{pmatrix} \tag{6}$$

where $v_{1,2} \equiv \langle 0|H^0_{1,2}|0\rangle$. Analytic expressions for the mixing coefficients $(\alpha_i, \beta_i, \gamma_i, \delta_i)$ obtained for the mass eigenstates of the matrix (6) would be neither practical nor illuminating. However, it is useful to note that in the limit $\epsilon \to 0$ the lightest state χ is approximately a Higgsino

$$\tilde{S} \equiv \frac{v_2\tilde{H}_1^0 + v_1\tilde{H}_2^0}{v} : m_{\tilde{S}} \approx \frac{2v_1v_2}{v^2}\ \epsilon \tag{7}$$

where $v \equiv \sqrt{v_1^2+v_2^2}$, while in the limit $M_2 \to 0$ the lightest state χ is approximately a photino

$$\tilde{\gamma} = \frac{g_1\tilde{W}^3 + g_2\tilde{B}}{\sqrt{g_2^2+g_2^2}} : m_{\tilde{\gamma}} \approx \frac{8}{3}\ \frac{g_1^2}{g_1^2+g_2^2}\ M_2 \ . \tag{8}$$

For moderate values of ϵ and $M_2 = O(M_W)$ these states mix with each other and with the $\tilde{Z}^0$ and the other $\tilde{H}^0$ combination. Contours for the masses of the two lightest neutrals χ and χ' are shown in fig. 1. We see that at least one and often both have masses less than 30 GeV, except when $|\epsilon|$ and M_2 are both $\geq M_W$. Mixing between gaugino and $\tilde{H}$ components is significant in regions where the mass contours curve and/or where those for the χ and χ' approach each other. The eigenstates mix instead of crossing, except when $v_1 = v_2$ in which case the $\tilde{S}$ state (7) decouples from the rest of the mass matrix (5).

Next we impose on the (ϵ, M_2) parameter space the cosmological constraint that the mass density of relic χ particles must not now exceed 2×10^{-29} gm/cc.[14,11,4,7] The force of this constraint depends on the assumed masses of the sfermions $\tilde{f}$: figs 2,4 (figs 3,5) correspond to the choices $m_{\tilde{f}} = 20$ (40) GeV. We find that if $m_{\tilde{f}} = 20$ GeV the $\tilde{\gamma}\tilde{\gamma}$ annihilation rate is such that the $\tilde{\gamma}$ must weigh more than about 1/2 GeV,[7] whereas it must weigh more than about 2 GeV [11] if $m_{\tilde{f}} = 40$ GeV. Because of their smaller annihilation cross-sections, $\tilde{H}$ states must weigh more than the $\tilde{\gamma}$,[4,7] forcing us above the solid diagonal lines in the figures, except that if $m_{\tilde{H}} \geq 0$ (5) GeV the $\tilde{H}\tilde{H} \to \bar{b}b$ annihilation rate becomes sufficient to suppress the cosmological abundance to acceptably low levels, and if $m_{\tilde{H}} \leq 0(100)$ eV the cosmological mass density again becomes acceptably small. Figures 2 to 5 also include dashed lines expressing the constraint that $m_{\chi^\pm} > 20$ GeV, based on the non-observation of $e^+e^- \to \chi^+\chi^-$ at PEP and PETRA. It has been argued [3] that a light charged gauge fermion might have escaped detection, in which case this constraint could be relaxed.

The cross-sections for associated production events $e^+e^- \to \chi\chi'$ are shown for $\sqrt{s} = 30$ GeV in figs. 2 and 3 for $m_{\tilde{e}} = 20$ GeV and 40 GeV, respectively. The kinematic limits of the reaction when $m_\chi + m_{\chi'} = \sqrt{s} = 30$ GeV are clearly discernible, and would be somewhat expanded when $\sqrt{s} = 40$ GeV as at PETRA. The overwhelming majority of the cross-section in both figs. 2 and 3 is due to $\tilde{e}$ exchange [eq. (2b)], and is significant because the χ' is not a very pure $\tilde{H}$ state except in the small domain where $M_2 \ll |\epsilon| \ll M_W$. The cross-section for $e^+e^- \to \chi\chi'$ events is naturally very sensitive to $m_{\tilde{e}}$, and at $\sqrt{s} \leq 40$ GeV becomes unobservably small [$< 0.1 \times \sigma(e^+e^- \to \bar{\nu}_\mu \bar{\nu}_\mu)$] if $m_{\tilde{e}} \geq 50$ GeV.* Raising the center-of-mass energy from $\sqrt{s} = 30$ to $\sqrt{s} = 40$ GeV changes the cross-section for $e^+e^- \to \chi\chi'$ away from threshold in fig. 2 (3) by a factor 0.9 (1.2) for $m_{\tilde{e}} = 20$ (40) GeV, and by a larger factor near threshold [see eq. (3)].†

The cross-section for $e^+e^- \to \chi'\chi'$ events is shown in figs. 4 and 5 assuming $m_{\tilde{e}} = 20$ GeV and 40 GeV, respectively. We find that the cross-section is substantial throughout most of the kinematically allowed regions, and we note that in this case both $\tilde{e}$ and Z^0 exchanges make important contributions, albeit in different regions of

* If the constraint $m_{\chi^\pm} > 20$ GeV is relaxed, $\sigma(e^+e^- \to \chi\chi')$ can still be of order $\sigma(e^+e^- \to \bar{\nu}_\mu \bar{\nu}_\mu)$ even when $m_{\tilde{e}} \sim M_W$.

† In figs. 2-3, cases (a) and (c) correspond to $\epsilon > 0$ and have $\eta\eta' = +1$ in the kinematically allowed regions. Cases (b) and (d) correspond to $\epsilon < 0$ and have $\eta\eta' = -1$.

parameter space. The exchange of a Z^0 dominates when χ' is mainly a $\tilde{H}$, as it is above the diagonal line where $m_\chi = m_{\chi'}$ (see fig. 1), and $\tilde{e}$ exchange dominates when χ' is mainly a $\tilde{\gamma}$, as it is below this diagonal line. This complementarity means that the $\tilde{e} - Z^0$ interference [eq. (2c)] is only important close to this diagonal line. It also means that the cross-section for $e^+e^- \to \chi'\chi'$ is largely independent of $m_{\tilde{e}}$ when χ' is approximately a $\tilde{H}$.

The χ' may decay into $(\ell^+\ell^-$ or $\bar{q}\,q)+\chi$ via either its gaugino or its $\tilde{H}$ components. Any admixture α or $\beta \geq m_f/M_W$ for the χ' in eq. (1) ensures that the decay $\chi' \to \bar{f}\,f\chi$ is dominated by the gaugino components, because of the small Higgs fermion couplings. If the gaugino components in the χ' are large enough for $e^+e^- \to \chi\chi'$ events to have an observable cross-section, this dominance condition is necessarily satisfied. Even in the region $M_2 \ll |\epsilon| \ll M_W$ where the $e^+e^- \to \chi\chi'$ cross-section is small, the gaugino admixture is large enough to dominate the $\tilde{H}$ contribution, for the light fermions which are kinematically accessible in χ' decays.

Decay modes are in the ratio

$$\frac{\Gamma(\chi' \to \ell^+\ell^-\chi)}{\Gamma(\chi' \to u\,\bar{u}\,(d\,\bar{d})\chi)} = \left(\frac{m_{\tilde{u}(\tilde{d})}}{m_{\tilde{e}}}\right)^4$$
$$\left\{\frac{1}{3}\;\frac{(\beta_1 \sin\theta_W + \alpha_1 \cos\theta_W)^2(\beta_2 \sin\theta_W + \alpha_2 \cos\theta_W)^2 + 16\sin^4\theta_W \beta_1^2\beta_2^2}{(\frac{\beta_1}{3}\sin\theta_W \overset{+}{\underset{(-)}{}} \alpha_1\cos\theta_W)^2(\frac{\beta_2}{3}\sin\theta_W \overset{+}{\underset{(-)}{}} \alpha_2\cos\theta_W)^2 + 16Q^4_{u(d)}\sin^4\theta_W\beta_1^2\beta_2^2}\right\} \quad (9)$$

and the parenthesized { } factor never falls below 0.6 (1.5) in the domain of interest. Since we expect that $m_{\tilde{q}} \geq m_{\tilde{\ell}}$, and indeed $m_{\tilde{q}} \approx 2\frac{1}{2}m_{\tilde{\ell}}$ for many models in the literature,[13] it seems reasonable to expect that

$$\Gamma(\chi' \to \ell^+\ell^-\chi) \geq \text{ (possibly} \gg) \; \Gamma(\chi' \to \bar{q}\,q\chi) \quad . \quad (10)$$

Interesting one-sided $e^+e^- \to \chi\chi'$ event signatures are therefore

$$e^+e^- \to \chi\chi' \quad \hookrightarrow \chi + (e^+e^- \text{ or } \mu^+\mu^-) \quad : \; e^+e^- \to \left.\begin{matrix} e^+e^- \\ \text{or} \\ \mu^+\mu^- \end{matrix}\right\} + \begin{matrix}\text{missing} \\ \text{energy-} \\ \text{momentum}\end{matrix} \quad (11)$$

while $e^+e^- \to \chi'\chi'$ has two-sided signatures:

$$e^+e^- \to \chi'\chi' \begin{cases} \to \chi + (e^+e^- \text{ or } \mu^+\mu^-) \\ \to \chi + (e^+e^- \text{ or } \mu^+\mu^-) \end{cases} : \; e^+e^- \to \left.\begin{matrix} (e^+e^-)(e^+e^-) \\ \text{or} \\ (e^+e^-)(\mu^+\mu^-) \\ \text{or} \\ (\mu^+\mu^-)(\mu^+\mu^-) \end{matrix}\right\} + \begin{matrix} \text{missing} \\ \text{energy-} \\ \text{momentum} \end{matrix} \tag{12}$$

With these decay signatures in mind we have computed the lepton energy spectrum $1/\sigma \; d\sigma/dE_{\ell\pm}$ for $e^+e^- \to \chi' + \cdots \to \ell^\pm + \cdots$:

$$\begin{aligned} \frac{1}{\sigma}\frac{d\sigma}{dE_{\ell\pm}} = \frac{1}{C(\frac{m_1}{m_2})}\frac{1}{m_2^6 k} \Bigg\{ & -\frac{2}{9}\,(X_+^3 - X_-^3) \\ & + (X_+^2 - X_-^2)\left[\frac{7m_2^2 + 15m_1^2}{24} - \eta_1\eta_2\,\frac{m_1 m_2}{4}\right] \\ & + (X_+ - X_-)\left[\frac{m_2^4 - 6m_1^4 - 15m_1^2 m_2^2}{12} + \eta_1\eta_2\,\frac{m_1 m_2}{2}\,(m_2^2 + 2m_1^2)\right] \\ & + \ell n\,\frac{X_+}{X_-}\left[\frac{3}{4}m_1^4\left(m_2^2 - \frac{m_1^2}{9}\right) - \frac{1}{2}\eta_1\eta_2 m_1^3 m_2 (m_1^2 + 2m_2^2)\right] \\ & + \left(\frac{1}{X_-} - \frac{1}{X_+}\right) m_1^4 m_2^2 \left(-\frac{m_1^2 + 3m_2^2}{12} + \eta_1\eta_2\frac{m_1 m_2}{2}\right) + \left(\frac{1}{X_-^2} - \frac{1}{X_+^2}\right)\frac{m_1^6 m_2^4}{12} \Bigg\} \end{aligned} \tag{13}$$

and also the invariant mass distribution $1/\sigma \; d\sigma/dM^2_{\ell^+\ell^-}$ for each $(\ell^+\ell^-)$ pair:

$$\begin{aligned} \frac{1}{\sigma}\frac{d\sigma}{dM^2_{\ell^+\ell^-}} = \frac{1}{C'(\frac{m_1}{m_2})}\frac{1}{m_2^8} & \left[(m_1^2 + m_2^2 - M^2_{\ell^+\ell^-})^2 - 4m_1^2 m_2^2\right]^{\frac{1}{2}} \\ & \times \left\{\frac{1}{6}\left[(m_2^2 - m_1^2)^2 + M^2_{\ell^+\ell^-}(m_1^2 + m_2^2) - 2M^4_{\ell^+\ell^-}\right]\right. \\ & \left. + \eta_1\eta_2 m_1 m_2 M^2_{\ell^+\ell^-}\right\} \end{aligned} \tag{14}$$

where $C(m_1/m_2)$ and $C'(m_1/m_2)$ are constants, $X_+ \equiv m_2^2 - 2E_2 E_{\ell\pm}(1 - k/E_2)$ and $X_- \equiv m_2^2 - 2E_2 E_{\ell\pm}(1 + k/E_2)$ for $E_{\ell\pm} \leq (m_2^2 - m_1^2)/2(E_2 + k)$ while $X_- \equiv m_1^2$

for $(m_2^2 - m_1^2)/2(E_2 + k) \leq E_{\ell\pm} \leq (m_2^2 - m_1^2)/2(E_2 - k)$. We have neglected the Z^0-exchange contribution for the reasons discussed earlier. Knowing these distributions, experimentalists may be able to devise cuts to enhance any gauge fermion signal relative to the QED and other backgrounds. *

Finally, it is worth noting that there are other possible χ' decay modes which might be kinematically accessible and could also lead to events with visible lepton pairs or hadrons in the final state:

$$\chi' \to \chi + H^0 \quad \text{(physical neutral Higgs), if } m_2 > m_1 + m(H^0)$$

$$\quad \llcorner\!\!\longrightarrow \left\{ \begin{array}{l} c\bar{c} \\ \text{or} \\ b\bar{b} \end{array} \right.$$

$$\chi' \to e^+ + \tilde{e}_L^- \to e^+ + \left\{ \begin{array}{l} \chi + e^- \\ \text{or} \\ \nu_e + \chi^- \end{array} \right. \quad \text{if } m_2 > m_{\tilde{e}_L}$$

$$\qquad\qquad \llcorner\!\!\longrightarrow e^- + \chi + \bar{\nu}_e \text{ if } m_{\tilde{e}_L} > m_{\chi^-}$$

$$\chi' \to e^+ + \tilde{e}_R^- \to e^+ + e^- + \chi \quad , \text{if } \; m_2 > m_{\tilde{e}_R} \quad .$$

If allowed energetically, $\chi' \to \chi + H^0$ could be the dominant decay mode of χ' since it proceeds via the $\tilde{H}$ component of χ' and the gaugino component of χ.

The search for neutral gauge fermions in e^+e^- annihilation at present energies is obviously more challenging than the search for charged gauge fermions. Furthermore, the model-dependences of the cross-sections are greater and there is no guarantee that any neutral gauge fermions are even kinematically accessible. Nevertheless, SUSY could easily be standing under this lamp-post and we urge our experimental colleagues to look here.

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* It is important to note that there is no forward-backward asymmetry in the differential cross-section for $e^+e^- \to \chi\chi'$. This follows from the Majorana nature of χ and χ', and is valid even if the mass matrix (6) is complex and the χ and χ' couplings do not conserve CP.

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Figure Captions

1. Masses of neutral gauge fermions in the minimal supersymmetric extension of the Standard Model, with the assumptions and notations explained in the text. Case (a) is $v_1 = 2v_2$, $\epsilon > 0$; (b) $v_1 = 2v_2$, $\epsilon < 0$; (c) $v_1 = 4v_2$, $\epsilon > 0$; and (d) $v_1 = 4v_2$, $\epsilon < 0$. We generally expect $v_1 \geq v_2$ and have made these representative choices since there are no cross-sections of interest when $v_1 = v_2$ or when $v_1 \geq 0(8)v_2$. Graphs A exhibit the lightest neutral gauge fermion mass m_χ, while graphs B exhibit the second smallest mass $m_{\chi'}$. Solid, dashed, dash/dotted and dashed/double-dotted lines denote 30, 15, 5 and 2 GeV contours, respectively.

2. Cross-sections for associated production events $e^+e^- \to \chi\chi'$ when $\sqrt{s} = 30$ GeV and $m_{\tilde{e}} = 20$ GeV, subject to PEP/PETRA bounds on $m_{\chi^\pm}$ (indicated by arrows) and to the cosmological constraints with $m_{\tilde{f}} = 20$ GeV, for the four cases (a) to (d) described in the caption to fig. 1. This and all subsequent cross-sections are given in units of the cross-section $\sigma(e^+e^- \to \bar{\nu}_\mu \nu_\mu)$ which is equal to 0.38 pb at $\sqrt{s} = 30$ GeV. In this and in the subsequent figures, in the lined regions $(1/10)\sigma(e^+e^- \to \bar{\nu}_\mu \nu_\mu) < \sigma < \sigma(e^+e^- \to \bar{\nu}_\mu \nu_\mu)$, while in the dotted regions $\sigma > \sigma(e^+e^- \to \bar{\nu}_\mu \nu_\mu)$.

3. Cross sections in units of $\sigma(e^+e^- \to \bar{\nu}_\mu \nu_\mu)$ for $e^+e^- \to \chi\chi'$ events when $\sqrt{s} =$ 30 GeV and $m_{\tilde{e}} = 40$ GeV, subject to the cosmological constraints with $m_{\tilde{f}} =$ 40 GeV for the four cases (a) to (d) described in the caption of fig. 1.

4. Cross sections for $e^+e^- \to \chi'\chi'$ events with the same assumptions as in fig. 2.

5. Cross sections for $e^+e^- \to \chi'\chi'$ events with the same assumptions as in fig. 3.

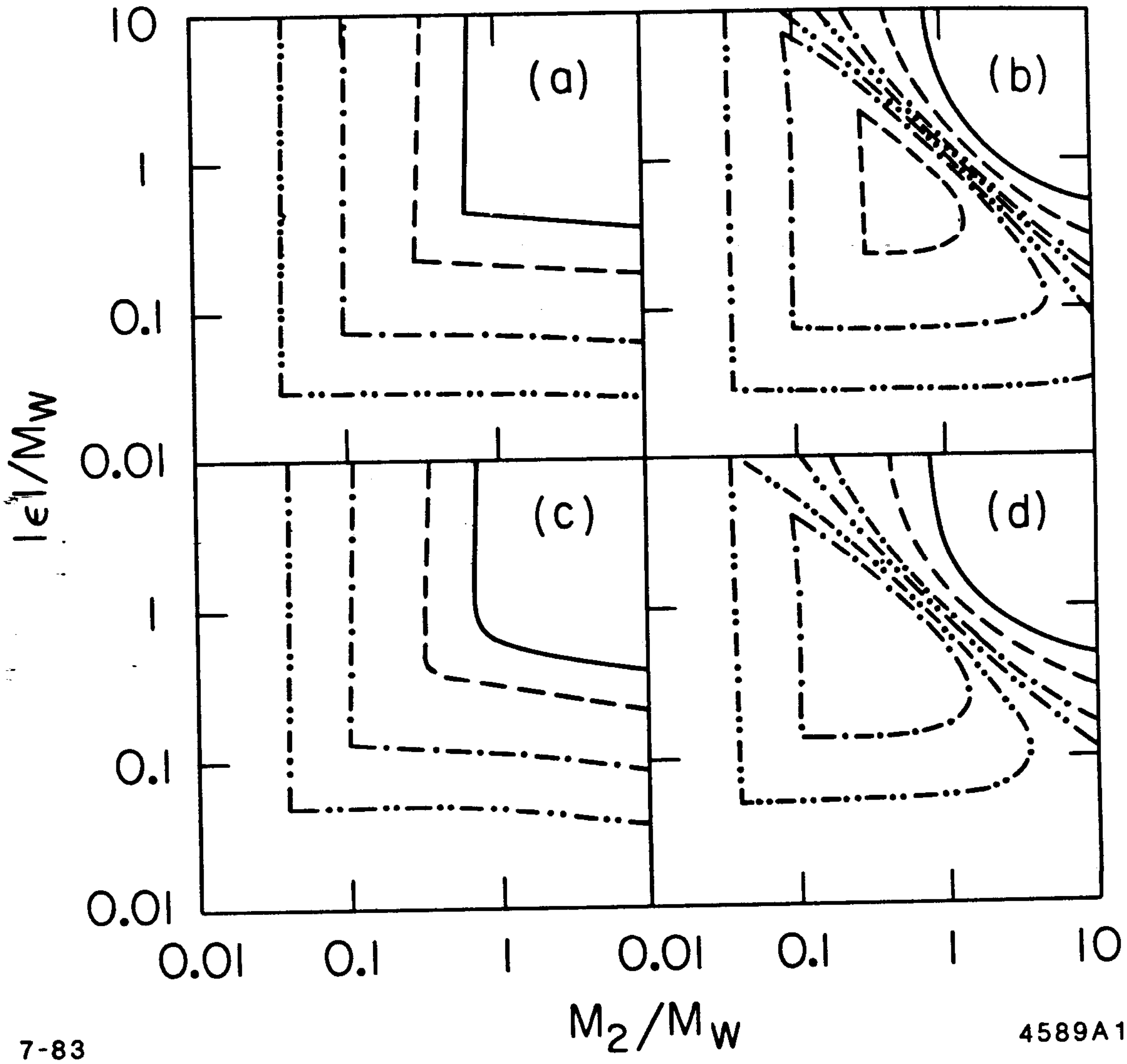

Fig. 1A

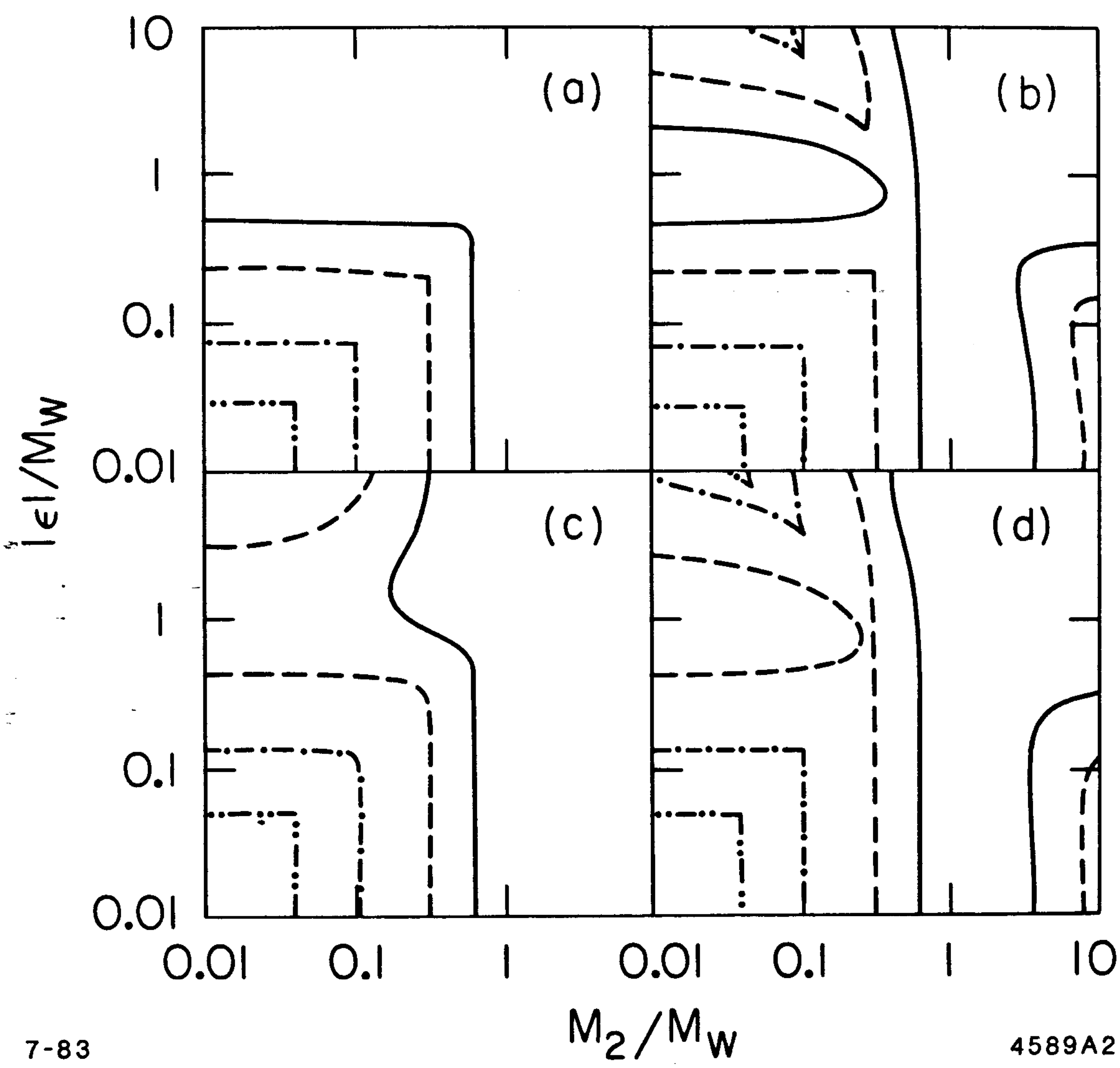
10
1
0.1
0.01
1
0.1
0.01
$|\epsilon|/M_W$
(a)
(b)
(c)
(d)
0.01
0.1
1
0.01
0.1
1
10
M_2/M_W
7-83
4589A2

Fig. 1B

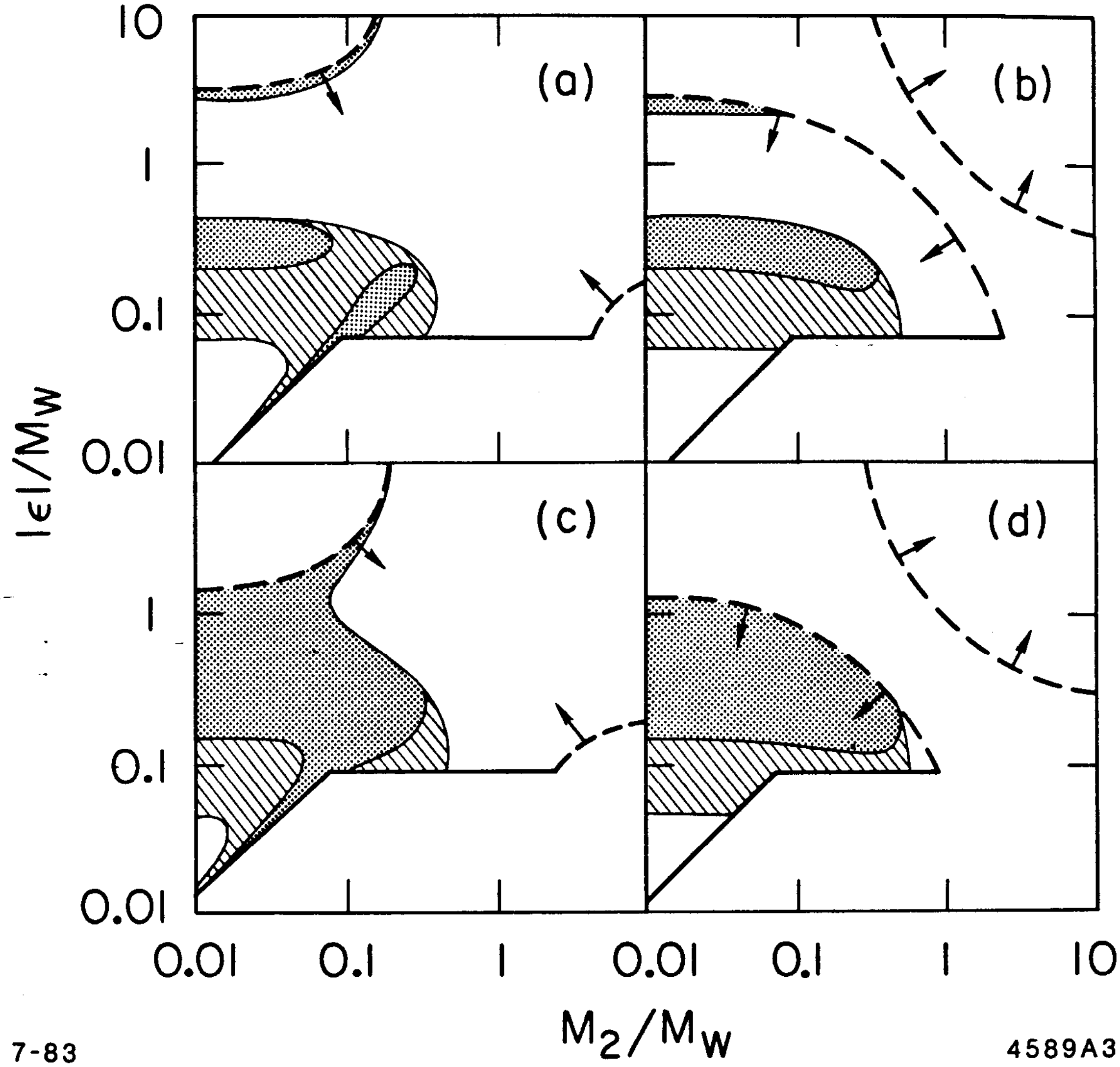

Fig. 2

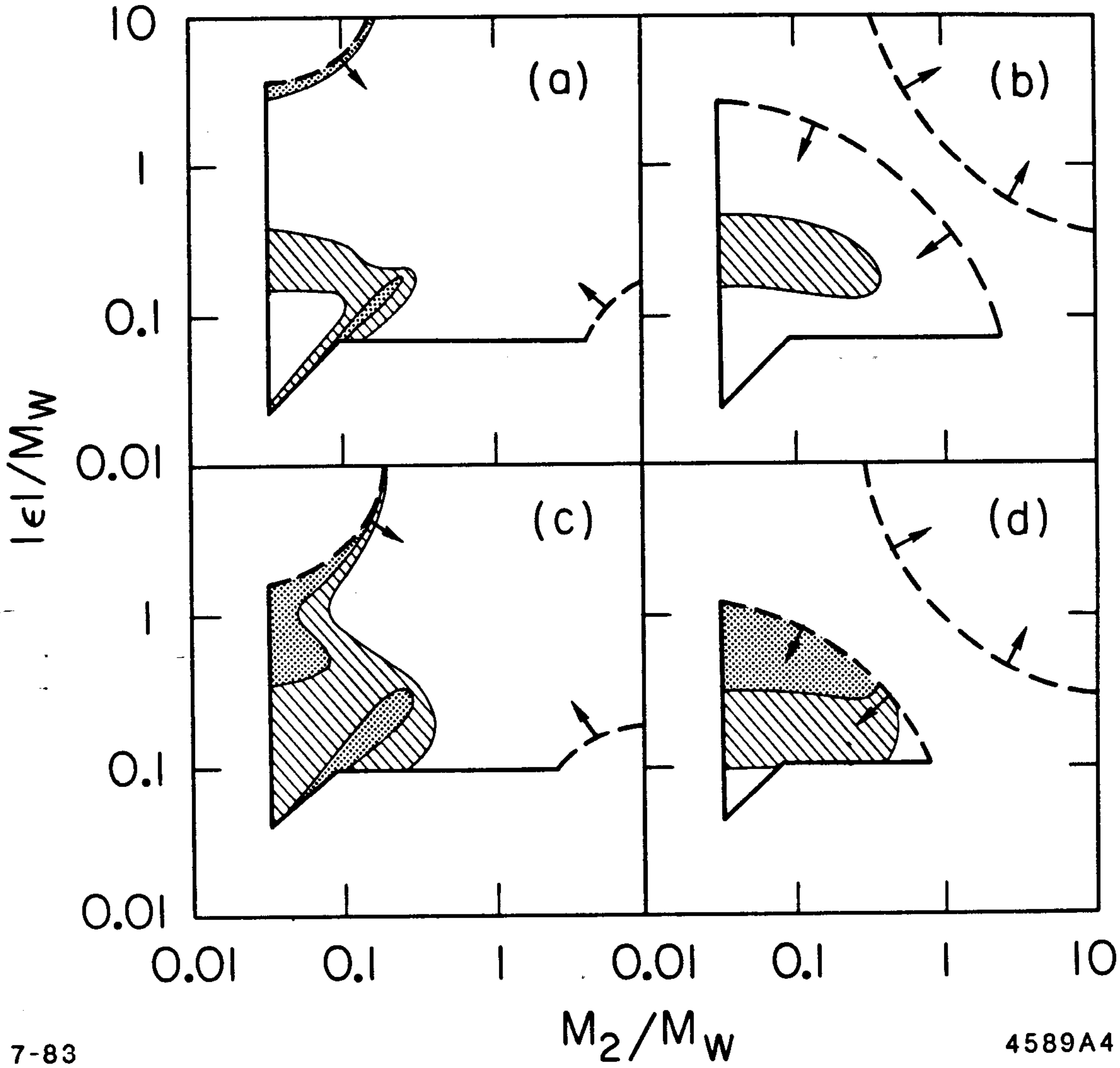
10
1
0.1
0.01
0.01
1
0.1
0.01
(a)
(b)
(c)
(d)
|ε|/M_W
0.01
0.1
1
0.01
0.1
1
10
M_2/M_W
7-83
4589A4

Fig. 3

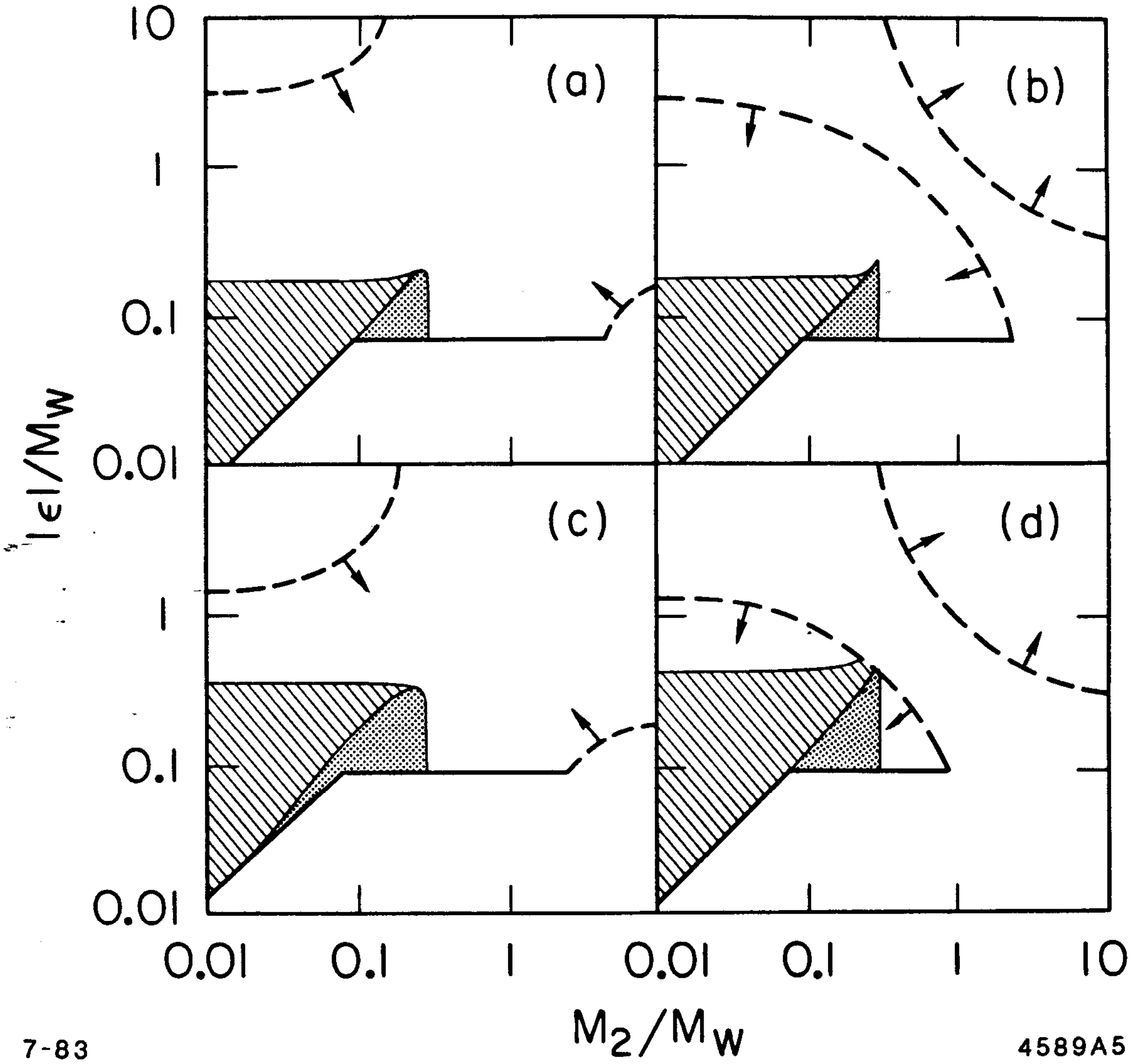

Fig. 4

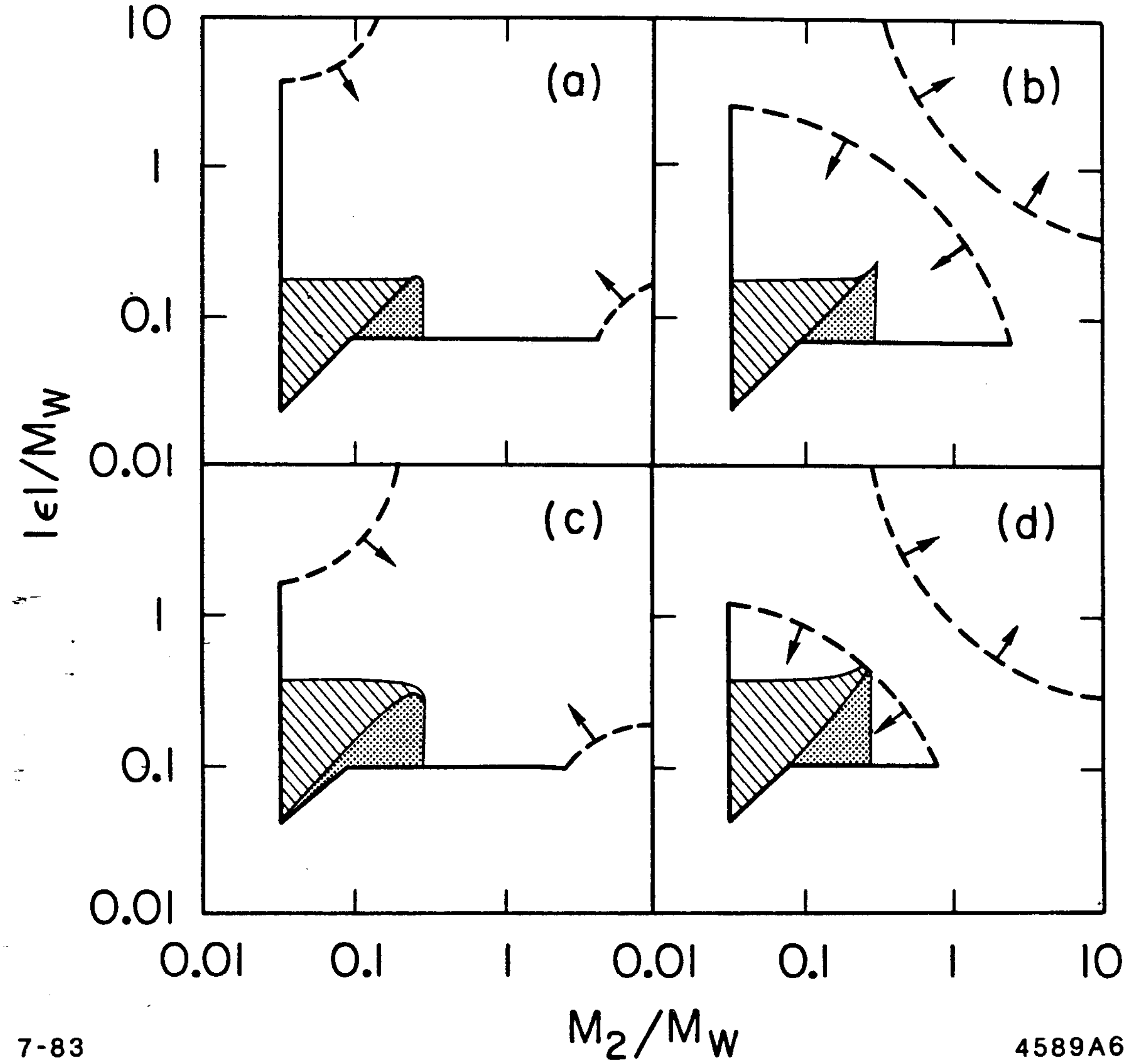

Fig. 5