SLAC-PUB-3127
May 1983
(T/E/A)

e^+e^- Collisions in the Multi-TeV Region*

JOHN ELLIS
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

1. Introduction

A leading role in the elucidation of the Standard Model during the last few years has been played by e^+e^- colliding beam experiments. They have advanced our understanding more than have hadron-hadron collision experiments during the same period. The e^+e^- discoveries have been made possible by the cleanliness of the experimental conditions and the ability to tune the centre-of-mass energy with precision to the desired value, thus avoiding less interesting background events. Whereas the great challenge in hadronic experiments has been to devise techniques for locating the needle in the haystack, in e^+e^- experiments there is little chaff to separate from the wheat.

We expect history to repeat itself in the next step of elucidating physics beyond the Standard Model. Just as past e^+e^- machines such as SPEAR, DORIS and CESR have uncovered physics inaccessible to hadron-hadron collisions with a centre-of-mass energy several times higher, so we feel that future e^+e^- colliders will provide information that could not be duplicated by hadron colliders with much larger centre-of-mass energies. It has been a great entrepreneurial feat to conceive and operate[1] the CERN $\bar{p}p$ collider years ahead of any e^+e^- machine able to make the $W^\pm$ or Z^0. However, the SLC and LEP will provide us with much more information about the $W^\pm$ and Z^0 than can the present-day CERN $\bar{p}p$ collider. The accident of history that permitted the earlier operation of a hadron-hadron collider need not recur for the next generation of accelerators.

There is a general consensus that the next interesting energy range is likely to be

$$E = O\left(\frac{m_W}{\sqrt{\alpha}}\right) = O(1)\ TeV\ . \qquad (1)$$

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

Invited Talk at the XIVth International Symposium on
Multiparticle Dynamics at High Energies
Lake Tahoe, June 22-27, 1983

It is in this energy range that whatever physics provides and stabilizes the masses of the intermediate vector bosons must be revealed. Unravelling this mass generation mechanism takes us beyond the gauge principle of the Standard Model which has been so triumphantly vindicated in recent months.[2] Therefore we discuss here the capabilities and attributes of an e^+e^- collider with at least 1 TeV energy per beam. We believe that by enabling an important new energy domain to be explored in detail, such an e^+e^- collider provides physics opportunities which cannot be paralleled by hadron-hadron colliding rings with centre-of-mass energies several times higher.

Section 2 of this report reviews some theoretical models which provide yardsticks for measuring physics reach: a compilation[3] of cross-sections is shown in Table 1. We find that a multi-TeV e^+e^- collider permits detailed studies to be made of an interesting range of physics questions. Section 3 uses the physics models of Section 2 to guess what e^+e^- luminosity may be desirable as a function of the beam energy spread required to investigate different structures. Section 4 discusses briefly some essential features of an existence proof for a multi-TeV e^+e^- collider which was recently generated[4] in response to a request from the HEPAP subpanel. Section 5 tries to estimate how much larger a hadron-hadron collider centre-of-mass energy would be necessary for probing physics at comparable energy scales. We find that the comparable hadron-hadron collider energy increases *faster than linearly* with that of an e^+e^- collider in the multi-TeV region, and must be at least an order of magnitude higher.

2. Physics Possibilities

We can now regard the gauge principle as well-established, with all the vector bosons of the Standard Models now observed.[2] The next item on the agenda is to ascertain the mechanism whereby the weak gauge symmetry is broken. Presumably it is broken spontaneously with some elementary or composite Higgs field acquiring a non-zero vacuum expectation value. Loop corrections render the masses of elementary scalar fields very unstable unless there is some mechanism to protect them. The only known protection mechanism is supersymmetry[5] which is discussed in Section 2.1. Perhaps there are no elementary Higgs scalar fields, but only composite spin-zero bosons manufactured from new fermionic constituents. This is the basis for models of dynamical symmetry breaking such as technicolour[6] discussed in Section 2.2. If Higgs fields may be composite, why not quarks and leptons and perhaps gauge bosons as well? Such preon models[7] are discussed in Section 2.3. None of these models is sufficiently well-developed to be taken literally as a theory of the world, but they may be sufficiently diverse to guide us through the analyses of Sections 3 and 5.

Table 1

Compilation of Cross-Sections in the TeV Region

	$e^+e^- \to$	Cross-Section in Units of σ_{pt}	Remarks
Weak Vector Bosons	W^+W^-	$\sim$ 20	Background reactions Peaked forward-backward
	Z^oZ^o	$\sim$ 20	
	$Z^o\gamma$	$\sim$ 20	
Higgs Bosons	Z^oH^o	0.16	Best way to look for heavy neutral Higgs?
	H^+H^-	0.26 β^3	Useful for $H^\pm$ which are not superheavy.
Fermions	$\mu^+\mu^-$	1.19	Includes Z^o contribution as well as γ.
	$Q(2/3)\bar{Q}(-2/3)$	2.04	
	$Q(-1/3)\bar{Q}(1/3)$	1.17	
	3 generations of $q\bar{q}$	9.6	
Resonances	New Z^o (Ω)	$\sim$ 5000?	Assuming couplings similar to first Z^o.
	Technicolor ρ	O(10)	Assuming couplings similar to ordinary ρ^o.
Super-symmetry Continuum	$\tilde{W}^+\tilde{W}^-$	1.99	Partners of $W^\pm$.
	$\tilde{Q}(2/3)\bar{\tilde{Q}}(-2/3)$	0.37	Partners of charge −2/3 quarks.
	$\tilde{Q}(-1/3)\bar{\tilde{Q}}(1/3)$	0.11	Partners of charge −1/3 quarks.
	$\tilde{\ell}^o\bar{\tilde{\ell}}^o$	0.60	Partners of neutral leptons.

2.1 – Supersymmetry

Let us suppose there are scalar fields H responsible for the weak interaction gauge symmetry breaking. They must have masses $m_H = O(m_W) = O(100)$ GeV. These masses should include the effects of radiative corrections such as those shown in Fig. 1a, which are of magnitude

$$\delta m_H^2 = O(\alpha)\Lambda^2 \qquad (2)$$

where Λ is a cut-off on loop momenta. In order for m_H to be $O(m_W)$, Eq. (2) tells us that

$$\Lambda = 0\left(\frac{1}{\sqrt{\alpha}}\right) m_W = 0(1)\ TeV\ . \qquad (3)$$

This cut-off Λ could be provided by the scalar fields being composite on a distance scale $O(1/1\ TeV) \approx 10^{-17} cm$, which is the technicolour idea[6] to be pursued in Section 2.2. If the scalar fields are elementary, the loops of Fig. 1a can only be cancelled by other loops as in Fig. 1b which involve particles with different statistics (fermions $\leftrightarrow$ bosons), similar masses:

$$|m_F^2 - m_B^2| \leq 0(1)\ TeV^2 \qquad (4)$$

and identical couplings. This requires doubling up all the particles of the Standard Model with supersymmetric partners – alas, no known pair of conventional particles can be supersymmetric partners of each other.

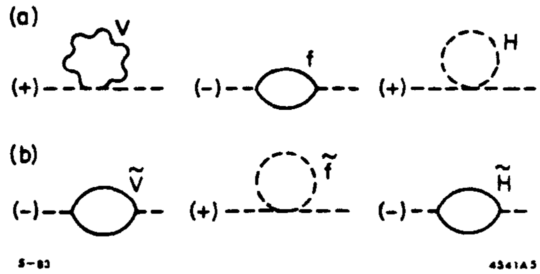

Fig. 1. (a) Diagrams which make quadratically divergent contributions to the masses of elementary scalar Higgs fields, and (b) diagrams which can cancel these divergences in supersymmetric theories.

The master formula (4) suggests that the supersymmetric particles may have masses which allow them to be experimentally accessible. Unfortunately, formula (4) is somewhat vague, and in particular particles which are very weakly coupled to scalar fields, such as the supersymmetric partner of the electron, could in principle be much heavier than 1 TeV. Our ignorance is reflected by the existence of many different phenomenological models of broken supersymmetry with different mass spectra in the generic range (4). Rather than discuss them in detail, we have made a histogram (fig. 2) of all the supersymmetric particle masses in a collection of different models recently published.[8],[9] Also indicated are the ranges of particle masses which can be produced at e^+e^- colliders such as the SLC, LEP and a multi-TeV e^+e^- collider. We see that whereas the SLC and LEP may have the luck to reach some supersymmetric particles, it is only a multi-TeV e^+e^- collider which offers a reliable guarantee of pair-producing most supersymmetric particles.

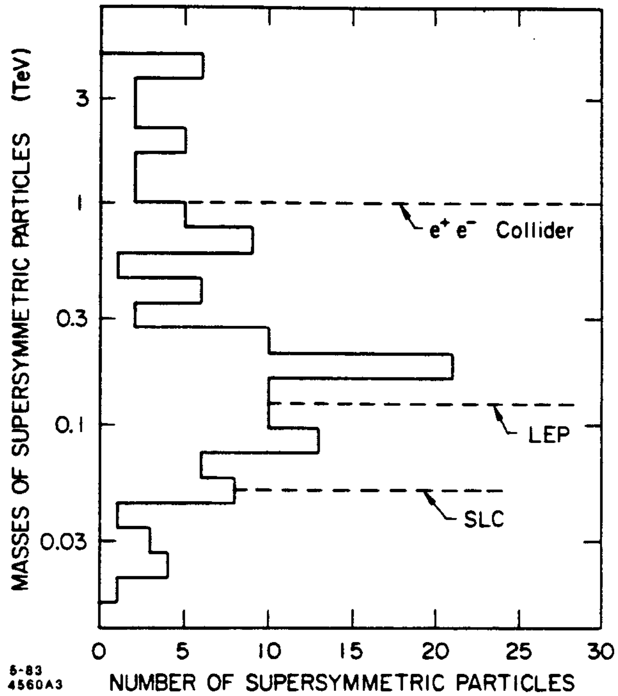

Fig. 2. Histogram of the number of supersymmetric particles predicted[8,9] in various mass ranges, compared with the beam energies of the SLC, LEP and a e^+e^- collider with 1 TeV per beam.

All the charged supersymmetric particles will be pair-produced with cross-sections that are comparable to the canonical point-like electromagnetic cross-section for $e^+e^- \to \gamma^* \to \mu^+\mu^-$:

$$R \equiv \frac{\sigma}{\sigma_{pt}} = 0(1) : \sigma_{pt} = \frac{4\pi\alpha^2}{3E_{c.m.}^2} \approx \frac{87nb}{E_{c.m.}(GeV)^2} \cdot \tag{5}$$

Table 1 includes some of the supersymmetric particle pair-production cross-sections, along with cross-sections for other particles both new and old.[4] In many models[8],[9],[10] the first two (and possibly three) generations of squarks or sleptons of the same charge have very similar masses,[11] while the cross-sections rise from threshold as β^3, which means that the thresholds for different generations may not be easily distinguishable. In many models the dominant decays of squarks or sleptons are expected to be into a light neutral gaugino plus the corresponding flavour of quark or lepton:

$$(\tilde{q} \text{ or } \tilde{\ell}) \to (\tilde{\gamma} \text{ or} \ldots) + (q \text{ or } \ell) \tag{6}$$

which means that only about 50% of $E_{c.m.}$ may be visible. This is not a problem in e^+e^- collisions, but could be a problem in hadron-hadron collisions since it means that these $\bar{\tilde{q}}\tilde{q}$ or $\bar{\tilde{\ell}}\tilde{\ell}$ pair-production events must be dug out from a much larger background of events with lower transverse energy.

We deduce from the above discussion that the a multi-TeV e^+e^- collider beam energy should be sufficient to produce supersymmetric particles, and that many which have masses less than the beam energy of a multi-TeV e^+e^- collider should be detectable.

2.2 – Technicolour

This is the name we use for theories of composite Higgs fields whose constituents (techniquarks and technileptons) are confined by new strong interactions due to an exact non-Abelian technicolour gauge group such as SU(4).[6] These interactions must become strong at an energy scale

$$F_T = O\left(\frac{m_W}{\sqrt{\alpha}}\right) = O(250)\,GeV \tag{7}$$

in order for the $W^\pm$ and Z^0 to acquire the correct masses by "eating" some of the light spin-zero technipion bound states. The simplest technicolour models[12] contain a complete generation (U, D, E, N) of technifermions which parallel conventional (u, d, e, ν) quarks and leptons, having the usual $SU(3) \times SU(2) \times U(1)$ transformation properties as well as sitting in a fundamental representation of the technicolour group. There are many technipions which are not eaten by the $W^\pm$ gauge and Z^0 bosons, but acquire masses between a few and 250 GeV, as seen in Table 2. The heaviest technipions are triplets and octets of the conventional $SU(3)$ colour interactions. Pair-production cross-sections for technipions,[13] neglecting direct-channel resonance effects, are shown in Fig. 3 to be substantial. Also noteworthy is the colour singlet techni-η' which is expected[14] to have a mass

$$m_{\eta'_T} = \left(\frac{4}{N}\right) \times 970\ GeV \tag{8}$$

where N is the dimensionality of the fundamental technicolour representation, e.g. $N = 4$ for $SU(4)$ technicolour. Beyond the spin-zero bosons there should be higher spin particles, analogous to the $q\bar{q}$ bound state spectroscopy of QCD. Of particular interest to e^+e^- colliders are the colour singlet technivector mesons such as the ρ_T. It is expected to have a mass

$$m_{\rho_T} = \sqrt{\frac{4}{N}} \times 900\ GeV \tag{9}$$

Table 2

Technimeson Masses

Particle	Description		Mass
$P^{0,3}$	e.m. neutral color singlet	Technipions	≤ 3 GeV ?
$P^{\pm}$	e.m. charged color singlet		≤ 15 GeV ?
P_{LQ}	leptoquarks color triplet		150 GeV
$P_8^{0,\pm}$	color octet		250 GeV
ρ_T	color singlet		$\sqrt{4/N} \times 900$ GeV
ω_T	color singlet		$\sqrt{4/N} \times 900$ GeV
η_T'	e.m. neutral color singlet		$(4/N) \times 970$ GeV
f_T, etc.	color singlet		$\sqrt{4/N} \times 1500$ GeV, etc.
B_T	technibaryons		$\sqrt{N/4} \times 1500$ GeV

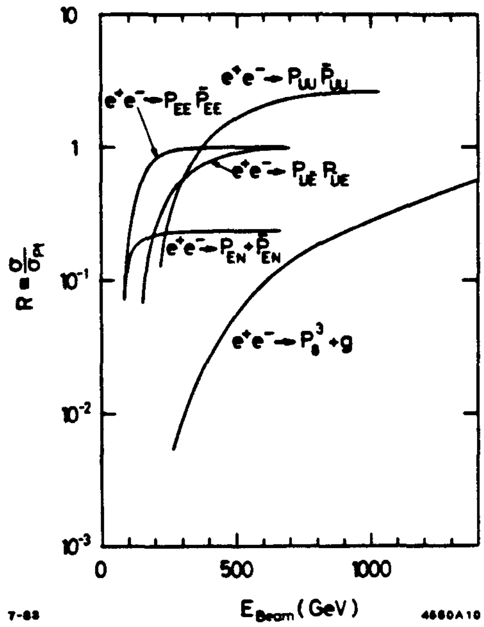

Fig. 3. Cross-sections for the pair-production of different species of technipions.

and a substantial coupling to e^+e^- through a virtual photon:

$$\Gamma(\rho_T \to \gamma^* \to e^+e^-) \approx 60\sqrt{\frac{N}{4}}\ MeV\ . \qquad (10)$$

The total decay width of the ρ_T is expected to be $O(300)\ GeV$, but the line-shape will be shifted from a naive Breit-Wigner shape centered at the nominal mass (9), because of the opening up of the $\rho_T \to 2$ technipion decay channels. In the simple model described earlier with a single technigeneration there is also a colour singlet ω_T which does not couple to e^+e^-. However, it is possible to modify the model in such a way that the ω_T can also be produced in e^+e^- annihilation. Figure 4 depicts the line-shapes of the ρ_T and ω_T to be expected in both the simplest and the modified technicolour models. Beyond these peaks we may expect to find radial excitations ρ_T', ω_T', etc. with masses in the range of 1 to 2 TeV. Clearly a detailed probe of technicolour dynamics would require a careful mapping-out of these putative structures.

It is also possible to produce technimesons which are not vectors by $\gamma\gamma$ collisions. Figure 5 exhibits the specific integrated annual luminosity for $\gamma\gamma$ collisions as a function of $m_{\gamma\gamma}$ for $E_{c.m.} = 4\ TeV$. The number of events for $e^+e^- \to e^+e^-\eta_T$ is about 8 $\times$ this curve evaluated at $m_{\gamma\gamma} = m_{\eta_T} \approx 1\ TeV$. Similarly one can estimate rates for tensor meson $e^+e^- \to e^+e^- f_T$ production, etc.

The technicolour theories we have outlined so far are incomplete in the they do not contain a mechanism for providing conventional quark and lepton masses.

The most complete proposal[15] for furnishing these masses involves extending technicolour by adding massive gauge bosons E_f whose exchanges (see Fig. 6) generate fermion masses

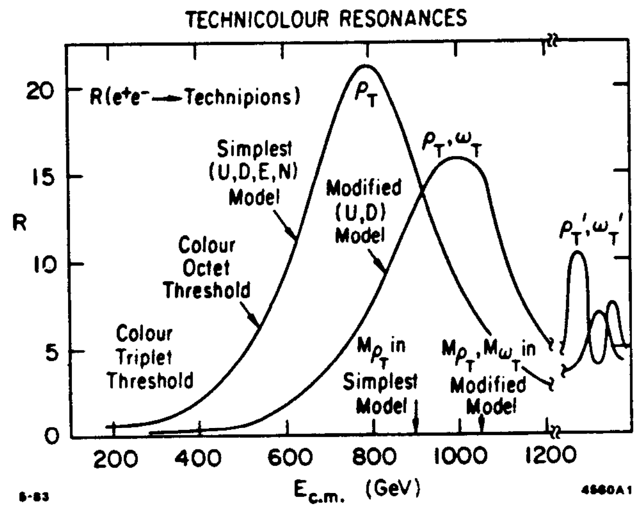

Fig. 4. The line-shape of the ρ_T in the simplest one-generation technicolour model, and of the ρ_T and ω_T in a modified model.

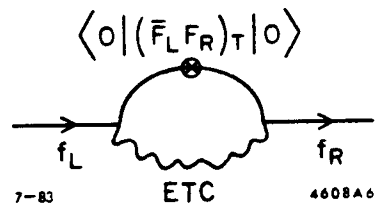

Fig. 5. Diagram responsible for fermion mass generation in extended technicolour theories.

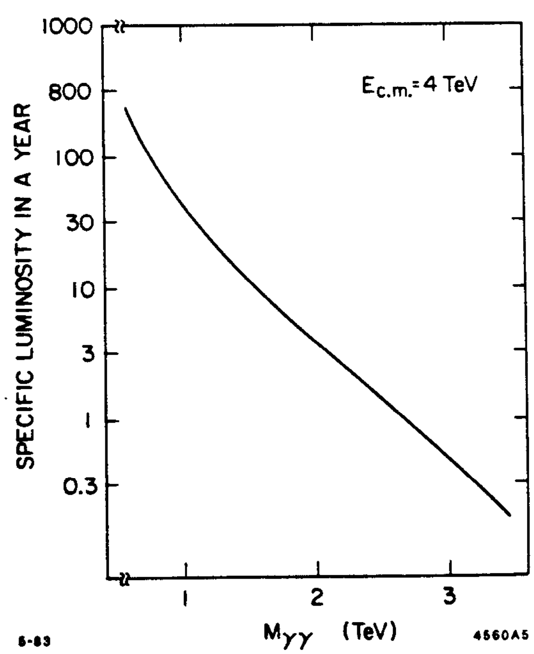

Fig. 6. Specific integrated annual luminosity for $\gamma\gamma$ collisions in an $E_{c.m.}$ = 4 Tev e^+e^- collider, obtained by operating at $10^{33} cm^{-2} sec^{-1}$ for a "year" of $10^7 sec$, and including a generic point-like cross-section factor of $4\pi\alpha^2/3m_{\gamma\gamma}^2$.

$$m_f \approx \frac{F_T^3}{m_{E_f}^2} \,. \tag{11}$$

Because of the wide range of the known fermion masses ($m_e \approx \frac{1}{2}\, MeV$ to $m_t > 20\, GeV$) there must also be a wide range of extended technicolour boson masses m_{E_f}. Estimates[16] suggest that the bosons responsible for the b and t quark masses may have accessibly low masses:

$$m_{E_t}^2 \simeq \frac{1}{2}\, TeV^2 \,, \quad m_{E_b}^2 \simeq 2\, \frac{1}{2}\, TeV^2 \,. \tag{12}$$

The E_t boson could be produced via the reaction $e^+e^- \rightarrow \gamma^*$ or $Z^* \rightarrow t\bar{t} \rightarrow E_t +$ techniquark $+ \bar{t}$ in e^+e^- collisions. The existence and nature of extended technicolour interactions are considerably more controversial than the basic technicolour model. As seen from the estimates (12), they can be probed by an e^+e^- collider with a centre-of-mass energy somewhat higher than 1 TeV.

Theorists have found it difficult to devise clear signatures of technipion production in hadron-hadron collisions,[13],[17] while their pair-production cross-sections are expected to be large in e^+e^- collisions, and we anticipate no difficulty in detecting them. Furthermore, we see no way in hadron-hadron collisions of probing the details of technicolour dynamics suggested in Fig. 4.

2.3 – Preons

If Higgs fields can be composite, why not also quarks and leptons, and possibly gauge bosons? Many theorists are motivated by the proliferation of quark and lepton flavours to pursue models in which these particles have constituents called here preons.[7] Model-dependent limits on the scale of compositeness come for example from the anomalous magnetic moments of the electron and muon.[18] They do not exclude the possibility pursued by many authors that quarks and leptons may be composite on a distance scale corresponding by the uncertainty principle to an energy scale $\mu \sim O(1)\ TeV$. Among the novel phenomena to be expected at this energy scale we can include the existence of excited quarks q^* and leptons ℓ^*. We should also expect new low-energy non-renormalizable effective interactions scaled by inverse powers of the compositeness scale μ, such as

$$O\left(\frac{1}{\mu^2}\right)(\bar{\ell}\ell\bar{\ell}\ell\,, \bar{\ell}\ell\bar{q}q\,, \bar{q}q\bar{q}q) \tag{13}$$

with different and possibly novel ($\neq V, A$) Lorentz structures. It is easy to search for composite quarks or leptons in e^+e^- annihilation produced via

$$e^+e^- \rightarrow \bar{\ell}\ell^* + \bar{\ell}^*\ell \,\text{or}\, \bar{q}q^* + \bar{q}^*q\,, \bar{\ell}^*\ell^* \,\text{or}\, \bar{q}^*q^* \tag{14}$$

with the excited states subsequently decaying via

$$(\ell^* \text{ or } q^*) \rightarrow (\ell \text{ or } q) + (\text{gauge boson}) \ . \tag{15}$$

One can also look for new interactions (13) of the $\bar{\ell}\ell\bar{\ell}\ell$ or $\bar{\ell}\ell\bar{q}q$ types, whereas hadron-hadron collisions could only probe for $\bar{q}q\bar{q}q$ interactions. Precision low energy experiments place severe constraints on interactions which violate conventional lepton or quark flavour conservation laws, but they do not impose such severe constraints on possible new interactions which do conserve lepton number, etc. Such new interactions can interfere with conventional interactions and provide deviations from conventional cross-sections[19],[20] as seen in Fig. 7. The observability of such effects is discussed in more detail in Section 3 of this report.

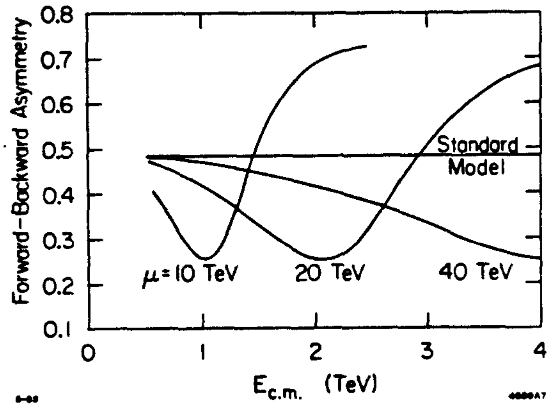

Fig. 7. Deviations from conventional expectations for the forward-backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$, due to purely left-handed interactions generated in a class of composite models (13) with the indicated values of the compositeness scale μ.

Preon models are not as fully developed as the theories discussed in previous parts of this section. Indeed, no phenomenologically satisfactory preon model exists which satisfies reasonable requirements of theoretical consistency. Nevertheless, preon models reflect a basic physical intuition about the structure of matter and deserve serious consideration as physics yardsticks. It is therefore encouraging that an e^+e^- collider capable of attaining centre-of-mass energies of a few TeV can test significantly such preon theories.

3. Luminosity and beam energy spread

Since interesting cross-sections in e^+e^- annihilation fall as $1/E^2_{c.m.}$, it is clear[4] that the conventional target luminosity of $10^{32} cm^{-2} sec^{-1}$ is not going to be adequate forever. A complicating feature of e^+e^- colliders is that the beam-beam interaction causes the beam energies to spread, and that the attainable luminosity L varies with the beam energy spread $\Delta E/E$ that one is prepared to tolerate. Much of e^+e^- physics concerns the continuum, and a large beam energy spread $\Delta E/E = O(1)$ is experimentally acceptable. However, often one wants to focus on a narrow structure such as a resonance or a threshold, and a smaller $\Delta E/E$ is required. In this case the peak cross-section is often larger than the generic σ_{pt}, and a lower luminosity is acceptable when the collider is run in such a "factory" mode. In this section we present guesses as to what design luminosity is desirable as a function of the permitted $\Delta E/E$. This we do by considering a handful of reactions found in the grab-bag of theories discussed in the previous section.

3.1 - Continuum Reactions

First suppose that we are looking for a new continuum reaction without a sharp threshold ($S =$ supersymmetric particle pair-production?) with a relative cross-section $R = 1$ at a centre-of-mass energy of 2 TeV:

$$N_S = \frac{8.7}{E_{c.m.}(GeV)^2} \times \left(\frac{\mathcal{L}}{10^{32}cm^{-2}sec^{-1}}\right) sec^{-1} . \tag{16}$$

We will be content with $\Delta E/E \simeq 1$, but ask for 2500 events in our "year" of 10^7 seconds. The required $\mathcal{L}$ is plotted in Fig. 8.

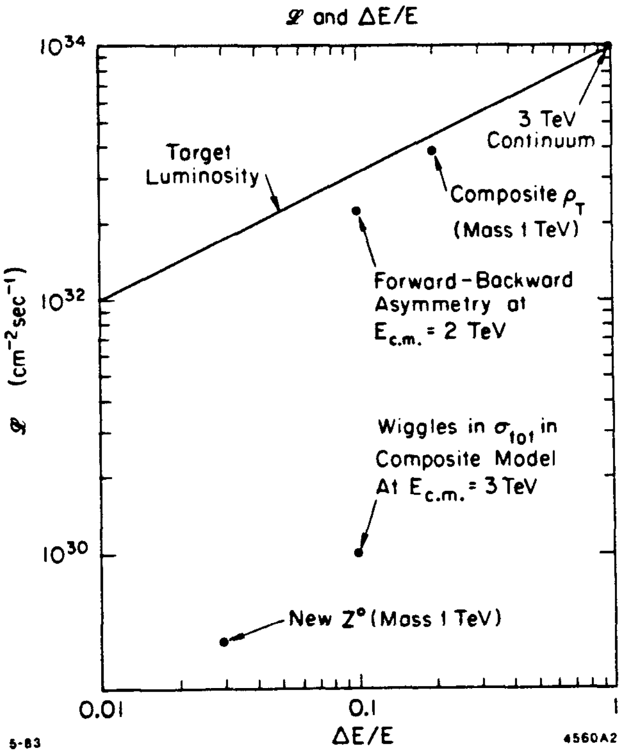

Fig. 8. Desirable e^+e^- luminosity $\mathcal{L}$ as a function of the attainable beam energy spread $\Delta E/E$, as determined using a selection of test reactions.

Alternatively, let us suppose composite models are correct, and we are interested in probing their predictions at a centre-of-mass energy of 3 TeV. Studies[19],[20] suggest that perhaps

$$\sigma \sim 10^{-1} nb \tag{17}$$

with wiggles $\Delta\sigma/\sigma = 0(1)$ on a change in energy scale $\Delta E/E \simeq 0.1$. The event rate is

$$N_C = \frac{1}{100} \times \left(\frac{\mathcal{L}}{10^{32} cm^{-2} sec^{-1}}\right) sec^{-1} \tag{18}$$

and if we demand 1000 events in 10^7 seconds we get another point shown in Fig. 8.

Finally, we take another suggestion[19],[20] of composite models that there may be an interesting deviation from the conventional forward-backward asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ due to interference with one of the new interactions (13). We will seek to measure such a deviation at the 10% level at an $E_{c.m.} = 2\ TeV$ with a beam energy spread $\Delta E/E = 0.1$, which we get with 10^2 events corresponding to a third point indicated in Fig. 8.

3.2 – Vector Resonances

Now let us focus on possible narrow structures in the cross-section. In general, at the peak of a vector resonance one has, neglecting radiative corrections,

$$R_V = \frac{\sigma(e^+e^- \rightarrow V \rightarrow X)}{\sigma_{pt}} = \frac{9}{\alpha^2} B(V \rightarrow e^+e^-) B(V \rightarrow X) \tag{19}$$

where the $B(V \rightarrow ?)$ denote decay branching ratios. Thus for the conventional Z^0 one expects $B(V \rightarrow e^+e^-) \simeq \frac{1}{30}$ implying $R_{Z^0} =$ 5600. The total Z^0 width is expected to be about 3 GeV, so any beam energy spread $\Delta E/E \leq 0.03$ is adequate to dissect the peak. We may anticipate either or both of two types of vector resonance in the TeV range: perhaps another elementary vector boson Ω or perhaps a composite vector meson ρ_T. In the elementary case we might be prepared to assume that $B(\Omega \rightarrow e^+e^-) \approx \frac{1}{30}$ again, and hence the number of events at the peak will be

$$N_\Omega = \frac{5 \times 10^4}{m_\Omega (GeV)^2} \times \left(\frac{\mathcal{L}}{10^{32} cm^{-2} sec^{-1}}\right) sec^{-1} \tag{20}$$

for any beam energy spread $\Delta E/E \leq \Gamma/m \simeq 0.03$. In the case of a composite ρ_T we take the generic values

$$B(\rho_T \rightarrow e^+e^-) \approx B(\rho \rightarrow e^+e^-) \simeq 4 \times 10^{-5}\ ; \frac{\Gamma}{m} \approx 0.2 \tag{21}$$

so that the peak

$$N_{\rho_T} \simeq \frac{60}{m_{\rho_T} (GeV)^2} \times \left(\frac{\mathcal{L}}{10^{32} cm^{-2} sec^{-1}}\right) sec^{-1} \tag{22}$$

as long as $\Delta E/E \leq \Gamma/m = 0.2$. We compute the desirable luminosity in the two cases (20,22) by assuming that $m_\Omega \simeq m_{\rho_T} \simeq 1$ *TeV* and asking for 10^4 events in a theoretical "year" of 10^7 seconds. The resulting $\mathcal{L}$ desired for $\Delta E/E \leq 0.03$ and $\Delta E/E \leq 0.2$ respectively are shown in Fig. 8.

3.3 - Discussion

We see from Fig. 8 that , as expected, the minimum acceptable luminosity decreases as $\Delta E/E$ decreases. We disregard the two lowest points on the graph, but rely on the three highest points. We infer that a luminosity of about $10^{33} cm^{-2} sec^{-1}$ at a $\Delta E/E \simeq 0.1$ is acceptable, and propose that for other beam energy spreads

$$\mathcal{L} \approx 10^{34}\left(\frac{\Delta E}{E}\right) cm^{-2} sec^{-1} \tag{23}$$

is a reasonable target to adopt.

4. Existence proof for a machine

You know I am not a machine physicist, and so do I, so nobody expects much from this section. You are encouraged to ask one of the people in ref. 4 for more information. All that has been done in the studies so far is to establish the possiblity in principle of constructing an e^+e^- collider with 1 to 2 TeV per beam. No serious attempt has been made to optimize the parameters, and all that we have is an existence proof. The basic philosophy followed has been to use conservative technology which already exists. Thus, for example, the field gradient is generally assumed to be similar to that presently available at SLAC, though there are also some supplementary remarks about what can be done with less conservative technology providing twice the field gradient.

Let me mention two basic equations of e^+e^- collider life. One tells you how the synchrotron radiation spreads the energy E of the other beam:

$$\frac{\Delta E}{E} = \frac{414 \times E(TeV) \times \mathcal{L}(10^{33} cm^{-2} \sec^{-1})}{\sigma_z(mm) \times f(Hz)} \tag{24}$$

where σ_z measures the bunch length and f is the collision frequency. Note that $\Delta E/E \; \alpha \; E\mathcal{L}$, which matches the desideratum of Fig. 8. The luminosity

$$\mathcal{L} = \frac{3.5 \times 10^{31} \times P(MW) \times D \times H(D)}{\sigma_z(mm)} cm^{-2} \sec^{-1} \tag{25}$$

where P is the power in Megawatts and D is the disruption parameter measuring how one beam gets distorted as the other passes through it. The enhancement factor $H(D)$ has a maximum value of about 6, attained when $D \approx 2$.

Table 3

Machine Parameters

	Conservative	Not So Conservative
Beam energy	1 TeV	
Frequency	2856 MHz	5712 MHz
Length of each linac L	50 km	25 km
Field gradient G	20 MV/m	40 MV/m
Repetition rate	185 Hz	
Bunches/pulse	12	
Dispersion σ_z	2 mm	
# particles N	1.4×10^{10}/bunch	
# klystrons	2500/linac	
Peak klystron power	330 MW	360 MW
Mean klystron power	23 KW	16 KW
Total power	390 MW	290 MW
# interaction points	6	
Cost: machine components	1.963 G$	1.525 G$
conventional facilities	1.207 G$	0.875 G$
total	3.170 G$	2.400 G$

Table 3 lists the parameters of 1 *TeV* × 1 *TeV* e^+e^- colliders using conservative (not so conservative) technology. The parameters of Table 3 are only preliminary and suggestive. Needless to say, the costs are even more uncertain than the technical specifications, but it is perhaps fair to assert that the costs are within one standard deviation of those for a circular hadron-hadron desertron. As one added bonus, the same linear e^+e^- collider could also do *ep* physics with 1 TeV per beam at a luminosity $\mathcal{L} \sim 10^{32} cm^{-2} \sec^{-1}$. It certainly seems to me that a multi-TeV e^+e^- collider should be regarded as a serious possibility for the next generation of machines.

5. Comparison between e^+e^- and hadron-hadron centre-of-mass energies

5.1 - Preliminaries

In view of the interest expressed[21],[22] in another generation of hadron-hadron colliders, it is important and instructive to devise measures of the relative physics reach of the two classes of machine in this energy range. In particular, we want to evaluate what centre-of-mass energy in hadron-hadron collisions would be required if one wishes to duplicate the energy and mass range accessible to a given e^+e^- collider.

The most intuitive and physically appealing argument runs as follows. We are interested in hard, large momentum transfer hadron-hadron collisions, not the peripheral junk that constitutes all but a minuscule fraction of the total cross section. The hardness means that we should consider parton-parton collisions. Nucleons contain three valence quarks plus a sea of gluons and $q\bar{q}$ pairs. Each of these components carries about one half of the momentum of a high energy nucleon:

$$p = \underbrace{3q}_{1/2} + \underbrace{gluons + q\bar{q}}_{1/2} \tag{26}$$

implying that each valence quark constituent carries about 1/6 of the hadron beam momentum. Accordingly, one might expect that as a rule of thumb

$$\left\langle E_{hh} \right\rangle \approx O(6) \times E_{e^+e^-} \tag{27}$$

is a reasonable conversion factor from $E_{c.m.}$ in e^+e^- to $E_{c.m.}$ in hh collisions.

This means of comparison is too naive for several reasons. One reason is that the parton distributions evolve with increasing energy, and appear softer in harder collisions. However, this effect is only log(logarithmic) and is not very important in the energy range of interest. A more serious point is that different types of particle have different production dynamics in hadron-hadron and e^+e^- collisions which modify the naive 1/6 factor. For example, reasonable quantities of $W^\pm$ bosons can only be produced in pairs in e^+e^- collisions, whereas they can be produced singly in hh collisions. Thus one needs the higher energies of LEP phase II to make W^+W^- pairs. Another remark follows from the different natures of the colliding particles: $q\bar{q}$ or gg collisions prefer to produce strongly interesting particles, whereas e^+e^- collisions democratically produce leptons and strongly interacting particles in comparable numbers as long as they have electromagnetic and/or neutral weak charges. Thus it was possible for SPEAR and DORIS to produce, detect and study the τ, which has never been seen in any hadron-hadron collisions. Another important comment follows from the geometric nature of hard cross-sections:

$$\sigma_{hard} \propto \frac{1}{M^2 \,\mathrm{or}\, E^2} \,. \tag{28}$$

This means that as one goes to higher energies one must have a higher effective luminosity $\mathcal{L}^{eff}(x)$ in order to study collisions with a fixed fraction x of the available centre-of-mass energy:

$$\sigma(x) = \frac{1}{E^2_{c.m.}} \frac{1}{x^2} \mathcal{L}^{eff}(x) \tag{29}$$

We have already explored this effect in the previous section during our discussion of e^+e^- luminosities, and a similar effect occurs in hadron-hadron collisions. However, because of the high total cross-section, no-one has yet demonstrated the feasibility of hadron-hadron experiments with a beam-beam luminosity above $10^{32} cm^{-2} sec^{-1}$ — unless one throws away low p_T particles which contain[23] considerable interest for new particle searches. This constant luminosity folded into Eq. (29) implies that one can only exploit an ever-shrinking fraction x of the available hadron-hadron centre-of-mass energy as one goes to ever-higher energies. Correspondingly, the hh $E_{c.m.}$ required to duplicate the physics reach of a high energy, high luminosity e^+e^- collider must increase more rapidly than the naive 1/6 "rule" (27).

5.2 – Model Calculations

To quantify these effects, we have exploited cross-section and sensitivity estimates made for high energy hadron-hadron colliders. We have mainly used figures from the Snowmass report,[21] and have checked the general conclusions using the recent Fermilab Dedicated Collider proposal.[22] The Snowmass calculations are most complete for colliders with $E_{c.m.} = 10$ TeV and 40 TeV, so we have taken them as our benchmarks. We have restricted ourselves to hh beam-beam luminosities of $10^{32} cm^{-2} sec^{-2}$. The relevant results are exhibited in Table 4.

Table 4

Physics Reach in Bellwether Reactions

Reaction	Accessible M (TeV) for $\mathcal{L} = 10^{32}\ cm^{-2}sec^{-1}$		Comparable $E_{c.m.}$ Required in e^+e^-
	$E_{c.m.} = 10$ TeV	$E_{c.m.} = 40$ TeV	
Jet Pair	3.6	8.0	same
$\mu^+\mu^-$ or heavy L^+L^-	0.3	0.4	same
Z^o	1.2	1.6	same
$W^\pm$	1.2	1.6	$\times$ 2
Technimeson η_T'	2.0	3.0	same
$\tilde{g}$	0.6	1.2	$\times$ (4 to 10)
Heavy $\bar{Q}Q$	0.9	2.3	$\times$ 2

We have mainly considered the bellwether reactions of Snowmass, modified in a few minor ways. We have discarded π^0 and γ production at large p_T, and we have added in heavy quark $\bar{Q}Q$ and lepton $\bar{L}L$ production. It is immediately apparent from Table 4 that the available physics reach increases less rapidly than the available centre-of-mass energy. The final column of Table 4 lists the correction factors that must be applied to determine the comparable $E_{c.m.}$ in e^+e^- collisions. For example, one needs twice as much $E_{c.m.}$ to produce W^+W^- pairs in e^+e^- collisions, and 4 to 10 times more energy to produce a gluino pair via the bremsstrahlung reaction $e^+e^- \rightarrow \bar{q}\,q\,\tilde{g}\,\tilde{g}$.

There is probably no fair way to distill this information down into a simple figure-of-merit comparison. Undaunted, we have computed in Table 5 the centre-of-mass energy fractions x to which each of the entries in Table 4 correspond.

Table 5

Effective e^+e^-/hh $E_{c.m.}$ fractions

	$E_{c.m.}$ = 10 TeV	$E_{c.m.}$ = 40 TeV
Jet Pairs	0.36	0.20
$\mu^+\mu^-$ or L^+L^-	0.03	0.01
Z^o	0.12	0.04
$W^\pm$	0.24	0.08
η_T	0.2	0.08
$\tilde{g}$	0.24	0.12
$\tilde{Q}Q$	0.18	0.12
Geometric Mean	0.16	0.07

We have then computed the geometric mean

$$\langle x\rangle = \left[\prod_{i=1}^{N} x_i\right]^{1/N} \tag{30}$$

and interpreted it as a mean energy conversion factor. We find

$$\langle x\rangle_{10\,TeV} = 0.17\ , \quad \langle x\rangle_{40\,TeV} = 0.07\ . \tag{31}$$

The mean conversion factor at 10 TeV is close to the naive estimate 1/6 of Section 5.1. However, the conversion factor at 40 TeV is considerably smaller. It indicates that increasing the hh centre-of-mass energy by a factor of 4 increases the physics reach by less than a factor of 2.

This deduction is depicted graphically in Fig. 9. The vertical axis displays the $E_{c.m.}$ in hadron-hadron collisions which is required to duplicate the physics reach of a given e^+e^- collider in the 1 to 4 TeV range of $E_{c.m.}$. We see that the naive 1/6 "rule" breaks down significantly in the energy range of interest, and that the physics reach of an e^+e^- collider with $E_{c.m.}$ = 2 to 4 TeV can be matched in the mean by an hh collider with $E_{c.m.}$ = 10 to 100 TeV. Since we have taken the geometric mean, hadron-hadron colliders win a few and lose a few. They have better physics reach for gluinos, but lose out on heavy leptons or other weakly interacting particles.

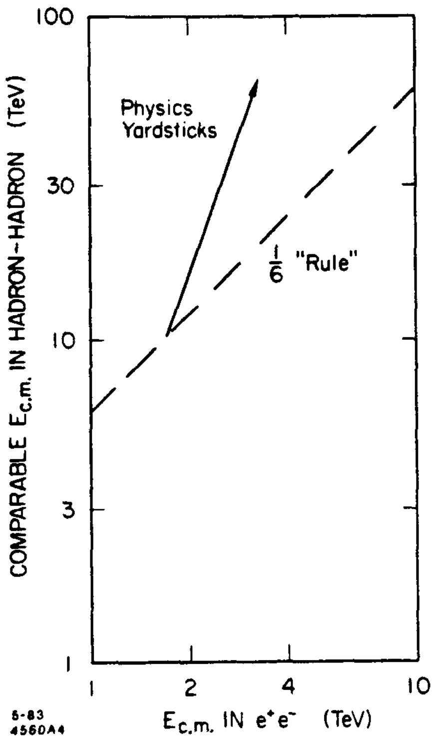

Fig. 9. The $E_{c.m.}$ required in hadron-hadron collisions to match the mean physics reach of a given e^+e^- collider.

5.3 – Discussion

While Fig. 9 is self-explanatory, there are two more points that should perhaps be re-emphasized. These are the relative cleanliness and precision in energy of e^+e^- collisions. The cleanliness permits detailed studies which are impossible in hadron-hadron collisions, as we know from the history of c and b quark exploration. This is an advantage not only in the detection of new particles such as squarks or technipions, but also in exploring their decay modes, which are vital clues to what exactly one has found. The ability to choose the centre-of-mass energy in e^+e^- collisions enables one to pick out a given feature of interest such as the $\psi''(3770)$ or the $\Upsilon(4S)$ and study it carefully. The wide spread of parton-parton scattering energies in hadron-hadron collisions precludes such concentration. It is difficult to imagine unravelling the line-shape of the ρ_T and ω_T by deconvoluting energies in hadron-hadron collisions. However, the wide range of parton-parton scattering energies does facilitate scanning experiments, such as those which found the J and Υ quarkonia. Nevertheless, it is also possible to scan in e^+e^- experiments, as witnessed by the discovery of the ψ simultaneously with the J (not to mention the follow-up on the discovery). The Υ family of resonances would have been found by scanning at CESR even if no hadron-hadron collision experiment had been available to reveal them earlier. These arguments are further reasons why Fig. 9, although eloquent, does not tell the full story about what hh centre-of-mass energy is necessary to duplicate the physics reach of a given e^+e^- collider.

I believe that it is premature to conclude that the next leap in high energy accelerators should be into a hadron-hadron collider (circular desertron). I believe that a high energy e^+e^- collider (linear desertron) deserves equal consideration, and think it would be unwise to foreclose this option without further study.

ACKNOWLEDGEMENTS

It is a pleasure to thank Michael Peskin for much advice and help with this presentation.

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