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PROBLEMS AT THE INTERFACE BETWEEN PERTURBATIVE AND NONPERTURBATIVE QUANTUM CHROMODYNAMICS*

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1. Introduction

Predictions based on perturbative QCD[1] rest on three premises: (1) that hadronic interactions become weak in strength at small invariant separation $r \ll \Lambda_{QCD}^{-1}$; (2) that the perturbative expansion in $\alpha_s(Q)$ is well-defined; and (3) factorization: all effects of collinear singularities, confinement, nonperturbative interactions, and bound state dynamics can be isolated at large momentum transfer in terms of (process independent) structure functions $G_{i/H}(x,Q)$, fragmentation functions $D_{H/i}(z,Q)$, or in the case of exclusive processes, distribution amplitudes $\phi_H(x_i,Q)$.[2]

The assumption of weak hadronic interactions at small separation is consistent with the presumed behavior of confining potentials at short distances, e.g., $V_{conf}(r) \sim \kappa r \lesssim 50$ MeV for $r < 10^{-15} cm$, and the asymptotic freedom property of perturbative QCD ($\beta_0 = 11 - 2/3\, n_f$):

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ell n \frac{Q^2}{\Lambda^2}} < 0.2 \quad \text{for } Q > 20\, GeV,\ r < 10^{-15}\, cm\ . \qquad (1.1)$$

The assumption that the perturbative expansion for hard scattering amplitudes converges has certainly not been demonstrated; in addition, there are serious ambiguities concerning the choice of renormalization scheme and scale choice Q^2 for

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the expansion in $\alpha_s(Q^2)$. We will discuss a new procedure[3] to at least partly rectify the latter problem in Section 2.

In the case of exclusive processes, the factorization of hadronic amplitudes at large momentum transfer in the form of distribution amplitudes convoluted with hard scattering quark-gluon subprocess amplitudes can be demonstrated systematically to all orders in $\alpha_s(Q^2)$.[2,4,5] In the case of inclusive reactions, factorization remains an ansatz; general all-orders proofs do not exist[6] because of the complications of soft initial state interactions for hadron-induced processes; thus far factorization has only been verified[7,6] to two loops beyond lowest order in a regime where the applicability of perturbation theory is in doubt. However, we shall show that a necessary condition[6] for the validity of factorization in inclusive reactions is that the momentum transfer must be large compared to the (rest frame) length of the target. We review the present status of the factorization ansatz in Section 3.

The basic form of the factorization ansatz for inclusive reactions at large momentum transfer is[8]

$$d\sigma(\{H\} \to \{H'\} + X) = \sum_{subprocesses} \int \prod_{H,H'} G_{i/H}(x_i, Q)\, D_{H'/j}(z_j, Q) \times d\sigma_{(a)}\left(x_i P_H, \frac{1}{z_j} P_{H'}; Q\right) \prod_{i,j} dx_i dz_j / z_j \tag{1.2}$$

where the structure and fragmentation probability distributions G and D for each hadron is a function of the parton light cone momentum fractions $x = (k^0 + k^z)/(p_H^0 + p_H^z)$ and the summation is an incoherent sum over all leading hard-scattering QCD subprocesses computed with the partons i, j collinear with the initial and final hadron directions. By definition the subprocess cross section has no collinear singularities so it can be expanded in powers of $\alpha_s(Q^2)$. In general the summation includes higher twist power-law suppressed subprocesses where the scattering partons are multiquark, multigluon, hadron or other QCD composite systems.[9,10] The power-law scaling of such processes, which can be important at the edge of phase-space, is determined by dimensional counting.[9,11] In addition one must allow for multiple collision processes,[12] which can be especially important for high transverse energy triggers.

Radiation collinear up to the scale Q is included in the definition of the structure functions, leading to evolution equations and moments of the form:[1]

$$\mathcal{M}^n_{i/H}(Q) = \int_0^1 dx\, x^{n-1} G_{i/H}(x, Q) = \mathcal{M}^n_{i/H}(Q_0) e^{-\gamma_n \xi(Q, Q_0)} \tag{1.3}$$

where

$$\xi(Q,Q_0)=\frac{1}{4\pi}\int\limits_{Q_0^2}^{Q^2}\frac{d\ell^2}{\ell^2}\alpha_s(\ell^2)\simeq\frac{1}{\beta_0}\ell n\left(\frac{\ell n\,Q^2}{\Lambda^2}\Big/\frac{\ell n\,Q_0^2}{\Lambda^2}\right). \tag{1.4}$$

and γ_n, the standard anomalous dimensions, are independent of the bound state hadron dynamics.

Similarly in the case of exclusive reactions one can isolate the long-distance confinement dynamics from the short-distance quark and gluon hard scattering dynamics — at least to leading order in $1/Q^2$. Hadronic amplitudes take the factorized form[2,4]

$$T=\int\prod_H \phi_H(x_i,Q)\;\hat{T}(x_i,p_H;Q)[dx_i]\;.$$

where $\phi_H(x_i,Q)$ is a universal distribution amplitude which gives the probability amplitude for finding the valence $q\bar{q}$ or qqq in the hadronic wave function collinear up to the scale Q, and $\hat{T}$ is the hard-scattering amplitude for scattering valence quark collisions with the incident and outgoing hadrons.

For example, the pion form factor at large Q^2 takes the form

$$F_\pi(Q^2)=\int\limits_0^1 dx\int\limits_0^1 dy\,\phi^*(x,Q)\;T_H(x,y;Q)\,\phi(y,Q) \tag{1.6}$$

where

$$\phi_\pi(x,Q)=\int^Q\frac{d^2k_\perp}{16\pi^2}\;\psi^Q_{q\bar{q}/\pi}(x,\vec{k}_\perp) \tag{1.7}$$

is the amplitude for finding the q and $\bar{q}$ in the valence state of the pion collinear up to scale Q with light cone longitudinal momentum fractions x and $1-x$, and

$$T_H=\frac{16\pi C_F\alpha_s[Q^2(1-x)(1-y)]}{(1-x)(1-y)Q^2}\left[1+\frac{O[\alpha_s(Q^2)]}{\pi}\right] \tag{1.8}$$

is the probability amplitude for scattering collinear constituents from the initial to the final direction. By definition, T_H contains only transverse momenta greater than Q in loop integrations so that T_H can be expanded in powers of $\alpha_s(Q^2)$. (The superscript Q in $\psi^Q_{q\bar{q}}$ indicates that all internal loop in $\psi_{q\bar{q}}$ are to be cutoff at $k_\perp^2 < Q^2$.) The log Q^2 dependence of the distribution amplitude $\phi(x,Q)$ is determined by the operator product expansion on the light-cone or an evolution equation; its specification at subasymptotic momentum requires the solution to the pion bound state problem. In general we have

$$\phi(x,Q) = x(1-x)\sum_n \phi_n(x,Q_0)\, e^{-\gamma_n \xi(Q,Q_0)} \tag{1.9}$$

where to one loop order, $\phi_n(x,Q_0) = a_n(Q_0)C_n^{3/2}(2x-1)$ are the eigensolutions of the evolution equation, the γ_n are anomalous dimensions[13] analogous to those that appear in Eq. (1.3), and the $a_n(Q_0)$ are determined by the bound state dynamics. The general form of $F_\pi(Q^2)$ is then

$$F_\pi(q^2) = \left|\sum_{n=0}^{\infty} a_n \log^{-\gamma_n}\frac{Q^2}{\Lambda^2}\right|^2 C_F\,\frac{\alpha_s(Q^2)}{Q^2}\left[1 + O\!\left(\frac{\alpha_s(Q^2)}{\pi}\right) + O\!\left(\frac{m}{Q}\right)\right]. \tag{1.10}$$

Similar calculations[14] determine the baryon form factors, decay amplitudes such as $\Upsilon \to B\bar{B}$ and fixed angle scattering processes such as Compton scattering, photoproduction, and hadron-hadron scattering, although the latter calculations are complicated by the presence (and suppression) of pinch singularities.[15] It is interesting to note that $\phi(x,Q^2)$ can be measured directly from the angular $\theta_{c.m.}$ dependence of the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ cross sections at large s.[16] In addition, independent of the form of the meson wave function, we can obtain α_s from the ratio[2]

$$\alpha_s(Q^2) = \frac{F_\pi(Q^2)}{4\pi Q^2 |F_{\pi\gamma}(Q^2)|^2}\left[1 + 0\!\left(\frac{\alpha_s(Q^2)}{\pi}\right)\right] \tag{1.11}$$

where the transition form factor $F_{\pi\gamma}(Q^2)$ can be measured in the two photon reaction $\gamma^*\gamma \to \pi^0$ via $ee \to \pi^0 ee$. Equation (1.11) is in principle one of the cleanest ways to measure α_s [see also Eq. (2.15)]. Higher order corrections in α_s are discussed in Refs. 5,17.

Thus an essential part of the QCD predictions is the hadronic wave functions which determine the probability amplitudes and distributions of the quark and gluons which enter the short distance subprocesses. Computation of the quark and gluon fragmentation function into hadrons require knowledge of the coherent amplitudes which form partons into hadrons. Thus hadronic wave functions provide the link between long distance nonperturbative and short-distance perturbative physics.[18] Eventually, one can hope to compute the wave functions from theory, e.g., from lattice or bag models, or directly from the QCD equations of motion, as we shall outline below. Knowledge of the hadronic wave function also allows the normalization and specification of several types of power law suppressed (higher twist) contributions, such as $1/Q^2$ contributions to the longitudinal structure function of mesons and baryons[17] at $x \to 1$, and direct meson and baryon production subprocesses.[20,21]

The wave function $\psi^{\pi}_{q\bar{q}}(x, k_\perp)$ which appears in Eq. (1.7). is related to the Bethe-Salpeter amplitude at equal "time" $\tau = t + z$ on the light-cone in $A^+ = 0$ gauge.[22] The quark has transverse momentum $k_\perp$ relative to the pion direction and fractional "light-cone" momentum $x = (k^0 + k^3)/(p^0 + p^3) = k^+/p^+$. The state is off the light cone $k^- = k^0 - k^3$ energy shell. In general a hadron state can be expanded in terms of a complete set of Fock state at equal τ:

$$|\pi\rangle = |q\,\bar{q}\rangle\psi_{q\bar{q}} + |q\,\bar{q}\,g\rangle\psi_{q\bar{q}g} + \cdots \tag{1.12}$$

with

$$\sum_n \int [d^2k_\perp]\,[dx]\,|\psi_n(x_i, \vec{k}_{\perp i})|^2 = 1\ .$$

(We suppress helicity labels.) At large Q^2 only the valence state contributes to an exclusive process, since by dimensional counting an amplitude (in a physical gauge) is suppressed by a power of $1/Q^2$ for each constituent required to absorb large momentum transfer. The amplitudes ψ_n are infrared finite for color-singlet bound states. The meson decay amplitude (e.g. $\pi^+ \to \mu^+\nu$) implies a sum rule

$$\frac{a_0}{6} = \frac{1}{2\sqrt{n_c}} f_\pi = \int_0^1 dx\, \phi_\pi(x, Q). \tag{1.13}$$

This result, combined with the constraint on the wave function from $\pi^o \to \gamma\gamma$ requires that the probability that the pion is in its valence state is $\leq 1/4$.[18] Given the $\{\psi_n\}$ for a hadron, virtually any hadronic property can be computed, including anomalous moments, form factors (at any Q^2), etc.

The $\{\psi_n\}$ also determine the basic form of the structure functions appearing in deep inelastic scattering ($a = q, \bar{q}, g$)

$$G_{a/p}(x, Q) = \sum_n \int^Q [d^2k_\perp]\,[dx]\,|\psi_n^Q(x_i, k_{\perp i})|^2\, \delta(x - x_i) \tag{1.14}$$

where one must sum over all Fock states containing the constituent a and integrate over all transverse momentum $d^2k_\perp$ and the light-cone momentum fractions $x_i \neq x_a$ of the spectators. The valence state dominates $G_{q/p}(x, Q)$ at the edge of phase space, $x \to 1$. All of the multiparticle x and $k_\perp$ momentum distributions needed for multiquark scattering processes can be defined in a similar manner. The evolution equation for the $G_a(x,Q^2)$ can be easily obtained from the high $k_\perp$ dependence of the perturbative contributions to ψ.

There are many advantages[22] obtained by quantizing a renormalizable local $\tau = t + z$. These field theory at fixed light-cone time include the existence of an orthornormal relativistic wave function expansion, a convenient τ-ordered perturbative theory, and diagonal (number-conserving) charge and current operators. The central reason why one can construct a sensible relativistic wave function Fock state expansion on the light cone is the fact that the perturbative vacuum is also an eigenstate of the full Hamiltonian. The equation of state for the $\{\psi_n(x_i, k_{\perp i})\}$ takes the form

$$H_{LC}\Psi = M^2\Psi \tag{1.15}$$

where

$$H_{LC} = \sum_{i=1}^{n}\left(\frac{k_\perp^2 + m^2}{x}\right)_i + V_{LC} \quad , \tag{1.16}$$

and V_{LC} is derived from the QCD Hamiltonian in $A^+ = 0$ gauge quantized at equal τ, and Ψ is a column matrix of the Fock state wave functions. Ultraviolet regularization and invariance under renormalization is discussed in Refs. 2,18.

A comparison of the properties of exclusive and inclusive cross sections in QCD is given in Table I. Given the $\{\psi_n\}$ we can also calculate decay amplitudes, e.g. $\psi \to p\bar{p}$ which can be used to normalize the proton distribution amplitudes. The constraints on hadronic wave functions which result from present experiments are given in Ref. 18. An approximate connection between the valence wave functions defined at equal τ with the rest frame wave function is also given in Refs. 7,8,23, so that one can make predictions from nonperturbative analyses such as bag models, lattice gauge theory, chromostatic approximations, potential models, etc. Other constraints from QCD sum rules are discussed in Ref. 24.

It is interesting to note that the higher twist amplitudes such as $\gamma q \to Mq$, $gq \to Mq$, $q\bar{q} \to M\bar{M}$, $q\bar{q} \to Bq$ which can be numerically important for inclusive hadron production reactions at high $x_\perp$ are absolutely normalized in terms of the distribution amplitudes $\phi_M(x,Q)$, $\phi_B(x_i,Q)$, using the same analysis as that used for form factors. In fact "direct" amplitudes[20,21] such as $\gamma q \to Mq$, $q\bar{q} \to Mg$ and $gq \to Mq$ where the meson interacts directly in the subprocess are rigorously related

Table I. Comparison of exclusive and inclusive cross section

Exclusive Amplitudes	Inclusive Cross Sections
$\mathcal{M} \sim \Pi\, \phi(x_i, Q) \otimes T_H(x_i, Q)$	$d\sigma \sim \Pi\, G(x_a, Q) \otimes d\hat{\sigma}(x_a, Q)$
$\phi(x, Q) = \int^Q [d^2k_\perp]\, \psi^Q_{val}(x, k_\perp)$	$G(x, Q) = \sum_n \int^Q [d^2k_\perp]\, [dx]'\, \lvert\psi^Q_n(x, k_\perp)\rvert^2$
Measure ϕ in $\gamma\gamma \to M\bar{M}$	Measure G in $\ell p \to \ell X$
$\sum_{i\epsilon H} \lambda_i = \lambda_H$	$\sum_{i\epsilon H} \lambda_i \neq \lambda_H$
	Evolution
$\frac{\partial \phi(x,Q)}{\partial \log Q^2} = \alpha_s \int [dy]\, V(x, y)\, \phi(y)$	$\frac{\partial G(x,Q)}{\partial \log Q^2} = \alpha_s \int dy\, P(x/y)\, G(y)$
$\lim_{Q\to\infty} \phi(x, Q) = \Pi_i\, x_i \cdot C_{\text{flavor}}$	$\lim_{Q\to\infty} G(x, Q) = \delta(x)\, C$
	Power Law Behavior
$\frac{d\sigma}{dx}(A + B \to C_D) \simeq \frac{1}{s^{n-2}} f(\theta_{c.m.})$	$\frac{d\sigma}{d^2p/E}(AB \to CX) \simeq \sum \frac{(1-x_T)^{2n_s-1}}{(Q^2)^{n_{act}-2}} f(\theta_{c.m.})$
$n = n_A + n_B + n_C + n_D$	$n_{\text{act}} = n_a + n_b + n_c + n_d$
T_H: expansion in $\alpha_s(Q^2)$	$d\hat{\sigma}$: expansion in $\alpha_s(Q^2)$
	Complications
End point singularities	Multiple scales
Pinch singularities	Phase-space limits on evolution
High Fock states	Heavy quark thresholds
	Heavy twist multiparticle processes
	Initial and final state intractions

to the meson form factor since the same moment of the distribution amplitude appears in each case.

At present there appear to be overwhelming evidence that perturbative QCD provides a viable theory of strong interactions at short distances. The evidence extends from e^+e^- annihilation (the scaling and normalization of $R_{e^+e^-}$, 3-jet events, $\psi \to 3$ jets), $\gamma\gamma$ annihilation ($\gamma\gamma \to$Jets, $F_{2\gamma}(x, Q^2)$), deep inelastic lepton scattering (structure function scaling and evolution), lepton pair production (normalization and scaling behavior, $Q_\perp$ growth), exclusive processes (dimensional counting, relative normalization), large transverse momentum hadron reactions (jets, charge correlations reflecting elementary QCD subprocesses), etc. The most interesting anomalies not readily understood in terms of the standard picture are

1. Charm production in hadronic collisions[25]. The $pp \to$ charm cross sections at ISR energies are much larger ($\sigma_c \sim 1\,mb$) and much flatter in x_L than predicted by the usual gluon fusion model ($gg \to c\bar{c}$). Indications for a significant charm quark distribution ($P_{c\bar{c}} \sim 1\%$) at large $x_{Bj} > 0.4$ increasing with W^2 are also suggested by EMC deep inelastic meson scattering measurements. The possibility that hadronic production of charm can be understood in terms of intrinsic charm states in the hadronic wave function is discussed in Ref. 26.

2. The $pp \to pX$ cross section at FNAL energies[27] scales roughly as $E\,d\sigma/d^3p \sim p_T^{-12}\,F(x_T, \theta_{cm})$, which is incompatible with the scaling laws predicted by quark fragmentation into protons derived from leading twist subprocesses. The approximate empirical scaling behavior[28] of the γ/π ratio $\sim p_T^2\,F(x_T, \theta_{cm})$ also hints at significant higher twist contributions for meson production.

3. EMC and SLAC measurements show that simple additivity $F_{2A}(x, Q^2) = A\,F_{2N}(x, Q^2)$ for nuclear structure functions breaks down at a significant level ($\pm 20\%$), a much larger deviation than that expected from shadowing and binding effects.[29] A range of possibilities have been suggested to explain this phenomena, such as anti-shadowing mechanisms,[30] anomalous isobar/meson degrees of freedom in the nucleus,[31] or physical changes of the nucleon quark wave function due to the nuclear environment.[32]

The above experimental anomalies do not really conflict with the basic premise that QCD is the correct theory of hadron interactions. A comprehensive comparison with experiment requires that one allow for all relevant QCD physical effects, including higher twist contributions and nonperturbative effects, particularly in jet fragmentation phenomenology, as well as initial and final state interactions and other non-leading contributions. It now seems apparent that these complications are preventing a detailed, quantitative check of the theory: e.g. determinations of $\alpha_s(Q^2)$ still have uncertainties at the 50% level.[33] Some of the complications which plague present QCD tests are listed in Table II.

Thus in order to really test QCD quantitatively we will need considerable information from nonperturbative dynamics. In particular, a detailed understanding of hadronic wave functions is needed in order to analyze the shape and Q^2 behavior of structure functions, the form of fragmentation distributions in $k_\perp$ and x, the effects of initial state interactions and how they control $k_\perp$ smearing effects, the form of distribution amplitudes needed for analyzing exclusive processes, as well as

Table II

Physics Measurements	QCD Complications
$[R_{e^+e^-}/3\Sigma e_q^2 - 1]$	Needs high precision; smeared data.
Structure function evolution	Higher twist terms; heavy quarks threshold effects; EMC effect.
$e^+e^- \to Jets$	Fragmentation model dependence.
$\Upsilon \to$ hadrons (3 jets)	Poor convergence of perturbation theory.
$p\bar{p} \to \ell\bar{\ell}X$	Poor convergence of perturbation theory (k-factor); no proof of factorization beyond two loop.
$pp \to HX,\ \gamma X$	Nonperturbative smearing corrections; initial and final state interactions; higher twist terms; k-factors.
$F_{2\gamma}(x, Q^2)$	Higher order QCD corrections; relation to vector meson dominated hadronic component not well understood.
$G_M^P(Q^2)$ and other exclusive channels	Higher order corrections not known; complications from end-point region; soft-wave function background; pinch singularities in hadron–hadron scattering.

calculating most power-law suppressed higher-twist contributions. Solutions to this problem await further progress in solving the light-cone equation of state or the equivalent. In the next section we will discuss a new approach for solving the scale and scheme ambiguities of perturbative QCD expansions. The present status of the factorization problem for inclusive hadronic reactions is discussed in Section 3.

2. Perturbative Expansions in Gauge Theories[34]

One of the most serious problems confronting the quantitative interpretation of QCD is the ambiguity concerning the setting of the scale in perturbative expansions. As an example, consider the standard perturbative expansion for the e^+e^- annihilation cross section in ($\overline{MS}$ scheme)

$$\left[\frac{R_{e^+e^-}(Q^2)}{3\sum e_q^2}-1\right]=\frac{\alpha_s^{\overline{MS}}(Q^2)}{\pi}\left[1+(1.98-0.115\,n_f)\frac{\alpha_s}{\pi}+O\left(\frac{\alpha_s^2}{\pi^2}\right)+\cdots\right]. \tag{2.1}$$

were n_f is the number of light fermion flavors with $m_f^2 \ll Q^2$. Note that if one chooses a different scale $Q \to \kappa Q$ in the argument $\alpha_s^{\overline{MS}}$ then the coefficient of all subsequent terms are changed. If this were a true ambiguity of QCD then higher order perturbative coefficients are not well-defined; furthermore, there is no clue toward the convergence rate of the expansion.

Is the scale choice really arbitrary? Certainty it is not arbitrary in QED. The running coupling constant is defined as

$$\alpha(Q^2)=\frac{\alpha(Q_0^2)}{1-\alpha(Q_0^2)[\pi(Q^2)-\pi(Q_0^2]} \tag{2.2}$$

where $\pi(Q^2)$ sums the proper contributions to the vacuum polarization. In lowest order QED

$$\pi(Q^2)-\pi(Q_0^2)\simeq n_f\frac{\alpha}{3\pi}\log\frac{Q^2}{Q_0^2}\,. \tag{2.3}$$

The use of the running coupling constant simplifies the form of QED perturbative expansions. For example, the light flavor contributions to the muon anomalous moment is automatically summed when we use the form

$$a_\mu=\frac{\alpha(Q^*)}{2\pi}+0.327\ldots\frac{\alpha^2(Q^*)}{\pi^2}+\cdots \tag{2.4}$$

where the scale Q^* is chosen such that[35]

$$\alpha(Q^*)=\frac{\alpha}{1-\frac{\alpha}{\pi}\left(\frac{2}{3}\log\frac{m_\mu}{m_e}-\frac{25}{18}\right)+\cdots}\,. \tag{2.5}$$

The scale Q^* in Eq. (2.4) is in fact unique; it is defined via Eq. (2.5) in such a way as to automatically sum all vacuum polarization contributions. The form of Eq. (2.4) is invariant as one changes the overall scale (e.g. $m_\mu \to m_\tau$) as we pass each new flavor threshold, if the vacuum polarization contribution of each new flavor is included in (2.5). Note, however, that the light-by-light contribution to a_μ, which appears in order α^3/π^3 from light-flavor box graphs, is not included in $a_\mu(Q^*)$ since this contribution is not part of the photon propagator renormalization and it does not contribute as a geometric series in higher order. Furthermore, for some QED processes, e.g. orthopositronium decay

$$\Gamma_{\text{orthopositronium}\to 3\gamma} \propto \alpha^3 m_e\left[1 - 10.3\frac{\alpha}{\pi} + \cdots\right] \tag{2.6}$$

there are no vacuum polarization corrections to this order, so the large coefficient cannot be avoided by resetting the scale in α. In QED, the running coupling constant simply sums $\alpha(Q)$ vacuum polarization contributions; in effect there are no scale-ambiguities for setting the scale. Similarly in QCD, it must be true that the vacuum polarization due to light fermions should be summed in $\alpha_s(Q)$. In fact, as we show below, this natural requirement automatically and consistently fixes the QCD scale for the leading non-trivial order in α_s for most QCD processes of interest.

In QCD the running coupling constant satisfies

$$\alpha_s(Q^*) = \frac{\alpha_s(Q)}{\left[1 + \frac{\beta_0}{2\pi}\alpha_s(Q)\ell n\left(\frac{Q^*}{Q}\right) + \cdots\right]} \tag{2.7}$$

where $\beta_0 = 11 - 2/3\, n_f$. Consider any observable $\rho(Q)$ which has a perturbative expansion at large momentum transfer Q. For definiteness we choose the $\overline{MS}$ renormalization scheme to define the renormalization procedure, and adopt the canonical form,

$$\rho(Q) = \frac{\alpha_{\overline{MS}}(Q)}{\pi}\left[1 + \frac{\alpha_{\overline{MS}}}{\pi}(A_{vp}n_f + B) + \cdots\right].$$

The second order coefficient can also be written as $-\frac{3}{2}\beta_0 A_{vp} + \left(\frac{33}{2}A_{vp} + B\right)$. The requirement that the fermion vacuum polarization contribution is absorbed into the running coupling constant plus the fact that $\alpha(Q)$ is a function of n_f through β_0 then uniquely sets the scale of the leading order coefficient:

$$\rho(Q) = \frac{\alpha_{\overline{MS}}(Q^*)}{\pi}\left[1 + \frac{\alpha_{\overline{MS}}}{\pi}C_1 + \cdots\right] \tag{2.8}$$

where $Q^* \equiv Q\, e^{3A_{vp}}$ and $C_1 = B + \frac{33}{2}A_{vp}$. For example, from Eq. (2.1) we have

$$\rho_R(Q) \equiv \left(\frac{R_{e^+e^-}(Q^2)}{3\Sigma e_q^2} - 1\right) = \frac{\alpha_{\overline{MS}}(0.71\,Q)}{\pi}\left[1 + 0.08\,\frac{\alpha_{\overline{MS}}}{\pi} + \cdots\right].$$

Thus Q^* and C_1 are determined unambiguously within this renormalization scheme and are each n_f-independent. Note that the expansion is unchanged in form as one passes through a new quark threshold. Given any renormalization scheme, the above procedure automatically fixes the scale of the leading order coefficient for the non-Abelian theory. In higher orders one must carefully identify the correct $n_f\, A_{vp}$ terms; e.g. distinguish light-by-light or trigluon fermion loop contributions not associated with the definition of $\alpha_s(Q)$.

If we apply the procedure (2.8) to the QCD interaction potential between heavy quarks, then one obtains

$$V(Q) = -C_F \frac{4\pi\, \alpha_{\overline{MS}}(Q^*)}{Q^2}\left[1 - 2\frac{\alpha_{\overline{MS}}}{\pi} + \cdots\right] \tag{2.10}$$

where $Q^* = e^{-5/6}\, Q \cong 0.43Q$. Thus the effective scale Q^* in $\overline{MS}$ is $\sim 1/2$ of the "true" momentum transferred by $V(Q)$.

The results (2.9), (2.10) suggest that $R_{e^+e^-}$ or $V(Q)$ can be used to define and normalize $\alpha_s(Q)$. Such empirical definitions serve as a renormalization scheme alternative to $\overline{MS}$. For example, in principle we can define

$$\frac{\alpha_R(Q)}{\pi} \equiv \left[\frac{R_{e^+e^-}(Q)}{3\Sigma e_q^2} - 1\right] \tag{2.11}$$

as a physical definition of $\alpha_s(Q)$ analogous to the Coulomb scattering definition of α in QED. Note then that $\alpha_R(Q)$ and $\alpha_{\overline{MS}}(0.71\, Q)$ are effectively interchangeable.

A further benefit of the "automatic scale fixing procedure" is that the physical characteristics of the effective scale can be understood. For example, the evolution of the non-singlet moments is uniquely written in the form

$$\frac{\partial}{\partial \ell n\, Q^2}\ell n\, M_n(Q^2) = -\frac{\gamma_n^0}{8\pi}\alpha_{\overline{MS}}(Q^*)\left[1 - \frac{\alpha_{\overline{MS}}(Q_n^*)}{\pi}C_n + \cdots\right] \tag{2.12}$$

with

$$\begin{aligned} Q_2^* &= 0.48\, Q\ , \quad C_2 = 0.27 \\ Q_{10}^* &= 0.21\, Q\ , \quad C_{10} = 1.1 \end{aligned} \tag{2.13}$$

and $Q_n^* \sim Q/\sqrt{n}$ for large n. This dependence on $\sqrt{n}$ reflects the physical fact that the phase space limit on the gluon radiation causing the Q^2-evolution decreases in the large n, $x \to 1$ regime.

In the case of Υ decay, the scale-fixed form of the Lepage-Mackenzie[36] calculation is

$$\frac{\Gamma(\Upsilon \to hadrons)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = \frac{10(\pi^2-9)}{81\pi e_b^2} \frac{\alpha_{\overline{MS}}(Q^*)}{\alpha_{QED}^2}\left(1 - \frac{\alpha_{\overline{MS}}}{\pi}(14.0 \pm 0.5) + \cdots\right) \quad (2.14)$$

where $Q^* = 0.157 M_\Upsilon$. Thus, just as in the case for orthopositronium, a large second order coefficient is unavoidable. Other procedures which reduce or eliminate this coefficient by an ad hoc procedure are clearly incorrect if they are invalid in QED.

As we have discussed in Section 1, there is presently no really reliable method for determining $\alpha_s(Q)$ to better than $\pm$ 50% accuracy. The $\Gamma(\Upsilon \to 3g)/\Gamma(\Upsilon \to e^+e^-)$ ratio appears to be unreliable in view of the poor convergence of the perturbative expansion. A somewhat more hopeful process is the direct γ branching ratio:

$$\frac{\Gamma(\Upsilon \to \gamma_D + hadrons)}{\Gamma(\Upsilon \to hadrons)} = \frac{36\, e_b^2}{5} \frac{\alpha_{QED}}{\alpha_{\overline{MS}}(Q^*)}\left[1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi}(2.2 \pm 0.6) + \cdots\right] \quad (2.15)$$

where again $Q^* = 0.157\ M_\Upsilon$.

The automatic scale setting procedure should have general utility for evaluating the natural scale in a whole range of physical processes. In the case of some reactions such as hadron production $H_A H_B \to H_C X$ at large p_T each parton structure function has its own scale $\sim Q^2(1-x_i)$. In addition each hard scattering amplitude has a scale determined by corresponding fermion loop vacuum polarization contributions.

3. Factorization for High Momentum Transfer Inclusive Reactions[37]

One of the most important problems in perturbative QCD in the last two years has been to understand the validity of the standard factorization ansatz for hadron-hadron induced inclusive reactions. Although factorization is an implicit property of parton models, the existence of diagrams with color exchanging initial state interactions at the leading twist level has made the general proof of factorization in QCD highly problematical.

To see the main difficulties from a physical perspective, consider the usual form assumed for massive lepton pair production [see Fig. 1(a)]

$$\frac{d\sigma}{dx_1 dx_2}(H_A H_B \to \ell\bar{\ell}X) = \times \frac{1}{3}\frac{4\pi\alpha^2}{3Q^2}\sum_i Q_i^2\left[q_A^{(1)}(x_i,Q)\,\bar{q}_B^{(2)}(x_2,Q) + (1 \to 2)\right] \quad (3.1)$$

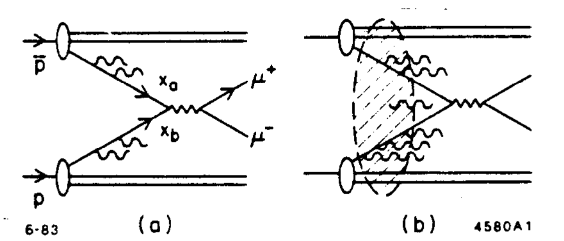


Fig. 1. (a) Gluon emission associated with QCD evolution of structure functions for the Drell-Yan process, $p\bar{p} \to \mu^+\mu^- X$. (b) Gluon emission associated with initial state interactions for the Drell-Yan process. The shaded area represents elastic and inelastic scattering of the incident quarks.

The factorization ansatz identifies the Q^2-evolved quark distributions q_A and $\bar{q}_A$ with those measured in deep inelastic lepton scattering on H_A and H_B. However, for very long targets the initial-state hadronic interactions occurring before the $q\bar{q} \to \ell\bar{\ell}$ annihilation certainly lead to induced radiation and energy loss, secondary beam production, transverse momentum fluctuations, etc. – i.e.: a profound modification of the incoming hadronic state [see Fig. 1(b)]. Since the structure functions associated with deep inelastic neutrino scattering are essentially additive in quark number even for macroscopic targets, Eq. (3.1) can obviously not be valid in general. At the least, an explicit condition related to target length must occur. The original proofs of factorization in QCD for the Drell-Yan process ignored the (Glauber) singularities associated with initial state interactions and thus had no length condition.

The potential problem and complications associated with "wee parton" exchange in the initial state were first mentioned by Drell and Yan[38] in their original work. Collins and Soper[39] have noted that proofs of factorization for hadron pair production in $e^+e^- \to H_A H_B X$ could not be really extended to $H_A H_B \to \ell\bar{\ell}X$ because of the complications of initial state effects. Possible complications associated with nonperturbative interaction effects were also discussed by Ellis *et al.*[40] More recently Bodwin, Lepage, and I[6,41] considered the effects initial state interactions as given by perturbative QCD and showed that specific graphs such as those in Fig. 2 lead to color exchange correlations as well as $k_\perp$ fluctuations. We also showed that induced hard collinear gluon radiation is indeed suppressed for incident energies large compared to a scale proportional to the length of the target. More recently, the question of the existence of color correlations on perturbative QCD has now been addressed systematically to two loop order by Lindsay *et al.*[7] and by Bodwin *et al.*[6] One finds that because of unitarity and local gauge invariance to two loop order the factorization theorem for $d\sigma/dQ^2\,dx_L$ is correct when applied at high energies to color singlet incident hadrons; more general proofs beyond two loop order await further work. We discuss the progress in this area at the end of this section.

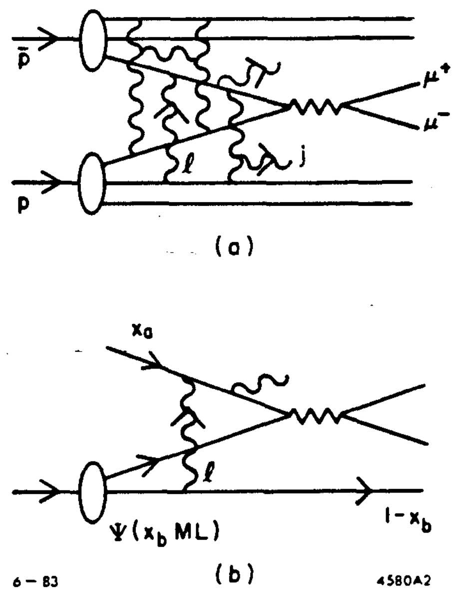


Fig. 2. (a) Representation of initial state interactions in perturbative QCD.
(b) Simplest example of induced radiation by initial state interactions in $q\pi \to \ell\bar{\ell}X$. Two different physical radiation process are included in this Feynman amplitude depending on whether the intermediate state before or after the gluon emission is on-shell. The two bremsstrahlung processes destructively interfere at energies large compared to a scale proportional to the target length L.

In addition to the above initial state interaction there are additional potential infrared problems in the non-Abelian theory associated with the breakdown of the usual Block-Nordsiek cancellation for soft gluon radiation. The work of Ref. 42 showed that any observable effect is suppressed by powers of s at high energies, again to at least two loop order.[43]

In addition to these problems the high transverse momentum virtual gluon corrections to the $q\bar{q} \to \ell\bar{\ell}$ vertex lead to relatively large radiative corrections of order $[1 + \pi^2 C_F(\alpha_s(Q^2)/\pi)]$.[44] It is usually assumed that such corrections exponentiate. As in the case of the $\Upsilon \to 3g$ problem, these corrections spoil the convergence of the perturbation theory and cannot be eliminated by choice of scale or scheme.

The remarkable feature of the QCD calculation is the fact that factorization is not destroyed by induced radiation in the target for high energy beams. This can be understood in terms of the "formation zone" principle of Landau and Pomeranchuk:[45] a system does not alter its state for times short compared to its natural scale in its rest frame. More specifically for QCD (in the Glauber/classical scattering region), consider the diagrams for induced radiation for quark-pion scattering shown in Fig. 2(b). Here $\ell^{\pm} = \ell^0 \pm \ell^3$, $y = \ell^+/p_B^+$, $x_a = p_a^-/p_A^-$ are the usual light-cone variables. The Feynman propagators of the line before and after radiation are proportional to $y - y_1 + i\epsilon$ and $y - y_2 + i\epsilon$, where the difference of the pole contribution is $y_1 - y_2 = M^2/x_a s$, and M^2 is the mass of the quark-gluon pair after bremsstrahlung. Using partial fractions, the gluon emission amplitude is then proportional to

$$\int_0^1 dy\, \psi[(x_b - y)M_nL]\left[\frac{1}{y - y_1 + i\epsilon} - \frac{1}{y - y_2 + i\epsilon}\right] \tag{3.2}$$

where we have indicated the dependence on the target wave function on target length. The two poles thus cancel in the amplitude if $(\mathcal{M}^2/x_a s) M_N L \ll 1$; i.e. the radiation from the two Glauber processes destructively interfere and cancel for quark energies large compared to the target length. If we take $\mathcal{M}^2 \sim \mu^2$ finite, then since $Q^2 = x_a x_b s$, the condition for no induced radiation translates to

$$Q^2 \gg x_b M_N L \mu^2 \,. \tag{3.3}$$

Taking $\mu^2 \sim 0.1\ GeV^2$, this is $Q^2 \gg x_b (0.25\, GeV^2)\ A^{2/3}$; thus one requires $Q^2 \gg x_b (10\ GeV^2)$ to eliminate induced radiation in Uranium targets.

Equation (3.3) is a new necessary condition for QCD factorization; it is also a prediction that a new type of nuclear shadowing occurs for low Q^2 lepton-pair production. If this condition is not met then the cancellations found in Ref. 7, for example, fail. The same length condition affects all sources of hard collinear radiation induced by initial or final state interactions of the hadrons or quarks in a nucleus; i.e., effectively hard collinear radiation occurs outside the target at high energies. In particular, the fast hadron production from jet fragmentation in $\ell p \to \ell HX$ occurs outside the target. In the case of very long or macroscopic targets the induced radiation destroys any semblance of factorization.

Although induced hard collinear radiation cancels at high energies, the basic processes of $k_\perp$ fluctuations from elastic collisions and induced central radiation [e.g. Fig. 2(a) with $j_z \sim m/\sqrt{s}$ in the CM] do remain. One expects that the main effects of initial state interactions can be represented by an eikonal picture where the hadronic wave functions are modified by a phase in impact space (see Fig. 3):

$$\psi_A(x_s, \vec{z}_{a\perp})\psi_B(x_b, \vec{z}_{\perp b}) \to \psi_A(x_a, \vec{z}_{a\perp})\psi_B(x_a, \vec{z}_{b\perp})U(\vec{z}_{\perp i}) \,. \tag{3.4}$$

Here

$$U(\vec{z}_{\perp b}) = P_T \exp\left\{-i \int_{-\infty}^{0} d\tau\, H_I(z_\perp, \tau)\right\} \tag{3.5}$$

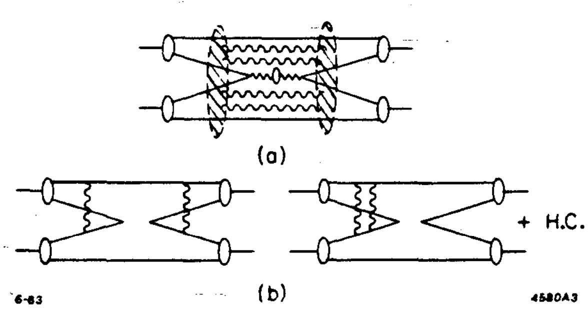


Fig. 3. (a) Representation of initial state interactions in the Drell-Yan cross section $d\sigma/dQ^2dx$. (b) Example of two-loops initial state interactions which cancel by unitarity in an Abelian gauge theory. In QCD these two contributions have different color factors.

includes elastic and soft inelastic collisions which occur up to the time $\tau = 0$ of the $q\bar{q}$ annihilation. The eikonal leads to an increased transverse smearing of the lepton pair and increased associated radiation in the central region proportional to the number of collisions ($A^{1/3}$) of the quark in the target. For a nucleus we thus predict

$$\Delta\langle Q_\perp^2\rangle \propto A^{1/3}\,, \quad \Delta\frac{dN}{dy} \propto A^{1/3} \tag{3.6}$$

In the case of an Abelian gauge theory the integrated cross section

$$\int \frac{d\sigma}{dQ^2\,dx_L\,d^2Q_\perp} = \frac{d\sigma}{dQ^2dx_L} \tag{3.7}$$

in unchanged because of unitarity, $U^\dagger(z_\perp)U(z_\perp) = 1$. See Fig. 3(b). Thus for an Abelian theory the increased production at large $Q_\perp$ from initial state interactions must be compensated by a depletion at low $Q_\perp$.

In general, initial state interactions will have a strong modifying effect on all hadron-hadron cross sections which produce particles at large transverse momentum simply because of the $k_\perp$ smearing of very rapidly falling distributions. The initial state exchange interactions combine with the quark and gluon $k_\perp$ distributions intrinsic to the hadron wave functions as well as that induced by the radiation associated with QCD evolution to yield the total $k_\perp$ smearing effect. The unitarity structure of the initial state eikonal interactions provides a finite theory of $k_\perp$ fluctuations even when the hard scattering amplitude is singular at zero momentum transfer.

In a non-Abelian theory the eikonal unitary matrix $U(z_\perp)$ associated with the initial state interactions is a path-color-ordered exponential integrated over the paths of the incident constituents. Since U is a color matrix it would not be expected to commute with the Drell-Yan $q\bar{q} \to \ell\bar{\ell}$ matrix element

$$U^\dagger \mathcal{M}_{DY}^\dagger \mathcal{M}_{DY} U \neq \mathcal{M}_{DY}^\dagger \mathcal{M}_{DY}\ .$$

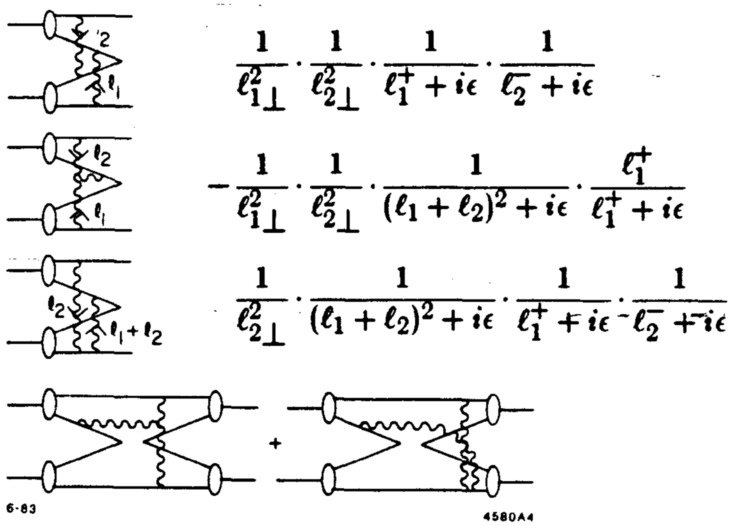
Thus unless U is effectively diagonal in color, the usual color factor $1/n_c$ in $d\sigma(q\,\bar{q} \to \ell\bar{\ell})$ would be expected to be modified. In principle, this effect could change $1/n_c$ to n_c or 0 without violating unitarity, although, as shown by Mueller,[46] the deviation from $1/n_c$ will be dynamically suppressed; hard gluon radiation at the subprocess vertex leads to asymptotic Sudakov form factor suppression of the color correlation effect.

Despite these general possibilities, it has now been shown that the color correlation effect actually cancels in QCD at least through two loop order, although it is present in individual diagrams. The cancellation in two loops was first demonstrated in perturbation theory by Lindsay, Ross, and Sachrajda[7] for scalar quark QCD interactions in both Feynman and light-cone gauge, and was subsequently confirmed in Feynman gauge by Bodwin *et al.*[6] A detailed physical explanation of the two-loop cancellation is not known; it seems to be a consequence of both causality at high energies and local gauge invariance, although neither by itself is sufficient. We also find that the cancellation breaks down at low energies or for long targets when condition (3.3) is not satisfied. It also fails in the case of spontaneous broken gauge theories with heavy gauge boson exchange because the trigluon graph is suppressed.

An example of the nature of the color correlation cancellations is shown in Fig. 4 for $\pi\pi \to \ell\bar{\ell}X$. The diagrams shown are a gauge-invariant distinct class which have a non-trivial non-Abelian color factor and involve interactions with each of the incident spectators. The generality of the pion wave function precludes shifting of the transverse momentum interactions to other graphs. The various virtual two-gluon exchange amplitudes interfering with the zero gluon exchange amplitude each produces a $C_F C_A$ contribution which cancel in the sum. On the other hand, the imaginary part of the virtual graphs gives a non-zero contribution which potentially could lead to a color correlation at four loops. However, we find that even the imaginary part is cancelled when one includes the real emission diagrams of Figs. 4(d) and 4(e). Explicitly the sum of all the virtual and real emission amplitudes is proportional to

$$\left(C_F^2 - \frac{C_F C_A}{2}\right) \frac{2\vec{\ell}_{1\perp} \cdot \vec{\ell}_{1\perp}}{\ell_{1\perp}^2 \ell_{2\perp}^2} \frac{1}{\ell_1^+ + i\epsilon} \frac{1}{\ell_2^- + i\epsilon} \times \frac{1}{\ell_1^+ \ell_2^- - (\vec{\ell}_{1\perp} + \vec{\ell}_{2\perp})^2 - i\mathcal{E}(-\ell_1^+)} \tag{3.9}$$

The integration over ℓ_2^- then leads to zero contributions for the leading power behavior.



$$\frac{1}{\ell_{1\perp}^2}\cdot\frac{1}{\ell_{2\perp}^2}\cdot\frac{1}{\ell_1^+ + i\epsilon}\cdot\frac{1}{\ell_2^- + i\epsilon}$$

$$-\frac{1}{\ell_{1\perp}^2}\cdot\frac{1}{\ell_{2\perp}^2}\cdot\frac{1}{(\ell_1+\ell_2)^2 + i\epsilon}\cdot\frac{\ell_1^+}{\ell_1^+ + i\epsilon}$$

$$\frac{1}{\ell_{2\perp}^2}\cdot\frac{1}{(\ell_1+\ell_2)^2 + i\epsilon}\cdot\frac{1}{\ell_1^+ + i\epsilon}\cdot\frac{1}{\ell_2^- + i\epsilon}$$

Fig. 4. Representative active spectator initial state interactions for $\pi\pi \to \ell\bar{\ell}X$ in QCD involving $C_F C_A$ evaluated in Feynman gauge. The real part of the two loop contributions represented by (a), (b), (c) (including mirror diagrams) vanishes at high energies. The imaginary parts cancel against the gluon emission contribution represented in (d) and (e).

More generally, the proof of factorization of the Drell-Yan cross section can be divided into two distinct steps, as indicated in Fig. 5. The first step is to prove that every contribution to initial state interactions in hadron-hadron scattering can be written as the convolution of two "eikonal-extended" structure functions as indicated in Fig. 5(a). This is the "weak-factorization" ansatz proposed by Collins, Soper, and Sterman[47] where each structure function has a eikonal factor attached which includes all of the elastic and inelastic initial state interactions of the corresponding incident annihilating quark or anti-quark. Explicitly, the eikonal-extended structure function of the target system A is defined as[47]

$$P_{q/A}(x,k_\perp) = \frac{1}{2(2\pi)^3}\int dy^- \int d^2y_\perp\, e^{i(xP_A^+ y^- - \vec{k}_\perp\cdot\vec{y}_\perp)} \times \langle A|\,\bar{\psi}_{DY}(0,y^-,\vec{y}_\perp)\gamma^+\psi_{DY}(0,0,\vec{0}_\perp)|A\rangle \tag{3.10}$$

where

$$\Psi_{DY}(y^\nu) = P\exp -ig\int_{-\infty}^{0} d\lambda\, n\cdot A(y^\nu+\lambda n^\nu)\psi(y^\nu)$$

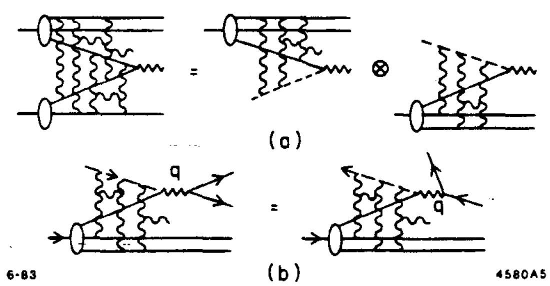


Fig. 5. (a) Schematic representation of the general decomposition required to prove weak factorization to general orders in QCD. The dotted line corresponds to the eikonal line integral of Eq. (3.10). Vertex corrections which modify the hard scattering amplitude are not shown. These provide a separate factor on the right hand side of 5(a). (b) The relationship between Drell-Yan and deep inelastic lepton scattering eikonal-extended structure functions required to prove factorization.

and n^μ is chosen such that $n \cdot \ell = 2\ell^3$ in the center-of-mass frame. The path-ordered exponential contains all of the interactions of the eikonal anti-quark line with the color gauge field along the incident $\hat{z}$ direction up to the point of annihilation.

Recently, we have in fact verified[6] that the weak factorization ansatz is correct through two loops in perturbation theory for $\mathcal{M}(A+B \to \ell\bar{\ell}X)$ despite the complicated color-topological structure of the contributing diagrams. The proof relies on splitting each Feynman amplitude into separate structure functions using identities of the form

$$\frac{1}{A\ell^+ + i\epsilon}\,\frac{1}{-B\ell^- + i\epsilon} = \left(\frac{1}{A\ell^+ + i\epsilon}\,\frac{1}{B} + \frac{1}{-B\ell^- + i\epsilon}\,\frac{1}{A}\right)\frac{1}{\ell^+ - \ell^- + i\epsilon} \tag{3.11}$$

and then analytically continuing each contribution out of the Glauber regime to either large ℓ^- or large ℓ^+, corresponding to exchange gluons collinear with the beam or target, respectively. Finally, the use of collinear Ward identities allows one to organize gauge-related diagrams into the desired weak factorization form. We are continuing efforts to try to extend the proof beyond two loop order in QCD.

The second step required to prove factorization is to show that the structure function (3.10) is actually identical to the corresponding eikonal-extended structure function for deep inelastic-lepton-hadron scattering which includes a post-factor for the final state interactions of the struck quark [see Fig. 5(b)]. This becomes intuitively obvious when one examines moments of the two structure functions. These moments differ only by terms proportional to powers of integral $\int_{-\infty}^{0} dz\, E_z(z)$, where E_z is the longitudinal component of the chromo-electric field along the eikonal line. In the center of momentum frame the hadron has ultrarelativistic momentum along the z axis, and consequently the Lorentz transformed longitudinal electric fields in the hadron are vanishing small. Thus all the moments and therefore the structure functions themselves become identical as $Q \to \infty$. Physically, the effective equality of the structure functions implies that the color fluctuations generated by initial and final interactions at high energies in massive lepton pair production and deep inelastic lepton scattering are basically equivalent.[48]

At this point there is no convincing counterexample to standard QCD factorization for hadron-induced large momentum transfer reactions; on the other hand there is no proof beyond two loop order for non-Abelian theories. Clearly if factorization is a general feature of gauge theories, then it is a profound feature which demands explanation in fundamental terms.[49] In any event, the initial state interactions lead to new physical phenomena for the $Q_\perp$ distributions, especially the nuclear number dependence as well as predictions for associated particle production. Furthermore, color correlations and breakdown of factorization explicitly occur for power-law suppressed contributions which are sensitive to the length scale of the target. Such effects should be measureable for heavy nuclear targets at moderate Q^2.

A detailed discussion including applications to other processes will be discussed in Ref. 50. Here we will only mention the following.

1. If color correlations exist at higher loop order, they will be inevitably suppressed by Sudakov form factors.[46]

2. Exclusive processes factorization is unaffected at the leading power law by initial or final state interactions.[6] Physically, the hard scattering exclusive amplitudes involve only that part of the hadron wave function which corresponds to valence quarks at separation $b_\perp \sim O(1/Q)$. Strong color cancellations eliminate strong interactions of these "small" color singlet configurations. This prediction can be tested experimentally by checking whether quasi-elastic large momentum transfer exclusive reactions occur in nuclei without target-induced $k_\perp$ smearing or radiation.[51] The color transparency of small color singlet systems can also be tested by observing interactions of heavy quark anti-quark states, and also in a very interesting manner using diffractive jet phenomena of hadrons in nuclei.[52]

3. It is important to note that aside from power-law suppressed contributions, "direct" photons or hadrons do not suffer initial or final state interactions.[6] For example direct mesons produced at large transverse momentum in the subprocess $gq \to M_D q$ or direct baryons produced by the subprocess $qq \to$

B_{Dq}, have suppressed color hadronic interactions to leading order in $1/p_T^2$. Thus one can use direct photon reactions, photoproduction, Compton scattering, and direct hadron interactions especially the A-dependence of the cross sections to eliminate and effectively isolate the effects of initial and final state interactions.

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