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# NOSONOMY OF AN UPSIDE DOWN HIERARCHY MODEL —II*

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## ABSTRACT

We study supersymmetry breaking in the spectrum of ordinary-energy particles induced by the inverse hierarchy mechanism. General techniques for computing non-supersymmetric interaction and mass terms are developed and illustrated on a toy model example. Application of these techniques to the upside down hierarchy model leads to the conclusion that the so-called "sliding singlet" mechanism does not work.

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## 1. Introduction.

In our previous paper[1] we described several serious difficulties which arise when one attempts to build a realistic grand unified model based on Witten's upside down hierarchy[2] idea. The difficulties are primarily connected with the necessity of including a light (mass $\sim$ 10 TeV) color octet superfield in the model in order to make the inverse hierarchy work. This changes the renormalization of the color coupling in a way that destroys many standard GUT predictions. The original purpose of the present paper was to describe another difficulty with the model which has to do with the breaking of the Glashow–Weinberg–Salam [GWS] symmetry by radiative corrections. We will show that although the GWS scale of $\sim$ 100 GeV is naturally generated by the model, actual $SU(2)\otimes U(1)$ breaking requires a fine tuning of parameters to accuracy of nine significant digits.

In the (rather long) period that it has taken us to prepare this paper for publication the inverse hierarchy mechanism has (justifiably) lost whatever appeal it originally had. However, the general idea of heaving large scale ( $>$ 100 TeV) supersymmetry breaking in a sector which is *almost decoupled* from presently observable physics has been incorporated into a variety of other models. Generally the coupling between the hidden sector and the ordinary world is non-renormalizable; in some models it comes through exchange of superheavy particles while in others it is supergravitational. In all such theories there will be radiative corrections to the effective Lagrangian of the light fields which depend logarithmically on the large ratios of energy scales in the model. The techniques that we have developed to sum these logarithmic effects in upside down hierarchy models are applicable (with small modifications) to this more general class of models.

Section 2 of this paper contains a general formalism for computing the radiatively induced supersymmetry breaking mass terms for light fields in inverse hierarchy scenario. In particular we show that scalar fields that are massless at the tree level develop radiative mass$^2$ terms only starting at the two-loop level. Nevertheless, we develop techniques that allows for summing all the leading-log radiative corrections while using the one-loop renormalization group equations.

In section 3 we apply our general formalism to a supersymmetric $SU(5)$ GUT in which $SU(2)\otimes U(1)$ is unbroken at the tree level. We show that for the range of parameters in the original Lagrangian, the correct pattern of $SU(2)\otimes U(1)$ breaking arises as a radiative correction. The model involves two Lagrangian-level scales — scale of an explicit supersymmetry scale and the Grand Unified Scale — which are both put in by hand despite a large ratio between them. There is also an "ordinary" fine tuning that makes the $SU(2)$ –doublet Higgses massless at the tree level. Although both the scale hierarchy and the fine tuning are stable against radiative corrections, they are artificial and we do not propose the model as an answer to all the problems in the world.*

Section 4 is devoted to the inverse hierarchy model. Although this model naturally produces a range of widely different mass scales, we show that the correct pattern of $SU(2)\otimes U(1)$ breaking can only be obtained by fine tuning which must be corrected in each order of perturbation theory. We also calculate the radiative masses of scalar quarks and leptons; the results add yet another disease to the model's nosonomy.

The nervous reader can now jump to the conclusion.

* It might however arise as an effective low energy approximation to one of the more ambitious models described above.

## 2. General Analysis.

In this section we will give a general discussion of supersymmetry breaking terms of dimensions 2 and 3 in the effective Lagrangian for the light sector of an inverse hierarchy model. We have shown in our previous paper that these models have four well separated energy scales:

- a Lagrangian scale $M$ which we expect to be about $10^{10}$–$10^{11}$ GeV (this is also the primary supersymmetry breaking scale of the model);
- a Grand Unified scale $y \sim 10^{16}$–$10^{18}$ GeV;
- a "mirror" light scale $M^2/y \sim 10\,\mathrm{TeV}$ at which supersymmetry breaking manifests itself in the particle mass spectrum;
- and finally the GWS scale $\alpha M^2/y \sim 100\,\mathrm{GeV}$.

The latter is the characteristic scale of radiatively induced mass terms for fields that happen to be exactly massless at the tree level. It is these supersymmetry breaking effects that we wish to calculate.

We will work in the superfield formalism.[3–4] Since we are interested in the light sector we would like to integrate out all the heavy fields in the model. The resulting effective theory is described by an effective superspace Lagrangian that is generally is given by

$$\mathcal{L}(\Phi, \Phi^\dagger, V) = K(\Phi, \Phi^\dagger, V) + W(\Phi) \cdot \delta^2(\bar{\theta}) + \overline{W}(\Phi^\dagger) \cdot \delta^2(\theta) \quad , \tag{2.1}$$

where $K$ is some $\theta$-local function of superfields $\Phi$, $\Phi^\dagger$ and $V$ and their covariant derivatives. By the famous non-renormalization theorem[5,3] the superpotential $W$ does not differ from its tree-level value

$$W \;=\; \tfrac{1}{2} M_{ij}\, \Phi_i \Phi_j \;+\; \tfrac{1}{3} \lambda_{ijk}\, \Phi_i \Phi_j \Phi_k \qquad . \tag{2.2}$$

At the tree level

$$K_0 = \Phi^\dagger\, e^{2V \cdot T}\, \Phi + Tr\Big[ \frac{1}{128 e^2} \big( e^{-2V} D^\alpha e^{2V} \big) \bar{D}^2 \big( e^{-2V} D_\alpha e^{2V} \big) \quad + \text{ H.c.} \Big] \tag{2.3}$$

which is just the gauge invariant kinetic term. In a renormalizable theory $K$ absorbs the wave function renormalization. Its gauge–kinetic part becomes

$$
\begin{aligned}
K_{kin} &= \Phi^{\dagger}\, Z_S\, e^{2V\cdot T}\, \Phi \\
&\quad + Tr\left[\frac{Z_V}{128e^2}\left(e^{-2V}\, D^{\alpha}\, e^{2V}\right)\bar{D}^2\left(e^{-2V}\, D_{\alpha}\, e^{2V}\right) \quad + \text{H.c.}\right] \quad ,
\end{aligned}
\tag{2.4}
$$

where the wave-function renormalization matrices $Z_S$ and $Z_V$ commute with the gauge group.

In the superfield formalism spontaneous supersymmetry breaking is taken into account by giving an expectation value to the auxiliary superpartner of the Goldstone fermion. The correspondent superfield therefore has a $\theta$-dependent VEV. For the Witten–O'Raifeartaigh mechanism of supersymmetry breaking Goldstino belongs to the chiral supermultiplet $Y$ whose *scalar* component has GUT-breaking VEV $y$.

The wave-function renormalization factors $Z$ that appear in (2.4) are generally complicated non-local functionals of $\Phi$ and $V$. However, we are interested only in momenta $\ll y$ so that $Z$-factors are simply functions of $\Phi$. On the other hand, $Z$-factors depend not only on the light fields $\Phi$, but on the large VEVs as well; in particular, they depend on the VEV of $Y$. Since the expectation value $< Y > = y + F_y\theta\theta$ is $\theta$-dependent, $Z$-factors also become $\theta$-dependent. It is this $\theta$-dependence of $Z$-factors that accounts for supersymmetry breaking in the effective Lagrangian for the light fields.

Let us now compute the explicit $\theta$-dependence of the $Z$-factors. Hermiticity of the Lagrangian of the chiral superfields $\Phi$ requires $Z_S$ to be a hermitian matrix, therefore

$$
\begin{aligned}
Z_S(\theta) &= Z_S\Big|_{\theta=0} + \frac{\partial Z_S}{\partial Y}\Big|_{\theta=0} \cdot F_y\, \theta\theta + \frac{\partial Z_S}{\partial Y^{\dagger}}\Big|_{\theta=0} \cdot F_y^{\dagger}\, \overline{\theta\theta} \\
&\quad + \frac{\partial^2 Z_S}{\partial Y\, \partial Y^{\dagger}}\Big|_{\theta=0} \cdot |F_y|^2\, \theta\theta\, \overline{\theta\theta} \qquad .
\end{aligned}
\tag{2.5}
$$

On the other hand, gauge invariance of the action for the vector superfields requires $Z_V$ to be chiral,[6] therefore

$$Z_V(\theta) = Z_V\Big|_{\theta=0} + \frac{\partial Z_V}{\partial Y}\Big|_{\theta=0} \cdot F_y\,\theta\theta \quad . \tag{2.5$^V$}$$

From now on we will omit the specification $\Big|_{\theta=0}$ in component decompositions and $x$-space expressions.

Let us now return to the effective action for the light sector. In order to understand the supersymmetry breaking effects one would like to have a conventional $x$-*space* Lagrangian. The latter can be obtained by substituting (2.2) and (2.4) into (2.1), expanding various superfields into components* and integrating over the fermionic coordinates $\theta$ and $\bar{\theta}$. The [rather cumbersome] result of such an integration can be found in *appendix A*.

The next step involves renormalization $A \mapsto Z_S^{-1/2} A$ , $\Psi \mapsto Z_S^{-1/2} \Psi$ , $\mathcal{V} \mapsto e Z_V^{-1/2} \mathcal{V}$ , $\chi \mapsto e Z_V^{-1/2} \chi$ and elimination of the auxiliary fields $F$ and $\mathcal{D}$. This leads us to a canonical Lagrangian

$$\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_3 + \mathcal{L}_2 \quad , \tag{2.6}$$

where $\mathcal{L}_d$ collects terms of dimension $d$. $\mathcal{L}_4$ is supersymmetric; it includes all kinetic terms and all gauge, Yukawa and 4-scalar interactions. The *renormalized* coupling constants are given by

$$e^r = e\,Z_V^{-1/2} \quad , \qquad \lambda^r_{ijk} = \lambda_{ijk}\,Z_i^{-1/2} Z_j^{-1/2} Z_k^{-1/2} \quad . \tag{2.7}$$

(Here we have diagonalized [the $c$-number components of] the $Z$-matrices.) On the other hand, $\mathcal{L}_3$ and $\mathcal{L}_2$ are not supersymmetric; generally they are given

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* $Z$-matrices are considered as explicit soft supersymmetry breaking parameters and are expanded according to (2.5).

by

$$
\begin{aligned}
\mathcal{L}_3 = & \left[ \tfrac{1}{2} M^r_{ij} \Psi_i \Psi_j \; + \; M^{r\dagger}_{il} \lambda^r_{ljk} A^\dagger_i A_j A_k \quad + \text{ H.c.} \right] \\
& + \left[ \tfrac{1}{2} \mathcal{M}_{\tilde{g}} \chi\chi \; + \; \lambda^r_{ijk} (\underline{m}_i + \underline{m}_j + \underline{m}_k) A_i A_j A_k \quad + \text{ H.c.} \right] \\
\mathcal{L}_2 = & - \left( M^{r\dagger}_{ik} M^r_{kj} + \delta_{ij} \mathcal{M}^2_i \right) A^\dagger_i A_j \; - \; \left[ M^r_{ij} (\underline{m}_i + \underline{m}_j) A_i A_j \quad + \text{ H.c.} \right]
\end{aligned}
\tag{2.8}
$$

where

$$
\begin{aligned}
\mathcal{M}^2_i &= - \frac{\partial^2 \ln Z_i}{\partial Y \, \partial Y^\dagger} \cdot F_y F^\dagger_y \\
\mathcal{M}_{\tilde{g}} &= \frac{\partial \ln Z_V}{\partial Y} \cdot \frac{F_y}{2} \\
\underline{m}_i &= \frac{\partial \ln Z_i}{\partial Y} \cdot \frac{F_y}{2} \qquad .
\end{aligned}
\tag{2.9}
$$

Here we have assumed that $Z$-matrices can be diagonalized together with their derivatives; when this is not the case, equations (2.8) and (2.9) become *very* complicated. We will return to the non-diagonal case in section 4.

Equations (2.9) may be further simplified when the scalar expectation value $y$ is hierarchically large, as in inverse hierarchy models. In such theories $Z(y) = Z(\ln y) + O(M/y)$. Systematically neglecting higher terms in $(M/y)$, we will consider only the logarithmic part. Then

$$
\begin{aligned}
\mathcal{M}^2_i &= - \frac{\partial^2 \ln Z_i}{\partial \ln Y \, \partial \ln Y^\dagger} \cdot \left| \frac{F_y}{y} \right|^2 \\
\mathcal{M}_{\tilde{g}} &= \frac{\partial \ln Z_V}{\partial \ln Y} \cdot \frac{F_y}{2y} \\
\underline{m}_i &= \frac{\partial \ln Z_i}{\partial \ln Y} \cdot \frac{F_y}{2y} \qquad .
\end{aligned}
\tag{2.10}
$$

Before proceding to a renormalization group analysis one should carefully separate $\ln y$ from $\ln y^\dagger$. The renormalization factor $Z_V$ is complex ( $Z_V(\theta)$ is chiral) and depends on $\ln y$ only. On the other hand, $Z_i$ are hermitian and depend on $\ln|y| = \frac{1}{2}(\ln y + \ln y^\dagger)$. Renormalization group equations involve as an independent variable

$$\tau = \ln\frac{|y|}{\Lambda} = \frac{1}{2}\ln\frac{yy^\dagger}{\Lambda^2} \tag{2.11}$$

where $\Lambda$ is an arbitrary mass scale. Reexpressed in terms of $\tau$, equations (2.10) become

$$\begin{aligned} \mathcal{M}_i^2 &= -\left|\frac{F_y}{2y}\right|^2 \cdot \frac{\partial^2 \ln Z_i}{\partial \tau^2} \\ \mathcal{M}_{\tilde{g}} &= \frac{F_y}{2y} \cdot \frac{\partial \ln|Z_V|}{\partial \tau} \\ \underline{m}_i &= \frac{F_y}{2y} \cdot \frac{\partial \ln Z_i}{\partial \tau} \quad . \end{aligned} \tag{2.12}$$

Since in perturbation theory the $l$-loop contribution to $Z$ or to $\ln Z$ is an $l$-degree polynomial in $\frac{\alpha}{\pi}\ln\frac{|y|}{M} = \frac{\alpha}{\pi}(\tau - \ln M/\lambda)$, $\mathcal{M}_{\tilde{g}}$ and $\underline{m}_i$ are of order $O\left(\frac{\alpha F_y}{2\pi y}\right)$ while $\mathcal{M}_i^2 = O\left(\left(\frac{\alpha F_y}{2\pi y}\right)^2\right)$. In particular, **there are no one-loop contributions to** $\mathcal{M}_i^2$. We see that all supersymmetry breaking terms in $\mathcal{L}_3 + \mathcal{L}_2$ belong to the same scale $O\left(\frac{\alpha F_y}{2\pi y}\right)$ that should be therefore identified with the GWS scale.

Let us now apply the renormalization group techniques to calculating the $Z$-factors. The absence of renormalization of $W$ makes it convenient to perform the renormalization group analysis using bare coupling constants and running $Z$-factors as renormalization parameters. This is especially convenient when there are non-diagonal $Z$-matrix elements leading to field mixing. From now on we will drop matrix notations and denote by $Z_a$ different non-trivial elements of $Z$-matrices (both $Z_S$ and $Z_V$). In this formalism the renormalization group equations become

$$\frac{dZ_a(\mu)}{d\ln\mu} = \Gamma_a(\{Z_b\}_b) \quad , \tag{2.13}$$

where $\mu$ is the renormalization scale at which $Z_a$ are evaluated; $\Gamma_a$ are of magnitude $O(\alpha/\pi)$.

At the GUT scale $O(|y|)$, the theory has a threshold. Generally the effective theories above and below the threshold look quite different. There are different sets of fields, different coupling constants, different renormalization factors ($Z_i^A$ above the threshold and $Z_a^B$ below it) and different $\Gamma$-functions $\left(\Gamma_i^A\left(\{Z_j^A\}_j\right)\right.$ above and $\Gamma_a^B(\{Z_b^B\}_b)$ below$\left.\right)$. At the threshold scale $y^*$ broken and unbroken theories match each other and $Z^B$ become expressable in terms of $Z^A$:

$$Z_a^B(\mu=y) \;=\; F_a\Big(\{Z_i^A(\mu=y)\}_i\Big) \quad . \tag{2.14}$$

These matching conditions are perturbative. At the lowest order most of them are trivial, *i.e.* $Z_a^B(y) = Z_{i(a)}^A(y)$. At higher orders they become polynomials in $\alpha/\pi$ but they do not involve large logarithms. Higher order corrections depend also on the choice of a renormalization scheme, but we need not worry about that in the leading-log approximation.

Before we to solve equations $(2.13)^{A,B}$ we have to establish boundary conditions. For the unbroken theory above $y$ they may be fixed at an arbitrary scale $\Lambda > y$. Obviously these conditions do not depend on $y$. Hence for any *fixed* scale $\mu > y$, $Z_i^A(\mu)$ do not depend on $y$. On the other hand, boundary conditions for the broken theory below $y$ (given by equations (2.14) ) do depend on $y$. Thus for any fixed scale $\mu < y$, $Z_a^B(\mu; y)$ depend on both $\mu$ and $y$. It is this $y$-dependence of $Z^B$ at fixed $\mu$ that will give us radiative supersymmetry breaking.

In most cases we cannot solve equations (2.13) analytically. Although one can always do it numerically, obtaining the second derivative of the numerical solution with respect to a parameter $[\tau]$ is a rather hopeless procedure. Instead we will obtain $\frac{dZ}{d\tau}$ and $\frac{d^2Z}{d\tau^2}$ at a scale $\mu$ just below $y$ and write down renormalization group equations for these derivatives. Those equations may now be solved numerically.

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* Actually the threshold scale is $C \cdot |y|$ where the coefficient $C$ depends on the renormalization scheme. For notational simplicity we will denote it by $y$ from now on.

Let us start with the renormalization group equations. Equation (2.13) $^B$ : $\frac{\partial Z_a^B(t;\tau)}{dt} = \Gamma_a^B(\{ Z_b^B \}_b)$ $(t \equiv \ln \mu/\Lambda)$ holds for any value of $\tau$. Differentiating it with respect to $\tau$ and interchanging $\frac{\partial}{\partial t}$ with $\frac{\partial}{\partial \tau}$ one obtains

$$\frac{\partial}{\partial t}\frac{\partial Z_a^B}{\partial \tau} = \frac{\partial}{\partial \tau}\Gamma_a^B\Big(\{ Z_b^B(t;\tau)\}_b\Big) = \frac{\partial \Gamma_a^B}{\partial Z_b^B}\cdot\frac{\partial Z_b^B}{\partial \tau} \quad . \tag{2.15}$$

Similarly

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\partial^2 Z_a^B}{\partial \tau^2} &= \frac{\partial^2}{\partial \tau^2}\Gamma_a^B\Big(\{ Z_b^B(t;\tau)\}_b\Big) \\ &= \frac{\partial^2 \Gamma_a^B}{\partial Z_b^B \partial Z_c^B}\cdot\frac{\partial Z_b^B}{\partial \tau}\frac{\partial Z_c^B}{\partial \tau} + \frac{\partial \Gamma_a^B}{\partial Z_b^B}\cdot\frac{\partial^2 Z_b^B}{\partial \tau^2} \quad .\end{aligned} \tag{2.16}$$

Having the renormalization group equations for $Z^B$, $\frac{\partial Z^B}{\partial \tau}$ and $\frac{\partial^2 Z^B}{\partial \tau^2}$ we need now boundary conditions for them. Equation (2.14) $Z_a^B(t;\tau)\ |_{t=\tau} = F_a(\{ Z_i^A(\tau)\}_i)$ holds for all $\tau, t = \tau$. This allows us to take its *full* derivatives with respect to $\tau$. The left hand side gives

$$\frac{d}{d\tau}Z_a^B(t;\tau)\Big|_{t=\tau} = \frac{\partial Z_a^B}{\partial \tau} + \frac{\partial Z_a^B}{\partial t} = \frac{\partial Z_a^B}{\partial \tau}\Big|_{t=\tau} + \Gamma_a^B\Big(\{ Z_b^B(t=\tau)\}_b\Big) \quad ,$$

while the right hand side gives

$$\frac{d}{dt}F_a\Big(\{ Z_i^A(\tau)\}_i\Big) = \frac{\partial F_a}{\partial Z_i^A}\cdot\frac{dZ_i^A}{d\tau} = \frac{\partial F_a}{\partial Z_i^A}\cdot\Gamma_i^A\Big(\{ Z_j^A(\tau)\}_j\Big) \quad .$$

Equating the left hand side with the right hand side we obtain

$$\frac{\partial Z_a^B}{\partial \tau}\Big|_{t=\tau} = \left[\frac{\partial F_a}{\partial Z_i^A}\cdot\Gamma_i^A - \Gamma_a^B\right]_\tau \quad , \tag{2.17}$$

where $\Gamma^A\ |_\tau$ and $F\ |_\tau$ are functions of $Z^A(\tau)$, and $\Gamma^B\ |_\tau$ are functions of $Z^B(t=\tau)$ given by (2.14) . In a similar way one may derive the boundary conditions for $\frac{\partial^2 Z^B}{\partial \tau^2}$:

$$\left.\frac{\partial^2 Z_a^B}{\partial \tau^2}\right|_{t=\tau} = \left[\frac{\partial^2 F_a}{\partial Z_i^A\, \partial Z_j^A} \cdot \Gamma_i^A \Gamma_j^A + \frac{\partial F_a}{\partial Z_i^A} \cdot \frac{\partial \Gamma_i^A}{\partial Z_j^A} \cdot \Gamma_j^A \right. \\ \left. + \frac{\partial \Gamma_a^B}{\partial Z_b^B} \cdot \Gamma_b^B - 2\, \frac{\partial \Gamma_a^B}{\partial Z_b^B} \cdot \frac{\partial F_b}{\partial Z_i^A} \cdot \Gamma_i^A \right]_\tau \quad . \tag{2.18}$$

Let us now reexamine our initial conditions for the unbroken theory. We had to keep $\Lambda$ fixed above $y$ to derive equations (2.17) and (2.18) . After we have evaluated $\frac{\partial Z^B}{\partial \tau}$ and $\frac{\partial^2 Z^B}{\partial \tau^2}$ and derived their renormalization group equations (2.15) and (2.16) , we no longer need to keep $\tau$ as a variable parameter. On the other hand, with $y$ fixed, solving the equation $(2.13)^A$ for the unbroken theory becomes totally redundant. Since $\Lambda$ is an arbitrary fixed scale we may simply set $\Lambda = y$ and set the parameters of the unbroken theory just above the threshold. If we now set $Z$-factors equal to their canonical values at this scale, then this is equivalent to defining our bare couplings as equal to their renormalized (running) values at the Grand Unified scale.

We may now formulate our recipe for obtaining radiative masses to a relative order $n$ in a given model.

1. Describe both unbroken and broken theories ( their sets of fields, couplings and relevant $Z$-factors ).

2. Choose the renormalization scheme (for $n > 0$). Fix the parameters of the unbroken theory at the threshold scale* .

3. Calculate the renormalization group $\Gamma$-functions for both unbroken and broken theories to $(n+1)$–loop order.

4. Derive the matching conditions (2.14) to order $n$† .

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* See the footnote on page 8.

† One should integrate out the heavy fields of the broken theory. For $n = 0$ this integrating out is mostly trivial (unless there is a field mixing), but it would not be trivial for $n > 0$.

5. Obtain $Z^B$, $\frac{\partial Z^B}{\partial \tau}$ and $\frac{\partial^2 Z^B}{\partial \tau^2}$ at $t = \tau$ for the broken theory from equations (2.14), (2.17) and (2.18).

6. Solve the renormalization group equations $(2.13)^B$, (2.15) and (2.16) and obtain $Z^B$, $\frac{\partial Z^B}{\partial \tau}$ and $\frac{\partial^2 Z^B}{\partial \tau^2}$ at $t = \tau$ at the low-energy scale.

7. Obtain radiative mass terms and 3-scalar couplings from equations (2.8) and (2.12); running renormalizable couplings (supersymmetric) at the low-energy scale are given by equation (2.7).

In the next section we are going to apply the formalism derived here to a simple toy model. The treatment of an actual inverse hierarchy model is postponed to section 4. We would like to finish this section with two comments:

1. The formalism we have derived here is easily generalized to the case of several thresholds (we will use it in section 4). The only necessary condition is that all tree-level supersymmetry breaking has to originate from a single O'Raifeartaigh mechanism.

2. Our formalism can be extended to models with *local* supersymmetry broken by the O'Raifeartaigh mechanism. In such theories both the Goldstone superfield and the chiral compensator of supergravity[7] are chiral superfields. We may use a linear combination of them for our $Y$ field. Our formulæ (2.14), (2.17) and (2.18) should now be replaced by appropriate supergravity formulæ, but the renormalization group equations $(2.13)^B$, (2.15) and (2.16) are still applicable.

## 3. Toy Model.

The renormalization group equations for Witten's inverse hierarchy model are exceedingly complicated and there is no limit in which they become analytically tractable. Rather than simply present the results of a numerical investigation of these equations, we have decided to illustrate our formalism with a toy model. Our toy model involves explicit soft supersymmetry breaking that mimics inverse hierarchy and fine tuning of the *"set it and forget it"* type,[8] but its phenomenology bears closer resemblance to the real world than that of the upside down hierarchy model. We do *not* present it as a viable model of particle physics but rather as an illustration of the mechanisms which determine scalar masses and $SU(2)\otimes U(1)$ breaking in SuSy GUTs.

The model has $SU(5)$ gauge symmetry and the following set of chiral supermultiplets:

- 3 generations of matter (*i.e.* quarks and leptons) $\psi$ [ each $(\mathbf{10}+\overline{\mathbf{5}})$ under $SU(5)$ ].
- $N$ pairs of [Weinberg-Salam] Higgs superfields $(H_i+\overline{H}_i)$ [ each $(\mathbf{5}+\overline{\mathbf{5}})$ under $SU(5)$ ].
- $SU(5)$–breaking Higgs $B$ [ $\mathbf{24}$ under $SU(5)$ ].
- $SU(5)$ singlet $Q$.

Its superpotential is given by

$$W=\tfrac{1}{3}\lambda_1\,(\,TrB^3+Y\,TrB^2\,)+\tfrac{1}{3}\lambda_2\,Q^3$$
$$+\sum_i G_i^{(1)}\,\overline{H}_i(B+\nu_iY)H_i+\sum_{ij}G_{ij}^{(2)}\,\overline{H}_iH_jQ \qquad (3.1)$$
$$+\ \text{Higgs–matter couplings}\ .$$

$Y$ is a non-dynamical superfield whose [constant] value $Y=y+F\theta\theta$ is an explicit supersymmetry breaking parameter. In a more realistic theory $Y$ would be dynamical. For phenomenological reasons $y\sim 10^{16}\,\text{GeV}$ and $m\equiv\alpha_{em}\,\cdot$

$(F/y) \sim 100$ GeV, so $F \sim (10^{10}\,\mathrm{GeV})^2$. Since we want some $SU(2)$–doublet Higgs to remain massless we should tune one of the $\nu_i$ to be

$$\nu = 2 + \frac{3F}{40\lambda_1 y^2} \quad . \tag{3.2}$$

This fine tuning ($\delta\nu \sim 10^{-14}$), however unwanted, is technically natural in the sense that once established it will survive radiative corrections to all orders of perturbation theory (there is no renormalization of $\nu$).

For the supersymmetric theory (for $F = 0$) there would be 3 degenerate vacua. When the supersymmetry is broken (*i.e.* $F \neq 0$) the degeneracy splits. The true vacuum (the one of the lowest energy) has

$$< B > = R \cdot \begin{pmatrix} 4/3 & 0 & 0 & 0 & 0 \\ 0 & 4/3 & 0 & 0 & 0 \\ 0 & 0 & 4/3 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix} \quad . \tag{3.3}$$

Many of the fields in the theory get masses $\sim M$. The surviving light superfields are:

- $SU(3) \otimes SU(2) \otimes U(1)$ gauge superfields;
- Matter superfields $\psi$;
- $Q$;
- $SU(2)$–doublet parts of some $(H + \overline{H})$ pair which we should denote by $(H_2 + \overline{H}_2)$; we suppose that only one such pair survives at low energies.

Their superpotential is

$$W^B = \tfrac{1}{3}\lambda_2 Q^3 + g_2 Q \overline{H}_2 H_2 + H\text{–}\psi \text{ couplings} . \tag{3.4}$$

and they do not feel supersymmetry breaking at this stage of the approximation.

Our first goal will be to calculate the masses of scalar quarks and leptons. The non-gauge couplings of the corresponding superfields are very weak. Indeed, even for a $t$-quark of mass $\simeq 35\,\mathrm{GeV}$ the one-loop effect of its Yukawa coupling to $H$ would be comparable to the two-loop effects of the gauge couplings. This allows us to neglect $H$–$\psi$ couplings entirely, so we are left with the gauge couplings only. In this approximation the *one-loop* renormalization group equations are exactly soluble.

The renormalization group functions for the unbroken $SU(5)$ theory are

$$\begin{aligned}\Gamma^A_{e_5} &= \frac{e^2}{8\pi^2}\cdot(10-N-2G)\\ \Gamma^A_{\psi} &= \frac{e^2}{8\pi^2}\cdot Z_\psi\, C_5(\psi)\, Z_{e_5}^{-1}\end{aligned} \tag{3.5$^A$}$$

while the broken $SU(3)\otimes SU(2)\otimes U(1)$ theory has

$$\begin{aligned}\Gamma^B_{e_3} &= \frac{e^2}{8\pi^2}\cdot(9-2G)\\ \Gamma^B_{e_2} &= \frac{e^2}{8\pi^2}\cdot(5-2G)\\ \Gamma^B_{e_1} &= \frac{e^2}{8\pi^2}\cdot\left(\frac{-3}{5}-2G\right)\\ \Gamma^B_{\psi} &= \frac{e^2}{8\pi^2}\cdot\left(C_3(\psi)\, Z_{e_3}^{-1} + C_2(\psi)\, Z_{e_2}^{-1}\, C_1(\psi)\, Z_{e_1}^{-1}\right) \quad .\end{aligned} \tag{3.5$^B$}$$

The matching equation (2.14) trivializes now to

$$Z^B_\psi = Z^A_\psi\,, \quad Z^B_{e_3} = Z^B_{e_2} = Z^B_{e_1} = Z^A_{e_5} \quad .$$

Here $\frac{e^2}{4\pi}$ is the bare gauge coupling constant while the running gauge couplings are given by $\alpha_i(\mu) = \frac{e^2}{4\pi}\cdot Z_{e_i}^{-1}$. $C_5$, $C_3$, $C_2$ and $C_1$ are $SU(5)$, $SU(3)$, $SU(2)$ and $U(1)$ quadratic Casimirs. Their values for different matter fields are given in

Table 1 below.

| Matter superfields and their Casimirs | | | | | | |
|---|---|---|---|---|---|---|
| fields | representations under | | Casimirs | | | |
| | $SU(5)$ | $SU(3)\otimes SU(2)\otimes U(1)$ | $C_5$ | $C_3$ | $C_2$ | $C_1$ |
| $Q$ | $\mathbf{10}$ | $(\mathbf{3},\mathbf{2},+1/3)$ | 18/5 | 4/3 | 3/4 | 1/60 |
| $U$ | $\mathbf{10}$ | $(\bar{\mathbf{3}},\mathbf{1},-4/3)$ | 18/5 | 4/3 | 0 | 4/15 |
| $D$ | $\bar{\mathbf{5}}$ | $(\bar{\mathbf{3}},\mathbf{1},+2/3)$ | 12/5 | 4/3 | 0 | 1/15 |
| $L$ | $\bar{\mathbf{5}}$ | $(\mathbf{1},\mathbf{2},-1)$ | 12/5 | 0 | 3/4 | 3/20 |
| $E$ | $\mathbf{10}$ | $(\mathbf{1},\mathbf{1},+2)$ | 18/5 | 0 | 0 | 3/5 |

Table 1.

Let us fix boundary conditions $Z_{e_5}(\Lambda) = Z_\psi(\Lambda) = 1$ at some scale $\Lambda \geq y$. Equations (3.5) then give us

$$
\begin{aligned}
Z_{e_5}(y) &= 1 + \frac{e^2\tau}{8\pi^2}\cdot(10-N-2G) \\
Z_{e_3}(\mu) &= 1 + \frac{e^2\tau}{8\pi^2}\cdot(1-N) + \frac{e^2 t}{8\pi^2}\cdot(9-2G) \\
Z_{e_2}(\mu) &= 1 + \frac{e^2\tau}{8\pi^2}\cdot(5-N) + \frac{e^2 t}{8\pi^2}\cdot(5-2G) \\
Z_{e_1}(\mu) &= 1 + \frac{e^2\tau}{8\pi^2}\cdot\left(\frac{53}{5}-N\right) + \frac{e^2 t}{8\pi^2}\cdot\left(-\frac{3}{5}-2G\right)
\end{aligned}
\tag{3.6}
$$

for the gauge renormalization factors and

$$
\begin{aligned}
\ln Z_\psi(y) &= \frac{C_5(\psi)}{10-N-2G}\cdot \ln Z_{e_5}(y) \qquad ; \\
\ln Z_\psi(\mu) &= \left[\frac{C_5(\psi)}{10-N-2G} - \frac{C_3(\psi)}{9-2G} - \frac{C_2(\psi)}{5-2G} - \frac{C_1(\psi)}{-\frac{3}{5}-2G}\right]\cdot \ln Z_{e_5}(y) \\
&\quad + \frac{C_3(\psi)}{9-2G}\cdot\ln Z_{e_3}(\mu) + \frac{C_2(\psi)}{5-2G}\cdot\ln Z_{e_2}(\mu) + \frac{C_1(\psi)}{-\frac{3}{5}-2G}\cdot\ln Z_{e_1}(\mu) \quad .
\end{aligned}
\tag{3.7}
$$

for the renormalization factors of the matter fields. Here $\mu$ is the low energy scale, $t \equiv \ln \mu/\Lambda$, and $\tau \equiv \ln y/\Lambda$ ($t < \tau \leq 0$). Differentiating $\ln Z_{\psi}(\mu)$ with respect to $\tau$ becomes now straightforward. Reexpressing the result with the help of the equation (2.12) we obtain mass$^2$ terms for scalar quarks and leptons.*

$$
\begin{aligned}
\mathcal{M}_{\psi}^{2} = \left|\frac{F\alpha_5^2(y)}{4\pi y}\right|^2 \cdot \Bigg\{ \Big[ & C_5(\psi)\cdot(10-N-2G) + C_3(\psi)\cdot(2G+2N-11) \\
& + C_2(\psi)\cdot(2G+2N-15) + C_1(\psi)\cdot(2G+2N-\frac{103}{5}) \Big] \\
+ \frac{\ln y/\mu}{2\pi}\cdot \Big[ & \frac{\alpha_3(\mu)\cdot(\alpha_3(\mu)+\alpha_5(y))}{\alpha_5(y)} \cdot C_3(\psi)\cdot(1-N)^2 \\
& + \frac{\alpha_2(\mu)\cdot(\alpha_2(\mu)+\alpha_5(y))}{\alpha_5(y)} \cdot C_2(\psi)\cdot(5-N)^2 \\
& + \frac{\alpha_1(\mu)\cdot(\alpha_1(\mu)+\alpha_5(y))}{\alpha_5(y)} \cdot C_1(\psi)\cdot(\frac{53}{5}-N)^2 \Big] \Bigg\} \quad .
\end{aligned}
\tag{3.8}
$$

We would like to remark that only the term in the first square bracket in (3.8) could have been obtained perturbatively (by a *tedious* 2-loop calculation). The terms in the second square bracket are the leading-log corrections; they are *not* small compared to the 2-loop term. The masses of the gauge fermions can be obtained from equations (2.12) and (3.6) :

$$
\begin{aligned}
\mathcal{M}_{e_3} &= \frac{F}{4\pi y}\cdot\alpha_3(\mu)\cdot(1-N) \quad , \\
\mathcal{M}_{e_2} &= \frac{F}{4\pi y}\cdot\alpha_2(\mu)\cdot(5-N) \quad , \\
\mathcal{M}_{e_1} &= \frac{F}{4\pi y}\cdot\alpha_1(\mu)\cdot(\frac{53}{5}-N) \quad .
\end{aligned}
\tag{3.9}
$$

---

* We have neglected contributions to $\mathcal{L}_2$ due to $M^r$. This approximation is justified for scalars that are much heavier than their fermionic partners, *i.e.* for all scalar quarks and leptons except $t$-quark.

Although the derivation of equation (3.9) was straightforward, its interpretation is not. $\mathcal{M}_{e_2}$ and $\mathcal{M}_{e_1}$ are Majorana masses of the $W$-ino and $B$-ino in a theory with *massless* $W$ and $B$. When the Glashow–Weinberg–Salam $SU(2) \otimes U(1)$ theory is spontaneously broken down to $U(1)_{em}$ gauge fermions mix with the Higgs fermions. Diagonalization of the gauge/Higgs fermion mass matrix requires knowledge of the pure Higgs masses which depend among other things on the accuracy with which (3.2) holds. As for the gluino mass, equation (3.9) gives us the *current* mass evaluated at the scale $\mu = O(100\,\mathrm{GeV})$. Calculating of the corresponding *constituent* mass and relating it to the masses of various oddballs† is a complicated $QCD$ problem.

To proceed with the evaluation of (3.8) and (3.9), we note that our toy model has the same gauge coupling renormalization as the minimal SuSy GUT. With 3 generations of matter, $\alpha_{em}^{-1}(0) = 137$ and $\Lambda_{QCD} \simeq 100\,\mathrm{MeV}$ as our input parameters we have $\alpha_{em}^{-1}(\mu) = 128$ and $\alpha_3^{-1}(\mu) \simeq 9$ in our "low"–energy world at $\mu = M_W \simeq 160\,\mathrm{GeV}$. This requires the scale "$y$"$\equiv 2M_X$ to be $\simeq 10^{16}\,\mathrm{GeV}$ and $\alpha_5^{-1}(y) = 24.6$, which in turn determines the low-energy values of $\alpha_1$ and $\alpha_2$. In particular $\sin^2\theta_W(\mu) = 0.233$. Thus our relevant couplings are:

$$\begin{aligned} \alpha_5(y) &= 1/24.6 \quad ; \\ \alpha_3(\mu) &= 1/9 \quad , \\ \alpha_2(\mu) &= 1/29.8 \quad , \\ \alpha_1(\mu) &= 1/59 \quad . \end{aligned} \tag{3.10}$$

The Casimirs of quarks and leptons are given in table 1. Straightforward substitution of (3.10) into (3.8) and (3.9) gives us the masses of the scalar quarks and leptons and of the gauge fermions. Table 2 summarizes them for $N = 1$ and $N = 2$.

---

† The name "oddballs" for the $R$-odd particles was suggested by S.Meshkov.

| Masses of $R$-odd fields | | |
|---|---|---|
| Superpartners of | Mass* | |
| | $N=1$ | $N=2$ |
| $Q$ | 1.03 | 1.18 |
| $U$ | 1.13 | 1.28 |
| $D$ | 0.78 | 1.11 |
| $L$ | 1.05 | 0.77 |
| $E$ | 1.41 | 1.12 |
| gluon | 0.00 | 1.21 |
| $W$ | 1.46 | 1.10 |
| $B$ | 1.77 | 1.59 |

* All masses are given in terms of $m \equiv \frac{\alpha_{em}(0)F}{y}$ .

Table 2.

Note that for $N = 1$ the gluino comes out massless. We may assume that the next-to-leading corrections would repair this defect, but they would give us $M_{\tilde{g}} \sim 0.05$ in our units which means a few GeV. The current lower limit on the gluino mass[9] is about 5 GeV, so our result is acceptable, but not entirely welcome. The obvious way out would be taking $N > 1$. Anyhow we would need an extra pair of $(\mathbf{5} + \bar{\mathbf{5}})$ Higgses for cosmological baryogenesis.

We have expressed the masses of $R$-odd fields in terms of $m$. In order to get an answer in GeV we must relate $m$ to $M_W$. Indeed, we must first show that $SU(2) \otimes U(1)$ breaking occurs so that $M_W \neq 0$. This requires us to evaluate the parameters of the effective Higgs potential at the low energy scale $\mu$. If relation

(3.2) holds exactly and the doublet Higgs mass vanishes exactly at the tree level, the Higgs potential becomes

$$
\begin{aligned}
V = e_{2r}^2 \cdot & \left\{ \frac{(|H|^2 - |\overline{H}|^2)}{4\cos^2\theta_W} + \left( |H|^2 \cdot |\overline{H}|^2 \; - \; |\overline{H} \cdot H|^2 \right) \right\} \\
& + \; g_r^2 \cdot |Q|^2 ( |H|^2 \; + \; |\overline{H}|^2 ) \; + \left| \lambda_r Q^2 \; - \; g_r \overline{H} \cdot H \right|^2 \\
& + \; \left[ g_r ( \underline{m}_Q \; + 2 \, \underline{m}_H ) \cdot Q \overline{H} H \; + \lambda_r \; \underline{m}_Q \cdot Q^3 \quad + \text{ H.c.} \right] \\
& + \mathcal{M}_H^2 \cdot ( |H|^2 \; + \; |\overline{H}|^2 ) \; + \; \mathcal{M}_Q^2 \cdot |Q|^2 \qquad ,
\end{aligned}
\tag{3.11}
$$

where $\lambda_r \equiv \lambda_2 / Z_H Z_Q^{1/2}$ , $g_r \equiv g_2 / Z_Q^{3/2}$ , $e_{2r}^2 \equiv e^2 / Z_{e_2}$ are the renormalized couplings and masses $\mathcal{M}_Q$ , $\mathcal{M}_H$ , $\underline{m}_Q$ and $\underline{m}_H$ are given by equation (2.12) . If the tree-level doublet Higgs mass did not vanish exactly, but were of magnitude $\sim 100\,\mathrm{GeV}$, the mass terms in (3.11) would become more complicated. We will not investigate this possibility here.

The renormalization group equations for $Z_H$ and $Z_Q$ are not solvable analytically and we would have to use the general recipe developed in the previous section. Relevant $Z$-factors of the broken theory are $Z_H$ , $Z_Q$ , $Z_{e_2}$ and $Z_{e_1}$ . The "initial" values of these factors and their derivatives at $t = \tau$ are given by equations (2.14) , (2.17) and (2.18) . Their actual values depend on the structure of the coupling matrices $G^{(1)}$ and $G^{(2)}$ (we have defined $H_i$ such that $G^{(1)}$ is diagonal). Let us now make a simplifying assumption that $G_i^{(1)} \equiv g_1$ and $G_{ij}^{(2)} \equiv g_2 \delta_{ij}$ ; it will not affect our conclusions. We would also like to assume that the Yukawa couplings of Higgses to matter are small enough compared to the other couplings and may be neglected in the lowest order.* Then at $t = \tau$ we have:

$$
Z_H \; = \; Z_Q \; = \; Z_{e_2} \; = \; Z_{e_1} \; = \; 1 \quad ; \tag{3.12$^I$}
$$

* This assumption is valid unless $t$-quark is too heavy.

$$8\pi^2\,\frac{\partial Z_H}{\partial\tau} = \tfrac{3}{2}\,e^2 - \tfrac{24}{5}\,g_1^2$$

$$8\pi^2\,\frac{\partial Z_Q}{\partial\tau} = (2-5N)\,g_2^2 \quad ; \tag{3.12$^{II}$}$$

$$\begin{aligned}64\pi^4\,\frac{\partial^2 Z_H}{\partial\tau^2} &= \left(\frac{159}{100}+\frac{3}{5}N\right)\cdot e^4 + \left(\frac{816}{25}\,g_1^2 - 3g_2^2\right)\cdot e^2 \\ &\quad + (5N-2)\cdot g_2^4 - \left(\frac{576+120N}{25}\,g_1^2 + \frac{252}{25}\,\lambda_1^2\right)\cdot g_1^2 \\ 64\pi^4\,\frac{\partial^2 Z_Q}{\partial\tau^2} &= \left(\frac{60N-39}{5}\,e^2 + \frac{5N-2}{2}\,\lambda_2^2 - \frac{24}{5}(5N-4)\,g_1^2\right. \\ &\quad \left. - (5N-1)(5N-2)\,g_2^2\right)\cdot g_2^2 \quad .\end{aligned} \tag{3.12$^{III}$}$$

Renormalization group equations for $Z_{e_2}$ and $Z_{e_1}$ are solved analytically (see (3.6) ). The renormalization group equations for $Z_H$ , $Z_Q$ and their derivatives with respect to $\tau$ look quite formidable; we have displayed them in *Appendix B.* These equations are supposed to be solved numerically with initial conditions given by (3.12) . Instead of solving them numerically for general values of couplings, we restrict our attention to the analytically tractable region $\lambda_2, g_2 \ll e$. In that limit everything can be worked out exactly and

$$\begin{aligned}\mathcal{M}_H^2 &= \mathcal{M}_L^2 + \left(\frac{F\,\alpha_5(y)}{4\pi\,y}\right)^2\cdot\frac{12}{5}\left(\frac{g_1}{e}\right)^2\cdot\left[(96+10N)\left(\frac{g_1}{e}\right)^2 + 21\left(\frac{\lambda_1}{e}\right) - 98\right] \quad , \\ \underline{m}_H &= \underline{m}_L - \frac{F\,\alpha_5(y)}{4\pi\,y}\cdot\frac{24}{5}\cdot\left(\frac{g_1}{e}\right)^2 \quad ,\end{aligned} \tag{3.13}$$

while $\mathcal{M}_Q^2 = O(g_2^2/e^2)\cdot m^2$, $\underline{m}_Q = O(g_2^2/e^2)\cdot m$ with all couplings normalized at $\Lambda = y$. The minimal value of $\mathcal{M}_H^2$ is obtained for $\lambda_1 \to 0$ and $(g_1/e)^2 = \frac{49}{96+10N}$. For $N=1$ it becomes $-1.03\,m^2$ while for $N=2$ it is $-1.36\,m^2$. Both of them are negative. Corresponding values of $\underline{m}$ are $0.93m$ and $0.71m$.

For negative $\mathcal{M}_H^2$, $e_{2r} \gg g_r \gg \lambda_r$ and $|\mathcal{M}_Q^2|, \underline{m}_Q^2 \ll |\mathcal{M}_H^2|, \underline{m}_H^2$ the potential (3.11) has its minimum at $< \overline{H} > = < H^\dagger >$ (up to a phase) with

$|\langle H \rangle|^2 = (\underline{m}_H^2 - \mathcal{M}_H^2)\cdot(1/g_r)^2$. This gives us the Glashow–Weinberg–Salam breaking of $SU(2)\otimes U(1)$ to $U(1)_{em}$ with

$$M_W^2 = (\underline{m}_H^2 - \mathcal{M}_H^2)\cdot(e_{2r}^2/2g_r^2) \quad .$$

This in turn determines $m$ to be

$$m = 95\,\mathrm{GeV}\cdot(g_r/e_{2r}) \qquad \text{for } N=2\,. \tag{3.14}$$

Since we have assumed $g_2 \ll e$, this probably means the scalar quark and lepton masses will come out small ($O(10\text{–}20\,\mathrm{GeV})$). However, one should not take too seriously the case $\lambda_2 \ll g_2$, $\lambda_1 \ll e$, $g_1/e \simeq \sqrt{49/96+10N}$ ; it was introduced for analytical convenience only. There is clearly a finite region of the parameter space in which our toy model gives the right qualitative low energy physics. It is a straightforward numerical exercise to map out the extent of this region and find the corresponding range of masses of the scalar quarks and leptons as well as the gaugini. However, since we are *not* advertising our toy model as a true picture of the origin of $SU(2)\otimes U(1)$ breaking, we will not burden the reader with a set of numbers to compare with experiment.

## 4. Upside Down Hierarchy Model.

In the previous section we have illustrated our techniques on a simple toy model. Now comes the time to apply them to a complete inverse hierarchy model and to add some new diseases to its already long nosonomy. We will make it in three steps, therefore three subsections of this section. The first subsection serves to describe our model and to explain how it *presumably* works. The second one is devoted to renormalization group analysis of the gauge couplings and to computing masses of the scalar quarks and leptons. The last subsection deals with the ***sliding singlet* mechanism** and its troubles.

### 4.1 DESIGN OF THE MODEL.

Except for notational changes, our choice of a particular inverse hierarchy model is the same as in our previous papers.[10,1] Its superpotential is given by

$$\begin{aligned} W &= \lambda\, Tr(X \cdot B^2) + \lambda' X' \cdot (\, Tr(B^2) - M^2\,) \\ &+ g\, \overline{H} B H + g' Q \overline{H} H \\ &+ \textit{Higgs–Matter couplings}\,, \end{aligned} \tag{4.1}$$

where chiral superfields $B$ and $X$ belong to **24** of $SU(5)$, $X'$ and $Q$ are $SU(5)$ singlets and $(H + \overline{H})$ form $(\mathbf{5}+\bar{\mathbf{5}})$ Higgs pairs. Since the fields $X$ and $X'$ couple only to $B$, the inverse hierarchy mechanism works exactly as described in [1]. The $SU(5)$ and supersymmetry breaking VEVs are given at the tree level by

$$\begin{aligned} < B > &= \frac{\lambda' M}{\lambda} \cdot \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \\ < X' > &= \frac{\lambda}{\lambda} \cdot < Y > \\ < X > &= \frac{\lambda'}{\lambda} \cdot < Y > \cdot \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} \tilde{\lambda} &\equiv \sqrt{\lambda^2 + 30\,\lambda'^2} \quad , \\ <Y> &= y \, + \, F_y\,\theta\theta \quad , \\ F_y &= \frac{\lambda\,\lambda'\,M^2}{\tilde{\lambda}} \quad , \end{aligned} \tag{4.3}$$

while $y$ is undetermined at the tree level, but is provided with a huge value ($\gg M$) by the inverse hierarchy mechanism.

Let us take a moment to explain our reasons for introducing the *"sliding singlet"* $Q$. $SU(5)$ is broken primarily by the large ($O(y) \sim 10^{18}\,$GeV) vacuum expectation value of the $X$ field; the expectation value of the $B$ field points in the same direction in the group space but has much smaller magnitude ($O(M) \sim 10^{11}\,$GeV). However it is the expectation value of $B$ that determines the mass splitting between the $SU(2)$-doublet Higgses and their $SU(3)$-triplet partners. The role of $Q$ is to provide a common offset for these Higgs masses, so they become

$$\begin{aligned} M_{H_2} &= g' < Q > \, - \frac{3}{\sqrt{30}}\, g < B > \quad , \\ M_{H_3} &= g' < Q > \, + \frac{2}{\sqrt{30}}\, g < B > \quad . \end{aligned} \tag{4.4}$$

If $Q$ was a parameter, then it could be fine tuned to exactly cancel the mass $M_{H_2}$ of the Weinberg–Salam doublets and thus allow them to develop $SU(2)$-breaking VEVs. Note that this would not be a *"set it and forget it"* type of fine tuning. The desired value $Q_2$ of $Q$ is determined in terms of an expectation value $<B>$ which is corrected in each order of perturbation theory.

Instead of keeping $Q$ as a [fine tunable] parameter, we chose it to be a dynamical scalar superfield whose expectation value is indeterminated at the tree level. If for some reasons $< Q >$ is close to $Q_2$, then $M_{H_2}$ is small and the doublet Higgses may be considered as light fields. In such case we may apply the techniques of section 2 to obtain their effective potential:

$$\begin{aligned} V_H &= e_2^2 \cdot \left\{ \frac{(|H|^2 - |\overline{H}|^2)}{4\cos^2\theta_W} + (|H|^2 \cdot |\overline{H}|^2 - |\overline{H} \cdot H|^2) \right\} \\ &\quad + g'^2 \cdot |\Delta Q|^2 (|H|^2 + |\overline{H}|^2) + g'^2 \cdot |\overline{H} \cdot H|^2 \\ &\quad + [2g' \, \underline{m}_H \cdot \Delta Q \, \overline{H} H + \text{H.c.}] + \mathcal{M}_H^2 \cdot (|H|^2 + |\overline{H}|^2) \quad , \end{aligned} \tag{4.5}$$

where $\Delta Q \equiv Q - Q_2$ and $H$ denotes the doublet part of the Higgs multiplet. For $g' < e_2$ the minimum of $V_H$ occurs at $\overline{H} = H^\dagger$ (up to a phase) and $V_H$ reduces to:

$$V_H = g'^2 \cdot |H|^4 - 2\left(|\underline{m}_H|^2 - \mathcal{M}_H^2 - g'^2 \cdot |\Delta' Q|^2\right) \cdot |H|^2 \quad , \tag{4.6}$$

where $\Delta' Q = Q - Q_2' \equiv Q - Q_2 - (\underline{m}_H / g')$ . If $\mathcal{M}_H^2 - |\underline{m}_H|^2$ is negative, then for $\Delta' Q$ small enough $H$ develops a non-zero VEV and breaks the low-energy gauge symmetry $SU(3) \otimes SU(2) \otimes U(1)$ down to $SU(3)_c \otimes U(1)_{em}$.

The range of $< Q >$ permitting further symmetry breaking is finite, but quite narrow: its width ($\sim m/g'$) compared to the natural scale $O(M)$ of $Q$ is very small. Why should the VEV of $Q$ fall into this narrow margin? To explain such a lucky choice one has to remember that the tree-level potential for $Q$ is completely flat. For large $\Delta' Q$ $V_H$ is positive definite, vanishes at its minimum, and thus is also indifferent to $Q$. However, when $Q$ comes within the narrow margin around $Q_2'$, which realizes $SU(2)$ breaking, $V_H$ is no longer positive definite. $V_H$ reaches it absolute minimum for $Q = Q_2'$ that allows the maximal VEV of $H$. So the vast plateau of the effective potential develops a dip centered at precisely the right point. The dip is narrow ($\sim m/g'$) and shallow ($\sim m^4/g'^2$), but as long as it remains the only topographical feature of $V(Q)$ the VEV of $Q$ has no choice but to fall into it.

One should note, however, that exactly the same mechanism can produce a second dip for $Q$ near $Q_3'$ which corresponds to color breaking. With two dips separated by a flat terrain, the choice becomes twofold, and the actual outcome seems to resemble Russian roulette, which we would rather avoid. Since the existence of each dip is determined by the sign of ( $\mathcal{M}_H^2 - |\underline{m}_H|$ ) for the corresponding parts of $H$, we would like to have negative ( $\mathcal{M}^2 - \underline{m}^2$ ) for doublet Higgses while keeping the corresponding value for triplet ones safely positive. We have seen in section 3 (and will see again in the next subsection) that scalar quarks tend to be somewhat heavier than scalar leptons, so one might expect to have only the "right" dip for certain values of parameters.

This mechanism was invented independently by several authors,[10,11] most of whom have also realized that it does not work. The problem is that equation (4.6) is incomplete: In addition to $V_H$, radiative corrections induce a curvature for the effective potential for $Q$ itself. While this curvature is comparable to that at the dips, $V_Q$ has a much larger range. Hence its characteristic magnitude is of order $O(M^2 m^2)$ as opposed to the $O(m^4/g'^2)$ magnitude of the dips. The true value of $< Q >$ depends mainly on the detailed shape of $V_Q$, much more so than on the radiative corrections to $V_H$!

This disaster will be avoided only if $V_Q$ happens to have it minimum precisely at $Q'_2 \pm O(100\,\mathrm{GeV})$.* We will argue that this can be achieved by a fine tuning of parameters (with 9-digit accuracy) which must be corrected at each order of perturbation theory. Clearly this model does not solve the hierarchy problem! However, since we have already done the calculations, we will not allow this minor problem to prevent us from presenting our results.

### 4.2 Gauge Couplings and Their Effects.

In section 2 we have developed the techniques allowing us to calculate the radiative masses of scalars and gauge fermions. We have illustrated these techniques on a toy model example and promised to deal with the real inverse hierarchy model later. The goal of the present section is to repay this debt.

---

* This is the *overly optimistic assumption* we have made in our cosmological paper.[10]

Unlike the toy model of section 3, the inverse hierarchy model has not one but three thresholds, and it causes additional complications. The highest of the thresholds is associated with the dynamically produced scale $y$ which we would like to be $\sim 10^{16} - 10^{18}\,\mathrm{GeV}$. In the $SU(5)$ model the Grand Unified symmetry is broken at this scale all the way down to $SU(3) \otimes SU(2) \otimes U(1)$; in bigger models (like $SU(6)$, $SO(10)$, $E_6$, *etc.* ) one might expect to have a somewhat bigger symmetry group surviving until the next threshold. The second threshold in the inverse hierarchy theories is associated with the original (Lagrangian) scale $M \sim 10^{10} - 10^{11}\,\mathrm{GeV}$. Even if this threshold does not affect the gauge group itself (as it does not in the $SU(5)$ model), it does affect the Higgs multiplets and thus affects the $\beta$–functions of the gauge couplings.

The third threshold at the "mirror" scale $\overline{m} \equiv \frac{F_y}{y} \simeq 10\,\mathrm{TeV}$ is due to light components of the $X$ multiplet. The only mass terms available for the components of $X$ and $X'$ are the Dirac (*i.e.* off-diagonal) masses$\sim M$ between them and the superheavy components of $B$. The resulting mass matrix looks like

$$\left( \begin{array}{c|c} y & M \\ \hline M & 0 \end{array} \right)$$

with eigenvalues $y$ and $M^2/y \simeq \overline{m}$ that correspond to $B$ and $X$ superfields. (For more details see [1].)

The unbroken theory has $SU(5)$ gauge superfields and its set of non-singlet chiral superfields consists of: two superfields ($X$ and $B$) in the adjoint representation $(\mathbf{24})$, $G \quad (\mathbf{10} + \bar{\mathbf{5}})$ generations of matter and $N \quad (\mathbf{5} + \bar{\mathbf{5}})$ Higgs pairs. All broken theories have the same set of gauge superfields — that of $SU(3) \otimes SU(2) \otimes U(1)$ — since the gauge symmetry is not broken at the second or at the third threshold. The non-singlet chiral superfields relevant in the energy range $M$ to $y$ are: $X_8$ and $X_3$ which together constitute the adjoint representation of $SU(3) \otimes SU(2)$, and all matter and Higgs superfields. In the energy range $\overline{m}$ to $M$ we still have $X_8$, $X_3$ and all matter superfields, but only one pair of $((\mathbf{1}, \mathbf{2}, +1/2) + (\mathbf{1}, \mathbf{2}, -1/2))$ Higgs superfields survives below the second threshold. Finally at the third threshold we lose the $X$ superfields, and the effective theory in the 100 GeV to 10 TeV range has only matter superfields and one pair of Higgs doublets.

The above information is sufficient to compute the gauge Γ–functions in each intermediate energy domain. (We use here the notations of section 2 in order to simplify the calculation of the radiative masses later in this section.) Since the computation is straightforward we simply present the results in the Table 3 below.

TABLE 3

| **Γ–functions of the gauge fields** | | | | |
|---|---|---|---|---|
| FIELD | | RANGE | | |
| group | Γ–function | $(M_W$ to $\overline{m})$ | $(\overline{m}$ to $M)$ | $(M$ to $y)$ |
| $U(1)$ | $\Gamma_1^e = \frac{e^2}{8\pi^2} \times$ | $(\frac{-3}{5} - 2G)$ | $(\frac{-3}{5} - 2G)$ | $(0 - N - 2G)$ |
| $SU(2)$ | $\Gamma_2^e = \frac{e^2}{8\pi^2} \times$ | $(5 - 2G)$ | $(3 - 2G)$ | $(4 - N - 2G)$ |
| $SU(3)$ | $\Gamma_3^e = \frac{e^2}{8\pi^2} \times$ | $(9 - 2G)$ | $(6 - 2G)$ | $(6 - N - 2G)$ |

Corresponding $\beta$–functions are given by

$$\beta_{e_i} = -e_i \cdot \frac{\Gamma_i^e}{Z_i^e} \quad .$$

A conventional next step of the renormalization group analysis of any GUT is computation of the GUT scale from the low-energy values of the gauge coupling constants. As we have already reported in [1], the results of such computations are disastrous: with three generations of matter QCD loses its asymptotic freedom and becomes non-perturbative before the Grand Unification scale is achieved. Restricting the model to two generations only makes the perturbative analysis valid, but the resulting GUT scale is 4 orders of magnitude above the Planck mass.

Therefore, we are forced to give up the phenomenology and consider the model as a semirealistic prototype. The least painful deviation from reality is to give up $\alpha_{em}(0) = 1/137$. We have arbitrarily decided to set $y \simeq 10^{18}$ GeV while keeping $\Lambda_{QCD} \simeq 100$ MeV. This choice leads to the following values of the gauge couplings at $\mu \simeq 2M_W$

$$
\begin{aligned}
\alpha_3(\mu) &= 1/9.5 \quad , \\
\alpha_2(\mu) &= 1/25.4 \quad , \\
\alpha_1(\mu) &= 1/49.3 \quad ,
\end{aligned}
\tag{4.7}
$$

which in turn imply $\alpha_{em}(0) = 1/114$.

We are now ready to compute the radiative masses of scalar matter fields and gauge fermions. The procedure we have now to follow is an exact copy of the one we have followed when exploring the toy model of section 3. All we need is to compute the renormalization factors $Z_\chi$ and $Z_V$ and to differentiate them with respect to $\tau$ ($\tau$ dependence of $\overline{m} \equiv M^2/y$ should be treated as explicit — that accounts for supersymmetry breaking in the spectrum of $X_{3,8}$). After manipulating a few rather lengthy equations* we obtain the radiative masses of the gauge fermions

$$
M_{e_i} = \frac{F_y}{4\pi y} \cdot \alpha_i(\mu) \cdot \left( D_i^H + D_i^L \right) \tag{4.8}
$$

* Our intermediate results are similar to equations (3.6) and (3.7), but the abundance of different scales and couplings makes them about three times longer.

and the scalar matter fields

$$
\begin{aligned}
\mathcal{M}_\psi^2 = & \left|\frac{\alpha_5^2(y)\cdot F_y}{4\pi y}\right|^2 \cdot \Big( C_5(\psi) - \sum_i C_i(\psi) \Big) \cdot (5 - 2G - N) \\
& + \left|\frac{F_y}{4\pi y}\right|^2 \cdot \sum_i C_i(\psi) \cdot \Big\{ -D_i^H \alpha_5^2(y) + D_i^L \alpha_i^2(\overline{m}) \\
& \qquad + \frac{\ln y/M}{2\pi} \cdot \Big[ \alpha_i^2(M)\,(\alpha_5(y) + \alpha_i(\overline{m})) \\
& \qquad\qquad + \alpha_i(M)\,(\alpha_5^2(y) + \alpha_i^2(\overline{m}))\Big] \cdot \left(D_i^H\right)^2 \\
& \qquad + \frac{\ln \overline{m}/\mu}{2\pi} \cdot \alpha_i(\mu)\,\alpha_i(\overline{m})\,(\alpha_i(\mu) + \alpha_i(\overline{m})) \cdot \left(D_i^H + D_i^L\right)^2 \Big\} \quad .
\end{aligned}
\tag{4.9}
$$

Here the index $i$ runs over the set $\{1,2,3\}$ and

$$
\begin{aligned}
D^H(1,2,3) &= (5,1,-1) \quad , \\
D^L(1,2,3) &= (0,2,3) \quad .
\end{aligned}
$$

Note that masses of the gauge fermions do not depend on the number of Higgs pairs $N$ or even the number of generations $G$. On the other hand, masses of the scalar quarks and leptons depend on $N$ and $G$ both explicitly (via the coefficient in the first term in (4.9) ) and implicitly, since the values of the gauge couplings at different scales depend on the numbers of generations and of Higgses. The latter (implicit) dependence makes it hard to see directly from (4.9) the overall effect of changing $N$ or $G$ on the scalar masses. Therefore, we have simply evaluated this equation for several values of $G$ and $N$.

The results of this evaluation look disastrous. For $G = 2$ and $N = 1$ the superpartners of the right-handed leptons come out with *negative mass*$^2$. Obviously one cannot allow non-zero vacuum expectation values for any electrically charged field, so we have to drop this case. Increasing $N$ makes things even worse: not only the mass$^2$ of "right sleptons" becomes more negative, but the mass$^2$ of the "left sleptons" (*i.e.* the superpartners of the left-handed leptons) becomes negative as well. For sufficiently large $N$ even scalar quarks get negative mass$^2$, although this may be questionable since in this case $\alpha_5(y)$ is too large to trust the lowest-order perturbation theory.

There appear to be only two ways to avert the disaster: one has to set either $N = 0$ or $G = 1$. The first choice is an obvious nonsense — one cannot make a light pair of Higgs doublets out of nothing. Therefore we have to deviate one step further from low-energy reality and give up the second generation of matter as well as the third. Table 4 below summarizes the masses of scalar matter and gauge fermions for $G = 1$ and $N = 1$ or 2.

| **Masses of $R$-odd fields** | | |
|---|---|---|
| Superpartners of | Mass* | |
| | $N = 1$ | $N = 2$ |
| $Q$ | 1.53 | 1.51 |
| $U$ | 1.50 | 1.46 |
| $D$ | 1.45 | 1.43 |
| $L$ | 0.63 | 0.54 |
| $E$ | 0.67 | 0.49 |
| gluon | 1.92 | |
| $W$ | 1.07 | |
| $B$ | 0.92 | |

Table 4.

∗ All masses are given in terms of $m \equiv \frac{\alpha_{em}(0) F_y}{y}$ .

The troubles we have encountered in this section appear to have a common source with the renormalization diseases we have reported in [1]. A supersymmetric theory burdened with the $X$ multiplet above the GUT scale and its remnants $X_3$ and $X_8$ below, does not have enough asymptotic freedom. Although we could not prove the connection between asymptotic freedom and the scalar masses, the conjecture is tempting. The coefficient in the first term in (4.9) is proportional to the ($-$) $\beta$-function of $SU(5)$. Addition of extra Higgs fields or extra generations affects all scalar mass$^2$ terms in the same direction — it reduces them. Scalar quarks are always (in our model) heavier than scalar leptons. It might be that all of the above is a sheer accident, but most likely a model with enough asymptotic freedom to reproduce the phenomenological values of the gauge couplings would have positive mass$^2$ terms for the scalar matter fields. Of course the test of this conjecture will have to wait for a better model than $SU(5)$.

### 4.3 Sliding Disaster.

The original idea of the automatic adjustment of $< Q >$ to allow for the GWS symmetry breaking was based upon a [wrong] assumption that the effective potential for the sliding singlet $Q$ is dominated by (4.6) . Unfortunately, that equation is incomplete since radiative corrections to the renormalization factor $Z_Q$ also contribute to the effective potential $V_Q$. One may expect from equation (2.8) the magnitude of $V_Q$ to be $O(m^2Q^2)$ ($m$ was defined in section 3 as $\alpha_{em}(0)F_y/y$ and we use the same definition here) while (4.6) amounts to $O(m^2M_W^2)$ only. Since $<Q>\sim M \gg M_W$, we may neglect (4.6) entirely and compute $V_Q$ in accordance with the general recipe of section 2.

Let us begin our analysis with the description of the effective theories in different energy domains. The effective theory for energies above the highest threshold $y$ has unbroken $SU(5)$ as its gauge group while its superpotential is given by equation (4.1) in which we may neglect $M$ and Higgs–matter couplings.* The

* Since we have given up the third and even the second generation of matter, we are left with Higgs–matter couplings of order $O(10^{-5})$ only.

effective theory below $y$ has the gauge group $SU(3)\otimes SU(2)\otimes U(1)$. For energies ranging from $M$ to just below $y$ the effective superpotential is given by

$$
\begin{aligned}
W = {} & \frac{\tilde{\lambda}}{\sqrt{30}}\, S\cdot\left(B_1^2 - \tilde{M}^2\right) - F\cdot Y \\
& + \frac{g}{\sqrt{30}}\, B_1\cdot(2\bar{H}_3 H_3 - 3\bar{H}_2 H_2) + g' Q\cdot(\bar{H}_3 H_3 + \bar{H}_2 H_2) \quad ,
\end{aligned}
\tag{4.10}
$$

where our notations follow those of the previous sections. Matter superfields along with $X_3$ and $X_8$ do not appear in the superpotential although they continue to interact via $SU(3)\otimes SU(2)\otimes U(1)$ gauge couplings. At the energy scale $M$ the superfields $B_1$, $S$, and $(\bar{H}_3 + H_3)$ become massive. Superfields $\bar{H}_2$ and $H_2$ will also become massive unless the expectation value of $Q$ obeys

$$
\nu \equiv \frac{\sqrt{30}\, g' < Q >}{g < B_1 >} = 3 \pm O(10^{-9}) \quad . \tag{4.11}
$$

Whether this happens or not is exactly the question we are interested in.

Naturally we would like first to satisfy equation (4.11) at least approximately at the scale $M$. Only after we succeed we will be able to discuss lower energies, since the very field content of the effective theory below $M$ depends on equation (4.11). Therefore, let us begin with a close examination of the fields relevant at energy scales between $M$ and $y$. There are two $SU(3)\otimes SU(2)\otimes U(1)$ singlet superfields that couple to Higgses, namely $Q$ and $B_1$. Once $SU(5)$ is broken, there is no conserved quantum number that can distinguish between them. Any symmetry, discrete or continuous, under which $Q$ and $B_1$ transform differently, would not be a symmetry of the superpotential (4.10) (this includes the so called $R$-symmetries as well). Thus, the theory contains nothing that can prevent these fields from *mixing*.

This field mixing could complicate significantly the effective *renormalized* superpotential. Fortunately the no-renormalization theorems allow one to work instead with the unrenormalized superpotential and non-canonical kinetic terms.

In this formalism condition (4.11) should be interpreted in terms of the *bare coupling constants* $g$ *and* $g'$ and the *unrenormalized fields* $Q$ and $B_1$, but otherwise remains as stated. In fact, it was precisely for this reason that the renormalization group analysis of section 2 was developed in such formalism. However, equations (2.8) and (2.12) were derived for renormalized and unmixed fields and we would like to rewrite them. An expression for the scalar potential in terms of the unrenormalized fields can be derived by eliminating auxiliary fields from the general Lagrangian of appendix A. Omitting the gauge-induced quartic interactions as irrelevant to the following discussion we obtain the effective scalar potential

$$
\begin{aligned}
V = & \left|\frac{F_y}{2y}\right|^2 \cdot A^\dagger \left[ \frac{\partial Z_S}{\partial \tau} Z_S^{-1} \frac{\partial Z_S}{\partial \tau} - \frac{\partial^2 Z_S}{\partial \tau} \right] A \\
& + \left(\frac{\partial W}{\partial A}\right)^\dagger Z_S^{-1} \left(\frac{\partial W}{\partial A}\right) \\
& + \left[ \frac{F_y}{2y} \cdot \left(\frac{\partial W}{\partial A}\right)^\dagger Z_S^{-1} \frac{\partial Z_S}{\partial \tau} A + \text{H.c.} \right] .
\end{aligned}
\tag{4.12}
$$

In a supersymmetric limit $F_y/y \to 0$ the vacuum state of our model has $< B_1 >= \tilde{M}$ while $< Q >$ is arbitrary. Since the only field mixing in the model occurs between $Q$ and $B_1$ and the VEV of $B_1$ does not break supersymmetry, only the first term in (4.12) depends on $Q$. Let us denote by $U$ the two-by-two block of the matrix in the first square bracket in (4.12) that corresponds to $B_1$ and $Q$. Then the effective potential for $Q$ is given by

$$
V_Q = \left|\frac{F_y}{2y}\right|^2 \cdot ( U_{QQ} Q^2 + 2U_{BQ} Q B_1 ) \quad . \tag{4.13}
$$

Implications of equation (4.13) depend on the sign of $U_{QQ}$, therefore there are two cases. For $U_{QQ} > 0$ the potential (4.13) is unbounded from below. Since equation (4.13) breaks down for $|Q| \gg M$ this unboundedness is only artificial. What it actually means is that the minimum of the total potential is achieved for $< Q > \sim y$. Obviously we should reject this case as unacceptable since it implies that *all* Higgs fields become superheavy.

We have not completed the renormalization group analysis of $\nu$ or Higgs masses. In the case of our toy model we have filled a page (see appendix B) with the renormalization group equations for only two fields and their derivatives. In the present case, however, one has to deal with coupled equations for *five* different fields ($Q$, $B_1$, $S$, $H_2$ and $H_3$) plus one field mixing parameter ($Z_{BQ}$). The resulting set of equations looks really fearsome. We doubt that the $\tau\epsilon\chi$ program that was used in printing this thesis could handle such a monstrosity without exceeding the computer memory limitations. Even the initial point ($t = \tau$) values of some derivatives of $Z$'s were quite complicated due to non-trivial matching conditions for $S$. In contrast with the toy model the inverse hierarchy theory allows for no limit in which those horrible equations became simpler. The only exception was the case $g = g' = 0$ in which Higgs multiplets became indistinguishable from leptons or quarks. On the other hand, even the complete set of equations could be easely solved by putting them on a computer. That we have not done so reflects not only our laziness, but also our low opinion of the viability of the model.

The conclusion of the analysis we have presented in this section is sad, but unavoidable: the idea of a sliding singlet is stillborn despite all its beauty. The fine tuning involved in such an "automatic" adjustment is much worse than that involved in adjustment by hand. The only remaining hope (within the framework of inverse hierarchy) rests on models of the Grinstein[12] type. Unfortunately such models tend to involve huge representations that make the problem of insufficient asymptotic freedom even more acute.

On the other hand, for $U_{QQ} > 0$ the potential (4.13) has a minimum at $< Q > \sim < B_1 > \sim M$. This minimum leads to a finite value of $\nu$ given by

$$\nu = -\frac{\sqrt{30}\, g'}{g} \cdot \frac{U_{BQ}}{U_{QQ}} \quad . \tag{4.14}$$

The last equation, when combined with criterion (4.11), spells the disaster: **the sliding singlet "solves" the second hierarchy problem only if a ratio of two perturbatively calculated functions is tuned to 9-digit accuracy.** Indeed, $U_{QQ}$ and $U_{BQ}$ are both obtained by perturbative calculations. They both vanish at the tree level and their leading terms are proportional to $\left|\frac{\alpha F_y}{4\pi y}\right|^2$. Their ratio (and therefore $\nu$) depends on various couplings in the theory.

Worse than that, $\nu$ suffers from higher order corrections. Therefore, any attempt to tune parameters of the theory to achieve $\nu = 3$ will require at least six-loop calculations! Such calculations are clearly impossible with presently existing techniques, so we need not elaborate on the subtleties involved in the high-order renormalization group analysis. Fine tuning of *this* kind is no better than that occurring in non-supersymmetric theories. In any case this situation is much worse than the "set it and forget it" kind of fine tuning which is common in supersymmetric models.

In order to show that the necessary fine tuning is at least possible we have made a crude calculation of $U$ using our general formalism. The calculation is done at the scale $y$ and does not take into account renormalization effects. In terms of bare parameters we find

$$\begin{aligned} U_{QQ} &= (4\pi)^{-4} \cdot 2\, g'^2 \cdot (\, 6\, e^2 \; - \; 23\, g^2 \,) \quad , \\ U_{BQ} &= (4\pi)^{-4} \cdot g g' \sqrt{\tfrac{6}{5}} \cdot (\, -e^2 \; - \; \tfrac{1}{3}\, g^2 \,) \quad . \end{aligned} \tag{4.15}$$

In particular, for $g^2/e^2 = 3/14$ we find $U_{QQ} > 0$ and $\nu = 3$. It is interesting to note that if we had desired $SU(3)$ breaking (instead of $SU(2)$ ) we would have been disappointed.

## 5. Conclusions.

The upside down hierarchy model is severely and, probably, terminally ill. Its diseases can be classified into three major groups.

1. Renormalization syndrome.

   The gauge couplings of the model are insufficiently asymptotically free because of the existence of the light (mass $\sim$ 10 TeV) chiral superfields in the adjoint representation of $SU(3) \otimes SU(2) \otimes U(1)$. This fundamental disorder leads to the following symptoms:

   (a) with three [or more] generations of matter QCD loses asymptotic freedom and becomes strong before Grand Unification can be achieved;

   (b) *realistic values of the low energy parameters* $\alpha_{em}$, $\Lambda_{\rm QCD}$ *and* $\sin^2\theta_W$ *cannot be achieved for a GUT scale* $\leq M_{\rm Planck}$;

   (c) with more than one generation of matter scalar leptons are tachyonic.

   The problem appears to be rather general. For any $SU(5)$ model, regardless of any particular choice of a set of O'Raifeartaigh fields ($X$, $X'$, *etc.*), the intermediate energy theory would always contain an *incomplete* [$SU(5)$] multiplet of the type ( <complete $SU(5)$ multiplet> $-(\mathbf{3},\mathbf{2},+5/6)-(\bar{\mathbf{3}},\mathbf{2},-5/6)$ ) whose effect on the GUT scale is identical to the one of $(X_8 + X_3)$ in our model. Since all possible Grand Unified groups contain $SU(5)$ as a subgroup,* the same argument applies to any inverse hierarchy model whose gauge breaking from the GUT group down to $SU(3) \otimes SU(2) \otimes U(1)$ occurs at a single scale $y$.

   The general inverse hierarchy framework allows, however, for two-stage GUT breaking. At the highest threshold $y$ the Grand Unified group $G$ is broken to some intermediate group $G_1$, and the subsequent breaking of $G_1$ to $SU(3) \otimes SU(2) \otimes U(1)$ occurs at an intermediate threshold $M$.

* There are semisimple exceptions to this rule, but none of them can be incorporated into the inverse hierarchy scenario.

Fields participating in the second breaking need not be light. Therefore, fields with masses $\sim$ 10 TeV form a complete multiplet of $G_1$ of type (<complete multiplet of $G$ > $-$ < $G/G_1$ >), whose effect on the convergence rate of the gauge couplings is equivalent to the adjoint representation of $G_1$. Unfortunately, we have not been able to use this loophole to build a healthy inverse hierarchy model. We have considered several $SU(6)$, $SU(7)$ and $SO(10)$ models, but none of them fared any better than the original $SU(5)$ model.

2. Supergrave complications

   (a) An inverse hierarchy model inevitably involves a supersymmetry breaking scale which is on the borderline of Weinberg's cosmological bounds (with the most optimistic assumptions about **R**-symmetry breaking).

   (b) Supergravitational effects on the inverse hierarchy mechanism are *not* negligible. The detailed consequences of these effects have not really been studied. (However see ref.[13].)

   (c) Supergravitational effects on the low-energy effective Lagrangian lead to scalar mass terms (and also 3-scalar interactions) whose magnitude is controlled by the gravitino mass. For $y > \alpha M_{\text{Planck}}$ these effects are stronger than the radiative ones and our computations of the scalar masses lose their validity.

On the other hand, the superfield effective Lagrangian for the light fields absorbs the effects of supergravity in exactly the same way it absorbs the radiative corrections. Hence equations (2.8) and (2.12) can be used to analyze their combined effect provided we identify the $Y$ field with an appropriate combination of the O'Raifeartaigh fields and the chiral compensator of supergravity. Moreover, the renormalization group equations $(2.13)^B$, (2.15) and (2.16) also remain valid, although their *boundary conditions* (2.17) and (2.18) has to be corrected. This formalism may even be applied to computing renormalization effects in supergravity models without an inverse hierarchy.

3. The real hierarchy problem.

   Despite the fact that inverse hierarchy models naturally generate a wide range of scales ($10^{18}$, $10^{11}$, $10^{4}$ and 100 GeV), they do not solve the true naturalness problem of the GWS theory. $SU(2)$ breaking at 100 GeV can be only achieved by unnaturally fine tuning. We do not see how to solve this problem without a disastrous destruction of asymptotic freedom (as in ref.[12]).

As far as we are concerned, these problems effectively kill the upside down hierarchy model. Of the various ideas generated by the model, the one that seems most likely to survive is the notion that supersymmetry can be an important constraint on the low energy particle physics even if it is spontaneously broken at a scale above $10^{10}$ GeV. Theories based on this idea resemble the real world much more than those based on low energy spontaneous supersymmetry breaking. We believe that the calculational techniques that we have developed in this paper will be useful in this wider class of models.

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# APPENDIX A

This appendix is to display the $x$-space effective Lagrangian for the light fields. At this stage no renormalization is performed and auxiliary fields are left as such. We have worked in the Wess–Zumino gauge.

$$
\begin{aligned}
\mathcal{L} =\ & F^\dagger Z_S F \ + \ i\overline{\Psi}\, Z_S \, \not{D}\Psi \ - \ (D_m A)^\dagger \, Z_S \, (D^m A) \\
& + \ \sqrt{2}\ A^\dagger Z_S \, (\chi \cdot T)\Psi \ + \ \sqrt{2}\ \overline{\Psi}(\overline{\chi} \cdot T)\, Z_S A \ + \ A^\dagger Z_S \mathcal{D} A \\
& + \ \frac{Z_V}{8e^2} \cdot \left( \mathcal{D}^2 \ - \ 4i\, \chi \, \not{D} \overline{\chi} \ - \ F_{mn} F^{mn} \ + \ i F_{mn} \, \tilde{F}^{mn} \right) \qquad + \ \text{H.c.} \\
& + \left[ \ \Psi_i \Psi_j \, \frac{\partial^2 W(A)}{\partial A_i \, \partial A_j} \ + \ \chi\chi \, \frac{\partial Z_V}{\partial Y} \, \frac{\partial F_y}{\partial 4e2} \quad + \ \text{H.c.} \right] \\
& + \ F^\dagger \cdot \left[ \frac{\partial W(A^\dagger)}{\partial A^\dagger} \ + \ \left( \frac{\partial Z_S}{\partial Y} \, F_y \right) A \right] \qquad + \ \text{H.c.} \\
& + \ A^\dagger \left( \frac{\partial^2 Z_S}{\partial Y \, \partial Y} \cdot F_y^\dagger F_y \right) A
\end{aligned}
$$

We have omitted all unnecessary indexes.

## APPENDIX B

This appendix is to display the renormalization group equation for $Z_H$, $Z_Q$ and their derivatives with respect to $\tau$ (See section 3 for details).

$$8\pi^2 \frac{\partial}{\partial t} Z_H = \frac{3e^2}{4} \frac{Z_H}{Z_{e_2}} + \frac{3e^2}{20} \frac{Z_H}{Z_{e_1}} - g_2^2 \frac{1}{Z_H Z_Q}$$

$$8\pi^2 \frac{\partial}{\partial t} Z_Q = -\frac{\lambda_2^2}{2} \frac{1}{Z_Q^2} - 2g_2^2 \frac{1}{Z_H^2}$$

$$8\pi^2 \frac{\partial}{\partial t} \frac{\partial Z_H}{\partial \tau} = \left(\frac{\partial Z_H}{\partial \tau}\right) \cdot \left(\frac{3e^2}{4} Z_{e_2}^{-1} + \frac{3e^2}{20} Z_{e_1}^{-1} + g_2^2 Z_H^{-2} Z_Q^{-1}\right)$$

$$+ \left(\frac{\partial Z_Q}{\partial \tau}\right) \cdot g_2^2 Z_H^{-1} Z_Q^{-2} - Z_H \cdot \frac{e^4}{8\pi^2} \left(\frac{9}{4} Z_{e_2}^{-2} + \frac{129}{100} Z_{e_1}^{-2}\right)$$

$$8\pi^2 \frac{\partial}{\partial t} \frac{\partial Z_Q}{\partial \tau} = \left(\frac{\partial Z_Q}{\partial \tau}\right) \cdot \lambda_2^2 Z_Q^{-3} + \left(\frac{\partial Z_H}{\partial \tau}\right) \cdot 4g_2^2 Z_H^{-3}$$

$$8\pi^2 \frac{\partial}{\partial t} \frac{\partial^2 Z_H}{\partial \tau^2} = \left(\frac{\partial^2 Z_H}{\partial \tau^2}\right) \cdot \left(\frac{3e^2}{4} Z_{e_2}^{-1} + \frac{3e^2}{20} Z_{e_1}^{-1} + g_2^2 Z_H^{-2} Z_Q^{-1}\right)$$

$$+ \left(\frac{\partial^2 Z_Q}{\partial \tau^2}\right) \cdot g_2^2 Z_H^{-1} Z_Q^{-2} - 2\left(\frac{\partial Z_H}{\partial \tau}\right)^2 \cdot g_2^2 Z_H^{-3} Z_Q^{-1}$$

$$- 2\left(\frac{\partial Z_Q}{\partial \tau}\right)^2 \cdot g_2^2 Z_H^{-1} Z_Q^{-3} - 2\left(\frac{\partial Z_H}{\partial \tau}\right)\left(\frac{\partial Z_Q}{\partial \tau}\right) \cdot g_2^2 Z_H^{-2} Z_Q^{-2}$$

$$- \left(\frac{\partial Z_H}{\partial \tau}\right) \cdot \frac{e^4}{8\pi^2} \left(\frac{9}{2} Z_{e_2}^{-2} + \frac{129}{50} Z_{e_1}^{-2}\right)$$

$$- Z_H \cdot \frac{e^6}{64\pi^4} \left(\frac{27}{8} Z_{e_2}^{-3} + \frac{3698}{250} Z_{e_1}^{-3}\right)$$

$$8\pi^2 \frac{\partial}{\partial t} \frac{\partial^2 Z_Q}{\partial \tau^2} = \left(\frac{\partial^2 Z_Q}{\partial \tau^2}\right) \cdot \lambda_2^2 Z_Q^{-3} + \left(\frac{\partial^2 Z_H}{\partial \tau^2}\right) \cdot 4g_2^2 Z_H^{-3}$$

$$- \left(\frac{\partial Z_Q}{\partial \tau}\right)^2 \cdot 3\lambda_2^2 Z_Q^{-4} - \left(\frac{\partial Z_H}{\partial \tau}\right)^2 \cdot 12 g_2^2 Z_H^{-4}$$