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DISTRIBUTIONS FOR $Z^0 \rightarrow e^+e^-\gamma$ *

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ABSTRACT

Simple analytical expressions are derived for $Z^0 \rightarrow e^+e^-\gamma$ and $q\bar{q} \rightarrow Z^0 \rightarrow e^+e^-\gamma$ at the Z^0 peak. The integrated decay rate is computed in a configuration where one of the leptons has a momentum much larger than the other. The propability of such an internal bremsstrahlung is found to be between 1-2% depending on the experimental cuts.

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The purpose of this letter is to derive simple analytical expressions for the radiative Z^0 decay into electrons. The motivation is clear; there is a candidate for the Z^0 at the CERN $p\bar{p}$ collider where however one of the electrons has a momentum of about 50 GeV while the other charged track has only 8 GeV.

The decay rate for $Z^0 \to e^+e^-\gamma$ is obviously suppressed by a factor α/π with respect to the non-radiative decay but there could be an enhancement due to the emission of a hard collinear photon giving a factor $\ell n(m_e^2/M_0^2)$. Indeed it is well known[1] that large logarithmic terms involving the lepton masses cancel in the total decay rate, including virtual plus real corrections, only when the momenta of the two leptons and photon are unrestricted. Hence we compute the integrated rate for $Z^0 \to e^+e^-\gamma$ within the standard $SU(2) \times U(1)$ model assuming for definiteness that no charge is measured and imposing kinematical cuts such that one of the fermions has energy greater than some threshold value while the other has energy lower than some other value.

$\Gamma_0(Z^0 \to e^+e^-)$ is given by

$$\Gamma_0 = \frac{\alpha}{48}\,\frac{v^2+1}{s_\theta^2 c_\theta^2}\,M_0 \qquad v = 4s_\theta^2 - 1$$

where $s_\theta(c_\theta)$ denotes the sin (cos) of the weak mixing angle. For $Z^0(q) \to e^+(p_1) + e^-(p_2) + \gamma(k)$ we compute the double differential decay rate

$$\frac{d^2\Gamma}{ds\,dt} = \frac{\alpha}{\pi}\,\Gamma_0 F(s,t)$$

where $s\,M_0^2 = -(p_1+p_2)^2, t\,M_0^2 = -(q-p_1)^2$ and $u\,M_0^2 = -(p_1+k)^2 = (1-s-t)M_0^2$, $F(s,t)$ can be cast in the form

$$F(s,t) = F_+(s,t) + F_-(s,t) + F_{int}(s,t) + F_m(s,t)\ .$$

$F_\pm$ denotes the contribution from the square of the diagram where $e^\pm$ emits the photon, F_{int} is their interference and F_m contains terms proportional to $m_e^2/(p_{1,2}\cdot k)^2$.

$$F_+(s,t) = 2\,\frac{1-s}{u} + s - 2\ ,\ F_-(s,t) = F_+(s,u)$$

$$F_{int}(s,t) = 4\,\frac{s}{1-s}\Big(\frac{1}{t}+\frac{1}{u}\Big) - 2s\ , F_m(s,t) = -4\Big(\frac{m_e}{M_0}\Big)^2\Big(\frac{1}{t^2}+\frac{1}{u^2}\Big)$$

If $E_\pm$ is the energy of $e^\pm$ we get

$$E_+ = \frac{1}{2}(1-t)M_0, \quad E_- = \frac{1}{2}(1-u)M_0\ .$$

The requirement that one of the fermions has $E \geq e_m M_0$ while the other has $E \leq e_M M_0$ together with the symmetry of F under $t \leftrightarrow u$ gives an integrated decay rate

$$\Gamma(e_m, e_M) = 2\,\frac{\alpha}{\pi}\Gamma_0 \int_D ds\,dt\,F(s,t)$$

where assuming $e_m + e_M > \frac{1}{2}$

$$\int_D ds\,dt = \int_0^{s_2-s_1} ds \int_0^{s_1} dt + \int_{s_2-s_1}^{s_2} ds \int_0^{s_2-s} dt \qquad s_1 = 1-2e_m,\ s_2 = 2e_M\ .$$

$F(s,t)$ is singular for $t=0$. When we include mass effects the lower boundary of the photon phase space ($t=0$) is replaced by $t = (\mu^2/s) + O(\mu^4)$ with $\mu = m_e/M_0$. This is the origin of the collinear mass singularity.

Performing the s,t integrations we find

$$\int_D ds\,dt\,F(s,t) = A(s_2)\ell n\,\mu^2 + B(s_1,s_2)\ell n\,s_1 + 2C(s_1,s_2)\ell n(1-s_2)$$
$$+ 2L(0, s_2-s_1; -1, 1-s_1) - 2L(0, s_2; -1, 0)$$
$$- 2L(s_2-s_1, s_2; -1, s_2) + R(s_1,s_2)$$

where

$$A(s_2) = s_2^2 - 6s_2 - 4\,\ell n(1 - s_2)$$

$$B(s_s, s_2) = (s_1 - s_2)(2 + s_2 - s_1) - 4\,\ell n(1 - s_2 + s_1)$$

$$C(s_1, s_2) = 2 + 2s_1 s_2 - s_1^2 + 2\,\ell n\,\frac{1 - s_2}{1 - s_2 + s_1}$$

$$R(s_1, s_2) = -\frac{1}{2}\,s_1^2 + s_1 s_2 + 4s_2$$

and

$$L(a, b; A, B) = \int_a^b dx \left(1 + x - \frac{2}{1 - x}\right) \ell n(Ax + B)$$

only the last integral is non trivial

$$\int_a^b dx\,\frac{1}{1 - x}\,\ell n(Ax + B) = s(b) - s(a)$$

$$s(x) = \ell n(Ax + B)\ell n\,\frac{A(1 - x)}{A + B} + Li_2\left(\frac{Ax + B}{A + B}\right)$$

where $Li_2(x)$ denotes the dilogarithm function. Using $s_\theta^2 = 0.22$ and $M_0 = 93\ GeV$ [2] we computed

$$R(e_m, e_M) = \frac{1}{\Gamma_0}\,\Gamma(e_m, e_M)$$

for few values of e_m, e_M. The result is $R = 2\%$ for $e_m = 0.45$ and $e_M = 0.1$, $R = 1.5\%$ for $e_m = 0.48$ and $e_M = 0.15$, $R = 1.7\%$ for $e_m = 0.45$ and $e_M = 0.15$. Including a 1% for external bremsstrahlung we can estimate that a total of 2-3% of the events may look this way.

We also considered the cross section for $q\,\bar{q} \to Z^0 \to e^+e^-\gamma$ at the Z^0 peak.

$$d^5\sigma = \frac{\alpha^3 Q^2}{2\pi M_0^2}\,|A|^2\,d\Omega_+\,dE_+\,dE_-\,d\phi_\gamma$$

where Q is the quark charge in units of e. Using the results of Ref. [3] we get

$$|A|^2 = -2\,\frac{m_e^2}{M_0^2\Gamma^2}\left[c_-\left(\frac{t^2}{k_-^2}+\frac{t'^2}{k_+^2}\right)+c_+\left(\frac{u^2}{k_+^2}+\frac{u'^2}{k_-^2}\right)\right]$$
$$+\frac{1}{\Gamma^2 k_+ k_-}\left[c_-(t^2+t'^2)+c_+(u^2+u'^2)\right]$$

with

$$c_\pm = (V_e^2+A_e^2)(V_q^2+A_q^2) \pm 4V_eA_eV_qA_q\ .$$

V, A being the vector and axial couplings of e and q to Z^0. Also $t = -2p_1\cdot p_4$, $t' = -2p_2\cdot p_3$, $u = -2p_1\cdot p_3$, $u' = -2p_2\cdot p_4$, $k_+ = -2p_4\cdot k$ and $k_- = -2p_3\cdot k$ with the convention that all the momenta flow inwards. $\Gamma = 2.92\ GeV$ [2] is the Z^0 width. $|A|^2$ is given by a very compact expression. One could use it directly for a numerical integration and any further achievement in the analytical approach is a matter of taste. In this case however we can show that the final answer is again very simple. Following Ref. [4] we find

$$E_+ = \frac{1}{2}(1-k_-)M_0\ ,\quad E_- = \frac{1}{2}(1-k_+)M_0$$

where from now on all the dimensional quantities have been scaled to M_0^2. The photon phase space for $m_e = 0$ is bounded by $k_+ = k_- = 0$, $k_+ + k_- = 1$. The variables t, t', u and u' are not independent but fulfill

$$t = t_0(1-k_-)\ ,\quad t' = t_0 + k_+ - t_0k_- + k_q$$
$$u = u_0 + t_0k_- - k_q\ ,\quad u' = u_0(1-k_-)$$
$$t_0 = \frac{1}{2}(\cos\theta - 1)\ , u_0 = -\frac{1}{2}(\cos\theta + 1)\ , k_q = -2p_1\cdot k$$

where θ is the scattering angle between $\bar{u}\,(d)$ and e^+. In a frame where e^+ is along the third direction with $\bar{u}\,(d)$ in the 1-3 plane the ϕ_γ integration can be carried on giving

$$\int_0^{2\pi} d\phi_\gamma k_q = -\pi(k_+ + k_-)(1 - \cos\theta\cos\theta_\gamma)$$

$$\int_0^{2\pi} d\phi_\gamma k_q^2 = \frac{1}{2}\pi(k_+ + k_-)^2\Big[(1 - \cos\theta\cos\theta_\gamma)^2 + \frac{1}{2}\sin^2\theta\sin^2\theta_\gamma\Big]$$

$$(k_+ + k_-)\cos\theta_\gamma = k_+ + k_- - 2\frac{k_+}{1-k_-}$$

We now assume that the detected e^+ has an energy greater than some threshold value $\frac{1}{2}\,e_m M_0$ while e^- has an energy less than $\frac{1}{2}\,e_M M_0$. Thus

$$\frac{d\sigma}{d\Omega_+} = \frac{\alpha^3 Q^2}{4\pi}\,\frac{1}{\Gamma^2}\int_D dk_+ dk_- |A'|^2$$

where D is the constrained photon phase space and

$$|A'|^2 = -2\mu^2(c_- t_0^2 + c_+ u_0^2)\frac{1}{k_-^2} + \sum_{\substack{n=0,1\\ m=1,2}} a_n^m\,\frac{k_+^n}{(1-k_-)^m} + \sum_{\alpha\beta} b_{\alpha\beta}k_+^\alpha k_-^\beta$$

$$a_n^m = c_- f_n^m(t_0) + c_+ f_n^m(u_0)\ , \quad b_{\alpha\beta} = c_- g_{\alpha\beta}(t_0) + c_+ g_{\alpha\beta}(u_0)$$

$$f_1^2(x) = 1 + 6x + 6x^2\ , \quad f_0^1(x) = -2x(2+3x)\ , \quad f_1^1(x) = -1 - 2x$$

$$g_{1,-1}(x) = g_{-1,1}(x) = x^2\ , \quad g_{0,-1}(x) = g_{-1,0}(x) = -g_{-1,-1}(x) = -2x^2$$

The integration is now trivial; for $e_m + e_M < 1$ D is a triangle in the k_+, k_- plane

$$\int_D dk_+\, dk_- = \int_{1-e_M}^{1} dk_+ \int_0^{1-k_+} dk_-\ .$$

Again the singularity at $k_- = 0$ is controlled by the electron mass since the boundary $k_- = 0$ is replaced by $k_- = \mu^2(k_+/1 - k_+) + O(\mu^4)$. The final answer contains a collection of logs as well as a dilogarithm coming from the $1/k_+k_-$ term.

As a next step one should fold this cross section with the quark distributions inside the proton in order to compute the numerical answer for $p\bar{p} \to Z^0 + X \to e^+e^-\gamma + X$. We have made no attempt in this direction.

CERN has now announced 6 Z^0 candidates (CERN press PR 10/83), 5 $Z^0 \to e^+e^-$ and 1 $Z^0 \to \mu^+\mu^-$. For the best $Z^0 \to e^+e^-$ candidate the tracks carry 45 and 47 GeV of energy.

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