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ON THE CONSISTENCY OF THE WESS-ZUMINO MODEL*

C. T. SACHRAJDA
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

*Physics Department, The University***
Southampton SO9 5NH, ENGLAND

and

G. THOMPSON
Physics Department, The University
Southampton SO9 5NH, ENGLAND

ABSTRACT

We study the three point function, $\Gamma(p_1^2, p_2^2, p_3^2)$, of the Wess-Zumino model in the limit where $p_2^2 \simeq p_3^2 \to \infty$. Contrary to the conclusions of some recent publications, we find that this three point function can vanish sufficiently quickly in this limit for the Schwinger-Dyson equations to be consistent and without violating other known properties of the model.

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** Permanent address.

Recently Krasnikov[1] and Krasnikov and Nicolai[2] have claimed to have shown that the Wess-Zumino model[3] is inconsistent. Their arguments rely on a comparison of the conditions imposed by the non-perturbative Dyson-Schwinger equation, spectral conditions and supersymmetric Ward identities and the renormalization group and perturbation theory.

One begins by deriving the Dyson-Schwinger equation from the Wess-Zumino Lagrangian:

$$\mathcal{L}_{ren} = Z\mathcal{L}_0 + \mathcal{L}_I \tag{1}$$

with

$$\mathcal{L}_0 = \bar{A}\Box A + i\partial_\mu \bar{\psi}\bar{\sigma}^\mu \psi + F\bar{F} \tag{2a}$$

and

$$\mathcal{L}_I = m\left(FA - \frac{1}{2}\psi\psi\right) + g(FA^2 - A\psi\psi) + h.c. \tag{2b}$$

where all the fields and couplings are renormalised.[1] This is done by taking the quantum equation of motion for the $\bar{F}$ field obtaining

$$\begin{aligned}\left\langle F(p)\bar{F}(-p)\right\rangle^{-1} = Z + g\int \frac{d^4k}{(2\pi)^4}\, \Gamma_{FAA}\left(p^2,(p-k)^2,k^2\right) \cdot \\ \cdot\left\langle A(k)A(-k)\right\rangle \cdot \left\langle A(p-k)\bar{A}(k-p)\right\rangle\end{aligned} \tag{3}$$

which is represented diagrammatically in Fig. 1. Equation (3) is valid for $m = 0$, this will be sufficient for our discussion below where we study certain ultraviolet aspects of the theory. The supersymmetric invariance of the model allows one to

[1]No separate renormalisation for the coupling is requires (4). And it is assumed that there is no pathological behaviour i.e. $Z \neq 0$.

relate (via the corresponding Ward Identities), the $\langle F\bar{F}\rangle$ propagator to the $\langle A\bar{A}\rangle$ propagator, specifically

$$\left\langle F(p)\bar{F}(-p)\right\rangle^{-1} = \frac{1}{p^2}\left\langle A(p)\bar{A}(-p)\right\rangle^{-1}, \tag{4}$$

so that the Schwinger-Dyson equation becomes

$$\begin{aligned} 1 - Z\langle F\bar{F}\rangle = g\int \frac{d^4k}{(2\pi)^4}\, &\frac{\Gamma_{FAA}\Big(p^2,(p-k)^2,k^2\Big)}{k^2(p-k)^2} \\ \cdot\left\langle F(p)\bar{F}(-p)\right\rangle \cdot \left\langle F(k)\bar{F}(-k)\right\rangle &\left\langle F(p-k)\bar{F}(k-p)\right\rangle . \end{aligned} \tag{5}$$

By the Källen-Lehmann spectral inequality $0 \le Z \le 1$ the left hand side of the above equation is finite. Therefore for consistency the integral on the right hand side must converge. In reference [2] it was claimed that this is impossible and hence it was concluded that the model is inconsistent. We note that nothing is to be gained by analyzing the Schwinger-Dyson equations for the other propagators, as these equations are related directly to (5) via the supersymmetric Ward Identities.

In this letter we reexamine the behaviour of the one particle irreducible three-point function $\Gamma^{FAA}(p^2,(p-k)^2,k^2,m,g,\mu)$ as $k^2 \to \infty$ for fixed p^2. An immediate question which arises is whether the behaviour of Γ^{FAA} as a function of k^2 is given by the renormalisation group equations or whether there are singularities as p^2 and/or $m^2 \to 0$. At least in perturbation theory there are no such singularities. Consider for example the supergraph of Fig. 2(a). If we set p^2 and m^2 equal to zero then there are two propagators with denominators equal to q_2^2 suggesting a singularity as $q_2^2 \to 0$, however the numerator gives us a factor of $\overline{q_2^2}$. In each of the ordinary Feynman diagrams of Fig. 2(b) and 2(c) there is an infrared divergence, these divergences cancel in the sum. The absence of infrared divergences as p^2 and/or $m^2 \to 0$ persists two higher orders in perturbation theory and we assume that it is also true non-perturbatively so that we can proceed with renormalisation group arguments. For our purposes it will be sufficient to set $p = 0$. Using the renormalisation group equations one can show that

$$\Gamma^{FAA}(0, k^2, k^2, m(0), g(0), \mu) = \frac{g(0)}{g(t)}\Gamma^{FAA}(0, e^{-2t}k^2, e^{-2t}k^2, e^{-t}m(t), g(t), \mu) \,. \tag{6}$$

Nicolai and Krasnikov now take the limit $t \to \infty$ and combine it with the result of Iliopoulos and Zumino[4]

$$\Gamma^{FAA}(p^2 = k^2 = 0, g, m) = 2g \tag{7}$$

to argue that the right hand side of (6) is equal to $2g(0)$, and hence that the left hand side does not vanish as $k^2 \to \infty$. This step is delicate however since on the right hand side of equation (6), not only the momentum but also the mass is going to zero. Moreover if $g(t)$ approaches a fixed point as $t \to \infty$ then the momentum and mass in $\Gamma^{FAA}(0, e^{-2t}k^2, e^{-2t}k^2, e^{-t}m(t), g(t), \mu)$ go to zero in the same way, and hence one cannot fairly say that one is taking the zero momentum limit. In perturbation theory even the lowest non-trivial order graphs which contribute to Γ^{FAA} depend on the relative size of k^2 to m^2. For example in the limit $k^2 \gg m^2$ the only super graph which contributes is that of Fig. 2(a) and gives a contribution proportional to

$$k^2 \int \frac{d^4q_1\, d^4q_2}{(2\pi)^8} \frac{1}{\left[q_1^2 + i\epsilon\right]\left[q_2^2 + i\epsilon\right]\left[(q_1 - q_2)^2 + i\epsilon\right]\left[(k - q_1)^2 + i\epsilon\right]} \cdot \frac{1}{\left[(k - q_1 + q_2)^2 + i\epsilon\right]} = -\frac{3}{8}\,\frac{1}{(2\pi)^4}\,\zeta(3)^{(5)} \tag{8}$$

whereas in the case $m^2 \neq 0$, $k^2 = 0$ (relevant for equation (7)) we get zero contribution from all the two loop supergraphs. In intermediate cases we get still different results. Thus we conclude that the application of equation (7) to equation (6) in the limit $t \to \infty$ is not valid.

We would now like to see whether it is possible for $\Gamma^{FAA}(0, k^2, k^2, m(0), g(0), \mu)$ to vanish as $k^2 \to \infty$, and in particular to vanish faster than logarithmically so that the integral in the right hand side of (5) is convergent. The running coupling constant $g(t)$ does not vanish as $t \to \infty$, and we assume that it goes to an ultraviolet fixed point g^* (as do Nicolai and Krasnikov). We rewrite equation (6) as

$$\Gamma^{FAA}(0, e^{2t}k^2, e^{2t}k^2, m(0), g(0), \mu) = \frac{g(0)}{g(t)}\Gamma^{FAA}(0, k^2, k^2, e^{-t}m(t), g(t), \mu) \qquad (9)$$

and shall study this equation for sufficiently large values of momentum so that the masses can be neglected. Thus

$$\Gamma^{FAA}(e^{2t}k^2) = f(g(t)) \qquad (10)$$

where on the left hand wide only the dependence on the momentum is explicitly exhibited and f is some function of $g(t)$. Thus

$$\frac{\partial}{\partial t}\,\Gamma^{FAA}(e^{2t}k^2) = \beta(g)f'(g(t)) \qquad (11)$$

and therefore *a priori*, that is without knowing $f'(g(t))$ and in particular $\beta(g)$, it is clearly possible that Γ^{FAA} vanishes like an inverse power of k^2 as $k^2 \to \infty$. A simple possibility with this consequence is that near the fixed point (g^*) $\beta(g)$ vanishes linearly, i.e. $\beta(g) \simeq A(g^* - g)$.

We have shown that $\Gamma^{FAA}(k^2)$ can vanish sufficiently quickly as $k^2 \to \infty$ for the integrals in the Schwinger-Dyson equations for the propagators to be convergent. This behavior of $\Gamma^{FAA}(k^2)$ leads naturally to the possibility that $\gamma(g^*) = 0$ without the propagator bearing a free propagator and hence the theory being a free theory. There is no inconsistency with the lack of asymptotic freedom, nor with a renormalisation group analysis, for both the massless and massive Wess-Zumino models.

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FIGURE CAPTIONS

1. Schwinger-Dyson equation for the inverse of the propagator $\langle F(p)\bar{F}(-p)\rangle$.

2. (a) A two loop supergraph contributing to the vertex $\phi\phi\phi$ (where ϕ is the chiral superfield consisting of A, ψ and F). In the massless theory this is the only two loop graph.

   (b) and (c) Two Feynman graphs contributing to the vertex FAA.

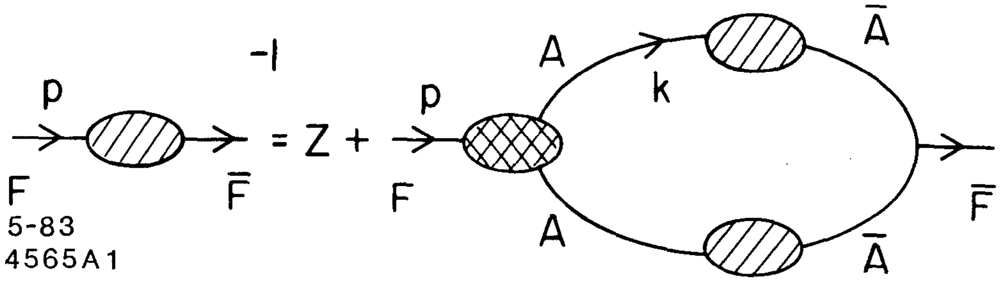

Fig. 1

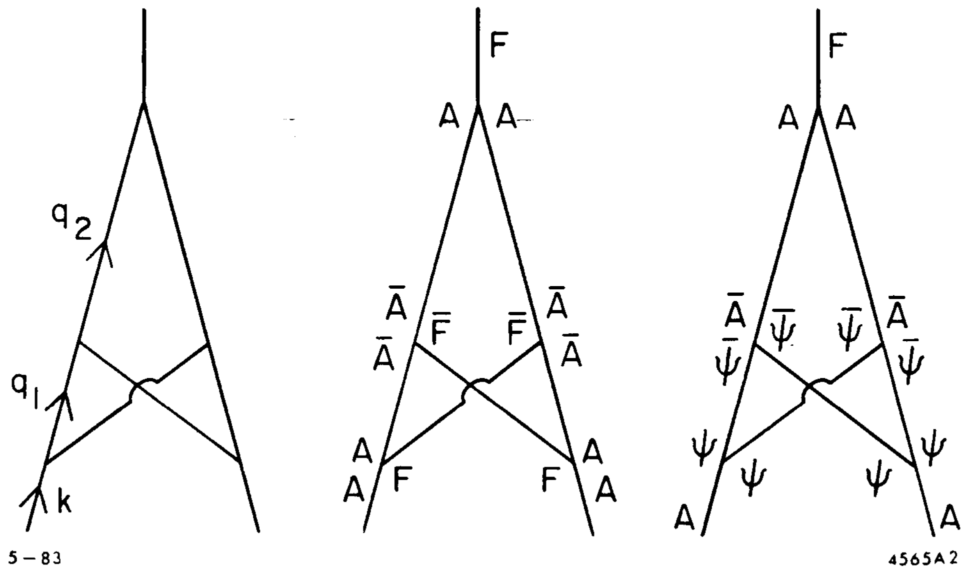

Fig. 2