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HEAT FLOW IN A HE II FILLED FIN*

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1. INTRODUCTION

The excellent heat removal capability of pressurized superfluid helium, He-IIp[1–9], has led in recent years to its use as a coolant in fusion[10] and accelerator[11] magnets where the operating temperature is typically in the range of 1.8 to 2. K. More recently, He-IIp has been proposed as the coolant for a polarized proton target at temperatures as low as $0.5K$[12]. In all cases the He-IIp acts as an intermediate fluid between the heat source (magnet winding or target bead) and the heat sink (saturated He-II in the case of the magnets or saturated helium-3 in the case of the polarized proton target). The heat sink fluid may be vaporized in a jacket external to the magnet vessel[13], in a heat exchanger which is immersed in the He-IIp[14], or in a separate bath in which is immersed an He-IIp filled fin which extends from the heated reservoir. In the latter configuration a single heat sink can serve multiple heat sources[15]. It is this latter arrangement which is considered here and which is pictured schematically in Fig. 1. We proceed to analyze the flow of heat in the cooled channel. This is in contrast to the considerable work which has been done for the insulated channel.[2,16–18]

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2. INTERNAL CONVECTION MECHANISM

The large apparent thermal conductivity of He-II (or He-IIp) is explained by the two fluid model as an internal convection in which there is a counter flow of the normal and superfluids with no net mass flow[19]. At very small temperature gradients the flow is laminar and the heat flow and temperature gradient are linearly related[20]. With significant temperature gradients, the flow becomes turbulent and is dominated by a mutual friction force which has been shown by Gorter and Mellink[21] to be proportional to the third power of the relative velocity between the two fluids and remarkably independent of channel size. When this term is incorporated into the Navier-Stokes equation and combined with an energy equation which states that the heat is transported in the fluid by the motion of the entropy carrier and with a continuity equation which enforces the zero net mass flow condition, one finds that the temperature gradient and heat flow are related as follows[19]

$$\nabla T = \frac{A\rho_n}{S}\left(\frac{q}{\rho_s ST}\right)^3 \tag{1}$$

Recently Kamioka, Lee and Frederking[22] have shown that the temperature dependence of the mutual friction parameter A is accounted for by

$$A = K_{GM}^{-3}\frac{\left(\frac{\rho}{\rho_s}\right)}{\eta_n} \tag{2}$$

and that the experimental data are well represented over a wide pressure range by $K_{GM} = 11.3$. The temperature gradient thus becomes

$$\nabla T = \left(\frac{1}{11.3}\right)^3 \frac{\rho_n}{S\eta_n}\left(\frac{\rho}{\rho_s}\right)\left(\frac{q}{\rho_s ST}\right)^3 \tag{3}$$

or conversely the heat flow is

$$q = 11.3\rho_s ST\left[\frac{\eta_n S\nabla T}{\rho_n}\frac{\rho_s}{\rho}\right]^{1/3} \tag{4}$$

3. HEAT TRANSFER MODEL

The arrangement of the source and sink are shown in Figure 1. A low temperature liquid volume at T_H is pressurized above the vapor pressure by a source of saturated liquid at one atmosphere pressure and cooled through heat exchange with a saturated liquid at T_B. Heat inputs to the liquid at T_B include internal heat generation Q_1, heat transfer via the pressurizing fluid Q_2, and heat transfer from the environment Q_3. All of this heat is transported to the saturated bath at T_B via a channel of circular cross section. Heat loss Q_R, from the channel to the bath at T_B, is in the radial direction and is limited only by the Kapitza resistances at the two surfaces. The thermal resistance of the channel wall is assumed to be negligible. The channel, therefore, behaves like a fin with the exception that the temperature gradient along its length is related to the third rather than the first power of the axial heat flow. A detailed heat transfer model is shown in Figure 2. A heat balance on the small element leads to

$$Q_x = Q_{x+\Delta x} + Q_R \tag{5}$$

All three of the terms in equation (5) are functions of the local temperature and in the case of the axial heat flows, its spatial derivative as well. The axial heat flows, Q_x and $Q_{x+\Delta x}$ are expressed by the product of equation (4) and the cross sectional area of the channel. The strong temperature dependence of the fluid properties is thus reflected in the heat flow.

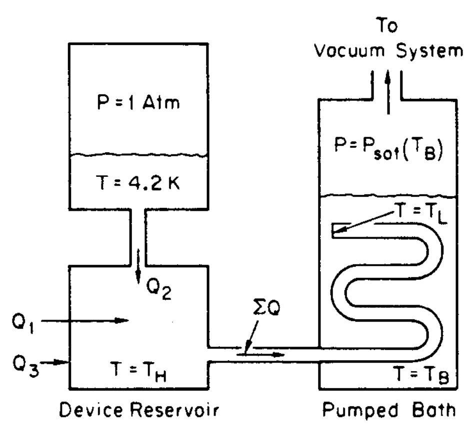

Fig. 1 Source Sink Arrangement

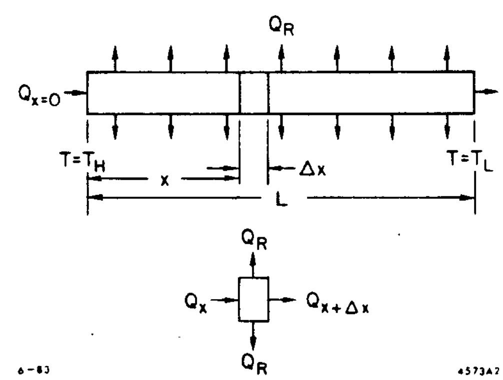

Fig. 2 Heat Transfer Model

The radial heat flow, Q_R, is the product of the overall conductance, the circumferential area, $2\pi r_0 dx$, and the local temperature difference, $T - T_B$. The overall conductance is

$$\frac{1}{\frac{r_0}{r_i}\frac{1}{h_{ki}(T)} + \frac{1}{h_{ko}(T_B)}} \tag{6}$$

where the temperature dependent Kapitza conductances, $h_{ki}(T)$ and $h_{ko}(T_B)$, are computed from

$$h_k = \left(\frac{T}{1.9}\right)^3 h_k(1.9) \tag{7}$$

3.1 Boundary Conditions

The temperature boundaries are at $x = 0, T = T_H$ and at $x = L, T = T_L$. An additional assumption must be made regarding the heat flow at x = L. The end may be insulated or as in the present case there may be a heat loss limited by Kapitza resistance.

3.2 Solution Procedure

The solution begins at x = L and proceeds in the negative x direction. At each element the temperature and its spatial derivative at $x + \Delta x$ are known from the values of the previous element at x. The temperature at x is

$$T_x = T_{x+\Delta x} - \left(\frac{dT}{dx}\right)_{x+\Delta x} \cdot \Delta x \tag{8}$$

The fluid properties and Kapitza conductances are evaluated at the average temperature of each element. The heat flow at x is computed from eq (5) and the spatial derivative of temperature at x from eq (3). The step size is varied to keep the temperature change at each step in a prescribed range, typically within 0.002 to 0.01 K. The calculation is terminated when the local temperature reaches T_H.

4. RESULTS

The source temperature T_H, was varied from near T_λ to 1.2 K which approaches the lower limit of eq (1). In all cases the sink temperature T_B, was chosen to be 0.1 K less than the source temperature T_H. This is representative of what one would choose for an optimized system. The end temperature T_L, was varied from near the source to near the sink temperature. Channel inside diameters were varied from 0.1 to 10. cm. For diameters less than one cm, the outside diameter was taken to be 0.1 cm greater than the inside diameter. For diameters one cm and larger, the outside diameter was assumed to be 0.2 cm greater than the inside. Results are shown for surfaces having Kapitza conductances at 1.9 K of 0.1, 0.2 and 0.8 W/cm^2K. In all cases the same values at 1.9 K are assumed for the inner and outer surfaces.

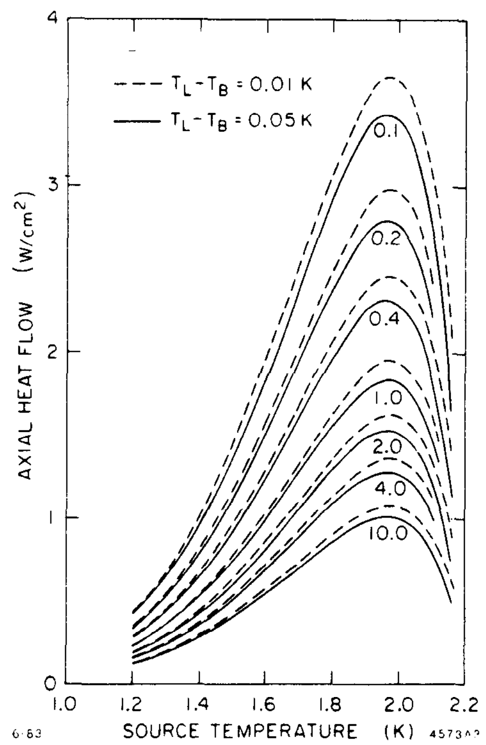

Fig. 3 Axial Heat Flow at Source End Kapitza Conductance at 1.9 K of 0.2 W/cm^2K

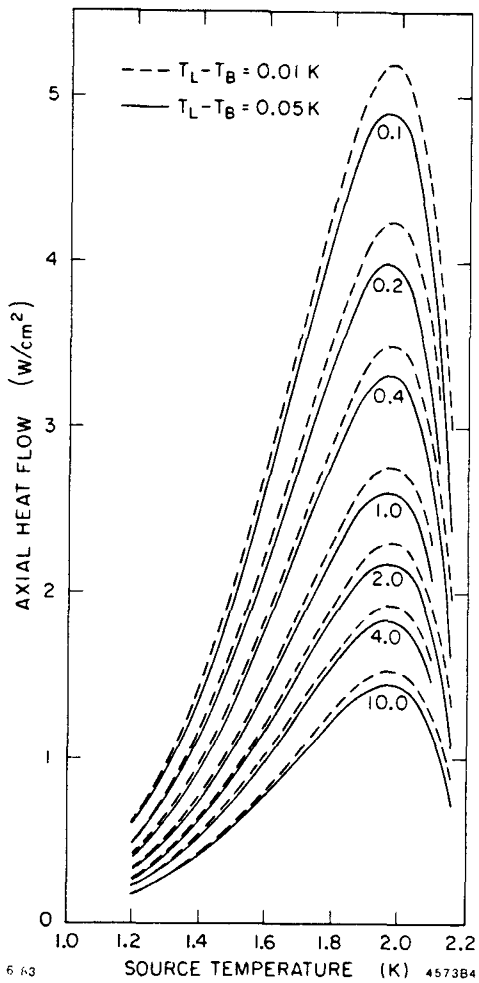

Fig. 4 Axial Heat Flow at Source End Kapitza Conductance at 1.9 K of 0.8 W/cm^2K (Numbers on curves refer to inside diameter in cm)

The heat flow density at x = 0 is plotted in Figures 3 and 4 as a function of source temperature for diameters from .1 to 10. cm and for two values of the Kapitza conductance. The strong dependence on source temperature is apparent particularly at the small diameters. For a 0.1 cm diameter the heat flow density changes by nearly an order of magnitude as the temperature changes from 1.2 to 2 K. At a 10 cm diameter the change is much less pronounced.

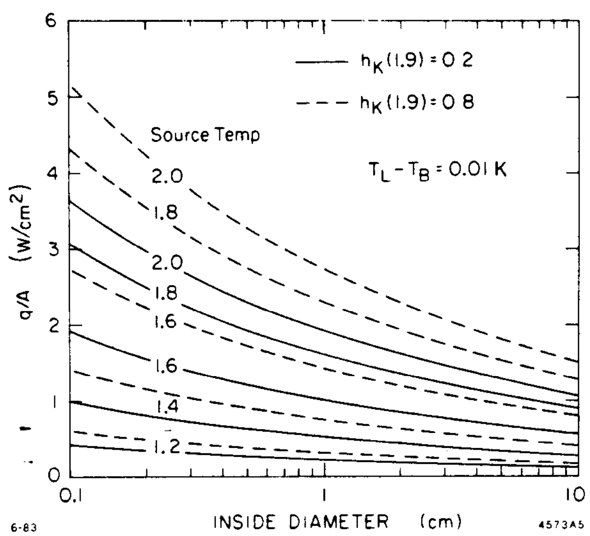

Fig. 5 Effect of Diameter on Axial Heat Flow at Source

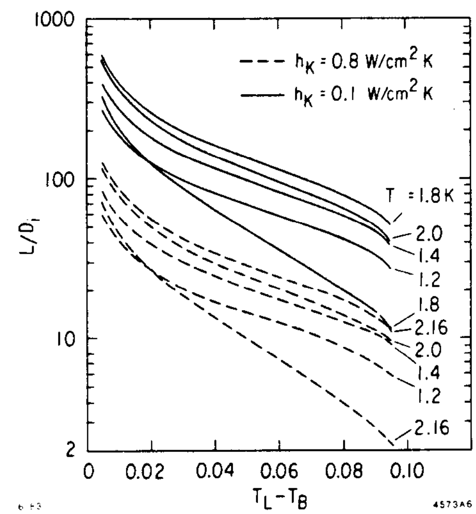

Fig. 6 Aspect Ratios For One Cm Diameter Channels

The much weaker dependence of the source heat flow on diameter is seen in Fig. 5 where $q_{x=0}$ is plotted versus inside diameter.

The length to diameter ratio needed to achieve a given approach of the temperature at x = L to the sink temperature is plotted in Fig. 6 versus $T_L - T_B$ for a few source temperatures and an inside diameter of 1.0 cm. For example at 1.8 K with $h_K(1.9) = 0.1/cm^2K$, 140 diameters are required to achieve an approach of 0.05 K to the sink temperature whereas 600 diameters are required for an approach of 0.005 K. The corresponding lengths for $h_K(1.9) = 0.8W/cm^2K$ are 29 and 130 diameters. Inspection of Figures 3 and 4, however, indicates that little is to be gained in terms of heat flow at the source from a close approach to the sink temperature.

5. CONCLUSION

The influence of diameter, length, Kapitza conductance and temperature on the heat carrying capacity of an externally cooled, circular He II filled channel with zero net mass flow and of negligible wall thermal resistance is shown. With these results one may choose appropriately sized channels for the extraction of heat from an He IIp cooled device operating in the temperature range of 1.2 K to T_λ.

6. NOMENCLATURE

A	Gorter-Mellink Constant
h_K	Kapitza Conductance
K_{GM}	Dimensionless constant as defined by eq (2)
L	Channel length
q	Heat flux per unit area
Q	Heat flux
r	Channel radius
S	Entropy
T	Temperature
x	Length coordinate
η	Viscosity
ρ	Density

Subscripts

B	Saturated bath
H	Source
i	Inside
L	Low temperature end of channel
n	Normal fluid
o	Outside
R	Radial direction
S	Superfluid
λ	Superfluid/normal fluid boundary

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