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BARYON WAVE FUNCTIONS AND NUCLEON DECAY*

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ABSTRACT

We apply to nucleon decay the knowledge about the short-distance structure of baryon wave functions gleaned from QCD form factor calculations and the $J/\psi \to \bar{p}\,p$ decay rate. We review the uncertainties arising when current algebra and PCAC are used to relate $N \to \bar{\ell} +$ meson decay rates to $\langle 0|qqq|N\rangle$ matrix elements. We show that the relevant matrix elements are not directly related to those of the leading twist operators "measured" in conventional high momentum transfer physics, but argue for an indirect relation based on models that fit both form factor and J/ψ decay data. We use these inputs to calculate the $p \to e^+\pi^0$ decay rate in minimal SU(5) and other grand unified theories (GUTs) for a specified value of the heavy vector boson mass m_X. Our results combined with the recent experimental lower limit on this mode indicate that $m_X > 2 \times 10^{15}$ GeV in the minimal SU(5) GUT, and we derive analogous bounds for supersymmetric GUTs. Our calculated lifetime for a given value of m_X is considerably shorter than previous estimates made using non-relativistic SU(6) or the bag model, a difference traceable to the different normalizations of 2 and 3 quark wave functions at short distances.

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1. Introduction

A crucial prediction of many GUTs [1,2] is that nucleons should decay: the next important questions are how often, and into what decay modes? Conventional calculations [3] of nucleon decay rates can be divided into 3 parts. The first is the specification of the baryon decay operator due to heavy particle exchange at short distances, which is very dependent on the GUT model invoked. The second part is the renormalization of this operator at short distances up to O(1) fermi, whose calculation is well-understood [4]. Finally there is the calculation of the hadronic matrix elements of the baryon decay operator renormalized at the long hadronic distance scale of O(1) fermi. This calculation is quite difficult, since it involves hadron dynamics in the non-perturbative strong coupling regime. Many different authors [3] have used many different approximations to estimate these hadronic matrix elements, with results that are not always identical. In this paper we propose a new way of calculating nucleon decay matrix elements which uses the understanding [5] of baryon wave functions at short and light-like separations obtained from QCD calculations of baryon form factors and the $J/\psi \rightarrow \bar{p}p$ decay rate [6].

Our starting point is the observation that these quantities are closely related: high-momentum transfer processes gives us knowledge of the baryon wave function at light-like separations $x^2 \rightarrow 0$, thereby giving us information about the nucleon decay amplitudes which are sensitive to physics as $x_\mu \rightarrow 0$. One might hope to short-circuit long-distance uncertainties by inter-relating directly the perturbative QCD calculations of high momentum transfer processes involving baryons to the nucleon decay amplitudes. Unfortunately, this does not turn out to be possible, since the two classes of process involve slightly different aspects of the baryon wave function. This could have been anticipated from the knowledge that baryon decay operators are of higher twist, whereas form factors, etc., are related to lowest-twist operators [7].

However, it is possible to make an indirect connection between the two short distance phenomena by using trial wave functions that fit [6] the form factor and $J/\psi \to \bar{p}p$ decay data. The results of exploiting this connection are somewhat surprising.

Section 2 of this paper states and discusses the assumptions of current algebra and chiral symmetry [8,9] which justify the relation of nucleon $N \to$ antilepton $\bar{\ell}+$ pseudoscalar meson P decay amplitudes to qqq annihilation amplitudes $\langle 0|qqq|N\rangle$. We thereby sidestep the uncertainties in $qq \to \bar{q}\,\bar{\ell}$ annihilation amplitudes which control $N \to \bar{\ell}+P$ decays in non-relativistic SU(6) and bag model calculations [3]. The chiral Lagrangian formalism is then used to tabulate $N \to \bar{\ell}+P$ decay rates as functions of the $\langle 0|qqq|N\rangle$ in a number of interesting GUTs such as minimal conventional SU(5) [2] and minimal supersymmetric SU(5) [10,11], as well as in a more general supersymmetric context [12]. In sect. 3 we discuss the structure of the $\langle 0|qqq|N\rangle$ operator matrix elements, and show that the leading contributions are proportional to the annihilating quark masses, while there are non-leading contributions related to antisymmetric parts of the nucleon wave function. Symmetry arguments and explicit diagrammatic calculations suggest that the quark mass factors should be interpreted as constituent quark masses rather than as short distance current quark masses. This same diagrammatic analysis points up the differences between the short (1 fermi $>$ d $>$ $1/m_X$) distance renormalization of the lowest twist qqq operators appearing in baryon form factors and the renormalization of the higher twist operators relevant to nucleon decay [7]. The different structures of these operators preclude a direct connection between baryon form factors and nucleon decay rates. However, an indirect connection can be made by using models [6] for hadron wave functions which fit high-momentum transfer data and enable one to calculate the matrix elements of higher twist operators, as is done in sect. 4. For any specified strength of the nucleon decay interaction, e.g., the value of m_X in the minimal conventional SU(5) GUT, we find a much larger nucleon decay rate than has previously been calculated [3] on the basis of non-relativistic SU(6) or the bag model. We find that the recent [13] experimental limit $\tau(p \to e^+\pi^0) > 10^{32}$ years implies that $m_X > 2 \times 10^{15}$ GeV in the context of the minimal SU(5) model [2]. This lower limit is profoundly embarrassing for this theory, in which $m_X \approx (1 \ to \ 2) \times 10^{15} \times \Lambda_{\overline{MS}}$, so that a value of $\Lambda_{\overline{MS}} > 1$ GeV would be required for consistency with the negative result [13] of the IMB experiment. In sect. 5 we discuss possible reasons for the discrepancy between our calculations and the previous non-relativistic SU(6) and bag model results. Is one or the other computation an incorrect deduction from the underlying formalism? Perhaps the nucleon decay rate cannot be related reliably to "known" aspects of the non-relativistic SU(6) wave function? Possibly the wave-function for three quarks annihilating at a point which we need, "know" from high momentum studies, and use here is not simply related to the two-quark overlap function "known" from non-relativistic SU(6) studies? Maybe the non-perturbative nucleon wave-function has an unexpectedly large high-momentum tail? Alternatively and more radically, perhaps the entire short-distance wave function programme is inapplicable to baryon processes at present-day momentum transfers [14]? This would require over 99% of the baryon form factor measured at $Q^2 = 0(10)\ GeV^2$ to be due to non-leading effects, and is an alternative we deem unpalatable. A less radical alternative is that we have overestimated the magnitude of the relevant quark masses, but the analyses of sections 2 and 3 strongly suggest that we should use constituent quark masses rather than current quark masses. The apparent discrepancy between the phenomenologies of non-relativistic SU(6) and of light-cone hadronic wave functions has an interest beyond the nucleon decay calculations which constitute the central thrust of this paper.

2. Current Algebra, PCAC and Nucleon Decay

Nucleon decay calculations [3] involve GUT dynamics modified by short distance ($d < 1$ fermi) corrections calculated within the Standard Model, and hadronic matrix elements whose values depend on the incompletely understood long-distance dynamics of QCD. Models for these matrix elements that have been considered in the literature include quasi-free $q \to \bar{q}\,\bar{q}\,\bar{\ell}$ decay [fig. 1(a)], two-particle $qq \to \bar{q}\,\bar{\ell}$ annihilation [fig. 1(b)] and three particle $qqq \to \bar{\ell}$ annihilation preceded by meson emission [fig. 1(c)]. These models are constrained by current algebra and PCAC which are model-independent but only apply directly to kinematic limits which may be too idealized to be useful. If one ignores this objection, current algebra and PCAC relate [8,9] all nucleon $\to$ antilepton + pseudoscalar meson ($N \to \bar{\ell} + P$) decay amplitudes to two basic three-quark annihilation matrix elements, illustrated in fig. 1(d):

$$\left\langle 0 \left| \left(\epsilon_{\alpha\beta}\, d^{\alpha}_{iR}\, u^{\beta}_{jR} \right) u^{\gamma}_{kL}\, \epsilon_{ijk} \right| p \right\rangle \equiv \alpha u^{\gamma}_{L}(\underline{p}, \underline{\sigma}) \tag{2.1a}$$

$$\left\langle 0 \left| \left(\epsilon_{\alpha\beta}\, d^{\alpha}_{iL}\, u^{\beta}_{jL} \right) u^{\gamma}_{kL}\, \epsilon_{ijk} \right| p \right\rangle \equiv \beta u^{\gamma}_{L}(\underline{p}, \underline{\sigma}) \tag{2.1b}$$

where α, β, γ are two component spinor indices and i, j, k are color indices. The existence of these current algebra and PCAC relations may appear surprising at first sight, since in the three-quark annihilation matrix elements (2.1) all three quarks in the nucleon must be at the same point, whereas they may be at different points in the models of figs. 1(a) and 1(b). Furthermore, the matrix elements (2.1) pick out the pure $3q$ Fock state component of the nucleon in the light-cone gauge, whereas components with extra gluons and $\bar{q}\,q$ pairs can contribute to figs. 1(a) and 1(b). In this section we discuss why the current algebra and PCAC relations may be reliable despite this apparent paradox, and tabulate nucleon decay rates as functions of α and β in some favoured GUT models.

The strategy for applying current algebra and PCAC to nucleon decay parallels closely the traditional calculations of nonleptonic hyperon decays $Y \to N + \pi$. The essential differences due to the replacement of the final state nucleon N by an antilepton $\bar{\ell}$ are twofold: (a) the extrapolation from the soft pseudoscalar limit to the physical region is much longer, and (b) the Lorentz structure of the decay amplitude is determined by the GUT model used and the point-like nature of the antilepton. In the case of nonleptonic hyperon decays the decay amplitude takes the general form

$$\bar{N}\,(a + b\gamma_5)\,Y \tag{2.2a}$$

with the terms proportional to a and b corresponding to S and P-wave decay amplitudes respectively. Current algebra and PCAC provide no useful information about P-wave amplitudes, but interrelate different S-wave amplitudes in the soft pseudoscalar limit $E_\pi / m_N \to 0$. This is likely to be a good approximation to the real world since $m_Y - m_N \ll m_N$. On the other hand, in the case of nucleon decay one has a decay amplitude

$$\bar{\ell}(A + B\gamma_5)N \tag{2.2b}$$

where the ratio of amplitudes A/B is fixed by the short distance GUT dynamics and is unaffected by long distance effects since the antilepton is assumed to be structureless. Therefore the calculation in the soft pseudoscalar limit can be carried over from the S-wave amplitude to the P-wave amplitude. Unfortunately this limit is *a priori* less relevant since $m_N - m_{\bar{\ell}} \approx m_N$ so that $E_\pi / m_N \approx 1/2$ for baryon decay. Therefore one may question whether current algebra and PCAC can give any more than qualitative impressions of the magnitudes of baryon decay amplitudes.

We can provide three arguments that the current algebra and PCAC calculation may in fact be at least semi-quantitative. The soft pseudoscalar limit would be *a priori* unreliable if at the end of the extrapolation we found a significant suppression factor in the matrix elements α and β (2.1) for some good symmetry or dynamical reason. However, the chiral $SU(3)_L \times SU(3)_R$ symmetry we are exploiting does not forbid couplings to baryons of the three-quark operators relevant to nucleon decay. These qqq operators will have definite $SU(3)_L \times SU(3)_R$ transformation properties but the nucleon only has definite transformation properties under $SU(3)_{L+R}$. If one uses the chiral Lagrangian formalism as in Claudson *et al.* [9] one can always write down an $SU(3)_L \times SU(3)_R$ invariant term which starts with a $\langle 0|qqq|N\rangle$ matrix element and continues with higher order pseudoscalar couplings, just as long as the qqq operator is an octet of conventional vectorial $SU(3)_{L+R}$. This argument for the absence of a group-theoretical suppression factor is buttressed by the perturbative calculations of sect. 3. There we find no suggestion that $\langle 0|qqq|N\rangle$ matrix elements vanish in the limit of vanishing current algebra quark mass. Instead they seem likely to be proportional to constituent quark masses which do not vanish in the chiral limit. Finally we note that in addition to these brave words there is a dynamical argument which supports the reliability of the current algebra estimate. Virtual contributions to decay amplitudes from radial recurrences of the nucleon are likely to be suppressed by factors in their wave-functions and by energy denominators. This argument is borne out by a specific non-relativistic SU(6) dynamical model [15].* Estimates of pole diagrams suggest that the two amplitudes of fig. 1(b) and fig. 1(c) should be in the ratio of 1:0.8, whereas the chiral limit yields a ratio of $1{:}g_A \approx 1{:}1.2$. Since we will only be interested in bounding

$$m_X \propto \left[\frac{\tau(N \to \bar{\ell} + P)}{(1 + \text{``}g_A\text{''})^2}\right]^{1/4} \tag{2.3}$$

this magnitude of this extrapolation error does not faze us.

* Reservations about other aspects of this model calculation are expressed in subsequent sections.

The current algebra and PCAC calculations require isolating the rapidly varying pole terms of fig. 1(c) and computing the remaining contact terms using soft pseudoscalar theorems. The most convenient formalism for deriving all these results systematically is the chiral Lagrangian framework developed in ref. [9]. We have used the general results of Claudson *et al.* [9] to obtain nucleon decay rates $N \to \bar{\ell} + P$ in terms of α and β (2.1) for the following three presently favoured classes of GUT models.

(A) Minimal GUTs such as SU(5) with the low energy effective Lagrangian [4]

$$\begin{aligned} \mathcal{L} = 2\sqrt{2}\,\tilde{G}_X\Big[&\epsilon_{ijk}\Big(\bar{u}^c_{iL}\,\gamma^\mu\,u_{jL}\Big)\Big(2\,\bar{e}^+_L\,\gamma_\mu\,d_{kL} - \bar{e}^+_R\,\gamma_\mu d_{kR}\Big) \\ &+ \epsilon_{ijk}\Big(\bar{u}^c_{iL}\,\gamma^\mu\,d_{jL}\Big)\Big(\bar{\nu}^c_{eR}\,\gamma_\mu\,d_{kR}\Big) \\ &+ \epsilon_{ijk}\Big(\bar{u}^c_{iL}\,\gamma^\mu\,u_{jL}\Big)\Big(2\,\bar{\mu}^+_L\,\gamma_\mu\,s_{kL} - \bar{\mu}^+_R\,\gamma_\mu s_{kR}\Big) \\ &+ \epsilon_{ijk}\Big(\bar{u}^c_{iL}\,\gamma^\mu\,d_{jL}\Big)\Big(\bar{\nu}^c_{\mu R}\,\gamma_\mu\,s_{kR}\Big) \\ &+ \text{herm. conj.} + \mathcal{O}(\sin\theta_c)\Big] \end{aligned} \tag{2.4a}$$

where i, j, k are colour indices and $\tilde{G}_X = AG_X$ where $G_X = g_X^2/4\sqrt{2}\,m_X^2$ and A is a short-distance enhancement factor.

(B) Minimal supersymmetric SU(5) GUTs with the dominant low energy effective Lagrangian [11,12]:

$$\begin{aligned} \mathcal{L} = 2\sqrt{2}\,\tilde{G}_S\Big[&\epsilon_{ijk}\Big(\epsilon_{\alpha\beta}\,d^\alpha_{iL}\,u^\beta_{jL}\Big)\Big(\epsilon_{\gamma\delta}\,s^\gamma_{kL}\nu^\delta_{\mu L}\Big) \\ &+ \epsilon_{ijk}\Big(\epsilon_{\alpha\beta}\,s^\alpha_{iL}\,u^\beta_{jL}\Big)\Big(\epsilon_{\gamma\delta}\,d^\gamma_{kL}\nu^\delta_{\mu L}\Big) + \text{herm. conj.}\Big] \end{aligned} \tag{2.4b}$$

where α, β, γ, δ are two component spinor indices and $\tilde{G}_S$ is described in sect. 5.

(C) The alternative form of low energy effective Lagrangian permitted by supersymmetry [12]:

$$\mathcal{L} = 2\sqrt{2}\,\tilde{G}_S^{\ell\ell}\Big[\epsilon_{ijk}\Big(\epsilon_{\alpha\beta}\, s_{iL}^{\alpha}\, u_{jL}^{\beta}\Big)\Big(\epsilon_{\gamma\delta} u_{kL}^{\gamma}\, \ell_L^{\delta} - \epsilon_{\gamma\delta}\, d_{kL}^{\gamma}\, \nu_{\ell L}^{\delta}\Big) + 2\epsilon_{ijk}\Big(\epsilon_{\alpha\beta}\, d_{iL}^{\alpha}\, u_{jL}^{\beta}\Big)\Big(\epsilon_{\gamma\delta} s_{kL}^{\gamma}\, \nu_{\ell L}^{\delta}\Big) + \text{herm. conj.}\Big] \tag{2.4c}$$

where $\ell = e, \mu$ and $\tilde{G}_S^{\ell\ell}$ are described in sect. 5. The amplitudes for $N \to \bar{\ell} + P$ in terms of α, β and $\tilde{G}_X$, $\tilde{G}_S$, $\tilde{G}_S^{\ell\ell}$ are given in Table 1. The task of the next two sections of this paper is to compute the coefficients α and β (2.1) using the technology of baryon wave functions at short distances.

3. "Short Distance Enhancement" and "Light-Cone Suppression" Factors

In this section we study the leading gluonic radiative corrections to the Born term contributions to both the proton decay amplitude and to the proton's magnetic form factor. These corrections lead to a "short distance enhancement factor" for the proton decay amplitude [4] and a "light-cone suppression" for the form factor [5]. The Feynman graphs which contribute to these factors are similar, particularly if one considers the pole contribution to the decay amplitude obtained from fig. 1(d), and hence we would like to explain why these factors are different.

We start by considering the magnetic form factor of the proton which has already been studied using light cone perturbation theory [5]. Asymptotically it can be written as

$$F(Q^2) = \int_0^1 [dx]\,[dy]\;\phi_{3q}^{*}(x_i, Q)\,T_H(x_i, y_i, Q)\,\phi_{3q}(y_i, Q) \tag{3.1}$$

where the $x_i \equiv (k_i^0 + k_i^3)/(p^0 + p^3) \equiv k^+/p^+ : 0 < x_i < 1$ are the quark light-cone momentum fractions, the $\underline{k}_{\perp i}$ are their transverse momenta relative to the nucleon momentum, and $[dx] \equiv dx_1 dx_2 dx_3\, \delta(x_1+x_2+x_3-1)$. Apart from the canonical $1/Q^4$ behaviour, the only dependence on the momentum transfer Q in T_H is through the two powers of $\alpha_s(Q^2)$ seen in fig. 2. The quantity $\phi_{3q}(x_i, Q)$ is related to the hadronic wave function $\psi_{3q}(x_i, \underline{k}_{\perp i})$ (strictly speaking, ψ is the Fourier transform of the positive energy projection of the Bethe-Salpeter wave function evaluated with the constituents at equal light-cone "time" $\tau \equiv t + z$) by

$$\begin{aligned}\phi_{3q}(x_i, Q) &= \left(\log\frac{Q^2}{\Lambda^2}\right)^{-3\gamma_F/2\beta} \int^{Q} \prod_{i=1}^{3}\left(\frac{d^2\underline{k}_{\perp i}}{16\pi^3}\right) \cdot 16\pi^3\delta^{(2)}\Big(\sum_i \underline{k}_{\perp i}\Big)\psi_{3q}(x_i, \underline{k}_{\perp i}) \\ &\equiv \left(\log\frac{Q^2}{\Lambda^2}\right)^{-3\gamma_F/2\beta} \int^{Q}\Big(\prod dk\Big)\psi_{3q}(x_i, \underline{k}_{\perp i})\end{aligned} \tag{3.2}$$

where $\beta = 11 - \frac{2}{3}n_f$ (n_f is the number of flavours) and $\frac{1}{2}\gamma_F$ is the anomalous dimension associated with the quark field renormalization. In the leading logarithmic approximation an integral equation can be written for ψ which leads to the solution

$$\phi_{3q}(x_i, Q) = x_1 x_2 x_3 \sum_{n=0}^{\infty} a_n\, \hat{\phi}_n(x_i)\left(\log\frac{Q^2}{\Lambda^2}\right)^{-\gamma_n} \tag{3.3}$$

where the $\hat{\phi}_n(x_i)$ are calculated functions of the (x_i) and the γ_n are calculated [5] positive anomalous dimensions. Asymptotically it is the lowest anomalous dimension $\gamma = 2/3\beta$ for which $\hat{\phi}(x_i) = 1$ which contributes, so that

$$\phi_{3q}(x_i, Q) \underset{Q^2\to\infty}{\longrightarrow} C\, x_1 x_2 x_3\left(\log\frac{Q^2}{\Lambda^2}\right)^{-2/3\beta} . \tag{3.4}$$

Thus we find that at large Q^2 the canonical $1/Q^4$ behaviour of the form factor is modified by factors of $\alpha_s^2(Q^2)$ (in T_H), and $\left(\log\frac{Q^2}{\Lambda^2}\right)^{-2/3\beta}$ from each of the two ϕ's in Eq.(3.1), so that* $F(Q^2) \sim \frac{1}{Q^4}\alpha_s^2(Q^2)\left(\log\frac{Q^2}{\Lambda^2}\right)^{-4/3\beta}$. The region of configuration

* There is an accidental cancellation of the leading term for the proton magnetic form factor which does not apply to other nucleon form factors.

space which is relevant for the leading behaviour is that in which the separations between the three quarks are light-like [17]. Peskin [7] has verified that the exponents γ_n of eq. (3.3) indeed correspond to the eigenvalues of the anomalous dimension matrix of the leading twist three-quark operators. All of these contribute to phenomenology at presently accessible momentum transfers, and are taken into account in the calculations presented in this paper.

In momentum space the leading behaviour of the form factor comes from the region where the transverse momenta are strongly ordered, so that for example in fig. 3(a) $k_{1\perp}^2 \ll k_{2\perp}^2 \ll \cdots \ll Q^2$. The helicity of the quarks is not changed by the radiative corrections, so that a typical factor in the integrand is [see fig. 3(b)]

$$\frac{\bar{u}_\uparrow(\ell_{i+1})\gamma^\mu}{\sqrt{\ell_{i+1}^+}} \frac{u_\uparrow(\ell_i)}{\sqrt{\ell_i^+}} \cdot \frac{\bar{u}_\downarrow(r_{i+1})\gamma_\mu}{\sqrt{r_{i+1}^+}} \frac{u_\downarrow(r_i)}{\sqrt{r_i^+}} \simeq \frac{2\ell_{\perp i+1}^2}{\ell_{i+1}^+ r_{i+1}^+} \tag{3.5}$$

where $\underline{\ell}_{\perp_{i+1}} \simeq -\underline{r}_{\perp_{i+1}}$ and $|\underline{\ell}_{\perp_{i+1}}| \gg |\underline{\ell}_{\perp_i}|$. In addition there is a factor of $1/\ell_{\perp i+1}^4$ from the energy denominators, so that the integrand over each transverse momentum is logarithmically divergent as expected. Note that in order to obtain the leading behaviour the masses may be neglected.

We now study the pole contribution to the proton decay amplitude $p \to e^+\pi^0$ using the same light-cone perturbation theory techniques. From the short distance technique study of this process [4] we expect to find that the canonical $1/m_X^2$ behaviour of this amplitude will be modified by an enhancement factor $O(\log(m_X^2/\Lambda^2))^{\gamma_D}$, where the exponent γ_D corresponds to the anomalous dimension of the 3-quark operators of lowest dimension. We shall try to understand, using light-cone perturbation theory, why different correction factors arise in the two processes we are studying.

A typical contribution to the proton decay amplitude $p \to e^+\pi^0$ is proportional to fig. 1(d), in which $\psi(x_i, \underline{k}_{\perp_i})$ has to be convoluted with the four-fermion baryon number violating vertex. Thus this vertex takes over the role played by T_H in the form factor calculation. We list the values of the four-fermion vertex in momentum space for the different helicities of the quark in Table 2. From this table we observe a number of differences from the form factor calculation. For example we have

$$\begin{aligned}&\frac{\bar{u}^c_\uparrow(p_1)}{\sqrt{p_1^+}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\frac{u_\downarrow(p_2)}{\sqrt{p_2^+}}\cdot\frac{\bar{v}^c_\downarrow(p)}{\sqrt{p^+}}\gamma_\mu\frac{1}{2}(1-\gamma^5)\frac{u_\downarrow(p_3)}{\sqrt{p_3^+}}\\&=\frac{2}{p^+p_1^+p_2^+p_3^+}\left[p^+p_{1\perp}^+-p_1^+p_\perp^+\right]\left[p_3^+p_{2\perp}^--p_2^+p_{3\perp}^-\right]\end{aligned} \tag{3.6}$$

where for any momentum k we define $k^\pm \equiv k^0 \pm k^3$ and $k_\perp^\pm \equiv k_x \pm ik_y$. If we keep the ground state SU(6) wave function for ψ it will be symmetric in momenta p_2 and p_3, which gives zero when convoluted with (3.6). This fact is reflected in the estimation of the decay rate in sect. 4. However, we can imagine convoluting (3.6) with a wave function which is antisymmetric in momenta p_2 and p_3, corresponding perhaps to an excited state of the proton. In that case we see that (from individual "time"-ordered graphs) we will obtain divergences which are quadratic, and not just logarithmic, since we obtain two powers of transverse momentum from the four-fermion vertex. These quadratic divergences must cancel in the sum of all "time"-ordered graphs, and we now present an explicit demonstration of such a cancellation.

As our example we take the Feynman diagram of fig. 4(a). We can derive the light-cone perturbation theory graphs by starting with the Feynman diagram and doing the k^- integration by contours. However, in addition to evaluating the residues of the various poles in k^-, each pole corresponding to a particular "time"-ordered graph, we have to evaluate the contribution from the contour at infinity since the k^- integration is logarithmically divergent. This last contribution has no analogue in light-cone perturbation theory. To avoid this problem we put back the X-boson propagator [see fig. 4(b)], i.e. we no longer treat the four-fermion vertex as local. Now

$$\int d^2k_\perp \to (m_X^2)\int\frac{d^2k_\perp}{k_\perp^2+m_X^2} \tag{3.7}$$

so that what we previously called quadratic divergences will now manifest themselves as real logarithmic ultraviolet divergences. The four "time"-ordered graphs corresponding to the Feynman diagram of fig. 4(b) are drawn in fig. 5. The coefficient of the ultraviolet divergence in each of the four graphs is as follows:

$$\text{diagram 5(a)} \sim -\frac{1}{p^+(p-p_3)^+}\,, \tag{3.8a}$$

$$\text{diagram 5(b)} \sim \frac{1}{p^+(p-p_3)^+}\,, \tag{3.8b}$$

$$\text{diagram 5(c)} \sim \frac{1}{p^+(p_1+p_3)^+}\,, \tag{3.8c}$$

$$\text{diagram 5(d)} \sim -\frac{1}{p^+(p_1+p_3)^+}\,, \tag{3.8d}$$

so that the divergences cancel as required. The leading terms which remain are $\propto \log m_X^2$ as expected. We see no reason to expect these to be the same as the $\log Q^2$ terms in the form factor calculation and they are not – hence the different anomalous dimensions for the two processes.

As we have already noted, the expression (3.6) has no overlap with the ground state SU(6) wave function. Taking the proton to be predominantly in this ground state, we are forced to take one of the other entries in Table 2 for our four-fermion vertex, one which has a component symmetric in the momenta. For example we can take

$$\frac{\bar{u}^c_\downarrow(p_1)}{\sqrt{p_1^+}}\gamma_\mu\frac{1}{2}(1-\gamma^5)\frac{u_\uparrow(p_2)}{\sqrt{p_2^+}}\;\frac{\bar{v}^c_\downarrow(p)}{\sqrt{p^+}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\frac{u_\downarrow(p_3)}{\sqrt{p_3^+}} = -\frac{2m_1m_2}{p_1^+p_2^+}\,. \tag{3.9}$$

The right-hand side of eq. (3.9) has no dependence on transverse momenta and hence one might have expected that integrating over the transverse momentum we would obtain $\phi(x_i, m_X^2)$ again and hence the same anomalous dimension as in the form factor calculation. However, this is not the case because we can no longer neglect the contributions in which the helicities of the quarks change. So for example in the diagrams of fig. 6 we not only have the combination of fig. 6(a) in which the helicities of the quarks do not change, but we also have the contributions from the graphs of figs. 6(b), 6(c), and 6(d) in which they do. In other words, since we cannot avoid picking up two factors of the quark masses we must keep all possible terms in which these factors arise. Indeed, the contribution from the diagrams of fig. 6(a) is cancelled by those of the other graphs of fig. 6 in which we keep only the masses in the k-dependent spinors, the contribution of fig. 6(a) being equal to that of fig. 6(b) and equal to minus each of the contributions figs. 6(c) and 6(d). An examination of the Feynman diagrams which sums all the contributions of fig. 6 shows that this had to happen. Thus the only remaining contribution is that in which we keep both factors of the mass from the *external* spinors in fig. 6(b) and neglect the masses everywhere in the loop. The coefficient of $\log m_X^2$ from this remaining contribution corresponds to the short distance enhancement factor and has no simple relation to the suppression factor in the form factor. This analysis can be iterated to more and more loops, and in each case the only surviving contribution (in the leading logarithm approximation) will be that in which the two factors of quark mass come from the external spinors. Iterating this procedure until the momenta in the external quarks are small, we find that the masses should be interpreted as masses $m_q(Q)$ appearing in the quark Dirac equation $\left(\not{Q} - m_q(Q)\right)q(Q) = 0$ at low momenta Q, and not as short distance quark masses. We therefore interpret these external quark mass factors as constituent quark masses, though this involves an intuitive leap from perturbation theory to include non-perturbative effects which we cannot justify formally. Note that analogous constituent quark mass factors appear implicitly in non-relativistic SU(6) model calculations, while some bag models prefer constituent quark masses considerably smaller than those favoured by these SU(6) models.

We have seen, within the context of light-cone perturbation theory, how it is that the correction factors due to gluonic radiative corrections are different in the proton decay amplitude and in the form factor. Nevertheless these factors are calculable in both cases, and in the next section we shall estimate the proton decay rate using a model wave function $\psi_{3q}(x_i, \underline{k}_{\perp i})$ for small $\underline{k}_{\perp i}$ which has already led to successful predictions for the proton form factor and other related quantities.

4. Light-cone Wave Functions and Nucleon Decay

As we have discussed in sect. 2, the calculation of nucleon decay within the chiral framework depends on the normalizations of the three-quark annihilation matrix elements α and β defined by eqs. (2.1). These matrix elements can conveniently be expressed as integrals over the three-quark valence Fock state wave-function of the baryon defined in the free quark-gluon Hamiltonian basis with quark helicities λ_i. As in sect. 3, we introduce an ultraviolet cutoff Q which is defined such that all intermediate states with $\underline{k}_{\perp}^2 > Q^2$ are excluded from the Fock state wave-function $\psi_{3q}(Q)$. Using the normalization convention of ref. [5], any amplitude involving the nucleon has the form

$$\sum_{\lambda_i} \int [dx] \prod_{i=1}^{3} \left(\frac{d^2 k_{\perp i}}{\sqrt{x_i}\, 16\pi^3} \right) 16\pi^3 \delta^2 \Big(\sum_{i=1}^{3} \underline{k}_{\perp i} \Big) \psi_{3q}(x_i, \underline{k}_{\perp i}, Q, \lambda_i)\, T(x_i, \underline{k}_{\perp i}, Q, \lambda_i) \quad (4.1)$$

where $T(x_i, k_{\perp i}, Q, \lambda_i)$ is the irreducible scattering amplitude with the nucleon replaced by three free quarks. By definition, $T(Q)$ contains no reducible qqq propagators with transverse momentum $\underline{k}_{\perp i}^2 < Q^2$, since such contributions are already included in the wave function $\psi_{3q}(Q)$. In the case of nucleon decay in a GUT the irreducible amplitude $T(Q)$ for $N \to \bar{\ell}$ is computed from the four-spinor matrix elements listed in Table 2. The gluon loop corrections to the tree graph amplitudes with $\Lambda^2 < \underline{k}_{\perp}^2 < m_X^2$ sum to the leading anomalous dimensions of the operators responsible for nucleon decay. To leading logarithmic order the matrix element acquires the enhancement factor

$$A = \left(\frac{\ell n\, m_X^2 / \Lambda_{\overline{MS}}^2}{\ell n\, Q^2 / \Lambda_{\overline{MS}}^2} \right)^{\gamma} \; : \; \gamma = \frac{2}{\beta_0} \quad , \quad \beta_0 = 11 - \frac{2}{3} n_f \; . \tag{4.2}$$

The central unknown in the nucleon decay amplitude in any given GUT is the form and normalization of the nucleon 3-quark wave-function $\psi_{3q}(x_i, \underline{\mathrm{k}}_{\perp i}, Q, \lambda_i)$ at a typical hadronic scale Q_H.

Eventually, $\psi(x_i, \underline{\mathrm{k}}_{\perp i}, Q_H, \lambda_i)$ may be computed from lattice gauge theory or some other method of solving QCD in the non-perturbative regime. However, one can already constrain [6] to some extent the normalization and size parameters of the valence wave function by using existing phenomenological information about high momentum transfer reactions. Especially useful are the $J/\psi \rightarrow \bar{p}p$ decay rate, the nucleon magnetic form factor, and the $x \rightarrow 1$ limit of the deep inelastic nucleon structure function. Recent preliminary data [18] on $\gamma\gamma \rightarrow \bar{p}p$ agree to within a factor of 2 with predictions [19] for $m_{\bar{p}p} = 2.3$ to 2.9 GeV based on a nucleon wave function consistent with the other phenomenological constraints.

The procedure used is to adopt the following parametrization for the spin, flavour and momentum space structure of the nucleon wave function at the hadronic scale Q_H:

$$\psi_{3q}(x_i, \underline{k}_{\perp i}, Q_H, \lambda_i) = B \exp\left[-b^2 \sum_{i=1}^{3} \left(\frac{\underline{k}_{\perp i}^2 + m_i^2}{x_i} \right) \right] \tag{4.3}$$

which falls off exponentially in the off-shell light-cone "energy" $\left(p^- - \sum_{i=1}^{3} k_i^- \right)$. If we ignore the $u-d$ quark mass difference the wave function (4.3) is symmetric in the quark light-cone momentum variables. This symmetry is natural if the nucleon quark wave function is purely S-wave in the centre-of-mass. By assumption, the parametrization (4.3) is also independent of the quark spins. Given a symmetric form in momentum space, the flavour-spin dependence following from SU(6) symmetry is [5]

$$\frac{1}{\sqrt{3}}\left\{\frac{d_\uparrow(1)u_\uparrow(3)+u_\uparrow(1)d_\uparrow(3)}{\sqrt{6}}\ u_\downarrow(2)-\sqrt{\frac{2}{3}}\,u_\uparrow(1)d_\downarrow(2)u_\uparrow(3)\right\} \tag{4.4}$$

multiplied by the antisymmetric colour factor $\epsilon^{abc}/\sqrt{3!}$.

The distribution amplitude corresponding to (4.3) is

$$\phi_{3q}(x_i, Q_H, \lambda_i) \equiv \int\left(\prod dk\right)\psi_{3q}(x_i, k_{\perp i}, Q_H, \lambda_i) \tag{4.5}$$

$$= B_\phi x_1 x_2 x_3 \exp\left[-b^2m^2\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}\right)\right] \tag{4.6}$$

It was shown in ref. [6] that the parametrization

$$B_\phi^2 = 0.35\ GeV^4 \tag{4.7}$$

$$b^2m^2 = 0.012 \tag{4.8}$$

is roughly consistent with the $J/\psi \to \bar{p}p$ branching ratio, the normalization of the proton magnetic form factor $Q^4G_M^p(Q^2)$ at $Q^2 = O(10)\ GeV^2$, and the normalization of the deep inelastic nucleon structure function at $x \gtrsim 0.6$. The normalizations (4.7, 4.8) correspond to a probability $P_{3q} = 1/4$ for finding the nucleon in the valence $3q$ state, while the valence radius is $R_{3q} = 0.23\,fm$. Given the relatively small uncertainties in the empirical inputs [e.g. $B(J/\psi \to \bar{p}p) = (0.22 \pm 0.02)\%$] and the approximate success of predictions based on (4.6), it seems to us reasonable to believe that the parameters B_ϕ^2 (4.7) and b^2m^2 (4.8) may be uncertain by at most a factor of two.

Since the momentum space wave-function (4.3) is assumed to be symmetric, each of the 3-quark decay amplitudes listed in Table 2 gives zero contribution to $\langle 0|qqq|N\rangle$ (2.1) except for the matrix elements proportional to products of pairs of quark masses m_im_j. Thus the nucleon decay rate is proportional to the fourth power of the (constituent) quark mass. For example, in minimal conventional SU(5) [2] the interaction Lagrangian [4] (2.4a) contributing to $p \rightarrow e^+\pi^0$ gives an effective Lagrangian density

$$\mathcal{L}^{pe} = 4\sqrt{2}\,\tilde{G}_X\,\alpha\Big[2\,\bar{\psi}_{e^+}\Big(\frac{1+\gamma_5}{2}\Big)\psi_p - \bar{\psi}_{e^+}\Big(\frac{1-\gamma_5}{2}\Big)\psi_p\Big] \qquad (4.9)$$

where

$$\alpha = \frac{4m_q^2}{m_p}\, I \qquad (4.10a)$$

with

$$I \equiv \int\Big(\prod dk\Big)\, dx_i\;\; \psi_{3q}(x_i, Q_H, \underline{k}_{\perp i})\,\frac{1}{x_1 x_2} = \int dx_1 dx_2\,\frac{\phi(x_i)}{x_1 x_2} \qquad (4.10b)$$

The results of Table 2 indicate that the two matrix elements (2.1) are related by $|\beta| = |\alpha|$. We assume that the constituent quark mass $m_q = \frac{1}{3}\, m_p$. The parameters of equations (4.7, 4.8) yield

$$I = 0.07\; GeV^2\;, \quad \alpha = 0.03\; GeV^3\;. \qquad (4.11)$$

The decay rate for $p \rightarrow e^+\pi^0$ can then be computed using chiral symmetry [8,9] as in Table 1:

$$\Gamma(p \rightarrow e^+\pi^0) = \alpha^2\,\tilde{G}_X^2\,\frac{5m_p}{2\pi f^2}\,(1+g_A)^2\Big(1-\frac{m_\pi^2}{m_p^2}\Big)^2 \qquad (4.12)$$

Taking $A = 2.9$ * and $\alpha = 0.03\; GeV^3$ we find

$$\tau(p \rightarrow e^+\pi^0) = 5 \times 10^{30}\; yr \times \Big(\frac{m_X}{10^{15}\, GeV}\Big)^4 \qquad (4.13)$$

* Here we include the SU(2) gauge boson exchange contributions to the renormalization group scaling between m_X and m_W, since these are numerically significant [20]. We truncate the perturbative renormalization factor A at a hadronic scale $Q_H = 1\; GeV$, and evaluate it using $\Lambda_{\overline{MS}}$ (3 flavours) $= 140\; MeV$.

which means that

$$m_X > 2 \times 10^{15}\ GeV \tag{4.14}$$

on the basis of the experimental lower limit $\tau(p \to e^+\pi^0) > 1.0 \times 10^{32}$ years established by the IMB group [13].

An alternative way of stating the result (4.14) is

$$\tilde{G}_X = \frac{A g_X^2}{4\sqrt{2}\ m_X^2} < 3.5 \times 10^{-32}\ GeV^{-2}\ . \tag{4.15a}$$

Corresponding limits can be imposed on the coefficients $\tilde{G}_S$, $\tilde{G}_S'$ of the operators (2.4b, 2.4c) using the results of Tables 1 and 2 in conjunction with experimental lower limits on other nucleon decay modes:

$$\text{Ref.[22]} : \tau(n \to \bar{\nu} K^0) > 8 \times 10^{30}\ \text{yrs} \Rightarrow \tilde{G}_S < 0.4 \times 10^{-30}\ GeV^{-2} \tag{4.15b}$$

$$\text{Ref.[21]} : \tau(p \to \bar{\nu} K^+) > 2 \times 10^{30}\ \text{yrs} \Rightarrow \tilde{G}_S^{\prime \ell} < 0.8 \times 10^{-30}\ GeV^{-2} \tag{4.15c}$$

$$\text{Ref.[22]} : \tau(p \to \mu^+ K^0) > 2.6 \times 10^{31}\ \text{yrs} \Rightarrow \tilde{G}_S^{\prime \mu} < 0.9 \times 10^{-30}\ GeV^{-2} \tag{4.15d}$$

The credibility and significance of these results are discussed in the last section.

5. Discussion

Before considering the significance of the results (4.14, 4.15) for GUT models, it is well to assess their credibility. The estimate (4.13) gives a nucleon partial lifetime which, for a given value of m_X, is about a hundred times shorter than estimates [3] based on non-relativistic SU(6) and the bag model. This difference can largely be traced to the fact that the valence nucleon wave function is quite small and dense, as is manifested by the normalizations (4.7, 4.8) and the corresponding valence radius $R_{3q} \approx 0.23\,fm$. In contrast, non-relativistic SU(6) and bag models would suggest that a natural hadronic scale is O(100 MeV) or O(1) fm, which gives dimensional estimates differing greatly from the estimates based on high momentum phenomenology [5,6]. A possible attitude [14,23] is to dispute all the exclusive QCD phenomenology, and assert that while it is doubtless valid in principle, less than 1% of the nucleon magnetic form factor observed at $Q^2 = 0(10)\ GeV^2$ (or of the $J/\psi \rightarrow \bar{p}p$ decay rate) can be ascribed to the true short distance hadronic wave function. Such a viewpoint would fly in the face of the successful prediction [19] of $\gamma\gamma \rightarrow \bar{p}p$, as well as of the related successful predictions for exclusive high momentum processes involving mesons. These are successfully normalized by the known pion decay constant f_π, a parameter closely analogous to the α and β (2.1) discussed here. We question whether the non-relativistic SU(6) or bag technology has been so severely tested in the range of interest. Most constraints on the SU(6) wave function involve the overlap of at most 2 quarks in the nucleon, rather than the three relevant to nucleon decay. The baryon $3q$ valence wave function may be more concentrated in configuration space (i.e. have a larger high momentum tail) than might be credited on the basis of non-relativistic SU(6) phenomenology. An alternative possibility is that our estimate using light-cone wave function phenomenology is erroneous, most probably because we use a constituent quark mass $m_q = \frac{1}{3}\, m_p$ in our matrix elements. We have already mentioned in sect. 3 that graphical analysis in perturbation theory, as well as the symmetry arguments of sect. 2, suggest no motivation for using short distance current quark masses instead of constituent masses. Indeed, we have verified graphically in section 3 that the relevant quark mass factors are effectively low momentum masses, which it is natural to interpret as constituent masses. For these reasons we think our numerical estimates presented in sect. 4 and the bounds (4.14, 4.15) are credible. What are their implications for GUT models?

It has been calculated [3] that in minimal conventional SU(5)

$$m_X = (1 \text{ to } 2) \times 10^{15} \times \Lambda_{\overline{MS}} \tag{5.1}$$

where $\Lambda_{\overline{MS}}$ is defined through two loops in a momentum range with four operational quark flavours. Most data analyses place $\Lambda_{\overline{MS}} < 400$ MeV, while lattice QCD calculations suggest $\Lambda_{\overline{MS}} = 100$ to 200 MeV, with an uncertainty resulting from the non-inclusion of quark loops. The limit $\Lambda_{\overline{MS}} > 1$ GeV inferred from equations (4.14) and (5.1) certainly seems unacceptably high. While embarrassing for the conventional minimal SU(5) GUT [2], this limit would not exclude philosophically similar models such as those based on SO(10) [23] in which the linkage (5.1) between m_X and $\Lambda_{\overline{MS}}$ can be relaxed. Of course, the bound (4.14) does not apply to supersymmetric GUTs [10] in which nucleons prefer to decay into $\bar{\nu} K$ [11] and perhaps [12] into $\mu^+ K$ or $e^+ K$ as well. The constraint (4.15b) applies to minimal supersymmetric SU(5) in which [12]

$$2\sqrt{2}\,\tilde{G}_S \geq A_S \, \frac{g_2^2 m_c m_s m_{\tilde{W}} \sin^2\theta_c}{16\pi^2 \, m_{H_X}} \, \frac{G_F}{\sqrt{2}} \left[F\Big(m_{\tilde{q}}, m_{\tilde{q}}, m_{\tilde{W}}\Big) + F(m_{\tilde{q}}, m_{\tilde{\ell}}, m_{\tilde{W}})\Big)\right] \tag{5.2a}$$

where

$$F(m_1, m_2, m_3) \equiv \frac{1}{m_2^2 - m_3^2}\left[\frac{m_2^2}{m_1^2 - m_2^2}\, \ell n \frac{m_1^2}{m_2^2} - \frac{m_3^2}{m_1^2 - m_3^2}\, \ell n \frac{m_1^2}{m_3^2}\right]. \tag{5.2b}$$

and where A_S is an $SU(3) \times SU(2) \times U(1)$ renormalization group enhancement factor [11] equal to 0.41 for the choices $Q_H = 1$ GeV, $\Lambda_{\overline{MS}}$ (3 flavours) $= 140$ MeV. The significance of the bound (4.15b) is difficult to assess because we know neither the superheavy Higgs mass m_{H_X} appearing in eq. (5.2), nor the relevant supersymmetric particle masses $m_{\tilde{q}}, m_{\tilde{\ell}}, m_{\tilde{W}}$. If we make the plausible guess [12]

$$m_W m_{\tilde{W}}\Big[F(m_{\tilde{q}}, m_{\tilde{q}}, m_{\tilde{W}}) + F(m_{\tilde{q}}, m_{\tilde{\ell}}, m_{\tilde{W}})\Big] = O(1) \tag{5.3}$$

then (4.15b) requires

$$m_X \gtrsim 7 \times 10^{17}\ GeV \tag{5.4}$$

which is considerably larger than the calculation [3]

$$m_X = (4\ to\ 8) \times 10^{16} \Lambda_{\overline{MS}} \tag{5.5}$$

of the superheavy gauge boson mass in minimal supersymmetric SU(5), though one should bear in mind the uncertainties of the approximation (5.3). Finally if we apply the constraints (4.15c,d) to the estimate [12] of the coefficient of the operator (2.4c):

$$2\sqrt{2}\, G_S^{\ell\ell} = A_S^{\ell\ell}\ \frac{\lambda}{m_{Planck}}\ \frac{G_F}{\sqrt{2}}\ \frac{m_W^2 m_{\tilde{W}}}{16\pi^2} \left[F(m_{\tilde{q}}, m_{\tilde{q}}, m_{\tilde{W}}) + F(m_{\tilde{q}}, m_{\tilde{\ell}}, m_{\tilde{W}}) \right] \tag{5.6}$$

where $A_S^{\ell\ell} \simeq 12$, then the plausible guess (5.3) tells us that

$$\lambda \lesssim 5 \times 10^{-7} \tag{5.7}$$

whereas we might [12] have expected $\lambda = 0(1)$.

While there are intangible uncertainties involved in our calculations, our estimates of nucleon decay rates based on the phenomenology of hadronic wave functions at high momenta make nucleon decay seem even more embarrassingly overdue.

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Table 1: Amplitudes for nucleon decay into an antilepton plus pseudoscalar.

Here $\tilde{G}_X$, $\tilde{G}_S$ and $\tilde{G}_S^{\prime\ell}$ are the coupling constants for GUTs, minimal supersymmetric GUTs and alternative supersymmetric GUTs defined in eq. (2.4),while α and β are the three-quark annihilation matrix elements defined in eq. (2.1). The quantity $f = 139$ MeV is the pion (or kaon) decay constant, and $D = 0.76$ and $F = 0.48$ are are taken from a recent Cabibbo fit to hyperon decays [16]. The spinors p_L, $\bar{e}_R^+$, etc. are ordinary Dirac spinors multiplied by chiral projection operators $(1 \pm \gamma_5)/2$. Our results for conventional GUTs agree with that of Claudson *et al.* [9], and our results for minimal supersymmetric GUTs agree with those of Chadha and Daniel [9].

$p \to e^+\pi^0$	$\left(4\alpha\tilde{G}_X/f\right)\left(1+D+F\right)\left(\bar{e}_R^+ p_L - 2\bar{e}_L^+ p_R\right)$
$p \to e^+\eta$	$\left(-4\alpha\tilde{G}_X/\sqrt{3}f\right)\left(1+D-3F\right)\left(\bar{e}_R^+ p_L - 2\bar{e}_L^+ p_R\right)$
$p \to \bar{\nu}_e\pi^+$	$\left(-4\sqrt{2}\,\alpha\tilde{G}_X/f\right)\left(1+D+F\right)\bar{\nu}_{eR}^c p_L$
$p \to e^+K^0$	$\left(2\sqrt{2}\,\beta\tilde{G}_S^{\prime e}/f\right)\left(1-D+F\right)\bar{e}_R^+ p_L$
$p \to \bar{\nu}_e K^+$	$\left(4\sqrt{2}\,\beta\tilde{G}_X^{\prime e}/f\right)\left(1+F\right)\bar{\nu}_{eR}^c p_L$
$p \to \mu^+K^0$	$\left(-4\sqrt{2}\,\alpha\tilde{G}_X/f\right)\left(1+D-F\right)\left(\bar{\mu}_R^+ p_L - 2\bar{\mu}_L^+ p_R\right)$ $+\left(2\sqrt{2}\,\beta\tilde{G}_S^{\prime\mu}/f\right)\left(1-D+F\right)\bar{\mu}_R^+ p_L$
$p \to \bar{\nu}_\mu K^+$	$\left(-8\sqrt{2}\,\alpha\tilde{G}_X/3f\right)D\,\bar{\nu}_{\mu R}^c p_L$ $+\left(2\sqrt{2}\,\beta\tilde{G}_S/f\right)\left(1+D+F\right)\bar{\nu}_{\mu R}^c p_L$ $+\left(4\sqrt{2}\,\beta\tilde{G}_S^{\prime\mu}/f\right)\left(1+F\right)\bar{\nu}_{\mu R}^c p_L$
$n \to e^+\pi^-$	$\left(4\sqrt{2}\,\alpha\tilde{G}_X/f\right)\left(1+D+F\right)\left(\bar{e}_R^+ n_L - 2\bar{e}_L^+ n_R\right)$
$n \to \bar{\nu}_e\pi^0$	$\left(4\alpha\tilde{G}_X/f\right)\left(1+D+F\right)\bar{\nu}_{eR}^c n_L$
$n \to \bar{\nu}_e\eta$	$\left(4\alpha\tilde{G}_X/\sqrt{3}f\right)\left(1+D-3F\right)\bar{\nu}_{eR}^c n_L$
$n \to \bar{\nu}_e K^0$	$\left(2\sqrt{2}\,\beta\tilde{G}_S^{\prime e}/f\right)\left(1+D+F\right)\bar{\nu}_{eR}^c n_L$
$n \to \bar{\nu}_\mu K^0$	$\left(4\sqrt{2}\,\alpha\tilde{G}_X/f\right)\left(1+D/3-F\right)\bar{\nu}_{\mu R}^c n_L$ $+\left(4\sqrt{2}\,\beta\tilde{G}_S/f\right)\left(1+F\right)\bar{\nu}_{\mu R}^c n_L$ $+\left(2\sqrt{2}\,\beta\tilde{G}_S^{\prime\mu}/f\right)\left(1+D+F\right)\bar{\nu}_{\mu R}^c n_L$

Table 2: Matrix elements needed for the evaluation of the proton's lifetime.

The notation is that of Brodsky and Lepage, Ref. 5; λ_i and m_i are the helicity and mass of the quark with momentum p_i, and λ is the helicity of the positron (whose mass is neglected). In the massless limit helicity $\downarrow$ ($\uparrow$) correspond to chirality L (R). For all momenta k, $k_\perp^\pm \equiv k_x \pm i k_y$.

Let: $O_1 \equiv \bar{u}^c_{\lambda_1}(p_1)\,\gamma_\mu\,\frac{1}{2}(1-\gamma^5)\,u_{\lambda_2}(p_2)\;\bar{v}^c_\lambda(p)\,\gamma^\mu\,\frac{1}{2}(1-\gamma^5)\,u_{\lambda_3}(p_3)$

λ_1	λ_2	λ_3	λ	O_1
$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\frac{2}{\sqrt{p^+p_1^+p_2^+p_3^+}}\left[p^+p_{1\perp}^+ - p_1^+p_\perp^+\right]\left[p_3^+p_{2\perp}^- - p_2^+p_{3\perp}^-\right]$
$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$-\frac{2m_1m_2}{\sqrt{p^+p_1^+p_2^+p_3^+}}\,p^+p_3^+$
$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\frac{2m_1m_3}{\sqrt{p^+p_1^+p_2^+p_3^+}}\,p_2^+p^+$
$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\frac{2m_2p_3^+}{\sqrt{p^+p_1^+p_2^+p_3^+}}\left(p_1^+p_\perp^+ - p_{1\perp}^+p^+\right)$
$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\frac{2m_3p_2^+}{\sqrt{p^+p_1^+p_2^+p_3^+}}\left(p^+p_{1\perp}^+ - p_\perp^+p_1^+\right)$

(a)

Let: $O_2 \equiv \bar{u}^c_{\lambda_1}(p_1)\,\gamma_\mu\,\frac{1}{2}\,(1-\gamma^5)\,u_{\lambda_2}(p_2)\;\bar{v}^c_\lambda(p)\,\gamma^\mu\,\frac{1}{2}\,(1+\gamma^5)\,u_{\lambda_3}(p_3)$

λ_1	λ_2	λ_3	λ	O_2
↑	↓	↑	↑	$\frac{2}{\sqrt{p^+p_1^+p_2^+p_3^+}}\left(p_1^+p_{3\perp}^+ - p_3^+p_{1\perp}^+\right)\left(p_2^+p_\perp^- - p^+p_{2\perp}^-\right)$
↑	↑	↓	↑	$\frac{2m_2m_3}{\sqrt{p^+p_1^+p_2^+p_3^+}}\;p^+p_1^+$
↓	↑	↑	↑	$-\frac{2m_1m_2}{\sqrt{p^+p_1^+p_2^+p_3^+}}\;p^+p_3^+$
↑	↓	↓	↑	$\frac{2m_3p_1^+}{\sqrt{p^+p_1^+p_2^+p_3^+}}\left[p_\perp^-p_2^+ - p_{2\perp}^-p^+\right]$
↓	↓	↑	↑	$\frac{2m_1p_3^+}{\sqrt{p^+p_1^+p_2^+p_3^+}}\left[p^+p_{2\perp}^- - p_2^+p_\perp^-\right]$

(b)

Let: $O_3 \equiv \bar{u}^c_{\lambda_1}(p_1)\,\frac{1}{2}\,(1-\gamma^5)\,u_{\lambda_3}(p_3)\;\bar{v}^c_{\lambda}(p)\,\frac{1}{2}\,(1-\gamma^5)\,u_{\lambda_2}(p_2)$

λ_1	λ_2	λ_3	λ	O_3
$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\frac{m_2 m_3}{\sqrt{p^+ p_1^+ p_2^+ p_3^+}}\; p^+ p_1^+$
$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$-\frac{m_1 m_2}{\sqrt{p^+ p_1^+ p_2^+ p_3^+}}\; p^+ p_3^+$

(c)

Let: $O_4 \equiv \bar{u}^c_{\lambda_1}(p_1)\,\frac{1}{2}\,(1+\gamma^5)\,u_{\lambda_3}(p_3)\;\bar{v}^c_{\lambda}(p)\,\frac{1}{2}\,(1+\gamma^5)\,u_{\lambda_2}(p_2)$

λ_1	λ_2	λ_3	λ	O_4
$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\frac{m_2 m_3}{\sqrt{p^+ p_1^+ p_2^+ p_3^+}}\; p^+ p_1^+$
$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$-\frac{m_1 m_2}{\sqrt{p^+ p_1^+ p_2^+ p_3^+}}\; p^+ p_3^+$

(d)

FIGURE CAPTIONS

1. Models for nucleon decay matrix elements:

   (a) quasi-free $q \to \bar{q}\,\bar{q}\,\bar{\ell}$ decay,

   (b) two-particle $qq \to \bar{q}\,\bar{\ell}$ annihilation,

   (c) three-particle $qqq \to \bar{\ell}$ annihilation proportional to

   (d) the basic three quark annihilation matrix element.

2. Lowest order contribution to the magnetic form factor of the proton.

3. (a) Feynman diagram which contributes to the leading behaviour of $\phi_{3q}(x_i, Q)$.

   (b) Typical single gluon insertion.

4. Feynman diagram contributing to proton decay. In (a) we have a $qqq\ell$ local operator whereas in (b) we have put the X-boson propagator in explicitly.

5. Four light-cone perturbation theory diagrams contributing to proton decay. Each of the diagrams is quadratically divergent. This divergence cancels in their sum.

6. Four Feynman diagrams contributing to proton decay. In light-cone perturbation theory each of the diagrams has two "time" orderings corresponding to $0 < k^+ < p_2^+$ and $-p_1^+ < k^+ < 0$.

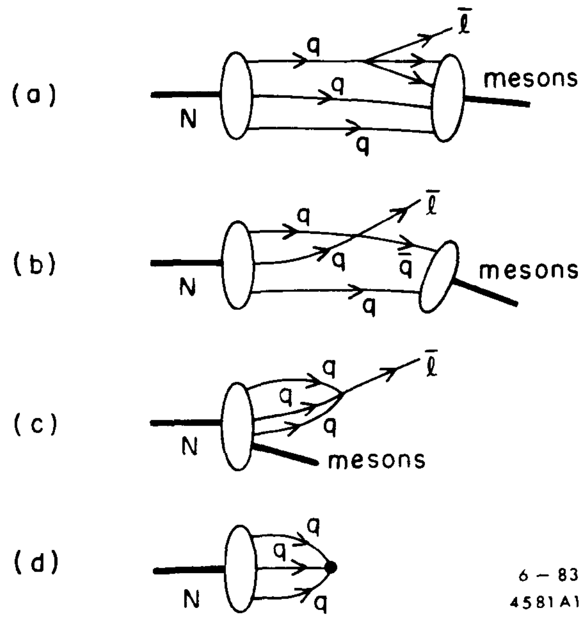
(a)
N
q
q
q
$\bar{\ell}$
mesons
(b)
N
q
q
q
$\bar{q}$
$\bar{\ell}$
mesons
(c)
N
q
q
q
$\bar{\ell}$
mesons
(d)
N
q
q
q
6 – 83
4581A1

Fig. 1

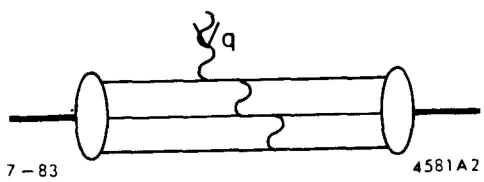

Fig. 2

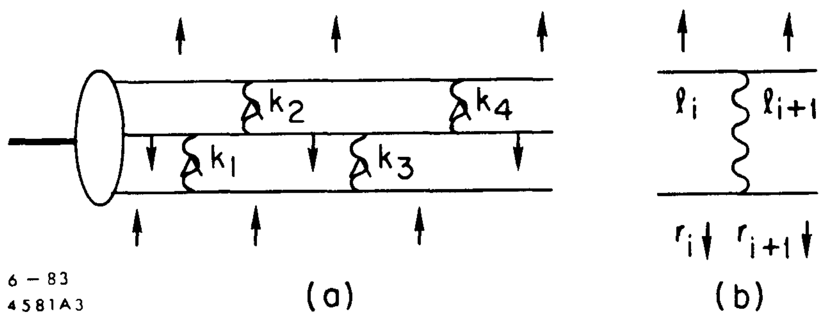

Fig. 3

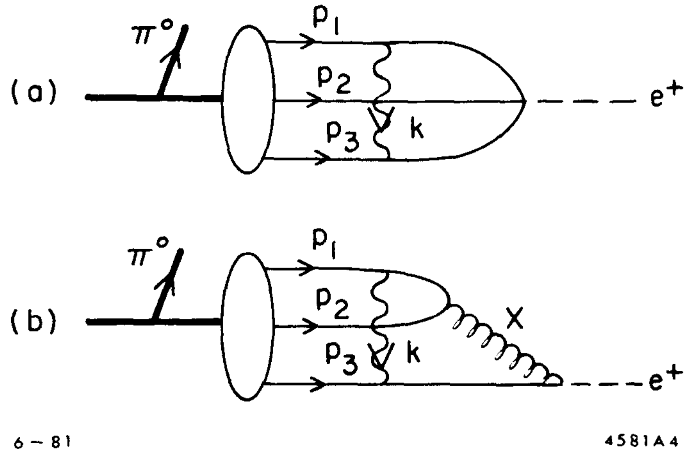

Fig. 4

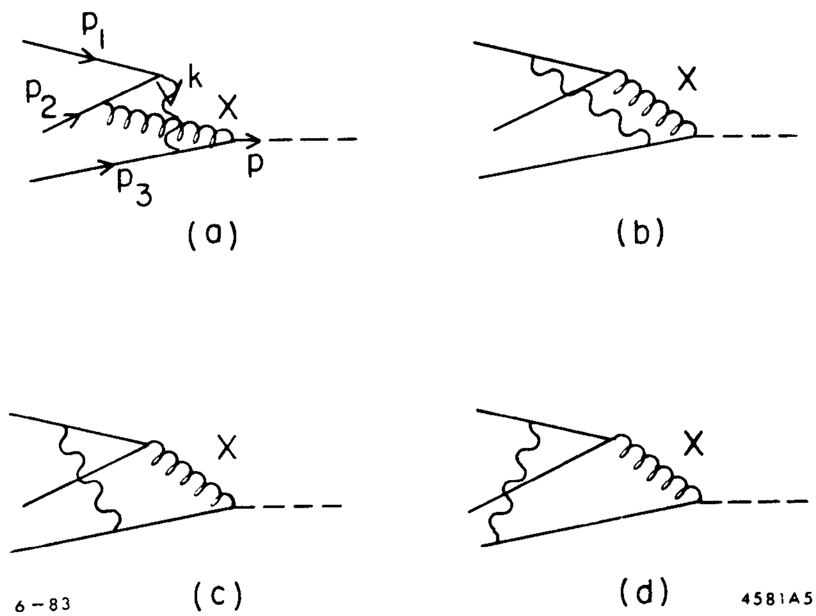

Fig. 5

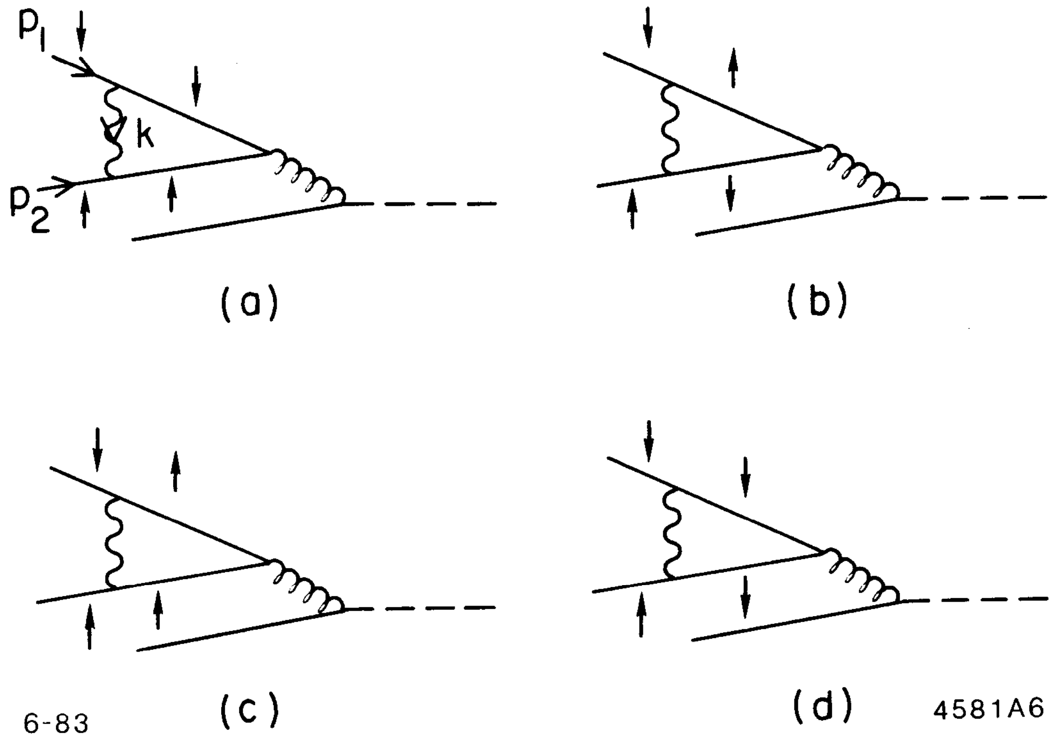

Fig. 6