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HEAVY FLAVOR PHOTOPRODUCTION IN SLAC LATTICE QCD*

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ABSTRACT

We use the SLAC lattice QCD theory to compute the large distance contribution to the process $\gamma + p \rightarrow q_H + \bar{q}_H + X$, where q_H is a heavy quark. In this way, we complete the usual short distance analysis of this process. The relationship between our completion and observation is discussed.

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At present, some of the more exciting observations in new particle phenomenology are those of the cross section for open charm photoproduction.[1,2] We have in mind, in particular, the recent measurement by the SLAC HF Photon Collaboration,[2] which finds (Here, $c\bar{c}$ refers to open charm particle production.) $\sigma(\gamma+p \rightarrow c\bar{c}+X) = 63\left(^{+32}_{-28}\right)$ nb at $E_\gamma \cong 19.5$ GeV, where E_γ is the energy of the photon in the proton rest frame. This observation should be compared with the popular γ-gluon fusion (γ-GF) result[3] (see Fig. 1), which at $E_\gamma \cong 19.5$ GeV is 38 nb in its more modern form as calculated by Phillips,[3] is 35 nb as calculated using the counting rules[4] by Duke and Owens,[3] and is 25 $\left(^{+12.5}_{-8.25}\right)$ nb as calculated by Novikov *et al.*[3] Thus, the result in Ref. 2, taken together with the data in Refs. 1, suggests that perturbative QCD in its most natural form[3] may not be sufficient to account for the entirety of the charm photoproduction cross section at 19.5 GeV, for example. Motivated by this, in what follows, we wish to present another mechanism for heavy flavor photoproduction.

Indeed, recently Halzen and Scott[5] have argued that the γ-GF model only accounts for the part of the $\gamma + p \rightarrow c\bar{c} + X$ cross section in which the $c\bar{c}$ pair carries away almost all of E_γ. These authors then propose a phenomenological model for the remaining part of $\sigma(\gamma + p \rightarrow c\bar{c} + X)$ in which the γ is viewed as fragmenting into a $c\bar{c}$ pair, one of which then scatters strongly with p with a quesstimated cross section of $\sim .2$ mb. Here, we wish to put the work of these authors in a more systematic theoretical framework by computing all aspects of $\gamma + p \rightarrow c\bar{c} + X$ in a complete non-perturbative and perturbative QCD analysis, where we will assume that the non-perturbative QCD part of the analysis can be represented by the SLAC lattice QCD theory.[6]

More specifically, for definiteness, we shall discuss open charm photoproduction in order to set the theoretical development. The application to other heavy flavors will then be immediate. In discussing open charm production, we shall take the work of Phillips[3] as representative of the perturbative γ-gluon fusion contribution to $\gamma + p \to c\bar{c} + X$ for $m_c = 1.5$ GeV. We would like to recall that Novikov *et al.* use $m_c = 1.25$ GeV and compute the leading order contribution to $\gamma + p \to c\bar{c} + X$ using the heavy quark expansion[7] together with dispersive techniques. However, in this dispersive analysis, perturbation theory is only used at the unphysical zero momentum point for the photon so that it is indeed expected to be reliable. Hence, we interpret the result of Phillips (which uses perturbation theory in the physical region and a gluon distribution $\propto$ $(1-x)^5$ near $x = 1$ as compared with $(1-x)^3$ for $x \to 1$ in the Novikov *et al.* analysis, where x is the lightcone momentum fraction) as indicative of the modifications in m_c and in the gluon distribution which are necessary if one wants to replace the calculation of Novikov *et al.* with the lowest order diagrams in Fig. 1. We intend to complete these γ-GF analyses with their non-perturbative analoga, which we presume to be described by the SLAC lattice QCD theory.[6]

To make contact with a process such as that in Fig. 1, we first note that the SLAC group has developed a reasonable phenomenology of the static properties of the low-lying hadrons by studying the fluxless light-hadron sector of the lattice Hamiltonian QCD theory described by the Hamiltonian

$$H(g,a) = \frac{1}{a}\Bigg\{ \sum_{\text{links}} \frac{1}{2} g^2 E^{\alpha^2}_{\vec{j},\hat{\mu}} - \sum_{\text{loops}} \frac{1}{g^2} \operatorname{tr}\left[\prod_{\substack{\text{around}\\ \text{loop}}} U_{\vec{j},\hat{\mu}} \right] - \left[i \sum_{\substack{\vec{j},\hat{\mu}\\ n>0}} \delta'(n)\, \psi^{\dagger\alpha f}_{\vec{j}}\, \alpha_\mu\, \psi^{\beta f}_{\vec{j}+n\hat{\mu}} \left[\prod_{m=0}^{n-1} U_{\vec{j}+m\hat{\mu},\hat{\mu}} \right]^{\alpha\beta} + \text{H.c.} \right] \Bigg\} \tag{1}$$

where a is the lattice constant, g is the QCD gauge coupling constant, and $U_{\vec{j},\hat{\mu}}$ is the link operator for the link connecting lattice site $\vec{j}a$ to site $\vec{j}a + \hat{\mu}a$ where $\hat{\mu}$ is the unit vector in the μ-direction. The operators $\vec{E}_{\vec{j},\hat{\mu}}$ measure the units of color flux created by the operators $U_{\vec{j},\hat{\mu}}$. The α_μ are Dirac matrices and will be represented here in the notation of Bjorken and Drell.[8] The quark spinor field $\psi_{\vec{j}}^{\alpha f}(t)$ at site $\vec{j}a$ carries color α and flavor f at time t. Finally, we note that the function $\delta'(n)$ is the truly defining characteristic of the theory of Drell *et al.* and is given by $\delta'(n) \to (-1)^{n+1}/n$ in the infinite volume limit in which we shall work. Here, we shall use (1) to analyze the large distance aspects of $\gamma + p \to c\bar{c} + X$.

Our strategy will be the standard effective Lagrangian technique,[9] in which, here, we use (1) at zero three-momenta to abstract, following Gell-Mann,[10] the relevant effective large distance interaction density for the process in question.[11] The relevant process is illustrated in Fig. 2. In order to determine the respective local effective interaction density $\mathcal{L}_{\text{eff}}$ which corresponds to $H_{\text{eff}}^{(2)}$ in Fig. 2, we follow the work of Symanzik[12] in his improvement program for lattice quantum field theory. The key result is that a lattice Hamiltonian such as $H(g,a)$ in (1), plus a mass term, is equivalent, insofar as its 1PI vertices are concerned, to a local effective Lagrangian $\mathcal{L}_{\text{loc}}$ such that

$$\begin{aligned}\mathcal{L}_{\text{loc}} =&\bar{\psi}(i\not{D}-m)\psi - \frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} + (Z_3-1)\left(-\frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}\right)\\ &+ (Z_2-1)\ \bar{\psi}(i\not{D})\psi + \sum_{n=1}^{\infty} a^{2n} \sum_{\{O^{(2n)}\}} C_{O^{(2n)}}(g,a,m)\ O^{(2n)}\end{aligned} \tag{2}$$

where

$$(iD_\mu)_{\alpha\beta} = i\partial_\mu\delta_{\alpha\beta} - g\left(\vec{\lambda}\cdot\vec{A}_\mu\right)_{\alpha\beta}\ ,\quad \vec{F}_{\mu\nu} = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu - g\epsilon_{\rightarrow bc}\,A_\mu^b\,A_\nu^c\ ,$$

and m is the quark mass matrix. Here, $\vec{A}_\mu$ is the QCD vector potential which carries the adjoined representation of the color group SU(3) and $\vec{\lambda}$ generate the fundamental representation of SU(3) with $[\lambda^e, \lambda^b] = i\epsilon_{ebc}\lambda^c$. Note that ψ is now the continuum limit of $\psi_{\vec{j}}^{\alpha f}$ so that we have suppressed the αf labels in (2). In (2), the Z_i are the usual renormalization constants[8] in the presence of our lattice cut-off and, for each n, the $\{O^{(2n)}\}$ represent a complete[12] set of local operators of dimension $4 + 2n$ so that the coefficient function $C_{O^{(2n)}}$ may be evaluated by computing the respective 1PI vertex at zero external momenta for example. (We should remind the reader that, at strong coupling, the existence of (2) requires that one expand the 1PI vertices of (1), using the Gell-Mann-Low formula, in negative powers of the energy denominator $: H_0 : -E$, where $: :$ denotes normal ordering, E is the respective energy variable and $H_0 = \frac{1}{2a}\sum_{\text{links}} g^2 E^{\alpha^2}_{\vec{j},\hat{\mu}} + \sum_{\vec{j}} \bar{\psi}_{\vec{j}} m \psi_{\vec{j}}$. This expansion will be meaningful for g^2/a large compared to E.) Thus, it is (2) from which we will derive $\mathcal{L}_{\text{eff}}$.

To this end, we note that, in parton model manipulations, one considers the effects of renormalization to have been implemented so that the masses in the quark mass matrix are the physical parton masses. Thus, the $a = 0$ limit of (2) describes the short distance interactions of partons in our $\gamma + p \to c\bar{c} + X$ analysis. The various lattice effective interactions $O^{(2n)}$ in (2), at $a \neq 0$, then represent the effects of large distance interactions as described by (1). For our application, we note that only color singlet flavor conserving operators $O^{(2n)}$ appear in (2). Further, any such operator involving the gluon field $\vec{A}_\mu$ will be suppressed in our application because of two reasons. Firstly, note that the non-trivial action of such an operator in our application would either imply that our initial $|p\rangle$ lattice state contained a gluonic excitation (i.e., involved the action of a $U_{\vec{j},\hat{\mu}}$

on the vacuum $|0\rangle$) or that at least two such operators were involved (to allow the creation and annihilation of the respective color flux). The latter situation would be higher order in $1/g^2$ than our leading order calculation and the former circumstance would be suppressed due to the small probability of finding such a flux containing state in the physical proton: the energy of a flux excitation is $\gtrsim \frac{1}{2}g^2 C_F/a \sim 1$ GeV (we will ultimately find $g^2 \sim 10$, $a \sim 5$ GeV^{-1}) and, since $s\bar{s}$ pairs, of current mass $\sim .3$ GeV, are already suppressed relative to $u\bar{u}$ and $d\bar{d}$ pairs in the parton vacuum, we expect the probability of a 1 GeV flux excitation to be extremely small. Thus, we need only to isolate, to the leading order in $1/g^2$, the dimension-six, parity conserving, pure quark, gauge invariant operators in $\{O^{(6)}\}$ which cannot be represented as contributions to the Z_i. These operators are

$$O_\Gamma = \sum_{a \in S} \bar{\psi}^{\alpha f}\, \Gamma \lambda^a_{\alpha\beta}\, \psi^{\beta f}\, \bar{\psi}^{\alpha' f'}\, \Gamma \lambda^a_{\alpha'\beta'}\, \psi^{\beta' f'} \quad , \quad S = \{0\}, \{1, \ldots, 8\} \ , \tag{3}$$

where λ^a, $a = 0, \ldots, 8$, may be identified with Gell-Mann's U(3) matrices with the normalization $\mathrm{tr}\, \lambda^a \lambda^b = \frac{1}{2}\delta^{ab}$, and where $\Gamma \in \{1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}\}$. Following the rules of Symanzik[12] for the evaluation of the C_Γ in (2), we find that the operators $\{O_\Gamma\}$ contribute the local effective Lagrangian

$$\mathcal{L}^{(6)}_{\mathrm{loc}}(\text{quark sector}) = -\frac{4a^2(2\varsigma(3))}{g^2\, C_F} \sum_{\substack{b=0 \\ f,f'}}^{8} \bar{\psi}^f\, \gamma_\mu\, \lambda^b\, \psi^f\, \bar{\psi}^{f'}\, \gamma^\mu\, \lambda^b\, \psi^{f'} \equiv \mathcal{L}_{\mathrm{eff}} \quad , \tag{4}$$

where ς is the Riemann zeta function and $C_F = (N_c^2 - 1)/2N_c$ with $N_c = 3$. This completes the derivation of $\mathcal{L}_{\mathrm{eff}}$.

From Symanzik's analysis we then conclude that (4) is applicable to $\gamma + p \to c\bar{c} + X$ so long as the momentum transfer through it resides in the first Brillouin zone. We should also comment on the value of g^2 to be employed in (4) in this first Brillouin zone scenario. We note, with the lore, that g^2 should be the value of the squared QCD running coupling constant at Q^2 if Q^2 is the respective squared momentum transfer through $\mathcal{L}_{\text{eff}}$. To compute this $g^2(Q^2)$, we will ultimately use a 5 flavor one-loop[13] formula of the Gross-Wilczek-Politzer type. This may seem somewhat suspect since we are expanding in $1/g^2$! However, since we will find that, in our typical momentum exchange through $\mathcal{L}_{\text{eff}}$, $g^2(Q^2) \sim 10$, $g^2/4\pi^2$, the expansion parameter for the QCD beta function, is indeed small: $g^2/4\pi^2 \cong .253$. Hence, we may expect the one-loop formula for $g^2(Q^2)$ to be a reasonable approximation. With these explanatory remarks, we may proceed with our analysis of $\gamma + p \to c\bar{c} + X$.

For the problem of charm photoproduction, we then apply (4) as indicated in Fig. 3. Since we are treating in (4) only the large distance effects, the diagrams obtained from those in Fig. 3 by attaching the photon line to the light quark lines are omitted here. Indeed, the restriction to large distance effects, using the kinematics in Fig. 3 with the photon momentum in the $-\hat{z}$-direction in the incident parton-photon center of momentum system, implies the restrictions .36 GeV$^2 \lesssim Q^2 \equiv -(p_q - p_q')^2 \leq 3(2\pi/a)^2$, $|p_q'^i| \leq \pi/a$, and $|p_f^i| \leq \pi/a$, $i = x, y, f = c, \bar{c}$. Here, we have anticipated our use[13] of a 5-flavor one-loop formula for $g^2(Q^2)$ with $\Lambda_{QCD} \cong .34$ GeV so that the interaction in (4) is essentially negligible for $Q^2 = .36$ GeV^2. Since the outgoing charmed hadrons (we will take them to be kinematically similar to $D\bar{D}$) are supposed to be consistent with the lattice spacing a, we further require that, in the outgoing $D\bar{D}$ center of momentum system, the 3-momentum of the $D(\bar{D})$ should be bounded by $\sqrt{3}\pi/a$ in magnitude – otherwise, it is difficult to justify the production of the respective particles on our lattice. Finally, we follow Phillips[3] and associate 5/6 of the rate for Fig. 3 where $(p_c + p_{\bar{c}})^2 < 4m_D^2$ with open charm production. This, then, defines the regime of applicability of the interaction (4). We shall refer to this region of the final particle phase space as the "restricted phase space."

To proceed, we use the standard manipulations to write the cross section corresponding to the diagrams in Fig. 3 as

$$\sigma(\gamma + p \to c\bar{c} + X) = \frac{\alpha(2\varsigma(3))^2 a^4 N_c}{9\pi^4 C_F^2} \left(\frac{a}{a_q}\right)^3 \sum_i \int_{x_{\min}}^{1} dx \, \frac{f_i(x)}{\hat{s}} \int dE_{\bar{c}} \, dE_c \, d\phi_c \, d\phi_{\bar{c}} \, dz_{\bar{c}} \, \frac{L^{\mu\nu} H_{\mu\nu}}{g^4(Q^2)} \tag{5}$$

(restricted phase space)

where $\alpha \cong 1/137$, E_j, ϕ_j, and z_j are respectively the energy, azimuthal angle, and cosine of the angle between the outgoing particle momentum $\vec{p}_j$ and the z-axis in the initial parton-photon center of momentum frame, $j = c, \bar{c}$, $\hat{s}$ is the square of the total subprocess energy in this latter frame, $f_i(x)$ are the parton distributions for the constituents of the proton (we take the f_i from Ref. 14 for definiteness), $g^2(Q^2) = 48\pi^2 / 23\ell n(Q^2/(.34GeV)^2)$ and the tensors $L^{\mu\nu}$ and $H^{\mu\nu}$ are $L^{\mu\nu} = p_q^\mu p_q^{'\nu} + p_q^\nu p_q^{'\mu} - p_q \cdot p_q' g^{\mu\nu}$ and

$$H^{\mu\nu} = h_0 g^{\mu\nu} + h_1 p_c^\mu p_c^\nu + h_2 p_{\bar{c}}^\mu p_{\bar{c}}^\nu + h_3 p_\gamma^\mu p_\gamma^\nu + h_4 \left(p_c^\mu p_{\bar{c}}^\nu + p_c^\nu p_{\bar{c}}^\mu\right) + h_5 \left(p_c^\mu p_\gamma^\nu + p_c^\nu p_\gamma^\mu\right) + h_6 \left(p_{\bar{c}}^\mu p_\gamma^\nu + p_{\bar{c}}^\nu p_\gamma^\mu\right) \tag{6}$$

with the condition $Q^\mu H_{\mu\nu} = 0$ for $Q = p_q - p_q'$ so that we need to list only $h_1, \ldots, h_4$:

$$h_1 = 4/p_c \cdot p_\gamma \ , \quad h_2 = 4/p_{\bar{c}} \cdot p_\gamma \ , \quad h_3 = -4m_c^2/p_c \cdot p_\gamma p_{\bar{c}} \cdot p_\gamma$$

and

$$h_4 = -2m_c^2/(p_c \cdot p_\gamma)^2 - 2m_c^2/(p_{\bar{c}} \cdot p_\gamma)^2 + 2(2p_c \cdot p_{\bar{c}} - p_c \cdot p_\gamma - p_{\bar{c}} \cdot p_\gamma)/p_c \cdot p_\gamma p_{\bar{c}} \cdot p_\gamma \ . \tag{7}$$

The value of $x_{\min}$ is the minimum value of the lightcone momentum fraction such that the subprocess can in fact produce a $D\bar{D}$ pair. Thus, $x_{\min} = 2m_D^2/m_p E_\gamma$ where $m_D = 1.863$ GeV and $m_p = .9383$ GeV. The factor $(a/a_q)^3$ arises because the lattice constant appropriate for D and F mesons,[11] $a \cong 4.4 \text{ GeV}^{-1}$, is different from that appropriate for a proton, which, in the M.I.T. bag model,[15] has quarks with internal momentum $\sim .408$ GeV; this corresponds to a lattice constant of $a_q \cong \pi/.408 \text{ GeV} \cong 7.7 \text{ GeV}^{-1}$ if we include, along a given direction, all momenta except those internal to the proton. Thus, the physical proton corresponds to a parton in the lattice state $|q, a_q\rangle$ whereas, since we are producing D mesons, the interaction (4) corresponds to a final state lattice constant of $a \cong 4.4 \text{ GeV}^{-1}$ and, hence, would require the parton lattice state $|q, a\rangle$. As we have argued in Ref. 16, the two states, to leading order in $1/g^2$, are related by $|q, a\rangle = (a/a_q)^{-3/2} \ |q, a_q\rangle$. Thus, presuming the initial state to be $|q, a_q\rangle$, we obtain the factor of $(a/a_q)^3$ in the cross section in (5). No such factor appears in (5) for the final state parton of type q since we assume, in a conventional sense, that, although it be restricted to lattice momenta, it hadronizes with the probability 1. With these explanatory remarks, we may proceed with the evaluation of (5).

We have evaluated (5) using approximate numerical techniques based on the Neyman method.[17] Our results are shown in Fig. 4 with the available data.[1,2] The agreement between curve (c) and the data is encouraging. Recall that, at SPEAR energies, the $D^0 : D^+ : F : \Lambda_c$ ratio is $\sim 2 : 1 : 1 : 1$.[18] The muoproduction data appear to be very close to curve (c). This is a little unexpected, since the muoproduction data have been analyzed assuming only $D\bar{D}$ production. Nevertheless, in summary, we feel we can say that both the muoproduction data and the real photoproduction data are not obviously inconsistent with our complete lattice γ-GF picture.

We should point out that, if we would replace the curve (c) by the sum of our lattice $m_c = 1.25$ GeV result and the $m_c = 1.25$ GeV result of Novikov *et al.*, curve (c) would change by less than 10% at $E_\gamma \cong 20$ GeV. Thus, we have added confidence that we have properly completed the perturbative γ-gluon fusion results near $E_\gamma = 20$ GeV. For this reason, when we note the value of curve (c) for 19.5 GeV, namely, ~ 87 nb, we feel we are consistent with the result in Ref. 2, in conjunction with the results in Refs. 1.

Although it is not shown in Fig. 4, we have also analyzed the lattice result for $\sigma(\gamma + p \to b\bar{b} + X)$. We obtained it from (5) by making the replacements $m_c \to m_b = 5.1$ GeV, $m_D \to m_B \cong 5.26$ GeV and dividing (5) by 4 to account for the difference in electric charge between b and c. What we find is that at $E_\gamma = 200$ GeV, the lattice contribution to $\sigma(\gamma + p \to b\bar{b} + X)$ is $\sim .11$ nb, which is $\sim 18\%$ of the prediction of Novikov *et al.*[3] At $E_\gamma = 500$ GeV, the respective lattice contribution is $\sim .098$ nb and is only $\sim 2.3\%$ of the result of Novikov *et al.*[3] Thus, the lattice contribution to open beauty production in the regime $200 \lesssim E_\gamma \lesssim 500$ GeV is entirely within the errors on the pure γ-GF result.

In conclusion, we note that we have proposed a lattice QCD completion of the perturbative γ-gluon fusion mechanism for open heavy flavor production. The available total charm cross section data are generally not inconsistent with this completion. Further tests of this complete heavy flavor photoproduction scenario will appear elsewhere.

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FIGURE CAPTIONS

1. γ-gluon fusion to lowest order in QCD. G is a gluon.

2. Lattice interaction for the process $c + q \to c + q$, where q is a light quark; $H^{(2)}_{\text{eff}}$ is the second order lattice interaction Hamiltonian implied by (1).

3. Large distance contribution to $\gamma + p \to c\bar{c} + X$.

4. Comparison of the lattice (+ γ-gluon fusion) result for $\gamma + p \to c\bar{c} + X$ with observation. Curve (a) is the $m_c = 1.5$ GeV $c\bar{c}$ production lattice result; curve (b) is the $m_c = 1.25$ GeV $c\bar{c}$ production lattice result; curve (c) is the sum of Phillips'[3] γ-GF prediction and curve (a). Curves (a) and (b) have $\pm 26\%$ numerical uncertainties.

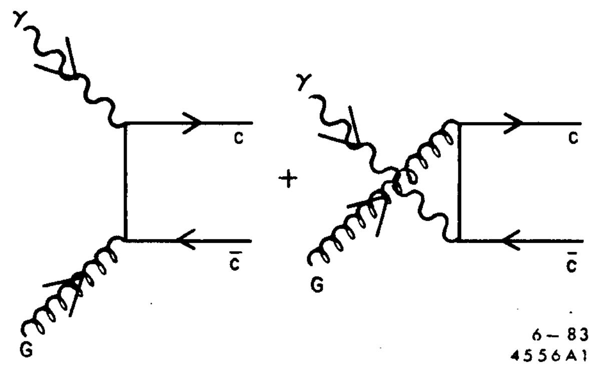

Fig. 1

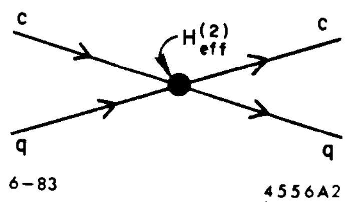

Fig. 2

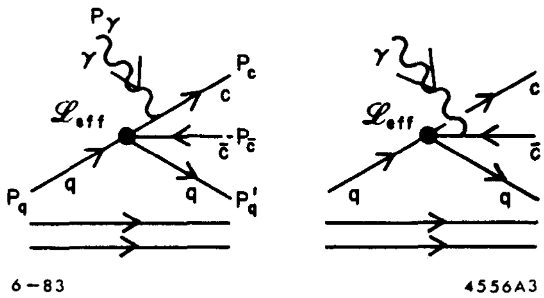

Fig. 3

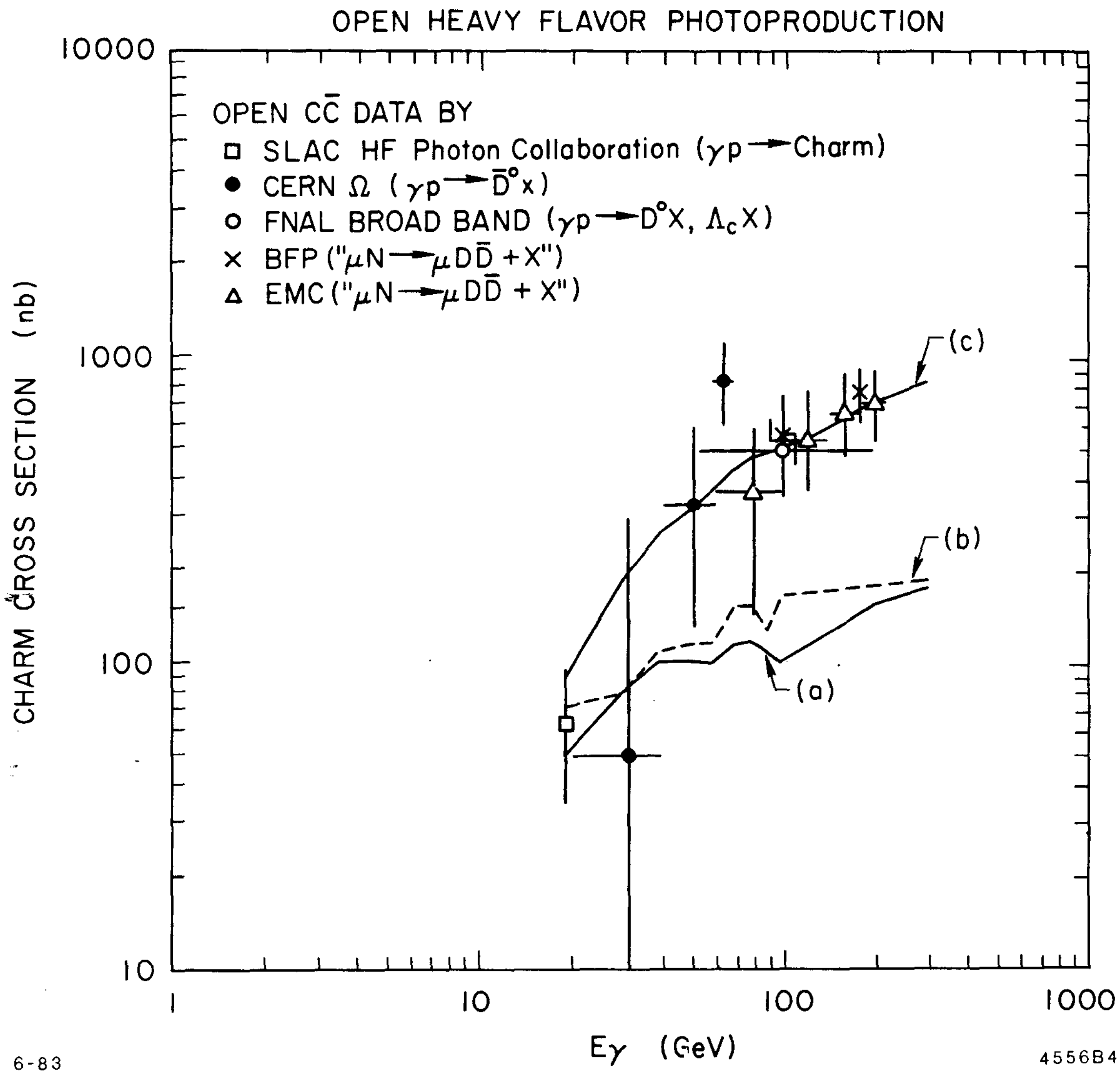
OPEN HEAVY FLAVOR PHOTOPRODUCTION
OPEN CC̄ DATA BY
SLAC HF Photon Collaboration (γp → Charm)
CERN Ω (γp → D̄°x)
FNAL BROAD BAND (γp → D°X, Λ_c X)
BFP ("μN → μDD̄ + X")
EMC ("μN → μDD̄ + X")
(a)
(b)
(c)
CHARM CROSS SECTION (nb)
10000
1000
100
10
1
10
100
1000
Eγ (GeV)
6-83
4556B4

Fig. 4