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TOWARD A CONSTRUCTIVE PHYSICS *

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ABSTRACT

We argue that the discretization of physics which has occured thanks to the advent of quantum mechanics has replaced the continuum standards of time, length and mass which brought physics to maturity by ***counting***. The (***arbitrary*** in the sense of conventional dimensional analysis) standards have been replaced by three dimensional constants: the limiting velocity c, the unit of action h, and either a reference mass (eg m_p) ***or*** a coupling constant (eg G related to the mass scale by $hc/(2\pi G m_p^2) \simeq 1.7 \times 10^{38}$). Once these physical and experiential reference standards are accepted, the conventional approach is to connect physics to mathematics by means of dimensionless ratios. But these standards now rest on counting rather than ratios, and allow us to think of a fourth dimensionless mathematical concept, which is ***counting integers.*** According to constructive mathematics, counting has to be understood before engaging in the ***practice*** of mathematics in order to avoid redundancy. In its strict form constructive mathematics allows no ***completed*** infinities, and must provide finite algorithms for the computation of any acceptable concept. This finite requirement in constructive mathematics is in keeping with the practice of physics when that practice is restricted to hypotheses which are testable in a finite time. In this paper we attempt to outline a program for physics which will meet these rigid criteria while preserving, in so far as possible, the successes that conventional physics has already achieved.

1. INTRODUCTORY REMARKS

We contend that the advent of quantum mechanics and the replacement of continuum standards of mass, length and time by standards based on counting integers has set the stage for a discrete physics based on finite, constructive mathematics. We offer a constructive algorithm which, starting from the empty string, leads to a growing universe of unique bit strings generated by discrimination and complementation. When neither operation generates novelty we increase the bit length of each string by adjoining a random bit at the growing end of the string. The only exception to this is the case when both strings, tested by discrimination, their complements, and the result of the discrimination are already in the universe, and in addition the string produced by discrimination contains an equal number of 0's and 1's. These unique events are identified both with the unique and indivisible quantum number and momentum conserving scattering events of quantum theory *and* with the events of particulate relativistic mechanics, once we have made clear the connection between our construction and laboratory space-time. In this way we achieve the unification of quantum mechanics and special relativity at an appropriately fundamental level.

During the construction we test the strings for linear independence and in this way construct sequentially 2, 3, 7, and 127 linearly independent vectors which can serve as the basis vectors for a representation of the four levels of the combinatorial hierachy. Once the basis is complete we allow the universe to continue to grow until all the discriminate closures of the basis vectors are present, completing the hierarchy. This gives us the scale constants 3, 10, 137 and $2^{127} - 1 + 137 \doteq 1.7 \times 10^{38}$, which are the cumulative cardinals of the sequentially completed levels.

Since the labeling capability of the combinatorial hierarchy scheme is now exhausted we define the bit string length when this occurs as the label length N_L. Since the length of the strings continues to grow, we define the portion of the string beyond the first N_L bits as the *address*. We group the strings hereafter into ensembles each of which carry the same first N_L bits, which we call the *label*, each member of the ensemble carrying a distinct address. It is clear that in this way we generate, eventually, all possible 2^{N_L} labels of length N_L, $2^{127} + 136$ of which can be further identified with the four levels of the combinatorial heirarchy. Because the matrix mapping construction of the hierarchy leads naturally to the restriction $N_L = 256$, we consider only that case in this paper. Thereafter all that can happen is that the number of members of each ensemble and the length of their addresses continue to grow.

We now focus on an event as previously defined and a second event one of whose labels is common with the first event, but occuring after the string length of the addresses has increased by b bits. We interpret the connection between the two events as a random walk of b steps, following a construction pioneered by Stein. Assuming a finite step length this automatically insures that we have a limiting velocity between events. Interpreting our criterion for an event– that the number of 0's in the address string for the intermediate state is equal to the number of 1's- - as defining zero "velocity" when the difference between these two numbers is zero, these two events define a "coordinate system". We then show that we can define coordinates for connected events in such a way that the intervals between events and relative velocities can be chosen in such a way that we have the same algebra and geometry as that described by the usual Poincaré transformations in 1+1 Minkowski space with restrictions imposed by the fact that our "time parameter" b is finite and integral. We then show that the construction can be extended to 3+1 Minkowski space, but no further because of the limitation of the hierarchy to four levels. In this way we demonstrate that the basic space for the description of events in our construction is 3-dimensional. We note that interactions in this space can be anticipated to have chiral properties.

We introduce the concept of mass by assuming that there is a correspondence between the labels and a parameter for each label with the physical dimensions of mass. We relate this to the finite step lengths in the random walks, and to the limiting velocity - which is now given physical dimensions and symbolized by c- by introducing a second parameter with the physical dimensions of action symbolized by h and defining the step length in a coordinate system where the velocity of the random walk is zero as the Compton wavelength $l_0 = h/mc$. This allows us to define conserved relativistic energy and momentum for free particles. To insure this connection we make the contact between our mathematical model and laboratory experience precise by requiring that any two events that can lead to space-time separated firings of counters (or their unobserved equivalent) conserve *vector* 3-momentum. We thus claim to have established a formal correspondence to relativistic particle kinematics within the framework of our theory.

We now construct coherent ensembles of these labeled sub-ensembles specified by a unique 3-vector momentum and demonstrate that these constructions exhibit a discrete version of deBroglie wave double slit interference. By identifying our events with the physical happenings that lead to the firing of counters in the laboratory, we can relate our model to the practice of high energy particle physics. We can then identify our limiting velocity c and our unit of action h with the laboratory definition of those physical parameters. Picking some reference particle, which we take to be the proton, we can then show that relativistic energy-momentum conservation allows us to measure mass ratios. The discrete interference phenomenon we derived then allows us to identify the step length of our random walk model in any arbitrary coordinate system with the physically defined deBroglie phase wave length and derive the basic Einstein-deBroglie quantization condition $E = hc/\lambda_{ph}$. This immediately leads to the deBroglie wavelength $\lambda = h/p$ and relates the universal constant h to our digital model. We now have the dimensional constants c, m_p and h; this validates our claim to have constructed a physical theory.

So far we have used only label conservation in the construction. Using the first two bits of each label to refer to some experimentally definable conserved quantum number such as charge or the particle-antiparticle dichotomy or helicity for spin 1/2 fermions, we construct a quantum mechanical scattering theory in momentum space for three particle systems. Our basic counter paradigm applied to the discrete coherent ensembles already introduced then leads us to the necessity of introducing probability *amplitudes* whose squares give us classical *probabilities*. We then recover free particle relativistic wave mechanics as a continuum *approximation*. The finite step length forces us to introduce complex numbers, and explains for us the ubiquitous $i0^+$ in the "propagators" of quantum mechanical scattering theory. This leads to the starting point of a "zero range scattering theory" which was initially derived within the conventional framework and is being vigorously pursued in that context. Connection to configuration space and "wave" phenomena then arises from Fourier transformation, as is customary for S-matrix theories which start in momentum space and use scattering boundary conditions.

Returning to the combinatorial hierarchy method for conserving the information content of discriminate closure and connecting it between levels, we find that level 1 describes a two-component neutrino theory, level 2 gives us the quantum numbers of quantum electrodynamics for electrons, positrons and gamma-rays, and the two combined give us the quantum numbers for weak-electromagnetic unification in the leptonic sector. Level 3 is then naturally interpreted in terms of baryon number, charged and neutral baryons, and isospin, providing the quantum numbers for $SU3$. The scheme might also support at level 4 a quark-heavy lepton-graviton interpretation, and the numerics suggest a connection to Harari's "rishons", but more work is needed before a choice is made.

Independent of the details of the quantum number assignment, we now claim to be on firm ground in interpreting Parker-Rhodes' successful calculation off the proton-electron mass ratio, as a calculation of the basic baryon-lepton mass ratio. Then the comparison with experiment is naturally to be made with the only known stable (to at least 10^{31} years) massive baryon and lepton. Since the baryon mass (recall that we are allowed *one* mass on dimensional grounds) is only approximately the proton mass (in the first order interpretation of the combinatorial result $hc/(2\pi G m_B^2) = 2^{127} + 136 \simeq 1.7 \times 10^{38}$) the calculation of the corrections needed to obtain the absolute value of the proton mass (or, equivalently the empirical value of G) and the empirical value of the fine structure constant remain a challenge for the theory. Once the dynamics, which is being explored in a more conventional context, allows us to compute ***unstable*** baryon and boson masses, this problem will provide a crucial test for the theory. Since we are allowed only one dimensional mass, there is no place in the theory for different gravitational and inertial masses. The problem of going from spin one photons and spin two gravitons to a continuum ***approximation*** in classical fields is basically the same as for any S-matrix theory which starts with a microscopic description based on quantum phenomena. Finally, some of the cosmological implications of the construction are briefly discussed.

2. BACKGROUND

Physics as formulated by Galileo in terms of length and time and completed by Newton, in the dimensional sense of physics, by the additional concept of mass used as its mathematical paradigm the continuum geometry of Euclid. Galileo in practice used the Eudoxian theory of proportions and hence could relate arbitrary laboratory standards of length and time to pure numbers, thus connecting dimensionless (in the physical sense) mathematics to the world of experience. The completion of this connection by Newton used, according to Mach's analysis, mass ratios and the Third Law (momentum conservation). So far physics has not found it necessary to introduce any dimensional standards other than length, time and mass, or three independent combinations of these units raised to integral or fractional powers. Hence classical physics is "scale invariant" and the Euclidean mathematical paradigm (extended by the calculus) appropriate.

The first break in this picture was forced by Planck's and Einstein's discovery that energy is quantized, and by Einstein's discovery of the universal limiting velocity, giving us the universal constants h and c. The empirical fact that electric charge is quantized still does not break the scale invariance since $hc/2\pi e^2 \doteq 137$ is a dimensionless number, but its universality cries out for explanation, at least according to Einstein. In his biography, Pais[1] says

"I conclude this time capsule with a comment by Einstein on the meaning of the occurance of dimensionless constants (such as the fine structure constant or the electron-proton mass ratio) in the laws of physics, a subject about which he knew nothing, we know nothing: 'In a sensible theory there are no [dimensionless] numbers whose values are determinable only empirically. I can, of course, not prove that...dimensionless constants in the laws of nature, which from a purely logical point of view can just as well have other values, should not exist. To me in my "Gottvertrauen" [faith in God] this seems evident, but there may well be few who have the same opinion.'[E9]"

The additional fact that the proton and the electron (stable for at least 10^{31}

years), as well as demonstrablly composite atoms and nuclei, have unique mass values breaks scale invariance in practice, but current theory does not have sufficient explanatory power to tell us why. Using the universal gravitational constant and the proton mass we can form the dimensionless combination $hc/2\pi G m_p^2 \doteq 1.7 \times 10^{38}$ which again cries out for explanation.

Meanwhile the arbitrary meter, second and kilogram have disappeared into history and have been replaced by a fixed number of wave lengths emitted by a monoisotopic atomic source which at one time approximately occupied the distance between the scratches on the standard meter, a fixed number of oscillations of an atomic clock which at one time approximated to what was then the standard second and (eg) the number of a specified type of atoms which at one time approximately balanced the standard kilogram; that is, all our current standards are based on counting integers. Yet current explanatory efforts based on quantum field theory start from continuum mathematics and, after considerable trial and error and experiment, attempt to "discover" the symmetries and non-linear "interactions" which will lead to the observed discreteness. Our contention is that current physics is ripe for an explanatory theory which starts from finite numbers and allows no completed infinities; hence we posit that the appropriate mathematical paradigm should be taken from finite constructive mathematics.

The key conceptual point here is that, in contrast to continuum mathematics which has to be connected to physics by ratios of physically measured quantities which can only be made suitable for mathematical analysis by taking out three arbitrary (historically speaking) units of MASS, LENGTH and TIME, constructive mathematics deals directly with integers, and hence is appropriate for introducing dimensionless integral (or rational) quantum numbers as a *new* type of link between mathematics and physics. In other words, quantization should be basic to the theory rather than derived. The problem is how to accomplish this.

2.1 Historical Background

This program has been pursued in various ways for a number of years, and is the product of several lines of development. Some have been brought together subsequent to the foundation of the Alternative Natural Philosophy Association in 1979. One strand of this work, of which Whitehead and Eddington were precursors, was the discussion of space time structure from an algebraic point of view, the dimensions being regarded as (we would say now) a combinatorial structure in fact isomorphic with the first level of the hierarchy as now known. This early work was published in a sequence of papers by Bastin and Kilmister about the *Concept of Order*[2–3] Another step was Bastin's hierarchical and multiple feedback loop model in which the points in spaces were built up in a hierarchical manner with the dimensional structure appearing at the simplest stage. Gordon Pask constructed a hardware form. In 1961 Parker-Rhodes made an algebraic formulation of the model in terms of binary variables. He invented the matrix mapping representation of the level connection (see Chapter 2) and the use of matrices as the new vector operands. He discovered the breakdown of the construction at the fourth level (when the successively completed structures are characterized by the integers $3, 10, 137, 2^{127} - 1 + 137 \doteq 1.7 \times 10^{38}$) and Bastin noted the connection to the scale constants of physics. Amson, in discussion with Bastin, in 1965 isolated the crucial notion of ***discriminate closure***, and then Kilmister showed that discrimination necessarily introduced an abelian group structure in each level of the hierarchy. Part of this collaborative work by Amson, Bastin, Kilmister and Parker-Rhodes was published in 1966[4] .

Meanwhile Noyes was becoming increasingly dissatisfied with the failure of hadronic theories using Yukawa type couplings to provide a quantitative and controlled description of the strong interactions. It is still possible to maintain that neither second quantized field theories nor S-matrix theory based on dispersion relations and crossing have met that challenge today. He proposed[5–6] a unitary relativistic scattering theory for three or more particles using as input only the on shell or "zero range" scatterings of the pairs. This program turned out to be difficult to articulate. By now the precise conditions under which the non-relativistic theory is self-consistent have been found.[7] The minimal relativistic three particle equations have been consistently developed by Lindesay[8] ; they go to the correct non-relativistic limit in that they predict quantitatively the Efimov effect (i.e. a logarithmic accumulation of three particle bound states) in the appropriate kinematic region.[9] Under the assumption that there is no direct particle-particle scattering but that a particle and a quantum can bind to form a state of the same mass and quantum numbers as the particle, the three body equations provide a covariant and unitary description of particle-particle scattering generated by single quantum exchange.[10] In contrast to the equations arising from conventional approaches these equations go uniquely and unambiguously to the non-relativistic scattering generated by a local Yukawa potential at low energy; the precise connection to quantum field theory is under investigation. This work is relevant to the current paper because the theory we construct leads most easily to this "zero range scattering theory" rather than to more conventional quantum mechanical formalisms.

Connection between Noyes' approach and the combinatorial hierarchy approach was first attempted some time ago. After becoming interested in the hierarchy work in the early '70's, Noyes attempted to survey some of the reasons why conventional approaches were felt by him to be inadequate[11] and presented the combinatorial hierachy work at one of the SLAC summer schools.[12] Work by Bastin and Amson was presented at the 1976 Tutzing Conference, and an attempted integration of the combinatorial hierarchy work with Noyes' ideas about scattering was presented by Noyes and Bastin at the 1978 Tutzing Conference; neither report appeared in the Proceedings, for reasons best known to the editors. A reasonably complete presentation of both the basic philosophical ideas and the technical achievements at that stage was subsequently published[13] ; see the introductory section of that paper for comparison with the *Ur* program of von Weizsacker and the *space time code* of Finkelstein. This was also the first occasion on which an attempt was made to integrate into the mainstream work the remarkable calculation of the electron-proton mass ratio by Parker-Rhodes[14] given in his *Theory of Indistinguishables.* A general description of his theory is given in Appendix I. Recent work by Kilmister on the foundations of the combinatorial hierarchy is given in Appendix II.

What was missing at that time was any explicit way to proceed from the individual concatenating processes (discriminations - see below) among bit strings, or *Schnurs*, and individual particulate scattering processes in space time. The connection was supplied by the work of Stein[15] developed initially from completely independent considerations. Extensive discussion of his work at the 2nd and 3rd annual meetings of the Alternative Natural Philosophy Association (1980, 1981) revealed both that there were close connections between Stein's thinking and Parker-Rhodes "indistinguishables"[14], and that Stein's construction of space-time and "particles" from random walks might be what we were looking for. Meanwhile Kilmister had realized that we were missing the initial constructive steps needed to get the scheme off the ground, and attempted to supply these[16] by modifying a construction of the integers due to Conway[17]. Noyes realized that the scheme proposed by Kilmister for generating bit strings would keep on going after the hierarchy scheme for preserving information was exhausted, and hence would provide labeled ensembles of strings which could be used for the Stein construction. A new presentation of the basic program making use of this insight was presented in April 1982 at the conference honoring Louis deBroglie's 90th birthday[18].

Discussion of the ever growing bit string universe at the 4th annual meeting of the Alternative Natural Philosophy Association and a refinement of Kilmister's treatment of generation and discrimination[19], followed by intensive work by Noyes, Bastin and Kilmister, allowed a complete overhaul of the paper for the de Broglie Symposium Proceedings prior to the submission of that paper[20] for publication. Other ideas presented at ANPA 4 have had considerable impact on the preparation of this paper. The presentation at ANPA 4 by Aerts of his thesis result[21] that the separability of classical physics is ***logically*** incompatible with standard quantum mechanics, and in particular with the von Neumann projection postulate, reinforced the conviction of Bastin and several other members of the Association that Bohr's concept of the correspondence principle is invalid and strengthened our case for the necessity of revision at the fundamental level. Bastin's views on complementarity are represented in this paper by an excerpt from his unpublished book *The Combinatorial Basis of the Physics of the Quantum* included as Appendix III. Manthey's contention[22] that concurrent communicating asynchronous digital systems necessarily generate randomness and that such systems necessarily have a conservation law and an uncertainty born of discreteness, exclusion, and asynchrony coupled with the fact that physical computing systems have a built in limiting velocity, reinforced the conviction that Stein's construction was the right place to start in order to obtain the Lorentz transformation, momentum-energy conservation and the uncertainty principle of quantum mechanics. Gefwert's discussion of constructive mathematics then and subsequently made it clear that it would be fruitful to make an attempt to simulate the whole construction by a computer program. A preliminary attempt to do just this by Noyes and Manthey led directly to the ideas presented in this paper.

We have been more complete than usual in presenting this historical background because no extant publication, other than a couple of paragraphs in the introduction to Parker-Rhodes' book[14], covers the many strands of thought on which the program relies. It is now time to turn to the ideas themselves.

2.2 Basic Ideas

Our basic postulate is that quantum events are unique and indivisible, but for reasons we wish to understand cannot be localized in the space-time of classical physics. We choose the high energy particle physics laboratory as the paradigm for the practice of physics. The basic data are the sequential firings of counters separated by macroscopic space and time intervals. We can use this type of data to measure mass ratios of particles relative to some standard type of particle using relativistic energy-momentum conservation. Given space and time enough we can refine the accuracy of these measurements as much as we like, at least for stable particles. This precise information is readily described by the relativistic kinematics of classical theory.

By using this descriptive framework we can derive from laboratory experience statistical information about the probability of the scatterings observed via the firings of the counters, or "cross sections". Conventional (and quantum) theories predict cross sections which exhibit interference phenomena reminiscent of the intensities in classical wave theory, but which can only be compared with theoretical predictions from quantum mechanics in the sense of the law of large numbers by the accumulation of a sufficient number of events starting from what, so far as we know, are the same initial conditions. For elementary particle phenomena what is missing compared to the classical situation is some independent means of measuring the real amplitudes whose squares predict the cross sections. The quantum theory puts these amplitudes beyond reach in principle, and not just in practice, by making them complex, but relative phases between amplitudes remain observable; it is only one overall phase that is always beyond reach, and a few more when there are "superselection rules".

Our problem is to construct a description or theory or "model" in which the situation described in the last paragraph arises as a natural consequence of the construction, and from which the experimental results successfully interpreted by elementary particle theories can be shown to follow. We assume, as is conventional for theorists who take an S-matrix point of view, that once we have constructed relativistic scattering amplitudes whose absolute squares predict cross sections, it is then possible to construct from them all of non-relativistic quantum mechanics, and the classical physics of particles and fields, under appropriately restricted circumstances. This contention will not be argued further here.

We start from very primitive finite mathematical structures (which we believe, but do not attempt to demonstrate here, can be grounded in the constructive mathematics of Bishop[23] and Martin-Löf[24], and provide a self-generating and self-organizing algorithm which leads to a universe of bit strings whose bounded size keeps on growing as long as we wish. The information content of this universe is partially organized into ensembles whose initial bits or "labels" are a representation of the combinatorial hierarchy. We provide as Appendix IV a specific computer program which will simulate this construction; a detailed discussion of the program will be presented elsewhere[25].

Our computer algorithm makes use of two processes which create new strings either by discrimination defined for two *ordered* bit strings of length N symbolized by $S_i = (..., x_i, ...)_N$, $x_i \in [0, 1]$ as

$$D_N S_i S_j = (..., x_i +_2 x_j, ...)_N \tag{2.2.1}$$

and complementation defined by

$$\neg S_i = (..., x_i +_2 1, ...)_N = D_N S_i I_N \tag{2.2.2}$$

where $+_2$ is addition, mod 2 or "exclusive or", and I_N is the *antinull* string containing N 1's. When neither of these operations succeeds in generating a string not already contained in the universe of bit strings we generate novelty by increasing the string length of all strings by appending a random bit at the growing end. There is one exception to this rule. When $D_N S_i S_j = S_3 = D_N \neg S_i \neg S_j$, all five strings are already in the universe, *and* the number of 0's in S_3 is equal to the number of 1's we do *not* increase the string length but simply continue. We call this happening an *event*. During the construction we organize the strings into the four levels of the combinatorial hierarchy [4,13]. Once the hierarchy is complete, we use the initial bits in the strings reflecting this organization as *labels* for ensembles of bit strings, labeled by these inital bits, and uniquely specified within each ensemble by the (growing) remainder of the bit string which we call the *address*. This construction is described in detail in Chapter 3.

In Chapter 4 we construct our discrete substitute for space-time "coordinates" by identifying explicitly chosen labeled subensembles connecting two events for which the the bit length of the address differs, but containing a common label, with Stein's random walks. We introduce our connection to physics by assuming that when we have two well separated counters of finite volume $\Delta x \Delta y \Delta z$ with a distance S between them greater than their spacial resolution which fire sequentially with a time interval T greater than their time resolution Δt that they define a velocity $v = S/T$ for some *object* which passed between them. The connection to the bit string universe is made by assuming that the label part of the string ensemble defining a random walk of b steps is to be identified later with the quantum numbers of the object. The dimensionless velocity of the object in the bit string universe is defined by $\nu = \langle N^1 - N^0 \rangle / (N^1 + N^0) = \langle N^1 - N^0 \rangle / b$. Here N^1 is the number of 1's and N^0 the number of 0's in the address strings of bit length b and $\langle\rangle$ is the average over the appropriate labeled ensemble. Clearly ν is bounded in absolute value by unity providing us with a limiting velocity for the connection between events. Hence we connect this aspect of the bit string universe to the limiting velocity of special relativity and laboratory experience by taking $v = \nu c$. This is done in such a way that, we claim, the events have the usual geometrical and transformation properties of 3+1 Minkowski space-time in an appropriate large number limit. For us the finite space-time volume of our counters makes special relativity into an approximate *macroscopic* theory. Our counter volumes cannot be allowed to shrink to points; we have prevented by our construction and interpretation any possibility of going to a *microscopic* continuum theory, and thus avoid the infinities of quantum field theory. Yet by the random walk paradigm and a set of specific algorithms we claim to show that this suffices to extract a limiting velocity and the usual observer-dependent coordinates of special relativity.

We now introduce dimensional units in the physical sense by identifying the random walk step length with the Compton wave length in the coordinate system in which two connected events have zero velocity and by postulating that the corresponding mass parameter is associated with one of our labels, which was the critical step taken by Stein. However, our treatment departs from his in that our basic counter paradigm compells us to see this length as Lorentz contracted in moving coordinate systems whereas he used it as a basic dimensional parameter. Our approach enables us to define relativistic energy and momentum for free particles correctly connected to the velocities we have already constructed. We now claim to have constructed a discrete version of classical relativistic particle kinematics which goes to the conventional continuum theory in the $h \to 0$ limit. We emphasize, as Stein did also, that this is only a mathematical approximation and *not* a "correspondence principle" limit in Bohr's sense. Our space time is a space time of discrete events connected by random walks of finite step length with *void*, not space, "in between". The events which we have now tied abstractly to coordinate systems related by the usual Poincaré transformations are our candidates for the finite, unique, and distinct happenings that lie at the core of quantum mechanics. Hence we will try to relate them to the processes which in conventional theory are called elementary particle scatterings and which are supposed to initiate the chain of happenings which, in the laboratory, are assumed to lead to the firing of a counter.

In Chapter 5 we try to to get from this reasonably familiar relativistic situation to quantum mechanics. We argue that our counter paradigm requires us to construct ensembles of ensembles with defined coherence properties. If this is done with care, we can then show that the underlying digital discreteness, which we have been careful to retain, allows us to anticipate interference phenomena, such as that found in the the double slit experiment, once we have succeeded in making the connection with our dimensionless (in the physical sense) mathematical structures and laboratory definitions of space, time and mass. We make the connection by first identifying the limiting " velocity" in the dimensionless theory with c and then assuming that some label can be put into correspondence with some laboratory particle such as the proton. We then can use the already established Lorentz invariance, which implies the relativistic kinematics of free particles, to relate the mass ratios of the theory to actual laboratory practice. The interference phenomenon mentioned above can now be connected to measurements of the deBroglie wavelength h/p. Having now shown how to connect our mathematics to the dimensional constants c, m_p, and h we can claim to have established a physical theory. By examining our counter paradigm in more detail we then show that we can recover the conventional continuum theory of deBroglie waves as an *approximation*.

We now concentrate on the first two bits in each label and show that these can be interpreted as conserved quantum numbers in such a way as to construct the elementary scattering amplitudes which drive the momentum space integral equations for a three particle relativistic quantum mechanical scattering theory identical in mathematical structure to the *zero range scattering theory* [5–10] initially developed from a more conventional starting point. Since this scattering theory can be shown to be generalizeable to systems containing any finite number of particles, we claim to have made contact with the practice of elementary particle physics at an appropriate level. In an S-matrix theory, the conventional approach is to connect the theory to wave phenomena by Fourier transformation. Whether this is a valid procedure is already a problem for S-matrix theorists, so we do not discuss it further in this paper; we rely on their competence as showing that a large body of practicing physicists are not too dissatisfied with this connection. We simply note that from their point of view, as for us, classical continuum wave phenomena and "fields" are not fundamental, but are derived consequences of elementary scatterings.

In Chapter 6 we discuss briefly a number of problems that remain to be solved before our theory can be expected to attract the interest of more than a few practicing elementary particle physicists. At the level of quantum numbers we have previously proposed [13] what looks like a promising interpretation in terms of charge, baryon number, lepton number, and helicity; here it is revised to bring it into closer contact with quantum numbers know from elementary particle experiments. This has the advantage of providing a rationalle for Parker-Rhodes remarkable calculation[26] of the proton-electron mass ratio. This argument is briefly reviewed. But we are also struck by the coincidence between the number of basic particles (8) we encounter at level 3 of the hierarchy and the basic "rishons" proposed by Harari[27]. The more detailed articulation of the theory to the point where we can make a choice would take us beyond the scope of this paper, so is given only cursory attention. We close with a few remarks on the cosmology implied by the construction.

3. CONSTRUCTING A BIT STRING UNIVERSE

In this paper we cannot discuss many of the basic philosophical issues raised by our program; some of them will be discussed by Christoffer Gefwert in his forthcoming thesis on constructivism[28]. We leave it open, here, whether it can be conclusively shown that a bit string universe constitutes the necessary conditions for explanatory closure consistent with the current practice of physics. "All" we need take from constructive mathematics are the symbols $0, 1, +_2, =$ with their usual significance, i.e.

$$0 +_2 0 = 0;\; 0 +_2 1 = 0;\; 1 +_2 0 = 1;\; 1 +_2 1 = 0$$

the "random" operator R which gives us 0 or 1 with equal probability, and ordered bit strings of the symbols 0 and 1. We take the symbols $0, 1, +_2$ to stand for primitive recursive functions. Now the expressions set out above can, essentially be seen as ***programs*** which give the information needed for their own evaluation[29]. By this strategy we aim at showing the expressions above to be self-explanatory vis-a-vis meaning; we do not have to embark on a reductionist strategy in order to justify the use of these expressions. From these symbols we then proceed to construct a universe of strings of the existence symbols 0 and 1 starting from the empty string. We show that this universe is self-organizing in a manner that can be labeled by the combinatorial hierarchy. Since the work has not appeared elsewhere, we include in Appendix II an earlier construction due to Kilmister[16] which starts from the empty set, and a refinement of this approach by him[19]; a third cut across this material is given in Appendix II.3. The construction discussed here is due to joint work with Manthey, and draws from the background provided by Kilmister.

3.1 Generation of the Strings

Our algorithm for creating a universe of bit strings starting from the empty string has been coded in *Pascal* by Manthey and is included here as Appendix IV. Since it uses concurrent programming, which may not be familiar to many readers of this paper, we will use here a less elegant approach presented using sequential programming flow charts and definitions. A full discussion of the concurrent program will be presented elsewhere[25]. Nevertheless, even in this sequential program the essential aspect of concurrency - which Bastin[30] maintains is a fundamental necessity in our construction - is already contained. The reason is that everything happens *between* TICKs, and cannot be ordered within that restriction; but this is getting ahead of our story.

Our initial algorithm is displayed in Figure 1. We see that $\mathcal{U}(N, SU)$ starts from the empty string, and sets the number of strings in the universe, and the length of the strings equal to zero. The critical operator we need for the construction is called R. The function of this operator is simply to pick randomly between the two bit symbols 0 and 1 with equal probability. We are investigating the problem of just where the idea of randomness enters constructive mathematics and how such an operator is to be constructed. For the moment we content ourselves, at least for the purpose of computer simulation, with the fact that pseudo-random number generators are a standard part of computer practice and are also used in high energy particle physics for "Monte Carlo" programs. The specific random number generator used by Manthey in the program assumes that a bit in a memory cell is flipped between 1 and 0 on a fixed local cycle time. A sampling process, which is necessarily *asynchronous* to the bit flipping process (necessarily so, to be consistent with the definition of process), therefore samples (reads) 1 or 0 with equal probability. Asynchronicity can be obtained in practice by driving the second cycle with (eg) a quartz oscillator detuned from the first. Whether this is the best way to achieve the result is irrelevant so long as some adequate random bit generator exists. The critical question for us is rather, since the program is more than exponential, whether pseudorandomness can *in principle* be extended rapidly enough to meet the requirements of the program no matter how large the finite size of the universe has become. We do not attempt to discuss that problem here.

Since our aim is to construct an ever growing universe of distinct symbols which is self-generating, we need to have a way of checking whether or not any symbol we turn up already is in this universe or not. Kilmister has shown[16,19] how to do this starting from the empty set and defining a "prediscrimination" operation which becomes equivalent to the discrimination operation used in earlier discussions of the combinatorial hierarchy[4,13], and already defined by Eq. (2.2.1). We assume that constructive mathematics can define for us what we mean by the ordered strings containing N of the existence symbols 0,1 needed in that definition. Clearly if the two strings are identical this operation gives us the null string 0_N; otherwise – for $N \geq 2$ – it must generate a string which differs from either. The first time through the program we pick a bit at random and call it the first string $U[1]$ in $\mathcal{U}(N, SU)$. Since discrimination requires two strings if it is to produce novelty we keep picking a bit a random, checking by discrimination whether it differs from $U[1]$ and when we succeed call it $U[2]$. For the purposes of the computer simulation we order the elements of $\mathcal{U}$ by the integers $\in [1, SU]$, the order being simply the order in which the new strings are generated. We will discuss shortly why this is only a simulation of the situation we actually envisage in which the order of the strings in the universe in the sense required by the simulation is forever beyond the reach of experiment.

Now that we have two strings we are ready to start the main program. Our first method of generating novelty in $\mathcal{U}$ is simply to pick two strings from $\mathcal{U}$ at random and discriminate them. If we unluckily have picked the same string twice, we try again. If we pass this test, we still may have generated a string already contained in $\mathcal{U}$ as the result of earlier operations, so we must test the candidate against all the strings in $\mathcal{U}$. If we pass this test, we adjoin it to $\mathcal{U}$ and continue. It should be remarked that here we are adopting Parker-Rhodes concept of "identity" as used in *The Theory of Indistinguishables*[14] with the implication of uniqueness. He differentiates this concept cleanly from the concept of indistinguishability, or the existence of "twins". Of twins one can can say that there are two of them but that they cannot be labeled; hence finite collections of twins can be assigned a cardinal number but cannot be ordered – they are "sorts", ***not*** "sets".

Our algorithm provides a simulation of his concept of identity,– necessarily a simulation because "twins" cannot be directly observed macroscopic objects or constructed using standard mathematical operations in a computer. Yet we can simulate the idea by requiring that the strings in $\mathcal{U}$ can only be accessed at random. We insure that they are distinct in that discrimination between any two of them gives a non-null result, thus telling us that we have two of them, yet they are "indistinguishable" in that we cannot tell which two we have. We thus claim to have constructed a simulation of a collection of twins of cardinality $S\mathcal{U}$. Of course the computer has to order the elements of $\mathcal{U}$ by the integers in order to function, and has to use bit strings ordered along the string by integers in order to carry out the discriminations. But we have arranged the program in such a way that this information is not available to us.

Although we have now insured that our universe contains only distinct strings, at any bit length N the operations defined solely by discrimination will eventually stop generating new strings because all the possibilities have been achieved. Hence we need a second operation to allow the universe to keep on growing. We provide this simply by putting in a branch which is used any time we have attempted to create a string already contained in $\mathcal{U}$. In our initial version of the program what we did then was simply to increase the string length by adding a random bit to each string in $\mathcal{U}$. Of course this could happen before we have created all possible strings of the bit length we are working with. Hence the universe we generate will have a random structure that is not predictable from the algorithm. The requirement that all the other strings are also augmented is simply a way of keeping the bit length of all strings in $\mathcal{U}$ the same. This might seem to be in violation of the principle of relativity, since in a sense this provides a "universal time". Actually such a time exists – the time since the "big bang" – and can be measured locally by the temperature of the background radiation, which is currently 2.7^0K. We therefore feel justified in allowing our simulation to contain this feature. The name we give to this string length increasing operation is "TICK".

The first time we enter the main program, we know that the universe consists of the two bits 0 and 1 and hence that we cannot create novelty by discrimination, so we enter the main program at TICK. From now on the program runs as indicated in the flow chart, Fig 1. The basic new operator we need at this point is one that will pick a string from $\mathcal{U}$ at random, and is called "PICK". Since the strings in $\mathcal{U}$ are indexed by integers all this amounts to is a random number generator for an integer $\in [1, SU]$ which is obviously easy to construct in binary notation using R. The explicit coding is given in Appendix IV. So now we pick a string, pick another which we check by discrimination is different, and if the result of the discrimination is novel we adjoin it to $\mathcal{U}$. (Ignore for now the box called CONSTRUCT LABELS... which in no wise impacts the running of the main program.)

In our first attempt at computer simulation we thought all we had to do at this point, if we had failed to produce novelty, was simply to TICK. Actually, we found that this left out of the universe a process that could later be identified with elementary scattering events, unless we put it in "by hand" at a later stage. We did not like this extreme form of "observer participation", and have come up with a simpler solution in terms of an operation we required at a later stage in any case. This operation is simply complementation, which we have already defined in Eq. (2.2.2). So the rest of the program simply creates the complement of one or the other of the strings we picked initially on this pass and adjoins it to $\mathcal{U}$ if it is not already present. This will have a lot of advantages later on, but we do not want to get too far ahead of our story. If the complement of the string produced by discrimination is not already in $\mathcal{U}$, we adjoin *it* to $\mathcal{U}$ and continue. At this point, if we have still failed to generate novelty, the careful reader will realize that we have achieved a situation in which

$$D_N S_1 S_2 = S_3 = D_N \neg S_1 \neg S_2 \tag{3.1.1}$$

At this point, since novelty has not been generated, the obvious thing to do is to TICK. Some of the time this is indeed what will happen, but we have decided to put a final branch in the main program at this point in the sequence. If the number of 0's in S_3 is equal to the number of 1's, we do not TICK; we simply continue.

To understand why this choice was made, we must look ahead. We will see later that for the label part of the string the presence of a 1 will correspond to the presence of a quantum number. In the particular case when L_i has (a) an even number of bits and (b) an equal number of 0's and 1's, $\neg L_i$ will necessarily have 1's where L_i has zeros and visa versa. Hence, once we have succeeded in assigning physical meaning to the ordered position of an entry in a label string as referring to a specific quantum number, these quantum numbers are paired as dichotomous variables for the case considered. Thus we have a natural interpretation of L_i as describing the quantum numbers of a particle, and $\neg L_i$ as describing the quantum numbers of the corresponding antiparticle. Which is "particle" and which is "antiparticle" will depend on an arbitrary choice, which cannot be given precise meaning until the dynamics of the scheme have been articulated and given physical interpretation. We will also find that when the address part of the string (a) has an even number of bits and (b) has an equal number of 0's and 1's, this will relate to the starting point for defining zero physical "velocity". Hence the criterion we have specified for not going to TICK will correspond, eventually, to two particles S_1, S_2 encountering their two antiparticles $\neg S_1, \neg S_2$ and producing and intermediate state S_3 with the quantum numbers of a particle-antiparticle pair and zero velocity.

For those familiar with Feynman diagrams we have just described a basic four-leg diagram in the coordinate system in which the total momentum is zero. Our intent is to construct all other scattering processes from this category of elementary events, and the "vertices" created by discrimination or complementation. Since we do not allow a TICK, only the occurance is specified in the sequential evolution of the universe. We have no way, without providing more "background" information about the event, of specifying which particles are "incoming" and which are "outgoing". Thus at this elementary level we are guaranteed what we need for "time reversal invariance" and quantum number conservation. We also know from earlier work [13] that, once an external time sequence has been established, the usual Feynman rules equating a particle moving "forward in time" to an antiparticle moving "backward in time" with opposite charge, helicity, and any other appropriate quantum numbers will follow. We further anticipate that our version of the CPT theorem will emerge in due course. But all of this is yet to come.

We emphasize that at this point in the construction we have simply inserted a simple rule that is readily understood at the bit string level. It provides us with a well specified definition of what we mean by an *event* in the universe of bit strings. That we can justify this term as corresponding to the unique and indivisible scattering events of quantum mechanics and to the point scatterings of classical particulate special relativity will constitute the main objective of this paper. In fact we can, we believe, with some justification claim that we are also talking about the individual collisions between hard and impenetrable atoms envisaged by Leucippus and Democritus.

We now have completed our construction of a growing universe of bit strings. We trust you will grant that it is a simple algorithm, which could be simulated on a computer in its early stages. Of course, since the program is more than exponential, any such simulation could not catch up with the current state of the universe by actual calculation. Thus we must turn to how it can be used for conceptual, and necessarily partial, interpretation. Physicists will probably be more comfortable with what we are doing if they view it simply as a model whose consequences can be supported or refuted by experiment. We suspect it might prove to be more than that, but will not address that deep question in this paper.

3.2 The Combinatorial Hierarchy Construction

The simple algorithm for generating novelty presented in the last section creates an amazing amount of structure. To keep track of the information we invoke the concept of discriminate closure, which leads to the combinatorial hierarchy [4,13]. We define a discriminately closed subset (DCsS) as a single non-null string or as that set of non-null strings which when any pair are discriminated yield another member of the set. If we start from linearly independent strings $a, b, c, \ldots$ (i.e. $a+b \neq 0, b+c \neq 0, c+a \neq 0, a+b+c \neq 0, \ldots$) we can clearly form the DCsS's

$\{a\}, \{b\}, \{c\}, \{a, b, a+b\}, \{b, c, b+c\}, \{c, a, c+a\}, \{a, b, c, a+b, b+c, c+a, a+b+c\}$ and so on. Here we have used + for discrimination; since $a+a=0$ the closure of the subsets is transparent. From j linearly independent strings we can obviously always form $2^j - 1$ DCsS's because this is the number of ways we can choose j distinct objects $1, 2, .., j$ at a time.

Starting from strings with two bits (N=2) we can form $2^2-1=3$ DCsS's, for example $\{(10)\}, \{(01)\}, \{(10), (01), (11)\}$. To preserve this information about discriminate closure we map these three sets by non-singular, linearly independent 2x2 matrices which have the members of these sets as eigenvectors. Rearranged as strings of four bits these form a basis for $2^3-1=7$ DCsS's. Mapping these by 4x4 matrices we get 7 strings of 16 bits which form a basis for $2^7-1=127$ DCsS's. We have now organized the information content of 137 strings into 3 levels of complexity. We can repeat the process once more to obtain $2^{127}-1 \doteq 1.7 \times 10^{38}$ DCsS's composed of strings with 256 bits, but cannot go further because there are only 256x256 linearly independent matrices available to map them, which is many to few. Thus when our generating and discriminating operations have gone on for a while the information carrying capacity of our information preserving mapping scheme is exhausted. We have in this way generated the critical numbers $137 \doteq hc/2\pi e^2$ and $1.7 \times 10^{38} \doteq hc/2\pi G m_p^2$ and a hierarchical structure with four levels of complexity.

The generation of this structure and its termination can be summarized by a very simple algorithm. Each level l is generated from a basis $\mathcal{B}(l)$ containing $B(l)$ linearly independent strings. From these we can construct a set $\mathcal{H}(l)$consisting of $H(l) = 2^{B(l)} - 1$ DCsS's, as we have already seen. If we have available another set $\mathcal{M}(l)$ which contains at least $H(l)$ linearly independent strings, we can map $\mathcal{H}(l)$ by $H(l)$ of them and use this mapping as the basis set $\mathcal{B}(l+1)$ for the next level. The matrix method discussed in the last paragraph gives a means by which the the mapping can be explicitly constructed and a cutoff rule coming from the maximum number of linearly independent strings available. However the actual termination of the sequence of levels does not depend on the origin of the rule.

This algebraic structure can be started by assuming a 0th "level" with $H(0) = 2$ and $M(0) = 2$ and the iterative rules

$$B(l) = H(l-1); \ H(l) = 2^{B(l)} - 1; \ M(l) = M(l-1)^2$$

The iteration stops when $M(l-1) < H(l-1)$. We are also interested in the number of strings in play at each level which is $C(l) = \Sigma_{j=1}^{l} H(j)$. the result is given in Table I.

Although we have constructed a universe of bit strings with many discriminations going on at random, it may not be immediately apparent that this already implies the existence of the hierarchical structure just described. However, John Amson[31] has shown that the whole of the hierarchy structure can be derived using only the general framework of group theory without ever mentioning the mapping matrices. Further, Kilmister has shown that if we use a minimal representation for the hierarchy, then any other representation, and the corresponding mapping matrices, can be constructed. Since this is not intuitively obvious, we give the details of this previously unpublished work, updated and simplified in presentation for the purposes of this paper, as Appendix II.3. Consequently, so far as organizing information in $\mathcal{U}$ goes, we can use any scheme which generates the correct cardinal numbers. A specific algorithm for doing this in conjunction with our initial algorithm is presented as a flow chart in Figure 2.and explicit coding provided in Appendix IV. It differs from earlier constructions in that it relies only on linear independence and discriminate closure, and makes no explicit use of mapping matrices. Nevertheless, the original matrix mapping scheme turns out to be important when we make explicit contact with quantum numbers later on. We give this more general approach here because we have not yet been able to settle on a ***unique*** quantum number interpretation drawn from first principles. Once we have solved that problem the coding for CONSTRUCT LABELS will be given a more precise form.

The case for using the mapping matrices is in fact considerably stronger than the appeal to application would indicate. As Kilmister points out[32] : "...(a) if you don't make linear operators correspond to DCsS, then why should we consider

DCsS in particular - any subset would do. (b)Without the matrix character of the correspondence, there is no reason to stop at level 4. (c)The matrix trick is the only one I know in which the coding of a DCsS by a single element at the next level is intrinsic to the element - I mean, given the high element, you don't need a code-book to determine what subset it is representing."

3.3 THE LABEL-ADDRESS SCHEMA

The important conceptual point to grasp at this stage of the construction is that the steps we now take to bring out the fact that the universe generated by our main program is *already* organized in a hierarchical fashion, whether we make use of that fact or not. Hence the coding we now develop is introduced for *our* convenience and in no wise affects the evolution of $\mathcal{U}$. In this sense it is like the observation process in classical physics which postulates a structure and makes use of that model to extract information from nature, or to set up experiments to obtain information which we did not previously possess. Where our scheme differs is that the mode of access, although for computer simulation it makes use of the ordering of the strings in $\mathcal{U}$, will only provide us in the end with structural information that does not allow us to actually determine that integer sequence. Thus we preserve the indistinguishability characteristics of the bit strings, and are debared from reifying them - a philosophical mistake which is all too often made by those who still think that classical physics describes the "real world".

The way in which we achieve this is to leave the universal memory untouched and to construct arrays of pointers which tell us what strings in $\mathcal{U}$ correspond to a particular representation of the hierarchy, without either extracting the specific indices from the machine, or the strings themselves. Thus we will in fact end up with a specific representation of the hierarchy inside the machine, but we will have no way at this stage in the construction of knowing which of the very large number of possible representations of the hierarchy we have in fact achieved.

The flow chart for our program is given in Figure 2, and the explicit coding for it, with the modifications needed to achieve concurrency, is given in Appendix

IV. As can be seen from Figure l, we enter this box each time we generate a novel string S either by discrimination or complementation, and only at that point in the sequence. The first time through we assign a pointer to that string calling it $BV[1,1]$ the first basis vector, $m = 1$ for level $l = 1$ of the hierarchy. The general notation for a basis vector will be $BV[l, m]$, $l \in [l, 2, 3, 4]$ and $m \in [1, .., B[l]]$. We have already seen that $B[1] = 2, B[2] = 3, B[3] = 7, B[4] = 127$, so this will be built into the logic. Since our basic algorithm guarantees that any string adjoined to $\mathcal{U}$ is unique, the next time we enter the box S will differ, and we can assign a pointer to it indicating that it is $BV[1,2]$.

It might seem logical at this point to immediately compute the discriminate closure of the first level, whose basis is now complete, But this would get us into trouble later on. All we have assigned is a *pointer*, not an explicit string. Each time we go through TICK, the string itself will acquire a new (random) bit at the growing end. However, this will not affect the bits which make $B[1,1]$ and $B[1,2]$ distinct. Hence they will always serve as basis vectors for level one, whatever their length. This is the basic point which has to be understood about our construction. We construct the basis vectors first, and only after the bases for all four levels are complete do we attempt to construct their discriminate closures.

We have already seen that, given any linearly independent set of n strings of the same length, we can form $2^n - 1$ DCsS's. Here by linear independence we mean that by forming all possible "sums" (i.e discriminations) of the strings taken 2, 3, up to n at a time, we never produce the string containing n zeros. Thus for each level separately it would seem that we need only test any new string S which comes into the box for linear independence within the level on which we are working, and go on to the next level when the basis at that level is complete, that is when we have $B[l]$ linearly independent strings. However, although our program guarantees that any string which comes into the box is not already in $\mathcal{U}$, it by no means guarantees that it is linearly independent of the basis vectors which have been assigned at lower levels. Thus our test must run over all levels, whether completed or not. It is this test which gives to our construction its hierarchical character; this is our replacement for the matrix mapping construction discussed in the last section. The coding is straightforward, as can be seen by looking at Appendix IV. It terminates when we have 2+3+7+127=139 linearly independent strings, organized as we go along into the four levels of the combinatorial hierarchy. We emphasize again that, even though the length of the actual strings in $\mathcal{U}$ continues to grow (every time we go through TICK) the pointer designation of each string we have assigned to the incomplete basis array is unchanged. Further, the linear independence already achieved, since it comes from the initial bits along the string, is not destroyed by the random bits which TICK keeps adding at the growing ends.

At this point in the program we must make another decision. We obviously could at this point simply compute all the discriminate closures of this basis and complete the hierarchy. Any that were not already in $\mathcal{U}$ could then be adjoined to it. But this would constitute an intervention, or glitch, in the main program for which we see no physical justification, and in fact no necessity. What we choose to do is simply to test whether in fact all the $2^{127} + 136$ strings which complete the discriminate closures are already in $\mathcal{U}$ or not. If they are, the hierarchy is complete, and we go on to start forming labeled ensembles. If they are not, we simply let $\mathcal{U}$ continue to grow until this happens. Thus, once the basis array is complete we have to continue to perform this mammoth calculation of the discriminate closures until our goal is achieved. The way the logic is set up we have to do this each time a new string enters the box, which is bad from the point of view of computer efficiency. It would be better to set a flag each time we go through TICK and only perform the mammoth calculation on the first pass through the box after that has happened. But conceptually this makes two intervention points rather than one in the main program. In practice this doesn't matter, except for efficiency, but we use the first alternative as conceptually cleaner. Clearly neither choice affects the actual structure of $\mathcal{U}$, which is basically all we require *until* we have a firmer grasp on how specific quantum numbers are generated.

Once the hierarchy is complete, that is not only the complete set of basis vectors but also all their discriminate closures, are all in $\mathcal{U}$ we are ready to construct labeled ensembles. We know in advance that the bit string length N at this point will be at least 139, since otherwise we could not have found that number of linearly independent strings. If the matrix mapping construction has a fundamental significance, we would anticipate that the actual length will be 256, but we will not investigate that question here. All we need do is to record the actual bit length at this point in the sequence, which we call N_L. This will be the length of our labels from now on. We now set up labeled ensembles for all the strings in $\mathcal{U}(N, SU)$. Each time we enter the box with some new string S We examine the first N_L bits. If the label already occurs, we assign a pointer which tells us that S is the next element in the ensemble with that label. If the label has not showed up yet, we make S the first string in a new ensemble with that label. Eventually it is clear that we will end up with 2^{N_L} labeled ensembles. Thereafter the number of members in each ensemble, and the length of the addresses in each ensemble $B = N - N_L$ will continue to grow. This is automatic, so we need not record the value of SU at the point where the labeling scheme is exhausted. Just when this occurs, and the size of the ensembles for each label when it occurs, is an interesting statistical question, which may ultimately have significance with regard to the cosmology implied by our construction. We leave this aside as a question for future research.

One interesting aspect of our construction is that, in contrast to the matrix mapping construction, the fact that we stop at four levels has become arbitrary. Clearly we could have let our routine run long enough so that we could get the $F = 2^{127} - 1 + 139$ linearly independent basis vectors needed to construct five levels, and the $2^{F-138}-1$ vectors which form their discriminate closures. Or could we? It may be that the procedure keeps throwing up strings in such a way that we never get there. If so, we would have an alternative to the matrix mapping construction stop rule. We leave this interesting question for future research. We also could stop at fewer than four levels. Such simplified universes might form useful models for simple physical situations, with small enough dimensionality so that an actual simulation might be attempted in practice.

We now claim to have shown explicitly that our simulation of $\mathcal{U}$ contains the combinatorial hierarchy, and that the hierarchy can be extracted from it. Further the extraction has been done is such a way that only the cardinals of the levels are known. We cannot know within a level which are the basis strings and which the discriminate closures, only that there are exactly $B(l)$ linearly independent basis strings in each level, and $H[l]$ vectors in each completed level. Consequently the appropriate indistinguishability properties are preserved by our simulation. This should make it clear that the information expressed by the combinatorial hierarchy is implicit in our original construction. Hauling it out to look at is an aid to our thinking, *not* a necessary part of the construction. In the next chapter we will not even need the explicit number of ensembles available (although the construction specifies them), only the fact that there are four classes of labels.

4. CONSTRUCTING SPACE TIME and PARTICLES

Since we aim at a fundamental theory, the manner in which we "break in" to the system to define subsystems and the (shifting) "observer-participator" boundary can be chosen only once and must be chosen with care. The paradigm we adopt is drawn from the practice of high energy particle physics where the usual fundamental data are discrete firings of counters separated by distances and time-intervals defined in the laboratory. Our strategy is to identify in $\mathcal{U}(N, SU)$ events as already defined which we can relate conceptually to the coordinates of conventional theories and the intervals between such events in an arbitrary laboratory coordinate system. As shown in Ch. 2, once $\mathcal{U}$ is large enough we can find in $\mathcal{U}$ ensembles of strings which have the same first N_L bits, which we will call the it label, followed by many different sequences of 0's and 1's called *addresses*. Since N_L has now been fixed by the point when SU reached (four level) hierarchy closure with N_L bits in the labels, from now on we talk about $\mathcal{U}(N, SL)$ with $N = N_L + B$. We now consider two strings S_1, S_2 with labels L_1, L_2 followed by B bits and define an *event*, as before, as the case when

$$D_N S_1 S_2 = S_3 = D_N \neg S_1 \neg S_2 \tag{4.0.1}$$

with $S_1, S_2, \neg S_1, \neg S_2$ already in $\mathcal{U}$ and the discrimination produces an S_3 which is *both* already in $\mathcal{U}$ *and* has an equal number of 0's and 1's. For this discussion we will also assume that either one of the four labels or one of the four addresses has an even number of bits; which of the eight possible choices satisfies the condition is obviously irrelevant, since the rest follow. We could insure this by requiring our cutoff criterion on "completion of the hierarchy" to occur only when N_L is even, but for the time being we are allowing only the minimal number of interventions in the main program. The simplest way to insure that both addresses and labels have an even number of bits is obviously to require N_L to be even. But we find it more interesting to leave the even-odd character of N_L open until we are compelled to do otherwise. For simplicity in what follows we require the "equal number of zeros and ones" criterion to apply only to B_3, that is to the *address* part of the string. Independent of this restriction of the definition of *event* we have already seen that an event defines, in the simulation memory of the computer, two integers $B = N - N_L$ and $B + 1$ the absolute string address lengths of the universe ***between*** which the event in question occurs. As we have seen, all that happens is that all five strings $S_1, S_2, S_3, \neg S_1, \neg S_2$, which by definition of ***event*** are already in $\mathcal{U}$ are untouched, and the program tries again to generate novelty; in other words nothing happens.

This "non-intervention" has a number of consequences which we might as well face right now. One is that the "same" event may occur more than once in the simulation (i.e. before the next TICK), without "anything happening". From our point of view this is good; it eliminates the basic source of the infinities which occur in relativistic quantum field theories basically because the uncertainty principle generates infinite energy at each space-time point. For us the virtual processes which generate these infinities occur in a Democritean ***void*** which is not part of space-time. They are finite and unique; repetitions of the same process simply do not occur in the sense that they do not change our basic $\mathcal{U}$. A second consequence is that there may be a number of "distant, simultaneous" events. This this can, of course, happen in any Galilean frame in special relativity. What appears to be disturbing from the point of view of special relativity is that our unique sequential definition of B implies a unique time frame which would seem to single out a special class of Galilean frames, and hence violate "the principle of ***special*** relativity". Here we believe that we are on firm *experimental* ground, and need not, to quote Phipps,[33] attempt to cover our

"..nakedness with a fog of blather about 'mind,' which could just as well be the 'God' whose sensorium provided Newton with such convenient cover in circumstances of like embarrassment."

What has changed since the time of Newton, and more particularly since Einstein is that, thanks to the 2.7^oK background radiation, we have an *experimentally* well defined coordinate system which defines both "zero velocity" and an absolute universal time scale. This might have pleased Newton, since it strengthens his case from the "bucket experiment" for an absolute space. Of course, since the background radiation is now believed to be understood on the basis of *general* relativity and particle physics, the background radiation is not usually considered an embarrassment for *special* relativity. But the absolute space for rotations looks somewhat embarrassing for general relativity in the light of the conceptual background of Mach's principle, which played a critical role in Einstein's creation of the general theory. His position on this point was[34]

"As you know the general theory is a field theory defined by differential equations, and any such theory must be supplied with boundary conditions. In the early days it was believed that the only solutions of the field equations far from gravitating matter were believed to be the flat space of special relativity, or an overall cosmological curvature; the uniqueness of these boundary conditions was believed to meet this problem. Since the discovery (Gödel, Taub) of solutions of the field equations with non-vanishing curvature everywhere in the absence of gravitating matter, this argument from uniqueness no longer applies. In a sense this is a violation of Mach's principle. But now that we have come to believe that space is no less real than matter, Mach's principle has lost its force."

Therefore we find it eminently satisfactory, and a real accomplishment of our theory, that we get both an absolute time frame, an absolute coördinate system for velocities, (thanks to our construction of special relativity below) and an absolute space for rotations as a direct consequence of our algorithm. Of course, this puts us under the obligation, eventually, to prove that our coordinate system has no experimental consequences (other than those of the order of magnitude of the background radiation) in conflict with current demonstrations of relativistic invariance in the laboratory. We believe our construction accomplishes this, but the reader will have to judge this himself. We anticipate that, if we can get the particle physics right, the background radiation will emerge in due course, but to discuss that question further here would take us beyond the scope of this paper.

4.1 The Construction of Space Time via a Random Walk Model

The basic model by which we go from the bit string universe to space-time was pioneered by Irving Stein[15]. Our current approach departs considerably from this, and has been adopted partly in response to detailed private criticism by Michael Peskin and Elihu Lubkin; we are much indebted to these physicists for the time and care they have taken in trying to understand an approach that is so far removed from the conventional continuum physics.

Stein's basic idea was to consider a random walk with a finite number of steps of finite length h/mc. In the context we have already established, this can be represented by an ensemble of bit strings which are subsegments of an ensemble of address strings with the same label starting between universal address string lengths B and $B+1$ and ending between address string lengths $B+b$ and $B+b+1$, where we consider only the last b bits in each string. Between B and $B+1$ we assume (see below) that there was an *event* involving label L_1 and any other label L_i, and at $B+b$ a second event involving label L_1 again and any other label L_j. At this point we have to look ahead to the interpretation we will ultimately give relating our construction to our version of Feynman diagrams. The only concept we need here is that in a basic event the label $\neg L_1$ which will occur, by our rules, as one of the legs of an event involving L_1 is the *antiparticle* to L_1. We further assume that, once we have provided sequential time (which cannot be done between TICKs, but only in relation to a sequence of TICKs, as we will explain in more detail below) that all legs in the basic reference diagram are *incoming*, and that an incoming antiparticle is, following Feynman, equivalent to an outgoing particle. Then L_1 is both incoming and outgoing in both events, and the ensemble of bit length b which connects the two events carries this label label between the two events; L_1 will eventually become a set of conserved quantum numbers.

We now start to introduce our basic interpretive paradigm by assuming that macroscopic laboratory *events* occuring in finite space-time volumes $\Delta x\Delta y\Delta z\Delta t$ will take place only when an *event* also occurs in the bit string universe. Consider in particular two *counters* separated by a macroscopic spacial interval S larger than their non-overlapping spacial resolutions which *fire* in sequence with a time interval T which is again larger than their time resolutions. Assuming (which can be checked experimentally, with enough effort) that the conserved quantum numbers which are related to the firing of the two counters correspond to some particle with those quantum numbers, we can then define the velocity of the particle between the two events as v=S/T. Correspondingly, in the bit string universe we can define the "velocity" connecting these two events by

$$\nu = < N^1 - N^{)} > /(N^1 + N^0) = < N^1 - N^0 > /b \tag{4.1.1}$$

where N^1 and N^0 are the number of 1's and 0's respectively in an address string in the ensemble and $<>$ is the ensemble average. We can now take the next step and assume that the ensemble represents a biased random walk of b steps with a probability

$$p = < N^1(\nu, b) > /b = (1/2)(1+\nu) \tag{4.1.2}$$

of taking a step in the positive velocity direction defined by our two counters and a probability

$$q = < N^0(\nu, b) > /b = (1/2)(1-\nu) \tag{4.1.3}$$

of taking a step in the negative direction. The velocity of the peak is given by Eq. (4.1.1) and is obviously bounded by

$$-1 \leq \nu \leq +1 \tag{4.1.4}$$

while the standard deviation from the peak is

$$\sigma(\nu, b) = (bpq)^{1/2} = (b/4)^{1/2}[1-\nu^2]^{1/2} \tag{4.1.5}$$

These relationships are exhibited graphically in Figure 3. Thus the random walk model, specified by the two parameters b, ν is equivalent to an ensemble of bit strings of length b, or to a binomial distribution specified by the same two parameters.

Stein's starting point was that the standard deviation of such a biased random walk, or binomial distribution, is algebraically suggestive of the Lorentz contraction. He then went on to relate this standard deviation to a space coordinate and derive the Lorentz transformations. Since he did not provide an operational paradigm like ours, many people found his derivation hard to follow, let alone accept it. We believe that his basic idea was correct, but have adopted the alternative being developed here as, hopefully, more convincing.

Once we have taken the basic interpretive step of *identifying* the two counter firings in the laboratory with two labeled events in the bit string universe connected by a random walk of b steps, a great deal follows. As already noted, the parameter ν can never exceed +1 or be less than -1. We can therefore make our first dimensional statement in a physical sense by claiming that our interpretive postulate introduces a limiting velocity for any connection between two events which is uniquely and unambiguously defined. Clearly we can identify it with the limiting velocity c of laboratory experience which, to our knowledge, has never been exceeded in any laboratory context where the sequential firing of counters was well understood.

Our next problem is that if we consider *all* the address strings labeled by L_1 in this segment of our evolving universe, there is no reason to expect that the ensemble average will have any particular value; in fact the randomness of our construction would seem to guarantee that the most probable value is zero! Thus we have to make it part of our interpretation that there are in $\mathcal{U}$ subensembles of the appropriate character to support our eventual dynamical interpretation in terms of physical scattering events. That we can select from $\mathcal{U}$ such ensembles for any value of ν and b we care to choose is easy to establish. The algorithm which does this is easy to construct; the coding is given at the end of Appendix IV. Thus our universe certainly contains binomial distributions, or random walks. Our problem is to construct a dynamics that tells us when they can be interpreted as physical scattering events. We have a lot to do before this is justified, so the reader is urged to be patient.

Accepting the first part of our basic interpretive paradigm, we are in much the same position as the kinematic theory of special relativity which does *not* specify the nature of events but treats them as *given*. Then, since our postulate specifies a limiting velocity and the possibility of "light signals", i.e. the address strings I_b corresponding to $+c$ and 0_b corresponding to $-c$, we can establish the Lorentz transformations in a conventional way in 1+1 Minkowski space; by taking as empirical the three dimensions of space these are then extended to the full Poincaré transformations in 3+1 Minkowski space. This was the point of view adopted in our preliminary report on this research[35] . But we believe it instructive to attempt to follow instead, so far as we can, our version of the Stein derivation.

Although our definition gives an absolute significance to B, as has been discused above, the current practice of physics for most purposes relies on relative rather than absolute coordinates. To construct these we consider three ***connected*** events, the first of which, symbolized by [12], occurs when $\mathcal{U}$ has acquired bit length B_{12} for the addresses, involves L_1 and L_2, and happens at a spacial coordinate which for the moment we call ξ_{12}. We identify this position macroscopically with the firing of a counter in the laboratory (or an equivalent basic event in nature) as already discussed. The second event [23] involves labels L_2 and L_3, occuring at a universal bit length $B_{23} = B_{12} + b_2$, is assigned coordinate $\xi_{23} = \xi_{12} + \xi_2$. The third event [31] involves label L_3 and again label L_1, completing the connection to the first event; its coordinates are $B_{31} = B_{23} + b_2$ and $\xi_{31} = \xi_{23} + \xi_3$. The geometrical situation this defines is illustrated in Figure 4. Clearly we have defined a "triangle" with sides labeled by $\neg L_1, \neg L_2, \neg L_3$ and spacial coordinate intervals

$$\xi_1 = \xi_{31} - \xi_{12}; \xi_2 = \xi_{12} - \xi_{23}; \xi_3 = \xi_{31} - \xi_{23} = \xi_1 + \xi_2 \tag{4.1.6}$$

where we have to use quotes on triangle because the vertices are volumes and ***not*** points. Similarly the bit length intervals between the three events are

$$b_2 = B_{23} - B_{12}; b_3 = B_{31} - B_{23}; b_1 = B_{31} - B_{12} = b_2 + b_3 \tag{4.1.7}$$

Our intuitive picture is to think of L_1, L_2, L_3 as labeling three "objects" that in some sense encounter one another in three connected scattering events. Eventually we will succeed in constructing from these objects (which we will find that we have to think of as labeled ensembles rather than as individual strings) ensembles which describe "free particles", and will discover that the labels for the sides of the triangle $\neg L_1$, $\neg L_2$, $\neg L_3$ can be thought of *either* as "antiparticles moving backward in time" *or* as "particles moving forward in time". For the moment it is less conterintuitive to take the second view, briefly introduced above in connection with the Feynman rules, and define their velocities in the usual way by

$$v_1 = \xi_1/b_1; v_2 = \xi_2/b_2; v_3 = \xi_3/b_3 \tag{4.1.8}$$

This is all familiar enough, except for the fuzziness of our vertices.

We now return to our space time construction, concentrating for the moment on the connection between the two events [12] and [31] labeled by $\neg L_1$ (cf. Fig.4). We assume that this connection is to be represented by a random walk of b_1 steps. The problem is to construct an ensemble labeled by $\neg L_1$ with the appropriate statistical properties to give us a random walk characterized by the parameter $v_1 = \xi_1/b_1$ that heretofore has only been defined geometrically. The point of view we adopt is that the basic program has run long enough so that there are an enormous number of strings in $\mathcal{U}$ all labeled by $\neg L_1$ when the address length is B_{12}. Consequently when their address length has increased by b_1 bits, there will be an enormous number of addresses in the ensemble containing b_1 *random* bits added by TICK at the end of each string. Therefore, even if b_1 is a small integer we can, by applying PICK to this ensemble a sufficient number of times and lopping off these last b_1 bits, construct an ensemble with the ensemble average

$$v_1 = < N^1 - N^0 > /b_l \tag{4.1.9}$$

with v_1 as close as we like to any preassigned value. A specific algorithm for doing this is given in Appendix IV. In this way we can actually construct a random walk, characterized by the parameters b_1, v_1 and labeled by $\neg L_1$. Following Stein, we will call this ensemble an *object*. As in our construction of the hierarchy, we claim that this information is already contained in $\mathcal{U}$ whether we extract it or not. Clearly our universe contains an enormous number of objects, each characterized by essentially any velocity we wish to consider between -1 and +1.

A more critical question than the existence of objects, which we believe we have now demonstrated, is whether in fact event [12] occured at string length B_{12} while "time is standing still" between TICK's *and* event [31] in fact occured when the string length had increased by b_1 bits to B_{31}. Since B_{12} and B_{31} are both unknown and unknowable at this stage in the construction, the question is not whether any one event has happened or will happen, since clearly this happens many,many times in the evolution of the universe. We can start our consideration with some event of the class we are considering as a reference point, and then ask whether the second connected event will occur after only b_1 steps in the random walk. To this we can only give a statistical answer as follows.

Returning to Figure 4, we see that although the most probable "position" for finding a member of the ensemble is at νb, we have a 50-50 probability of finding it anywhere within $\sigma(\nu, b) = (b/4)[1-\nu^2]^{1/2}$ of the peak. Since our whole analysis is predicated on the assumption that the event [31] did in fact occur, we take account of this statistical uncertainty by *defining* the spacial coordinate interval b_1 between the two events by

$$\xi_1 - v_1 b_1 = \sigma(v_1, b_1) = (b_1/4)^{1/2}[1 - v_1^2]^{1/2} \tag{4.1.10}$$

Note that in this definition we have been careful to use the geometrically defined parameter $v_1 = \xi_1/b_1$ rather than the parameter ν used in the preliminary discussion. This a critical step, which we claim follows from our statistical analysis of the situation we are attempting both to describe and to understand. With this definition of velocity and position understood, we have a similar defining equation for the connection between spacial interval, number of steps and velocity connecting the remaining events.

4.2 The Relationship between Different Coordinate Systems

Up to now we have relied on the fact that there is a unique coordinate system, given by our constructive algorithm for $\mathcal{U}$, in which the velocity associated with the intermediate string in each event is, by definition, zero. By associating each of the three connections between each of the three events we are considering with three spacial intervals ξ_i, three finite bit string lengths b_i, and three velocities

$v_i = \xi_i/b_i, i \in [1,2,3]$ and by invoking the random walk model as our basic interpretive device we have succeeded in arriving at the the three basic relations

$$\xi_i^0 - v_i^0 b_i^0 = (b_i^0/4)^{12}[1-(v_i^0)^2]^{1/2} \tag{4.2.1}$$

where we have added to the notation the superscript "0" to remind us that so far this relationship is justified only in a particular (universal and defining) coordinate system.

Our next critical step is to relate the three events, and more significantly the intervals between them, to descriptions in different coordinate systems. Consider first the description of the situation in which we wish to assign to object 1 a zero velocity. This could happen to be the case already for some class of three connected events of the type events of the type we are considering. In that case the spacial interval our rule requires us to assign to the connection between [12] and [31] is $\sigma(0, b_1^0) = (b_1^0)^{1/2}$.

The thoughtful reader may already have wondered why in our basic definition we took the position of the event to be a standard deviation beyond the peak of the distribution rather than on the near side. The answer is that in the case of zero velocity, this would specify a *negative* direction for the random walk excursion, which would not make sense when we are talking about zero velocity with no reference sense for + or - direction. When we are through, only *relative* and not absolute direction will survive for the small (in this case 3) event numbers we are now considering. The same will happen with time, in spite of our unique (complexity increasing) "time's arrow" sequential character for the bit string universe as a whole. But this is getting ahead of our story.

Having recognized this implication for the constructions/definitions already established, we are now in a position to explain what we mean by coordinates in a "coordinate system" in which object 1 is "at rest". Referred to the basic coordinates in which it took b_1^0 steps, it will have wandered a distance

$$\xi_1' = \sigma(0, b_1^0) = (b_1^0/4)^{1/2} = (\xi_1^0 - b_1^0)/[1-(v_1^0)^2]^{1/2} \tag{4.2.2}$$

where we have made an obvious algebraic use of Eq. (4.2.1).

Whether the non-conventional route by which we have obtained the Lorentz contraction, and the first half of the Lorentz transformations in 1+1 Minkowski space is in fact a "derivation", as we are inclined to believe, or a "definition" is a question which the reader will have to decide for himself. In any case these were the critical steps for what follows. We have included the derivation since it follows in a "straightforward" (which as usual in the jargon means after many recursive iterations and much agony) way from our bit string universe. More significantly, as was already foreseen by Stein, the same random walk model will allow us to get a new insight into the foundations of quantum mechanics. But this is yet to come. What is important to realize at this point is that once we have achieved the result, the precise statistical formula by which it was achieved drops out; for instance, it would not matter if we had used probable error rather than standard deviation. What *is* critical is the proportionality between the statistical uncertainty and $[1 - v^2]^{1/2}$. The general features of Stein's insight are therefore, from our point of view, (a) that any random walk has a built in limiting velocity, which by *some* route is more or less guaranteed to end up in special relativity, and (b) that the narrowing of the peak in a *biased* random walk as it approaches the limiting velocity has the same algebraic form as the Lorentz contraction factor. Therefore we are convinced that these general features will survive in any *successful* attempt to put constructive physics on a digital basis whether or not the reader finds our particular route convincing.

But we have more work to do before we can arrive at the Lorentz transformations for the basic triangle which we wish to relate to laboratory coordinates. For the quantities already under consideration, we adopt the notation ξ_i^1, b_i^1, v_i^1. We have seen that according to internal reference to the events [12] and [31] object 1 wanders by an amount ξ_1'. But this wandering is not a laboratory phenomenon. If we wish to take object 1 with zero laboratory velocity as the reference system, and take event [12] as the origin of coordinates, then we must have that $\xi_1^1 = 0$ and hence that $v_1^1 = \xi_1^1/b_1^1 = 0$. But because of our initial argument, we insist that we can also take

$$\xi_1^1 = (\xi_1^0 - v_1^0 b_1^0)/[1 - (v_1^0)^2]^{1/2} \tag{4.2.3}$$

which is consistent since it does insure that $\xi_1^1 = 0$ thanks to our initial assumption that $v_1^0 = \xi_1^0/b_1^0$.

This still leaves the quantity b_1^1 in this new coordinate system undefined. Here we must return to the fact that we are "breaking in" to the system at a stage where the universe has been evolving for a long time, and recognize that whatever the actual content of the memory in the simulation, we have no immediate way in the laboratory to access the "universal time" B. We are allowed to make use of the structural information which comes from the fact that events occur, and that in *some* coordinate system the intermediate states have zero velocity. As we have seen, by invoking the random walk model this allows us to construct the first half of the Lorentz transformation for spacial coordinates. At this point we must recognize that our choice of the 1 bits as representing steps in the + direction was arbitrary; we could just as well have chosen the 0 bits, since this would not alter our fundamental assumption of zero velocity for the intermediate strings in the events. More than that, since we cannot access the bit string universe directly, our formalism must not only be indifferent to the algebraic sign of velocities, but must not allow us at this stage to determine anything other than *relative* velocities. We recognize this fact by asserting that our description of the interval between the two events [12] and [31] has to be able to be constructed starting from the coordinate system in which object 1 is at rest, and constructing a random walk which will connect this system to the one in which we started where object 1 had velocity v_1^0. Since, as we have already seen, the universe has a sufficiently large number of appropriately labeled ensembles so that we can construct a binomial distribution, or random walk, for *any* choice of the parameters v, b we can clearly construct the ensembles we need. But if we are to delete any reference to the absolute universal coordinate system, the number of steps b_1^1 must be defined in a new way, as already noted. We choose to do this by noting that the relation between the two coordinate systems now must have the relative velocity $-v_1^0$ rather than $+v_1^0$, a familiar requirement. Consequently, we claim, that by applying the same analysis as before, we can *define* b_1^1 by

$$\xi_1^0 = (\xi_1^1 + v_1^0 b_1^1)/[1 - (v_1^0)^2]^{1/2} \tag{4.2.4}$$

This is the last critical step we require for establishing the space-time kinematics of special relativity as a consequence, or more modestly as consistent with, our bit string universe, since we now have the connection between intervals and *relative* velocity. Further, it now follows algebraically that

$$b_1^0 = (b_1^1 + v_1^0 \xi_1^1)/[1 - (v_1^0)^2]^{1/2} \tag{4.2.5}$$

and that

$$b_1^1 = (b_1^0 - v_1^0 \xi_1^0)/[1 - (v_1^0)^2]^{1/2} \tag{4.2.6}$$

completing our derivation of the Lorentz transformation.

So far it seems that we made little use of objects 2 and 3, but in fact they provided the critical zero velocity intermediate states that got us off the ground. Clearly we can now go on and derive the Lorentz transformations referring to their velocities.

One further point deserves mention. Since, as already noted, the algebraic sign of our velocities has now only a relative significance, we have not only lost any reference to "universal time" but also to the the unique evolutionary sense of time in the underlying model. Hence, we can treat our "time parameters" b as negative or positive without affecting the formalism we have established. Relative time sense can be established for events connected by macroscopic intervals, but (as is appropriate in kinematic special relativity) the absolute time sense has been lost. Thus, at this stage, we claim to have demonstrated "time reversal invariance" for this piece of the formalism. At a later stage, when we have developed quantum mechanics, we will recover time irreversability. Our point of view, with which Lee concurs,[36] is that the "time irreversability" which leads to the second law of thermodynamics is correctly identified with the irreversibility of quantum mechanics, as we have discussed long ago[6]. In this respect we are essentially on the same footing as conventional theories. Further, when we come to cosmology, the universal time sense is ready to hand, without having to go to general relativity. We believe this to be a strength of our approach.

Since we have already insured that the triangle connecting the three events closes, and our random walk derivation insures that no velocities can exceed one, we trust that it is obvious we have now derived the full geometry of 1+1 Minkowski space in the limit of such a large number of steps that b can be treated as a continuous variable. But the underlying integral character of the steps will become critical for us in the next section.

To proceed from the 1+1 Minkowski space which we have now constructed to 2+1 and 3+1 space is straightforward. We first consider, as in Figure 5, appropriate line intersections with velocities along the lines for which we can construct appropriate velocity ensembles and make the appropriate transformations for 2+1 space. It hardly seems necessary to spell out the algebra here. The transformations we have already established suffice to define the invariant intervals

$$s_i^2 = (\xi_i^j)^2 - (v_i^j b_i^j)^2 \qquad (4.2.7)$$

not only for j referring to any of the three particles or the coordinate system 0 with which we started but for *any* coordinate system in which object i has any *arbitrary* velocity v bounded by ± 1 with respect to the coordinate system in which object i has zero velocity. The fact that we have defined our connected events in such a way that the triangle (now in 2+1 space) closes allows us to prove algebraically that the coordinate perpendicular to the direction of the velocity transformation must be unaltered. Of course now the random walk ensembles must be constructed in such a way that the addresses for each label contain *two* subensembles referring to the vector velocity components, but the construction is obvious and will not be spelled out here.

To go on to 3+1 space is equally straightforward, using the paradigm given in Figure 6. Obviously we must now use *three* subensembles with the same label to refer to the three vector components, but that is a detail. There is only one subtlety, namely that we have to stop with 3+1 space! The reason is that so far all the labels within a particular level are indistinguishables. Hence we are only allowed four distinct lines at this stage in the construction. When we go on in the next section to assign parameters (in fact masses) which distinguish different labels, we can go on to construct multidimensional configuration spaces. But our basic space of description remains 3+1.

We note also that since our lines and intersections are necessarily always labeled, the space can immediately acquire chiral properties once we have any way of generating interactions. Again this should be obvious from Figure 6, where the fact that the vertices carry distinct labels required us to draw two figures rather than one. In structural chemistry they would be referred to as *stereoisomers*. If, as in classical chemistry, the basic interactions (electromagnetic) are non-chiral, the energy levels of the two isomers are identical, and the chiral properties can only show up in dynamical interactions where the *macroscopic* geometry defines the chirality. But if our bit strings labels turn out to have chiral properties (and they had better when we come to "weak interactions") we see that the fact that our space is defined in terms of *labeled* events rather than in terms of an achiral background will make "parity non-conservation" a natural consequence of the construction. We find it pleasant that this possibility emerges so early in the construction.

To summarize what we claim to have shown, we start from our basic bit string universe, subdivided into growing ensembles labeled by the levels of the combinatorial hierarchy, and show that from these we can always construct subensembles corresponding to a random walk with a specified velocity bounded by a universal limiting velocity. By assuming that this random walk represents an object whose coordinates are defined by three appropriately chosen events, we then show that this allows us to describe the relationship between the coordinates of the objects engaging in the events with specified relative velocities and derive the Lorentz transformations. Here we use the contraction factor of the biased random walks and the fact that our definition of velocity necessarily implies a universal limiting velocity. The transformations are algebraically identical to the usual Lorentz transformation, except that the derivation requires the "time" coordinates to be integers. We then show that this suffices to construct the full geometry of 1+1, 2+1, and 3+1 Minkowski space, and that our basic space of description must stop there. We leave it up to the reader as to whether we have "derived" space-time from our bit string universe, or defined it starting from that basis. What we do claim is that our procedure is self-consistent, and provides an adequate basis for what follows.

4.3 Construction of Momentum Space

Our next problem is to go from the mathematical coordinates we have succeeded in constructing to dimensional coordinates that can be used for the purposes of physics. Since we have a universal limiting velocity, so far simply unity, we obviously equate this to c, the limiting velocity of special relativity. Since we have a random walk model the obvious way to make this dimensional is to associate with each label some invariant step length l_0. This will be justified if we can connect the labels by some specified procedure to laboratory events. For that purpose it is more convenient to use the concept of mass as the identifier of objects. We then can introduce a second universal constant h and connect this to step length by taking $l_0 = h/mc$. Clearly we can identify this with the step length in the coordinate system in which the object carrying this label, step length, and now mass, is at rest. But then the Lorentz transformation properties we have already established require that in a coordinate system with velocity v, the step length be Lorentz contracted, i.e. that $l = l_0[1 - v^2]^{1/2}$. This in turn allows us to define a second coordinate system dependent quantity $E = mc^2/[1-v^2]^{1/2}$. This is then related to the step length in any coordinate system by

$$l = hc/E \tag{4.3.1}$$

Thus, when we have done a lot more work, we will find that our discrete step length is the basic Einstein-deBroglie quantization condition connecting energy to phase wave length or (for light) frequency.

So far this step is purely definitional and only on dimensional grounds are we justified in calling E "energy", or for that matter calling m "mass". However, if we take the second step of defining $\vec{p} = m\vec{v}/[1-v^2]^{1/2}$, where $\vec{v}$ has the spacial significance already established, these definitions and our Lorentz invariance yield immediately the invariant relation

$$E^2 - p^2c^2 = m^2c^4 \tag{4.3.2}$$

We now have constructed the usual Lorentz invariant coordinate description of a momentum space for free particles.

It is at this point that we "break in" to the system of description by providing a basic definition of a new process in $\mathcal{U}$ which can be associated with those happenings which initiate the chain of happenings that lead in the laboratory to the firing of a counter. For this purpose we need to connect our mass parameters to each other in such a way that they can be measured in the usual sense. Mach realized long ago that Newton's Third Law, or momentum conservation, is the critical component in the observational definition of mass ratios. As he showed, this allows us to define these ratios relative to some standard reference mass and that this works because empirically mass ratios so defined are (within experimental error) scalars and independent of the order in which they are measured. This remains true in special relativity if we take care to use the definition of momentum we have introduced above. What we need is a process in our bit string universe that can be identified with a momentum conserving collision. The feature of such a collision that we pick is that in a system in which the vector sum momentum of the the two particles is initially zero, the intermediate state formed by the collision has zero velocity. Since we have in effect a particular coordinate system available to us from the construction (with cosmological interpretations already mentioned) and our definition of event in that coordinate system does have zero velocity for the intermediate states S_3 because the address part of string S_3 has an equal number of zeros and ones, we have already accomplished this.

As a matter of fact, if we return to our basic definition Eq. (4.0.1) and refer to Fig. 4 we seem to have done too much. Each object which enters our paradigmatic triangle leaves with its velocity unaltered, if we assume, as was done above in going from the specific situation to the general Lorentz transformations, that we keep on constructing ensembles with the same velocity parameter as the bit length b increases. We have "momentum conservation" all right, because our events as defined up to now do not lead to scattering; they are simply "crossings". What we have at this point is simply the correct relativistic kinematics for a system of "free particles".

The situation can be rectified as follows. We enter the main program by inserting a flag which tells us what labels are involved in events following some particular TICK and before the next TICK occurs. We allow the universe to run for b TICKS, and then enter the program again looking for any two labels (which occured the first time in two ***different*** events), and then keep looking for an event in which either of these two labels occur. This may not happen before the next TICK, in which case we keep on looking between each subsequent pair of TICKs until it does, and record the bit length b_s counting from the TICK when the first two events occured. We now can form a vector velocity ensemble for each of the two labels which meets the criterion

$$m_1 \vec{v}_1 /[1-(v_1)^2]^{1/2} + m_2 \vec{v}_2 /[1-(v_2)^2]^{1/2} = 0 \tag{4.3.3}$$

Clearly this defines the initial legs for a momentum conserving collision. In the same way we can follow the two labels after the collision and look for two subsequent events, and construct ensembles for the final state legs which again conserve momentum. In this way we demonstrate that our universe does indeed contain not only crossing events, but momentum conserving elementary scattering events. To calculate the probabilities for such scatterings will take a lot more work. We content ourself in this chapter with having, we believe, demonstrated that our bit string universe has been shown to contain the usual kinematics of conventional relativistic particle mechanics, in spite of its digital basis. Since we already have the correct Lorentz transformation properties for velocities – a concept defined in both coordinate and momentum space– we trust it is now obvious that our elementary scattering events will conserve momentum in any coordinate system, and have the needed properties for connecting up to laboratory scattering events. But we have to do a lot more work before this can be made convincing.

We now claim that we have shown our basic random walk model to lead to the usual relativistic kinematics of free particles and momentum-energy conserving "point" collisions – points in the sense that we can assume all magnitudes we need consider large compared to the inverse number of steps $1/b$. In fact we now have a formal way of taking that limit simply by letting our universal constant $h \to 0$. We emphasize that this approximation is just that and nothing else. It explains for us why physics was able to get so far using continuum models, but it does not mean that, even conceptually, our space time is the continuum space time of special relativity. Ours is a space of discrete events with discrete random walks in between, a point which has also been emphasized by Stein. Hence we do *not* have a "correspondence principle" in Bohr's sense. In fact, we claim that, contrary to his basic assumption, we have shown that it *is* possible to construct physics without assuming a continuum space-time background. Parker-Rhodes has a different, but conceptually similar, way to achieve the same result.[14] In this approximation we can use rods and clocks in the laboratory to connect up the firing of counters to particular sources of particles, measure mass ratios, cross sections, and so on. Thus at this point we have the kinematic basis for a classical relativistic particle physics connected to laboratory practice. This theory is, of course "scale invariant" because of our approximation that the step length is zero. To go on to the quantum theory we must obviously retain the discrete aspect of our bit string universe and not throw it away in this fashion. We have done so here only to establish contact with macroscopic experience. In the next chapter we will show that the underlying discreteness also has macroscopic consequences in agreement with experience.

5. CONSTRUCTING QUANTUM PARTICLES and SCATTERING THEORY

If we had a way of "reaching into" the universe and identifying the precise integers between which an event occurs and then counting the steps in the random walk to the next event, the "objects" constructed in the last chapter could serve as our basic particulate description. But these events occur at the sub-microscopic level which our hands and eyes can never reach. The closest way that has been found so far to approximate what we are looking for is to construct a "counter" of macroscopic dimension Δz and time resolution Δt which will "fire" when an event of specified type (learned from experience, and theoretical analysis) occurs somewhere within this space-time volume. The counter is constructed so that the initial event leads to a chain of events (usually some sort of ion cascade) which magnifies the effect of the initial happening to the point where it can make a macroscopic record - an audible click, a bit on magnetic tape, a developable grain in a photographic emulsion,... Our problem is to relate this macroscopic result to the underlying bit string universe.

The counter technology just described is already enough to accomplish a great deal. In the approximation in which the space-time volume of the counter can be considered to be a "point", we have already seen that we have the full particle kinematics of special relativity. By finding (eg. radioactive) sources of particles in nature, or constructing them using vacuum and electromagnetic technology (accelerators), we can give a laboratory definition of a source of particles as anything which fires a counter. We can discover "absorbers" which when interposed between source and counter keep the counter from firing. Using these we can construct a sequence of slits or holes which define a beam of particles. Using counters in the beam, we can measure their velocity, or velocity distribution, and calculate the experimental uncertainty in these quantities arising from the finite size and time resolution of the counters. Since this procedure has been discussed elsewhere[37] , we refer the reader to that publication for details. Given collimated beams of particles, we can set up two in-two out elastic scattering experiments and measure mass ratios relative to any particle chosen as a standard using relativistic energy-momentum conservation. From this we can go on to study more complicated situations in which novel types of particles are produced in the interaction.[38] This suffices conceptually for understanding much of the *experimental* practice of high energy particle physics. We see that this type of measurement is essentially classical, once we have learned from Einstein that particles can be created out of energy. That fact itself can be understood thanks to Wick's profound analysis[39] of Yukawa's meson theory,[40] as an inescapable semiquantitative consequence of the coupling of relativity to quantum mechanics. But this still does not suffice for us to construct a scattering theory for quantum particles.

5.1 "Free Particle" Basis States

Returning to the bit string universe, all we have so far is that when two counters separated by a macroscopic space and time interval larger than the volumes and time resolutions of the counters have fired, some random walk connecting those two volumes has occured. But we do not know within those macroscopic volumes where this random walk started and ended. To meet this problem, we construct an ensemble of objects (which are themselves ensembles) all characterized by the same vector velocity $\vec{v}$ and the same label (or mass) chosen in such a way that, after k steps, each of length $l = (h/mc)[1-(v/c)^2]^{1/2}$, the peak of the random walk distribution will have moved a distance l in the direction of $\vec{v}$. We take as our unit of time the time to take one step, $\delta t = l/c$. It is important here to realize that we are debarred from using any other definition. Our steps are digitized, and we have no way as yet of assigning meaning to fractions of a step. We *do* have a clear understanding of what we mean by a *sequence* of steps, which justifies our use of them as specifying a "time sequence", even though we do not carry with that many of the customary concommitents of the concept of "time". Once "time" is understood in this digital sense, the velocity of the peak of each subensemble in this coherent ensemble has a velocity c/k. We call this *coherent* ensemble of ensembles a *free particle* of mass m, velocity $\vec{v}$, and momentum $\vec{p} = m\vec{v}/[1-(v/c)^2]^{1/2}$. We assume that the size of the counter Δz in this direction and in the plane perpendicular to this direction is so large that we can ignore end effects; we return to these below.

There is a second "velocity" associated with this ensemble of ensembles, namely that with which something moves at each step always in the direction $\vec{v}$. We call this v_{ph}; clearly $v_{ph} = kc$, and $vv_{ph} = c^2$. Associated with each of the two velocities and the label (or mass) there is a characteristic length

$$\lambda_{ph} = l = hc/E; \lambda = kl = h/p \qquad (5.1.1)$$

Our next step is to show that these coherent ensembles of ensembles, which we can clearly construct algorithmically from our bit string universe by extending procedures already developed, has *experimental* consequences that can be exemplified in the laboratory.

5.2 The Double Slit Paradigm

We now consider our coherent ensemble of ensembles specified by $\vec{v}$ and m incident on a "screen" perpendicular to $\vec{v}$ made of absorbers containing two holes (or slits in the two dimensional approximation in which the distances perpendicular to the line between the holes and to $\vec{v}$ are so large as not to produce appreciable end effects) a distance d apart. This geometry is illustrated in Figure 7. This is all well and good in the laboratory where we have established the meaning of absorbers. In the bit string universe the absorbers can be thought of as containing so many events that their consequences are so diffuse as not to affect the progress of the experiment. Our coherent ensemble will pass through these two holes dividing into two subensembles without loosing its coherent properties.

This is our answer to the old question of "which slit" the "particle" goes through. So long as the coherence is not destroyed, it goes through *both* slits. This is possible for us because our "particle" is a coherent ensemble of ensembles of indistinguishables, and not a single entity. But if there is a counter in the slit *and* a scattering occurs, the coherence is destroyed; in that case we know that the particle went through that slit. More detailed analysis reveals that this class of events will lead to a single slit interference pattern. Thus we are led to the same conclusion as the wave theory when it is analyzed in this way[37] even though we have used a digital basis.

At some large distance D behind the screen we set up a counter array in a plane perpendicular to $\vec{v}$. We further assume that the source is a distance S on the other side of the array, and is equipped with a counter which fires when the particle leaves the source. Calling the time interval between when source and detector fire T, the velocity between source and detector is $v = (D+S)/T$. By making D and S large enough, and assuming that the source has a velocity spectrum which includes v, we can select in this way particles whose v is as precisely known as we like[37]. This step is necessary to insure that *all* elements in the coherent ensembles we consider have the same v to requisite precision. Only such pairs of events will provide data for the experiment.

It is important to realize that our precision is now no longer limited *in principle* by the finite resolution $\Delta z \Delta t$ of the counters. All we need to do is make the experimental setup long enough. It is this fact that makes the concept of velocity rather than space-time fundamental for a quantitative development of scattering theory, as was realized long ago by the S-matrix theorists. We have also seen that, once our bit string universe contains a large enough number of labeled ensembles, we can also construct the appropriate binomial distributions describing any value of velocity to arbitrary precision. So we are making contact at the appropriate point. But the random walks still enter into the picture when we now go on to find out where we are most likely to have the detectors fire in the counter array as a function of the distance x away from the center line. Because of the coherence properties we have built in to our definition of "free particle", this will be most probable when the two path lengths to the detector are an integral number of coherence lengths λ apart, since this is the only place where all the peaks of the distribution line up. At any other geometrical configuration, some of the distributions will have lower probability amplitude, and the occurance of the event will be less likely. Hence our bit string universe and definition of free particle predict that we will find maxima in the distribution in x characterized by an integer n (counting away from the center line) which occur at positions x_n given by(cf. Fig.7)

$$n\lambda = x_n d/D \tag{5.1.2}$$

We now claim to have shown that our bit string universe contains something related to "deBroglie wave interference", and that by defining velocities and counting maxima under appropriate circumstances, we can measure h, which we are now justified in identifying with Planck's constant. We have also derived the deBroglie wave length and the relativistic phase wave length he introduced (Eq. 5.1.1). Hence in the limit of negligible mass, we have the basic Einstein-Planck quantization condition $E = hc/\lambda$ as well. The fact that energy is quantized is thus, for us, a direct consequence of our digitized step length.

It is important to realize that our theory is still, in principle, "scale invariant" because all we we have defined are mass ratios taken from experiment. If there were in nature stable elementary particles with arbitrarily large masses, we could with sufficient ingenuity find a way to measure arbitrarily short distances. In fact, all we know how to do is to give elementary particles like the electron and proton very large energies. But when we try to use these as probes, what we end up doing is to create more particles by the Wick-Yukawa mechanism, which frustrates any *direct* space-time description of the internal structure of "elementary particles". What is usually done is to assume that the second quantized theory of the matter field, which uses Lagrangian densities defined (mathematically) in terms of a continuum space-time, can meet this problem. But as was pointed out long ago by Bohr and Rosenfeld,[41] the second quantization of the matter field *cannot* be given an operational definition, making this whole conceptual framework suspect. Current research by quantum field theorists attempts to meet the problem by trying to calculate the quantized mass values found in nature from the nonlinearity of their fundamental theory, but we believe it is fair to say that this program has not yet succeeded. We will see in the next chapter an alternative way to get one stable mass ratio, and the absolute mass scale of our theory, from digital considerations. But before we do that it will be useful to show that our theory can be extended from free particles to a quantum scattering theory, and approximates free field theory in an appropriate continuum limit.

5.3 "PHOTONS"; WAVE MECHANICS

Now that we have seen that we can construct from our bit string universe basis states with the internal periodicities (but not yet the "continuum" wave structure) of the conventional relativistic deBroglie theory for free particles and asymptotic energy-momentum conservation, our next problem is to find our how the underlying statistical bit string structure leads to scattering. Although we could proceed to construct our scattering theory directly in an algebraic fashion, we choose to first set up the conventional wave theory limit in order to understand in a more familiar context the algebraic rules we invoke in momentum space. To do this we must return to the bit string universe and explore in more detail the connection between *label* and *address* which we have already built into the theory.

For the purposes of our preliminary discussion it will suffice to use only the simplest possible labels, those corresponding to level 1 of the hierarchy. These, as we have already seen, are (10), (01), and (11); they close under discrimination. Up to now we have concentrated on two in-two out events, which occur between TICKs, but if we return to the basic flow chart (Fig.1), we see that there are two other types of process going on between TICKs, namely discrimination and complementation. Discrimination between two strings gives us a third string which, if it is not already in the universe, is added to it. If that string is already present, and the complement of either of the initial strings chosen by PICK is not already present in the universe, that complemented string is added to the universe. If all five are present, we have what has been called an event and again the universe does not go TICK. Thus, so far as the labels we are now considering go, there are six cases illustrated in Figure 8. These occur between TICKs with equal *a priori* probability.

If we now think of the label as referring to a dichotomous quantum number such as charge, we can, for instance, think of the ensembles of ensembles labeled by (10) as a *particle* of positive charge, labeled by (01) as an *antiparticle* of negative charge, and labeled by (11) as a *quantum* which externally will appear to be neutral but internally contains the charges of a particle-antiparticle pair. This interpretation is reminiscent of the Fermi-Yang model for the pion.[42] For the events, since in the universal coordinate system the intermediate state will, by definition, have zero velocity, and the complemented strings reversed velocity, these are a primitive version of four leg Feynman diagrams with all particles incoming or all outgoing. But since the external time sense has to be established by linking these up to other events, as we have already seen in our construction of "space-time", we also have the usual Feynman rule that an antiparticle moving "backward in time" is equivalent to a particle moving "forward in time". In the conventional theory, this is derived from the CPT theorem. Here we claim to *derive* this basic theorem from our fundamental definition of event.

Of course there is a lot more to the CPT theorem than this primitive example. In particular, the reversal of velocity direction must not reverse the helicities for spin 1/2 particles. If we treat a second dichotomous pair of bits in the label as referring to the helicity state this will follow in due course, (cf. below). Further, again as we can see from the constructions in the previous chapter, these basic processes are momentum conserving. A little thought should convince the reader that the complementation rules will also allow us to guarantee momentum conservation at the vertices, and that in both cases this will continue to be true in any coordinate system. Thus we have the basic ingredients for a momentum space scattering theory. The remaining problem is to construct a dynamical theory by connecting up basic events and vertices in such a way that we can actually calculate scattering amplitudes for physically observable processes and compare the predictions with experiment.

The simplest case we can consider is one in which all the steps in the address are taken in the same direction, that is the address strings are all 1's or all 0's. It is clear in this case that the random walk has no dispersion and that our objects (or particles) will all move with $\pm c$, a fact already noted by Stein. Thanks to our identification of the step length $l = hc/E$ we clearly have no trouble in taking the zero mass limit, which is required for consistency with our relativistic kinematics. Whether our theory will actually predict that the labels associated with such particles have precisely zero mass, is too early in the construction to speculate about. Fortunately our theory will not be in conflict with experiment if the photon turns out[43] to have a mass $m_\gamma \sim m_e e^{-137}$ or if some or all of the neutrinos have small finite masses, for which there is some controversial experimental evidence. In that case, the present discussion refers to an approximate (and convenient) *model*, and is not fundamental. We hope this will become clearer as we go on.

For massless particles it will be simplest to think of our dichotomous variable as *helicity* and for the simple case at hand to assume that in dimensional units it will have magnitude $h/4\pi$. To justify this numerical value will take a lot of work, as will the demonstration that it is a pseudovector (i.e. has the transformation properties of an angular momentum). For the moment all we require is the dichotomous character. Then the label (10) with the address (1111...1) can be thought of as a neutrino with positive helicity and (01) with the address (1111...1) as referring to a neutrino with negative helicity. Then the reflection operation which takes (1111..1) to (0000...0) will indeed reverse the vector direction without reversing the helicity, showing that our "helicity" is indeed a pseudovector. It is important to realize that we can define pseudovectors in this way *between* TICKs because our definition of the direction of velocity is defined directly in terms of bit strings. However, to define time reversal we would require a sequence involving at least three ticks, and to get time irreversability many more than that. Once this is understood the Feynman rules we have already derived work in the conventional way. We conclude that if we start with only one type of neutrino, the antineutrino has opposite helicity and we get the usual two component theory in which neutrino and antineutrino have opposite chirality. Thus we do have the chiral properties we noted in the last chapter as implicit in our method for constructing space-time.

As is well known, neutrinos have no classical analog, so will not directly serve our purpose of constructing the photon. In our notation the four possible two component neutrino states are

$\nu_L =$ (01)(1111...1): left handed neutrino,+c

$\nu_L =$ (10)(0000...0): left handed neutrino,-c

$\bar{\nu}_R =$ (10)(1111...1): right handed anti-neutrino,+c

$\bar{\nu}_R =$ (01)(0000...0): right handed anti-neutrino,-c

According to our Feynman rules the antiparticle to a left handed neutrino is right handed, and the neutrality of the neutrino does not allow us the other possibility in this notation, which we will see in the next chapter will require us to assign additional slots for the helicity quantum numbers of charged particles. Of course our choice of the particle as left-handed is made to conform to the usual conventions which describe parity non-conservation in beta-decay. As in conventional theory, we cannot get the other variety by a Lorentz transformation, since a particle traveling with light velocity cannot be brought to rest.

We now extend our discussion to level 2 of the hierarchy, but for the moment need not use the full structure, which is discussed in the next chapter. What we need is two dichotomous variables and the the helicity we have already introduced extended to two spin 1/2 particles combined to make spin 1 states traveling with light velocity. By an obvious extension of the notation already introduced, the four photon states are

$\gamma_R^+ = (1010)(1111...1)$: right handed photon, +c

$\gamma_R^- = (1010)(0000...0)$: right handed photon, -c

$\gamma_L^+ = (0101)(1111...1)$: left handed photon, +c

$\gamma_L^- = (0101)(0000...0)$:left handed photon, -c

Again these states cannot be brought to rest by a Lorentz transformation, and the reversal of the velocity does not change the helicity, so the spin is again a pseudovector. The Feynman rules still apply.

It is important to realize that we *have* to go to this level of label complexity before we can construct a classical limit. Our two-component neutrinos are the simplest particles the scheme allows, but are intrinsically chiral and hence non-classical. Our"photons" have two internal states which provide us with a pseudovector polarization of (in units we have yet to justify) spin $h/2\pi$, *correlated* with the direction of propagation. For our current purpose it is only the existence of this internal dichotomous degree of freedom and not the subsequent interpretation which matters, as we now demonstrate.

We now have developed enough internal structure in our bit string universe to *explain*, with appropriate phenomenological input, the early nineteenth century wave theory for polarized light. Since our "photons" are composed of two coherent ensembles of ensembles (particles) with different dichotomous quantum numbers, all that need be added to the construction of the coherent ensembles previously discussed is that when they are combined coherently, a macroscopic meaning can be given to the internal spacing within a step length between the two ensembles; this parameter is called the *phase*. By a sufficiently detailed operational analysis (standard undergraduate physical optics, if taught from an operational point of view) we claim that, just as we were able to understand the "double slit experiment" and reduce it to measurements which can be refined, macroscopically, to any desired practical accuracy, we can give operational (laboratory) meaning to phase. In the undergraduate laboratory this amounts to the usual optical bench experiments using polarimeters and quarter wave plates and a monochromatic source to construct and analyze elliptically polarized light. As in classical physical optics, the overall phase of the system remains beyond experimental reach.

Long before the nineteenth century development of the wave theory of light, Newton had tried to understand the phenomenon of the rings in terms of "fits of transmission" and "fits of reflection", and tried to understand what we now call "polarization" in terms of light particles being rectangular ("having sides"). Thus his approach to optics was particulate, digital, and contained two internal states. One might say that we are returning to a Newtonian model in that sense, but must relate it to a continuum model because of the subsequent development of physics. Because of the success of the mechanistic interpretation of Newtonian physics as applied to vibrating strings, and later to elastic solids, it was natural for nineteenth century physicists to think of periodic phenomena in terms of wave motion. Ignoring for the moment the internal degree of freedom, what we have constructed so far from our bit strings, in the zero mass limit where $\lambda = h/p = hc/E = \lambda_{ph}$ is the coherent ***amplitude*** (we will justify this term below)

$$A(z,t;\lambda) = \sum_{n=0}^{N} \delta(z + n\lambda \pm ct) \tag{5.3.1}$$

Since this tells us that, within the moving region where the $\delta -$ functions occur,

$$A(z+\lambda, t;\lambda) = A(z,t;\lambda) + 0(1/N) \tag{5.3.2}$$

this allows us to assume in first order in that approximation that $A(z,t;\lambda) \simeq a[(z \pm ct)/\lambda]$. We have seen that the parameters z, t are ***macroscopically*** defined, and have computed and related to experiment in a macroscopic context a means of ***measuring*** the ***microscopic*** parameter λ in the laboratory by counting. So far we have only a start on the Newtonian description.

In the nineteenth century context, it was natural to interpret these discrete phenomena in terms of a continuum theory using the periodic functions ***sin*** and $cos 2\pi[(z/\lambda) \pm (t/T)]$ with $T = \lambda/c$, or more generally or more powerfully in terms of the solutions of the wave equation

$$(\partial/\partial z)^2 a(z,t) = (1/c^2)(\partial/\partial t)^2 a(z,t) \tag{5.3.2}$$

which are $a(z \pm ct)$. This left open what these amplitudes referred to. In the context the easiest thing to do was to think of them as the some physical displacement in an elastic solid. This led to a difficulty, since it was obvious from the experimental values of the wavelength and the velocity that what was measured must be a time average over many oscillations, and the time average of these periodic functions over many cycles approaches zero. But in the vibrating string or elastic solid analogy, it was also known that the energy stored in the oscillations is positive, and proportional to the time average of the square of the amplitude of oscillation. So again it was natural to assume that the ***intensity*** of the light as measured was proportional to the time average of the square of the amplitude.

The triumph of this continuum model came when it was realized that the two states of polarization of light could be modeled as two amplitudes transverse to the direction of propagation and at right angles to each other, and by choosing the phase between them appropriately could describe either linear or circular polarization, or any degree of elliptical polarization in between. Hence Hamilton was able to predict conical refraction and see it demonstrated in the laboratory, which settled the question of the adequacy of the wave theory of light for most physicists. The critical experiment for the nineteenth century was based on the fact that Newton's derivation of Snell's law required the velocity of light in a medium with index of refraction n to be nc, while the wave theory required c/n. To explain the experimental result in terms of a particle theory would have required coherent ensembles of particles, and a detailed discussion of the coherent scattering from atomic centers, as in the theory we are now constructing. The conclusive explanation of the lower propagation velocity in material media was achieved by Rayleigh using the wave theory, with propagation velocity c in the space between atoms. Thus in the absence of experimental evidence for the particulate nature of light, the wave theory appeared to rest on an unshakable foundation.

This long excursion into nineteenth century physics has been taken for two reasons: (a) first, to show that two internal discrete states, plus the assumption of a continuum model for coherent periodic phenomena gives the macroscopic-microscopic connection we seek, and (b) second to explain the origin of the amplitude squared rule for the interpretation of periodic phenomena. But from our point of view, this modeling can just as well apply to our bit string universe provided only the discrete, periodic phenomena we have constructed and now provided with an internal dichotomous degree of freedom allows us to introduce a measurable phase between these two degrees of freedom when they are are assumed to be averaged over time in macroscopic experiments. As we have argued before, and will continue to argue, this success of classical (and later quantum) field theory does *not* allow us to extrapolate this continuum model down to infinitesimal distance. What it *does* allow us to do is to claim that we have a right, when all counters in an experiment include a large number of steps, $l = hc/E$ measured in terms of laboratory standards of length and time, to *approximate* our bit string results by the conventional continuum wave theory model for *relativistic* deBroglie waves in terms of the real, complete set of basis functions $sin[2\pi(pz \pm Et)/h]$ and $cos[2\pi(pz \pm Et)/h]$.

[From here on we follow the convention of high energy physics of taking $c = 1 = h/2\pi$, which leaves the only physical dimensional parameter as mass, and dimensional analysis confined to establishing the mass of *one* reference particle, to which all *dimensionless* mass ratios of the *mathematical* theory are referred].

In order to justify this statement we consider the boundary condition provided by a counter of finite spacial resolution Δz in the wave theory, and prove that the same result can be derived from our digital model (Eq. 5.3.1) to order $(1/N)$ where N is the number of steps we need to consider in our basis states. Assume that the counter is centered at z and fires at $t = 0$; since the finite time resolution has been discussed elsewhere[37], and adds nothing conceptual to the discussion, we will assume that it is so good that only the spacial resolution matters. Then to insure that our particle was somewhere in this region at that time, we must make up a wave packet with different momenta of amplitude $f(p)$ such that

$$\int_{-\infty}^{+\infty} dp f(p) e^{ipz} = \Theta(z - \Delta z) - \Theta(z + \Delta z) \tag{5.3.3}$$

Therefore, by Fourier inversion

$$(1/2\pi) \int_{-\infty}^{+\infty} dz e^{ip'z} \int_{-\infty}^{+\infty} dp f(p) e^{ipz} = \int_{-\infty}^{+\infty} dp \delta(p - p') f(p') \tag{5.3.4}$$

and hence

$$f(p') = (1/2\pi p')[e^{ip'\Delta z} - e^{-ip'\Delta z}] = (i/\pi p') sin(p'\Delta z) \tag{5.3.5}$$

But if we apply the same boundary condition to the basis states of Eq. (5.3.1) noting that n must now run between $-N$ and $+N$, our boundary condition becomes

$$\int_{-\infty}^{+\infty} dpf(p)\ \Sigma_{-N}^{+N}\delta(z+n\lambda) = \Theta(z-\Delta z) - \Theta(z+\Delta z) \tag{5.3.6}$$

But the ***mathematical*** operation of Fourier inversion can just as well be applied to this formula as to Eq. (5.3.3). Doing so, we recover Eq (5.3.5) plus correction terms of order $(1/N)$, which proves our theorem. To extend our discussion to deBroglie coherence lengths for finite mass and hence to deBroglie waves we need only represent the bit string ensemble by $\delta(z+n\lambda-ct/\lambda_{ph}) = \delta[(pz+nh-Et)/h]$ We therefore claim to have ***derived*** wave mechanics as an ***approximation*** to our digital model in a form (laboratory boundary conditions based on counters of finite macroscopic size) which will serve for most of the practical applications of scattering theory. Further, we can now derive the Heisenberg uncertainty relations for continuum variables in the usual way. Thus we claim to have proved that we have constructed free particle quantum wave mechanics on a digital basis as an ***approximate*** theory.

While this paper was in the final stages of preparation, it was brought to our attention by I. Stein that W.H.Lehr and J.L.Park (*J.Math.Phys. 18*, 1235 (1977)) have developed a random walk or stochastic model as the basis for a derivation of the Klein-Gordon equation, thus getting the continuum limit in another way than our approach here. In their model they find that relativity requires them to digitize their time with a unit $\tau = 3\hbar/mc^2$, so while not identical to Stein, it is closely related. J.C. van den Berg informs us that yet another derivation of the Klein-Gordon equation from a stochastic basis by N.C.Petroni and J.P.Vigier appeared recently (*Foundations of Physics 13*, 253 (1983)). This reference contains a number of references to related work. In both cases the "particle" takes chaotic steps with the velocity of light, whereas, as we have seen above, in either Stein's approach or ours the velocity in the random walks can have any value bounded by c. We have also recently encountered work in the imaging problem in radio astronomy which clearly indicates that when one is dealing with information arriving through continuum waves analyzed by classical techniques, one cannot tell whether the original input was in fact discrete or not. In particular, the sequence of δ-functions which we have used above to relate our bit strings to a wave theory is known as the *shah* function, named for a letter in the Russian alphabet. The history of this imaging problem has been reviewed in a forthcoming paper by R.N.Bracewell, which will be published by the *Cambridge University Press*, and the function itself is discussed in his book, *The Fourier Transform and its Applications*, New York, McGraw-Hill, 1965. Also relevant is a paper by Bracewell on the discrete Hartley transform which has been submitted to the *Journal of the Optical Society.* This is of particular interest because it shows how the complex Fourier transform can be readily represented by two real functions that avoid the $\pm i$ ambiguity in a way that provides distinct computational advantages, This material makes it clear that the semi-quantitative approach used above to make the passage from the bit strings to a wave theory using the counter paradigm, which we believe is adequate for the purposes of this paper, can be given precise mathematical formulation in a well understood context. It is important to recognize that, although we get conventional scattering theory (see below) in this way, our digital basis cannot be thrown away. We will see in the next chapter that it allows us to calculate the proton-electron mass ratio in agreement with experiment, a result not yet achieved by the continuum theory. But we have a lot more work to do before this calculation can be justified.

Even more important than the justification of this continuum approximation in wave theory by recourse to well understood laboratory practice, is the realization that we must take the *squares* of amplitudes, appropriately averaged, in order to make contact with our laboratory paradigm taken from physical optics. Hence we claim to have justified our earlier contention that the basic entities derived from our bit string universe are properly called *probability amplitudes* and not *probabilities.* We reserve the term *probability* for a number $p \in [0, 1]$ where the value can be any rational fraction in that interval, or an approximation to some irrational or transcendental number in that interval, established by some well defined finite algorithm. They obey the usual rules of *classical* statistics and are to be interpreted in terms of conventional frequency theory and the "law of large numbers". In an appropriate context all their moments and correlations and the "law of large numbers". In an appropriate context all their moments and correlations can be defined in a conventional way. One of the purposes of our "operational analysis of the double slit experiment"[37] was to prove that the *counting statistics* of such an experiment, using deBroglie waves and counters in both slits, obey these classical rules. Of course the probability amplitudes of conventional quantum theory, or our own version of it, do *not*. This is the basic problem for statisticians, like Patrick Suppes, who try to understand quantum mechanics in terms of classical statistics. They are quite prepared to accept a degree of nonlocality which horrifies many practicing physicists, but find it hard to accept probabilistic concepts which do *not* allow all moments of a distribution to be defined, and "correlations" incompatible with the properties of probibilites bounded by 0 and 1.

As already discussed, a digital theory of light was not considered a viable option in the nineteenth century, in spite of Newton's early start in that direction and his continuing authority. The wave theory of light could be well modeled in terms of an "amplitude" taken by analogy from experience with elastic solids. This model got strong support from the connection Maxwell was able to forge between the phenomenological theory and the "obviously continuous" electromagnetic waves that Hertz succeeded in generating in the laboratory. Einstein got rid of the mechanical *aether* which served as such a useful and fruitful prop to the nineteenth century imagination, but clung to the continuum concept (again fruitfully). Yet his own work in 1905 destroyed, by his interpretation of the photoelectric effect, that continuum basis and brought the theory back to events that are discrete and localized, such as developable grains in a photographic emulsion. Careful experimental work proved that the concentration of energy required to produce these laboratory phenomena could not be accounted for by the continuum theory, except in an average sense.[44] We hope that our approach reduces the mystery connected with this fact.

Of course that was the beginning of the story, not the end. For us, that comes with the Bohr-Rosenfeld analysis[41] already cited. Their analysis of the measureability of the electromagnetic field makes use of complicated, classical apparatus within one wavelength of the radiation being studied (eg. a low frequency radio wave). Yet, assuming that the material apparatus is restricted by the Heisenberg uncertainty principle, they succeeded (after two years of effort[45] !) in showing that the usual commutation relations, more economically derived by second quantization, follow from a detailed operational analysis. Since we have already proved that we also get the uncertainty relations in the appropriate context, we can accept their analysis for the electromagnetic field. BUT, as already emphasized, they point out that the the analysis is only possible when there are only two dimensional constants (h and c), and *cannot* be extended to the "second quantized matter field" with m a fixed parameter; that theory is no longer scale invariant.

Now that we have free particle wave functions and the "amplitude squared" rule in contact with experiment in terms of our model, it might seem that we are through. At first sight one might quarrel with our extension from the electromagnetic case to matter waves, but as already noted, our theory goes through just as well for finite mass as for the particular limiting case we have invoked. To settle any unease on this score, consider the scattering of a spin 1/2 particle from a spin zero target with a p-wave resonance. For the $j = 1/2$ state, this divides an unpolarized beam into two beams in a manner that is precisely (mathematically speaking) analogous to a nicol prism. Further, if the particle has a magnetic moment, a permanent magnet with a gap containing constant magnetic field of appropriate length and strength will change transverse polarization to longitudinal polarization, which is precisely analagous to the action of a quarter wave plate. So the whole optical bench type of experiment with nicol prisms and quarter wave plates (*and* a digital detector for the photons) can be repeated for spin 1/2 particles. In any case, it would be inappropriate in a fundamental theory for us to introduce more than one paradigm for connecting the bit strings to the probability of registering counts in the laboratory.

What does not follow so easily is the use of *complex* rather than real ($\pm$) amplitudes in quantum mechanics. Of course it is convenient, as in electrical engineering to use $e^{\pm i(pz-Et)}$ wave functions and calculate intensities by taking the absolute square. But there is no ***necessity*** for doing this for one particle problems in quantum mechanics. In fact Bohm and Vigier have shown that it is quite possible to reproduce all the results of non-relativistic ***one particle*** quantum mechanics with a real, classical "hidden variable" theory,– although many physicists find their theory bizarre, and physically unmotivated. So we now turn to two particle scattering problems for our attempt to meet this problem.

5.4 SCATTERING THEORY

Since we now have standard relativistic particle wave mechanics for free particles, it would seem that we could now develop scattering theory in a conventional way. This true up to a point, but there is a critical conceptual difference. We have no Hamiltonian, so we ***cannot*** calculate scattering amplitudes as the matrix elements of such an operator between appropriate scattering states. This problem was met some time ago[46] by constructing a "Democritean scattering theory" starting from free particle wave functions and arriving at the standard Goldberger-Watson wave function[47] for N_A particles in and N_B particles out. The essential point is that the scattering amplitude then becomes a ***kinematic*** quantity describing any conceivable experiment of this type, including those which do not conserve flux. Then we are under the obligation of supplying ***dynamical*** equations for this amplitude which guarantee flux conservation, or in technical terms are unitary.

We consider first the elastic scattering of two particles in the usual geometry shown in Figure 9. Since the technical problem of using "wave packets" is adequately discussed in standard texts[47], we will ignore this complication and use the free particle basis with precisely known initial and final momenta, as is customary. The initial state starting from two particles with momenta $\vec{k}_i$ and energies $\epsilon_i = (k_i^2 + m_i^2)^{1/2}$ is then simply $e^{i(\vec{k}_1\cdot\vec{r}_1+\vec{k}_2\cdot\vec{r}_2-\epsilon_1 t-\epsilon_2 t)}$. Note that we are using an ***on shell*** or ***single time*** model consistent with our bit string universe. If we now define

$$\vec{K} = \vec{k}_1 + \vec{k}_2;\ E = \epsilon_1 + \epsilon_2$$

$$\vec{q} = (m_2\vec{k}_1 - m_1\vec{k}_2)/(m_1+m_2)$$

$$s^{1/2} = (q^2+m_1^2)^{1/2} + (q^2+m_2)^{1/2} = \epsilon_1(q^2) + \epsilon_2(q^2)$$

$$\vec{X} = (m_1\vec{r}_1 + m_2\vec{r}_2)/(m_1+m_2); \vec{x} = \vec{r}_1 - \vec{r}_2$$

the initial state wave function becomes

$$\Psi_i = e^{i(\vec{K}\cdot\vec{X}-Et)}e^{i\vec{q}\cdot\vec{x}} = [e^{i(\vec{K}\cdot\vec{X}-Et)}]\psi_{\vec{q}}(\vec{x}) \tag{5.4.1}$$

One advantage of this step is that only the first factor refers to the laboratory coordinate system, and is easily transformed to any frame; hence the remaining wave function has a Lorentz invariant significance. Further, in this zero momentum frame (where, as we have already seen it is most convenient to discuss our bit string universe) there is no explicit reference to time; we have "stationary state" wave functions.

So far our wave function assumes only the incident state. Both for simplicity and because we will develop an amplitude of this type from our bit string model in the next section, we will assume that the scattering is spherically symmetrical in the zero momentum coordinate system. Then the elastic scattering wave function will be

$$\psi_{\vec{q}}(\vec{x}) = e^{i(\vec{q}\cdot\vec{x})} + T(q^2)e^{iqx}/x \tag{5.4.2}$$

in the asymptotic region where the counters are located. Flux is conserved provided that

$$T(q^2) - T^*(q^2) = 2i\rho(q^2) \mid T(q^2) \mid^2 \tag{4.4.3}$$

where ρ is the appropriate density of states in momentum space. The probability of scattering, or *cross section*, is $\sigma(q^2) = 4\pi \mid T(q^2) \mid^2$, and can be directly compared with counts in detectors. Thus our descriptive job is complete. The task of the theory is clearly to calculate $T(q^2)$.

The unitarity condition clearly requires T to be a complex number, and leads to well known experimental consequences - in particular wave interference terms between the unscattered and the scattered wave function in appropriate angular regions. We could therefore fall back on this as the reason why we are required to use complex amplitudes in our theory, just as we justified the amplitude squared rule by comparison with experiment. But we claim there is a more fundamental reason connected with our bit string universe. If we make up wave packets in time such that for large times in the past (i.e. in the region of the collimators which define the beams) only the first term in the wave function is present. As was pointed out by Lippmann and Schwinger[48] an easy way to accomplish this within the formalism is to put into the time-energy factor e^{-iEt} multiplying the scattering part of the wave function the replacement $E \rightarrow E + i\eta$ where η is a small positive quantity, and the limit $\eta \rightarrow 0^+$ is implied. Then at large negative times this term is exponentially damped. In momentum space this leads to the wave function

$$\psi(\vec{q},\vec{q}')^+ = \epsilon(q^2)\delta^3(\vec{q}-\vec{q}') - T(q^2)/[\epsilon(q'^2 - \epsilon(q^2) - i0^+] \qquad (5.4.4)$$

where $\epsilon(q^2) = (q^2+m^2)^{1/2}$ is the proper factor for a free particle (*not* field) state to guarantee Lorentz invariant normalization, and since we will use it below we have assumed $m_1 = m = m_2$.

In the conventional theory the states with $q'^2 \neq q^2$ the q' states are called "virtual" and in the momentum space integral equations for the scattering amplitude are summed over. The factor $1/[\epsilon' - \epsilon - i0^+]$ then guarantees asymptotic energy-momentum conservation. Clearly we have to perform a similar sum over all possible bit strings when we describe the same situation in the bit string universe; we have now learned that this is the proper weighting factor in the continuum limit. The question is whether we can justify it in our own basic terms. The limit is easy, since we have the same asymptotic requirement. But we are summing over discrete, rather than continuous energies, thanks to the fact that our minimum step is $\delta t = l/c$ and the quantization condition $E = hc/l$. In fact we see that the minimum energy step $\delta E = h/\delta t$ and hence that for a spread in energy δE and time δt we have that $\delta E \delta t \geq h$. We emphasize that this is *not* the Heisenberg uncertainty principle, which we have already seen comes in a conventional way from limitations on measurement due to finite counter size. It is due to the fact that nothing happens between TICKs; we must take at least one step in a random walk for anything to happen. This fact will be important for us later when we see how our theory avoids the self energies of quantum field theory. It is certainly natural for us in the bit string universe to assume that the weighting factor as we move away from the asymptotic conservations stepwise by $E' = E + n\delta E$ should be proportional to $1/[E' - E]$. But then, since n=0 can occur in the sum we would produce an infinity, violating our basic finite assumptions. Therefore we argue that the best way to avoid this is to use instead $1/[E' - E - i\delta E]$ which indeed goes to the proper limit ($i\delta E = i0^+$) in the continuum momentum space theory. We hope at a later date to replace this plausibility argument by a calculation, but will not hold up this paper for that refinement. If this argument is accepted, we have now made the connection between conventional scattering theory and our construction, and can proceed to N particle scattering theory along the lines previously developed.

5.5 A MINIMAL UNITARY (RELATIVISTIC) SCATTERING THEORY

So far what we have done is to work up from the bit string universe to relativistic free particle wave functions, and in the last section to remind the reader that if we have unitary two particle amplitudes, no matter how obtained, we can from them construct a relativistic and unitary N-particle scattering theory using relativistic Faddeev-Yakubovsky equations. Our next step is to show that our construction provides us with the elementary driving terms from which this theory can be constructed. Returning to Fig. 8b we see that the bit string universe provides us with three basic events, so we start with these.

Since the intermediate state has zero velocity by definition and some mass which we will call μ the most probable value for its energy will be μ. The prescription used in scattering theory for the probability amplitude due to this intermediate state is to say that the energy as a function of the final momentum q' is proportional to $1/[\epsilon(q') - \mu - i0^+]$, which when integrated over all positive values of $(q')^2$ will give a delta function that insures energy conservation ($\epsilon(q^2) = \epsilon((q')^2)$. The simple prescription used here is independent of the direction of $\vec{q}$, so all directions of scattering are equally probable, and the scattering is spherically symmetric. The coefficient of this amplitude must be chosen such that the number of outgoing partices is equal to the number of incoming particles, which is summarized by saying that the amplitude is "unitary". In the jargon of relativistic quantum scattering theory such an amplitude would be the simplest version of an "s channel resonance". The $i0^+$ prescription is required so that the singularity $\epsilon = \mu$ only occurs in the specification of the integral, and is needed to specify which branch of the square root singularity in ϵ to take in that integration.

The situation we are now considering is an extension of the simple events pictured in Figure 8 to those possibilities in which the intermediate particle (of mass μ) occurs not just between two TICKs but engages in all random walks which, when summed, will lead to the energy-momentum conserving elastic scattering selected by our initial and final boundary conditions. Since we have already seen that our vertices conserve momentum, if the final particles have momenta $\vec{k}_1$ and $\vec{k}_2$ (from which the asymptotic final selection picks out $\vec{k}_1 = \vec{q}'$ and $\vec{k}_2 = -\vec{q}'$), the intermediate particle will have energy $\epsilon_\mu = [(\vec{k}_1 + \vec{k}_2)^2 + \mu^2]^{1/2}$. For the simple cases where the event is elementary (occurs between two TICKs with no intervening random walk) the vector sum of the final momenta, like that of the initial momenta, is zero and $\epsilon_\mu = \mu$; clearly this will also be true for any other case in which this vector sum vanishes. The problem is how to weight these cases relative to those when the energy differs from μ – the "off shell" states in the language of scattering theory.

As we have seen in Sec. 5.1, the time unit for the random walk that the intermediate state of mass μ engages in between the two vertices in an extended event (i.e. one that leads to asymptotic energy-momentum conservation when all possibilities are summed) is $\delta t = l/c$ and hence the minimum energy by which these intermediate energies of these random walks can differ from each other, called δE is given by Eq.(5.1.1) as $\delta E = h/\delta t$. We emphasize that this is *not* the "uncertainty principle" but simply the digitization of energy in a particular circumstance arising from our discrete time steps. Any extended event will therefore have an "off shell" energy $\epsilon_\mu(n) = \mu + n\delta E$ with $n \in [0, 1, ..., N]$, where as we have seen, if the universe has been ticking long enough, N can be as large as we like. As we go to more and more steps in the intermediate states, we will have a harder and harder time finding in the finite segment of $\mathcal{U}$ referring to our particular scattering experiment to find strings which will match up to our energy-momentum conserving boundary conditions, so it is clear that the weighting factor should fall off as we go farther and farther "off shell". The simplest choice would seem to be proportional to $1/[\epsilon_\mu(n)]$. But this will not do, because it is infinite when $n = 0$ which violates our absolutely basic requirement that the theory give only finite results. Since we cannot introduce fractions of a step, and as we have seen must include the $n = 0$ case, the solution we adopt is to use instead $1/[\epsilon_\mu(n) - \mu - i\delta E]$. Naturally, this choice of a small imaginary part to remove the singularity is motivated by our desire to reproduce conventional quantum scattering theory in the continuum limit, and we cannot at this stage claim that this introduction of imaginary amplitudes is *forced* on us by the construction we have been following. But we do believe it is a straightforward postulate consistent with what has gone before, and hope some day to give a more convincing argument.

Once this argument is accepted, and for convenience (because we have not yet gone to the work of reducing the whole theory to digital operations) we take the continuum limit, we still have the question of how to relate ϵ_μ to the laboratory variables in terms of which the scattering problem is actually formulated. This we do by simply equating it to the energy corresponding to the external particles when they are off shell, $2[(q')^2 + m^2] = (s')^{1/2}$, (where, for simplicity we have taken both incoming - or outgoing - masses m to have the same magnitude) and by adding these scattering processes to the initial state (Eq.(5.3.1)) obtain the basic momentum space wave function for two particle scattering

$$\psi_i(\vec{q}, \vec{q}') = \epsilon(q^2)\delta^3(\vec{q} - \vec{q}') - G(q, q')/[2[(q')^2 + m^2]^{1/2} - \mu - i0^+] \qquad (5.3.2)$$

The function G which actually determines the strength of the scattering has not yet been determined by our argument. It is restricted by the requirement that the overall normalization of the wave function be the same as if no scattering occured, and is the technical way in which the "unitarity" or "flux conservation" property of the theory is met. But this is familiar to scattering theorists, and of no conceptual importance for us at the moment.

This starting point for the minimal unitarity scattering theory (MUST) was shown by James Lindesay[8] to lead to a consistent, unitary relativistic three particle scattering theory including three particle bound states, elastic and rearrangement collisions and breakup. In the appropriate non-relativistic kinematic region this theory leads quantitatively[8,9] to the logarithmic accumulation of three particle bound states first found by Efimov[49] . In Efimov's treatment the logarithmic accumulation occurs when $|a|/R$ approaches infinity where a is the two particle scattering length and R is the finite range of forces. That this effect should emerge in a relativistic treatment which has only one free parameter (μ/m) is somewhat startling, particularly in what can reasonably be called a "zero range theory". Yet the lack of scale invariance in the relativistic theory provides the "range" parameter h/mc, which allows the quantitative results of the two calculations to be compared. This is the more remarkable in that the integral equations which provide the dynamics of the two theories are different in detail, and the way in which a finite result is obtained (except in the singular limit) is mathematically quite different; in particular one cannot go from one equation to the other by taking a "correspondence limit". This is fortunate, since the occurance of an arbitrary parameter R in a fundamental theory would be for us more than just an embarrassment.

The next critical step was taken by Noyes and Lindesay[10] who realized that this basic model could be brought into closer contact with elementary particle theory by assuming that the parameter μ is not arbitrary but connects a "particle" mass m to a quantum mass m_Q in a specific way. In particular, if the quantum and particle "bind" ***kinematically*** to make a "bound state" with the same mass $(m + m_Q \rightarrow \mu \equiv m)$ and quantum numbers as the "particle" the two particle driving term leads, via the relativistic Faddeev equations, to single quantum exchange (cf. Figure 10a). In technical terms, an s-channel bound state leads by this mechanism to the correct lowest order t-channel exchange, and the amplitude is unitarized in a covariant manner by the integral equations. Further, this covariant theory (which is simple enough not to require approximation for accurate numerical solution) goes in the non-relativistic approximation to the usual equations for scattering by a Yukawa "potential". Thus we have derived from our bit string universe a first approximation usable in nuclear physics, and in the small quantum mass regime an accurate approximation to Rutherford scattering and the Bohr hydrogen atom. By adding the postulate that two quanta can "bind" kinematically to a particle to form a state with the same mass and quantum numbers as the particle we can also describe quantum-particle scattering in the two particle sector of a three particle theory with the correct lowest order driving terms (cf. Figure 10b).

The extension of this approach to the four particle sector via relativistic Faddeev-Yakubovsky equations is beyond the scope of this paper. Since the theory can be developed from relativistic free particle deBroglie wave functions without invoking the digital basis, it is being pursued vigorously in that context. In particular, the connection between this relativistic quantum mechanics of finite particle number and quantum field theory (where the "kinematically bound" states of the finite theory and the corresponding "time inverse" vertices are represented by creation and destruction operators - with resulting infinities that have to be "renormalized") is being explored[50] . So we will not discuss this development further here. For the purposes of this paper, what is important is that we have made effective, and we claim mathematically and physically rigorous, contact between our bit string universe and current active research in elementary particle physics.

It is important to realize at this point what we have, and have not, claimed to accomplish so far. We claim that we have a definite algorithmic structure which can be connected by unambiguous rules to the practice of high energy particle physics, with the usual wave interference phenomena, including a practical approximation that is (so far as we can see) unlikely to get us into trouble with known experimental information. We have, in common with quantum field theory, two universal ***dimensional*** constants h and c with the same practical consequences. What we do ***not*** have is a third dimensional constant of the dimensions of mass, or energy, or a coupling constant not expressable in terms of h and c. This is, of course, also true of conventional theories. The current frontier of research in this area consists of attempts to use the phenomenological symmetry schemes and the non-linearities of quantum field theory to yield a single coupling constant producing a "grand unification" from which the particle masses can be computed. This hope rests on an analogy to the quantum mechanics of the solid state where many connected modes can produce spontaneous symmetry breaking and a ground state (with a gap) lower than the non-interacting free mode basis. Still more ambitious schemes (eg "supergravity") would take this single coupling constant to be deriveable from Newton's gravitational constant G, and to get all masses and coupling constants which are observed as deductive consequences of one or another symmetry scheme.

In spirit the current attempts at unification are in one sense not very different from ours, and have the historical advantage of having reduced an enormous amount of very complicated experimental data to understandable form along the way. But our basic approach requires us to view the attempts to generate order out of non-linearities (which were initially infinities) in the continuum as a mistake, or at least as a very complicated way to get at something that might prove to be much simpler. Since we are allowed one mass on dimensional grounds, and since the only stable baryon (or quasistable with a lifetime greater than 10^{31} years) – the proton – and the energetic scale for many high energy phenomena(1 Gev) that, superficially at least, do nor involve protons are approximately the same (1 Gev $\approx m_p c^2$), we try the simpler alternative of taking the baryon mass as our basic third dimensional unit. In the next section we will try to convince you that this gets us pretty far, and provides some justification for the constructive mathematical work which has been developed in this paper, and earlier.

6. STABILIZATION OF PARTICLES

We have now seen that our construction gives a complete *phenomenological* theory for relativistic N-particle scattering if we supply the masses and coupling constants from experiment. We took care in our original construction to show that the *label-address* schema was sufficient to construct the approximate theories of relativistic particle mechanics and relativistic quantum scattering theory *without* specifying the content of the labels. We thought this important because it shows how to carry through a reconstruction of quantum mechanics on a digital basis independent of the combinatorial hierarchy which gave it birth. Hence we can hope for acceptance of that aspect of the work without getting into the Einstein-Eddington program of understand how and why it might be possible to compute the masses and dimensionless constants of physics from first principles. While some physicists can see the point to getting rid of the continuum, which after all is never observable in physical practice, the idea that things which are clearly physical entities might also have a digital basis tends to stick in their craw.

But one motivation for taking the approach seriously came from the remarkable coincidence between the cardinals of the hierarchy and the scale constants of physics, and was strongly reinforced by Parker-Rhodes' success in computing the proton-electron mass ratio in agreement with experiment. It is time to face this problem head on and attempt to show in this chapter that, given the digital basis for quantum mechanics we have now firmly established, it is possible to obtain significant physics out of the combinatorial hierarchy labeling scheme itself. This is the objective of this chapter. The work is incomplete, since we have yet to get a scheme for quarks, larks (i.e. leptoquarks), heavy leptons and all that which is competitive with the grand unification schemes on which so much of current elementary particle theory and experiment is focused. But we believe we have gone far enough to show that we have exciting possibilities which, hopefully, will engage the imagination of theorists who come to our work with fresh eyes.

6.1 DICHOTOMOUS QUANTUM NUMBERS GIVEN BY THE HIERARCHY

In our previous discussion of the hierarchy we showed that the mapping matrix scheme connecting levels 1 and 2 starting from the basis (10), (01) is easy to construct. The explicit mapping matrices which have the three DCsS formed from this basis, rearranged as strings, are $a = (1110), b = (1101), c = (1100)$. From these we can form the $2^3 - 1 = 7$ DCsS's $\{a\}, \{b\}, \{c\}, \{a, b, a + b\}, \{b, c, b + c\}, \{c, a, c + a\}, \{a, b, c, a + b, b + c, c + a, a + b + c\}$. Recalling that (with + for discrimination) $a + a = 0$, we see that all seven sets are *closed* under discrimination.

Even at this level, there is an ambiguity in physical interpretation which has to date resisted definitive solution. Instead of taking the obvious basis given above, we could have replaced (10) or (01) by (11); we cannot use all three because, since (10)+(01)+(11)=(00), only two of the possible basis strings are linearly independent. Then the mapping would give us $a = (0011)$ in the first case or $b = (0011)$ in the second. These two alternatives are not distinct, since the rule by which we rearrange the mapping matrices as strings (so long as it preserves the cyclic order) is still arbitrary; further they both lead to the same maximal DCsS: $\{(0001), (0010), (0011), (1100), (1101), (1110), (1111)\}$. But they produce an alternative choice, not only in one of the basis vectors, as already indicated, but also in terms of two of the three DCsS with three members, i.e. between $\{(1110), (1101), (0011)\}$, $\{(1101), (1100), (0001)\}$ in one case, and $\{(1110), (0011), (1101)\}$, $\{(1100), (0011), (1111)\}$ in the other.

Nevertheless it is possible to reduce the ambiguity and obtain significant clues to physical interpretation. The simplest place to start is with the first representation. The three basis strings are of the form (11yz), which guarantee that the seven strings in the maximal DCsS are all of the form (wwyz). In contrast, the eight remaining possible non-null strings are of the form (wxyz) with $w \neq x$. Thus the only 4×4 matrix which has these seven as eigenvectors and none of the eight is the one illustrated as A in Figure 11. Thus the simplest approach to the problem is to leave the first two rows untouched. So far as we can see, the remaining six mapping matrices are unique up to one ambiguity, and are illustrated in Figure 11. This ambiguity is unimportant, since it corresponds simply to a relabeling of the rule which takes us from a 4×4 matrix to a string with 16 bits. That our choice of representation does indeed give us seven linearly independent strings, and hence a basis for level 3, is also illustrated in Figure 11. The simplest structural feature that emerges is that we can use no less than 10 slots to meet the problem, and that as already argued the remaining six slots must be null. Thus, using strings of length 16 we can represent the first three levels of the hierarchy by using the first two bits for level 1, the next four for level 2, and the last 10 for level 3. This will be used below for physical interpretation.

Construction of the mapping matrices using the alternative basis is a little more cumbersome, and we have yet to approach the uniqueness achieved in the last paragraph for the first basis. That such a representation can be achieved by using the methods explained in Appendix II.c is clear, but the details are still under investigation. We have gone far enough to have some confidence that the $2+4+10=16$ representation for the first three levels has a basic significance. But the reader is warned that the scheme we follow below for physical interpretation is tentative, and may have to be revised when the theory is further articulated.

In chapter 5 we showed that the two slot notation for level 1 supports an interpretation in terms of the starting point for a two component neutrino theory. We now go on to interpret the four slots provides for us at level 2 as refering to the helicity states of electrons and positrons according to the scheme given in Table II. We see that we now have the correct quantum number content and connections for lowest order QED, and can go on to a full lowest order dynamics once we supply the appropriate momentum factors and interpretation. We believe it possible to develop from this starting point and the minimal unitary scattering theory[8,10] (extended to Faddeev-Yakubovsky equations[7]) a finite particle number version of QED; results will be presented elsewhere[50]. Further, by combining levels 1 and 2 we have the basic six fermions $(\nu_L, \bar{\nu}_R, e_L^-, e_L^+, e_R^-, e_R^+)$ for Weinberg's[51] weak-electromagnetic unification in the leptonic sector, as well as the basic lowest order diagrams once we invoke the minimal unitary scattering theory; our explanation of mass differs from his, as mentioned above.

The full quantum number scheme which relates this construction to the labels in the bit string universe is still under investigation[50]. Our tentative scheme for the first three levels, making use of the mapping matrices is given in Table III. We see that at level 1 we have two component neutrino theory in which, when we add the address label corresponding to zero mass, has $\nu_L = (10...0)_{16}(111..1)$ establishing our helicity convention. At the combined levels 1 and 2 we have the two helicity states of the photon, coupling to electrons and positrons by the extension of Figure 2, W^+, W^-, Z^0 as vector bosons, and the longitudinal or coulomb photon. At this point the particles and quanta are still massless; reversal of velocity [i.e (111...1) $\rightarrow$ (000...0)] does not change the direction of spin, proving that it is indeed a pseudovector. At level 3 we find the baryons of strangeness 0 and ± 1 as the obvious interpretation, and the proper number of and quantum numbers for the usual pseudoscalar (because they are bound states of fermion-antifermion pairs) and vector quanta. We might seem to have a problem with the appearance of two longitudinal or coulomb photons. However if one takes the Wheeler-Feynman point of view that all quanta are ultimately absorbed, the unitarity condition in the minimal unitary scattering theory fixes the mass in terms of the coupling constant, or visa versa. J.V.Lindesay, A.Markevich and G.Pastrana[52] find that in the weak coupling limit for $e^2 \simeq 1/137$ the mass of the photon $m_\gamma \simeq m_e e^{-137}$ which is not in conflict with any known experiments as has already been noted[43]. Then the two $S_z = 0$ photons are simply the vector and scalar photons in a four-component theory, and the problem is solved. With some care, and free use of the minimal unitary scattering theory[7-10], it is possible to show that all the usual Feynman diagram rules apply, and hence that our theory is *CPT* invariant at level 3. At level 4 we will have 16×16 quantum numbers. The problem of getting quark quantum numbers, heavy leptons, or, as looks promising from the numerics, ***rishons*** will be studied after level 3 is under control.

6.2 THE MASS RATIO SCALE AND THE UNIT OF MASS

Independent of the details of this scheme, we see from the basic randomness of our construction that at level 3 the exchange of a "coulomb photon" will occur with probability 1/137 compared to all other alternatives. This allows us to calculate the electron mass as the expectation value of its coulomb energy in a coordinate system at rest by a statistical average $m_e c^2 = < e^2/r >$ using $e^2 = hc/2\pi \times 137$. The calculation itself was originally made by Parker-Rhodes[14] starting from a very different construction of space time and the combinatorial result; we provide here a modification of our previous discussion of this calculation[13]. Taking as our basic mass the baryon mass m_B (*because* of the connection to the gravitational constant G) and noting that the heaviest system to which the coulomb photon system couples directly is a baryon-antibaryon pair, the minimal distance we can consider in a system starting from rest is half a baryon Compton wavelength. We therefore scale r by $r = (h/2m_B c)y, 1 \leq y < \infty$. The charge in the lepton must separate by more than r into two lumps which by charge conservation we can write in terms of a dimensionless parameter x as ex and $e(1-x)$, where x is a statistical variable reflecting the fact that we have both charged and neutral leptons and baryons. Hence

$$< e^2/r > = (hc/2\pi \times 137) < x(1-x) > (2m_B/h) < 1/y > = m_l c^2 \quad (6.2.1)$$

and

$$m_B/m_l = 137\pi / < x(1-x) >< 1/y > \quad (6.2.2)$$

Since we have now established our space as necessarily three-dimensional, the discrete steps in y must each be weighted by (1/y) with three degrees of freedom; hence

$$< 1/y > = [\int_1^\infty (1/y)^4 dy/y^2]/[\int_1^\infty (1/y)^3 dy/y^2] = 4/5 \quad (6.2.3)$$

Since the charge must both separate and come together with a probability proportional to $x(1-x)$ at each vertex, the weighting factor is $x^2(1-x)^2$. For one degree of freedom this would give

$$< x(1-x) > = [\int_0^1 x^3(1-x)^3 dx]/[\int_0^1 x^2(1-x)^2 dx] = 3/14 \quad (6.2.4)$$

Once the charge has separated into two lumps each with charge squared proportional to x^2 or $(1-x)^2$ respectively, we can then write a recursion relation[13,14]

$$K_n = [\int_0^1 [x^3(1-x)^3 + K_{n-1}x^2(1-x)^4]dx]/[\int_0^1 x^2(1-x)^2 dx] \qquad (6.2.5)$$

and hence

$$K_n = 3/14 + (2/7)K_{n-1} = (3/14)\Sigma_{i=0}^{n-1}(2/7)^i \qquad (6.2.6)$$

Therefore, invoking again the three degrees of freedom, we must take $< x(1-x) >= K_3$ and we obtain the Parker-Rhodes result

$$m_B/m_l = 137\pi/[(3/14)[1+(2/7)+(2/7)^2](4/5)] = 1836.151497... \qquad (6.2.7)$$

Since the electron and proton are stable for at least 10^{31} years we identify this ratio with m_p/m_e in agreement with experiment, thus setting the basic mass ratio scale for the theory. Whether this mass ratio remains unchanged and we can calculate the masses of *unstable* baryons and bosons from our dynamical theory is under investigation[50].

As already noted, the absolute unit of mass in the theory must be approximately the proton mass because of our identification of $2^{127} + 136$ with the inverse gravitational coupling constant. Since the calculation given above is a mass ratio, its success is independent of the absolute value of this unit. The corrections which take us from our single dimensional mass parameter m_B to the empirical value for the proton mass, given G (or equivalently to the empirical value for G, given m_p) and to the empirical value of the fine structure constant will have to come from level four of the theory, where we must also find a place for the equivalent of quarks and heavy leptons. Since we will then have 256 quantum numbers to play with, this will be challenging but not obviously impossible. Other problems, such as building up the electromagnetic field from our photons and the gravitational field from gravitons (we can obviously make the latter – so far as quantum numbers go – from leptons as spin 2 helicity states) is similar to that of any theory which starts from the weak coupling limit.

The reader immersed in special relativity may be troubled by the ticking universe, which provides a universal time, and the fact that our zero velocity criterion which defines the basic momentum-conserving events ($v_3 = N^1 - N^0 = 0$) would seem to single out a particular coordinate system. We have been led to the construction which places scatterings *between* TICKs because we cannot allow our events to have a continuum limit in *points*; else we would get back to the agony of infinite energy at each point, which it has taken so much hard technical work for quantum field theory to deal with. *Our* "virtual" processes occur in the "void" as finite fluctuations which cannot be directly accessed by experiment. We claim this is a strength rather than a weakness. As to the special coordinate system, we claim to have shown that we can still define macroscopic velocities v to arbitrary precision, and derive (or, according to some like Michael Peskin, *define*) the Lorentz transformation, thus recovering special relativity as a *macroscopic* approximation. As to the special coordinate system we claim that empirically there *is* such a coordinate system which defines $v = 0$ by the 2.7^oK background radiation. This is no more an embarrassment for us than for special relativity; the fact that it occurs so naturally in our theory we again count as a strength rather than a weakness. Clearly we still have to show that we can get the particle physics right, and then go on to show that the big bang emerges from our initial generation operations. This is a problem for future research. We are encouraged by the fact that we have only one type of mass in the theory, and in that sense have no place for a difference between gravitational and inertial mass. Further, if we do indeed succeed in getting spin 2 gravitons in the weak coupling limit, we can hope to recover gravitational theory from that starting point, a problem already discussed by Weinberg[53] . As to the big bang itself, scattering events labeled by the full level 4 quantum number scheme can only start when the 256 bit hierarchy scheme closes off and we have $2^{256} - 1$ conserved labels in $\mathcal{U}$. If we can get our microphysics right, this is a reasonable estimate for the baryon number and lepton number of the universe.

Our final point is that by focusing on *velocity* rather than space and time as basic we believe we have the correct fundamental starting point for unifying macroscopic quasi-continuous measurement with a digital model, a point of view already stressed by S-matrix theorists. Further, our ticking universe allows us to fuse the special relativistic concept of *event* with the unique and indivisible events of quantum mechanics. Whatever else survives from this attempt to construct a digital model for the universe, we are convinced that this is the correct place to connect relativity with quantum mechanics in a fundamental way. We close by remarking that the cosmological implications of the model are not in conflict with experience.

7. SUMMARY and CONCLUSIONS

In this paper we have argued that the three dimensional constants which connect physics to mathematics are now, experimentally, defined by counting integers, and hence that a digital model for physics is more appropriate than the conventional continuum models. We provide a simple algorithm which leads to a growing universe of bit strings which contains unique happenings which we call ***events***. To relate these to laboratory experience we assume that when two spacially separated counters fire in an ordered and distinct sequence, there were two events in the bit string universe connected by a random walk representable by a labeled ensemble of bit strings. From this basic interpretive paradigm we conclude that our connections between events have a limiting velocity which we identify with the laboratory limiting velocity c. From this we claim that the kinematics of special relativity follows as an ***approximate macroscopic*** theory.

By postulating that our labels can be put into correspondence with masses measured by mass ratios to a standard mass, we identify our random walk step length with the Compton wave length and define energy by $E = hc/l$. Then our spacial kinematics allow us to define relativistic vector momentum and a second length h/p. We postulate, compatible with our bit string construction, that events which lead to the firing of counters conserve energy and momentum macroscopically. By constructing coherent ensembles of ensembles we find we can identify h/p with the deBrogile wave length in the double slit paradigm and hence measure the unit of action in our theory as Planck's constant. Further, we show that our basic counter paradigm then allows us to construct the deBroglie wave theory for free particles as a continuum ***approximation***. From these free particle states we then can construct a quantum scattering theory using relativistic Faddeev-Yakubovsky equations. The driving terms in these equations can be related to our bit string construction, completing the link between our theory and the practice of elementary particle physics at the ***phenomenological*** level.

At this point we claim to have provided a consistent and rigorous basis for the reconstruction of quantum mechanics on a digital basis. Like quantum mechanics, this theory so far contains two universal constants h and c, some arbitrary particulate reference mass such as m_p or m_e, and dimensionless mass ratios and dimensional or dimensionless coupling constants which have to be taken from experiment. In conventional theories this basis is used to construct quantum field theories and from them to attempt to identify unifying symmetries which reduce the number of empirical parameters. But these theories currently are bound up with continuum models and infinities which have to be manipulated away. We find this repugnant in a fundamental theory, and take another route to attack the common problem.

We explore in detail the label structure provided by the combinatorial hierarchy mapping matrices and make tentative identifications which at least have the quantum numbers for weak-electromagnetic unification and the lowest hardonic states described by *SU*3 when the first three levels are combined. This work is still in its infancy, and will not become convincing until the minimal unitary scattering theory has led to more detailed results. But we, at least, find the degree oif unification we have achieved exciting, and hope others may as well. Independent of the details of the scheme, we claim to have now put the Parker-Rhodes calculation of the proton-electron mass ratio on a firm basis thus providing the mass ratio scale for our theory. The previous identification of the terminal cardinal of the hierarchy with the gravitational coupling constant in terms of the proton mass then completes the dimensional content of the scheme. On dimensional grounds we then have no place for a difference between gravitational mass and inertial mass; in that sense the "equivalence principle" is already built into our scheme and is not a separate postulate. Getting spin 1 photons and spin 2 gravitons from the weak coupling limit is a task we anticipate will be completed in the foreseeable future. The construction of general relativity as a gravitational theory would then be on essentially the same footing as any attempt which starts from the same weak coupling limit. The cosmological implications of the theory do not, at this stage, give us any conceptual or experiential difficulty, and provide us with a preliminary estimate of the mass of the universe which is of the right order of magnitude. The critical task in that respect is obviously to first get the elementary particle physics right. The basic idea with which we wish to leave the reader is that by invoking a ticking universe in which everything happens *between* ticks, we avoid the infinities of the continuum theories and believe we have unified relativity and quantum mechanics at an appropriately fundamental level.

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Table I

The combinatorial hierarchy

hierarchy level	l	$B(l) = H(l-1)$	$H(l) = 2^{B(l)} - 1$	$M(l) = [M(l-1)]^2$	$C(l) = \Sigma_{j=1}^{l} H(j)$
	0	-	2	2	-
	1	2	3	4	3
	2	3	7	16	10
	3	7	127	256	137
	4	127	$2^{127} - 1$	$(256)^2$	$2^{127} - 1 + 137$

Level 5 cannot be constructed because $M(4) < H(4)$

Table II

Interpretation of the second level of the combinatorial hierarchy
in terms of electrons, positrons and gamma rays

		e_L^-	e_L^+	e_R^-	e_R^+	Q	H	outside	e_L^-	e_L^+	e_R^-	e_R^+	Q	H
inside the hierarchy														
basis	$\gamma_L + e_R^-$	1	1	1	0	-1	-1/2	$e_L^- + e_R^-$	-1	0	1	0	-2	0
	γ_L	1	1	0	0	0	-1	$e_L^- + \gamma_R$	1	0	1	1	-1	+1/2
	$\gamma_L + e_R^+$	1	1	0	1	+1	-1/2	e_L^-	1	0	0	0	-1	-1/2
								$e_L^- + e_R^+$	1	0	0	1	0	0
discriminate														
closure	e_R^-	0	0	1	0	-1	+1/2	$e_L^+ + e_R^-$	0	1	1	0	0	0
	e_R^+	0	0	0	1	+1	+1/2	e_L^+	0	1	0	0	+1	-1/2
	γ_R	0	0	1	1	0	+1	$e_L^+ + \gamma_R$	0	1	1	1	+1	+1/2
	γ_0	1	1	1	1	0	0	$e_L^+ + e_R^+$	0	1	0	1	+2	0

quanta $\gamma_L, \gamma_R, \gamma_0$

Table III

Interpretation of the first three levels of the combinatorial hierarchy
in terms of particles (fermions) and quanta (bosons)

		ℓ_L^o	ℓ_R^o	ℓ_L^-	ℓ_L^+	ℓ_R^-	ℓ_R^+	B	$\bar{B}$	B_L^{ch}	B_L^n	B_R^{ch}	B_R^n	i_z^+	i_z^-	i_z^+	i_z^-
Level 1:																	
particles	ν_L	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	$\bar{\nu}_R$	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
quantum	Z_0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Level 2:																	
particles	e_L^-	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	e_L^+	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	e_R^-	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	e_R^+	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
quanta:																	
basis	γ_L	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
	W_L^-	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
	W_0^+	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0
discriminate																	
closure	W_R^+	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	W_0^-	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	γ_R	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0
	γ_0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
Level 3:																	
particles	p_L	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0
	$\bar{p}_L$	0	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0
	n_L	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0
	$\bar{n}_L$	0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	0
	Σ_R^+	0	0	0	0	0	0	1	0	0	0	1	0	1	0	1	0
	Σ_R^-	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1
	Σ_0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0
—	$\bar{\Sigma}_0$	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	1

quanta π, ρ, ω, K, K^*, ϕ (1,2,3 level Coulomb (1111111111111111))

Appendix I. THE INCHOATIVE HYPOTHESIS

An Introduction to *The Theory of Indistinguishables*

by

A. F. Parker-Rhodes M.A. Ph.D.

[A version of this paper was presented at the second annual meeting of the Alternative Natural Philosophy Association, King's College, Cambridge, 1981.]

From time to time the suggestion has been put forward that the paradoxes, puzzles, and contradictions, which still plague theoretical physics despite its impressive record of successes, might perhaps be cleared up, if we had knowledge of a level of being anterior to the physical, which might furnish the raw material, so to say, out of which the known furniture of the universe, in particular the subatomic particles, could be seen as being made. Such suggestions have all, so far, come to nothing, for various reasons. The great difficulty in implementing the idea is that the contents of this new level, if it is to have any explanatory power, must be absent from our present world picture. Knowledge about it must therefore be gained, if at all, by using means of knowing which are themselves unknown.

This difficulty would disappear, if we could use another well-tried strategy of the scientific method, namely to postulate the existence of some entity, suitably described so that inferences could be drawn from its assumed existence, which could be tested against known or accessible phenomena. But if this entity is to be a new hypo-physical plane, it must be defined in such a way that its own non-observability is a plausible inference from the definition, and at the same time so that other more constructive inferences about what is observed might follow. I claim that such a definition can be found, by a principled search, and leads directly, by way of some difficult and at times surprising mathematics, to a tenable theory.

I do not claim however that the theory was discovered in any such principled manner. Having no grasp of the difficulties involved, nor any foresight of the length of time required to fill in the numerous gaps, it came about by serendipity, as do most successful and unsuccessful theories. But an autobiographical account of a mathematical theory, even a well-written one, is not the right thing if understanding it to be attained. I shall therefore proceed as if I were expounding a well-known system to intelligent students, except that most of the real mathematical bones of the theory, to which I offer here only an introduction, will be filleted out.

I.1 *Unorderables*

There is a well-known theorem in Set Theory, that any Set of n members, finite or infinite, can be simply-ordered. This is surprising, on two counts. That it should be provable from the axioms commonly used implies that orderability is tacitly concealed among them and might need to be extirpated; and that it might not be true is an affront to common sense, of the kind that might well have found it its own place in a revised set of axioms. Common sense tells us that any two things, or concepts, can be arbitrarily labelled as first and second. Common sense, however, refers to the world of experience, and if we are to venture into a realm where observations (and *a fortiori* common sense) cannot guide us, such ideas as this might be wrong.

My first step is then to admit unorderable things into my basic axioms. If the hypo-physical plane were to consist of such, our not having noticed it hitherto would be superficially accountable (at least), and the first major difficulty of defining what we are talking about would be overcome (there are plenty more!). This means however that we abandon Set Theory; and since so much of normal mathematics is based on Set Theory (or supposed to be), we shall be unable to get much help from the existing literature in constructing a mathematical system to describe the Inchoative Plane. But if not Sets, then what? It is of course well known that not all classes are Sets, and, assuming that we are dealing with a plurality, or something with aspects of plurality anyway, we shall at least have classes. When we come to setting out an axiom system, differing from that of Set Theory as adumbrated, I shall call any class to which the new axioms apply a Sort, and the whole system will then be Sort Theory. (I capitalize "Sort" to distinguish it from other sorts of sorts, and "Set", too, to set it apart form the set of colloquial "sets".

From this point of view, the main peculiarity of finite Sets (which can be considered as a special class of Sorts) is that their cardinals and ordinals are always numerically equal. A Sort, on the other hand, can have any ordinal not greater than its cardinal. If all the members of a Sort are mutually unorderable, they all occupy the same position in any ordering, which we call the first place, so that the ordinal is 1. If every pair is orderable, then the ordinal is n. These are classed "perfect" and "ordinable" Sorts respectively; in the general case of a "mixed" Sort S we have $1 <^0 S < n$, ${}^cS = n$.

I.2 *Triparitous Mathematics*

Suppose now that we have a class containing two entities. If these are *identical* the cardinal of the class, and therefore its ordinal, is 1. Were we then mistaken in saying that the class contained two entities?That would be too harsh; it might well be the natural way of reporting experiences we need to discuss, even though we know, as in a fairy-tale, that we cannot really experience them. If however our two entities were not really identical, but still unorderable, the class will have the cardinal 2, and the ordinal 1; it is a perfect Sort. There is no "mistake" there; but we have met up with *twin* entities. These don't exist in the real world, but of course we are assuming that they do so in the Inchoative Plane. And, of course, we might meet with entities which are not even twins, but orderable, and in that case, back in the everyday at last, we shall have a class with both cardinal and ordinal equal to 2. But we still shan't know whether to call it a Set or a Sort, until we know what else is likely to turn up.

We have therefore, when dealing with situations such as that described above, to reckon with three "parity-relations" among entities; they may be either identical, or twins, or distinct. In normal mathematics we have only two: equal or unequal; I shall call such a theory "biparitous", as opposed to "triparitous" mathematics, where we have three parity-relations. Strictly, of course, we are not three but six, for whereas not-equal is the same as unequal (and vice versa), not-identical means either twins or distinct, and likewise all the negations are disjunctions of the other two. In both systems, cases may arise where we do not know what parity-relation obtains between two things, but this of course does not count as an additional parity-relation.

Triparitous mathematics (as far as I am concerned) uses the same system of inference as classical, in which a proposition is either true or false (or undecidable) but never both at once and always (if decided) one or the other. To admit a third parity-relation is not to be confused with accepting a third truth-value (as in intuitionist mathematics where not-not-P does not entail P).

The imaginary entites among which the "twin" relation of unorderability holds are called "indistinguishables", or "ibs" for short. It is important, in entering on an unfamiliar field, not to cut corners; that is why I use separate terms for the relation of twinship and the things that exhibit it. The abstract relation, and the associated notations, exist in the mathematical sense, once they are located in a consistent theory, and no questions need be asked so long as we are doing pure mathematics. But in a theory that is to be applied, we must at some point pass over to thinking what the mathematics means, and in our case we say that " a and b are twins" means that there are two indistinguishables, denoted by the symbols a,b, or that a,b are ibs.

It is part of the hypothesis which I am examining that such a remark is allowed as sensible in relation to the Inchoative Plane. In the world commonly thought of as "real", there are of course no ibs. They exist, if at all, in a non-ordinary reality, and we speak of them (as in a fairy-tale) *as if* they exist in the same sense that ordinary objects do, hoping that in the end we shall come to conclusions which can be compared with actual experience. In fact, things do turn out thus, and so we shall be tempted perhaps to say that the ibs are real after all, even "more real" than electrons and protons; this would be nonsense. Reality is a different matter in each plane, and it would tend to clarity of thought to recognize this more clearly than has been customary hitherto. Some philosophers have long been pointing out that it makes a good deal of possibly unexpected sense of our sense-data to assume that there exists a physical world outside of us; in a similarly instrumentalist spirit, I claim that what we have to assume this physical world is like makes, here and there, more sense if we assume that there exists an inchoative world, of which the physical world, in part at least, is the observed expression. I don't need to claim that the Earth "really" moved round the sun (even today a geocentric view would make only a negligible dent in the cosmological principle).

I.3 *Indistinguishables or Unorderables?*

The only virtue of thus going beyond our familiar concepts is the promise of more simplicity than physical theories are currently coming up with. In fact, a lot follows from the root idea that at the "bottom" of the objective world there exists an infinite class of unorderable entities. But to prove it we have to reduce this idea to proper mathematical form, and there are many steps before we can even begin to look for possible empirical consequences; it is therefore important to keep in mind this initial simplicity of the concept, which it will be all too easy to lose sight of.

The first difficulty is that the property of "unorderability", easy though it is to grasp in the imagination, does not lead by itself to the more difficult but more productive idea of "relative identity". Two ibs separately encountered cannot be distinguished from one ib; the decision between identity and twinship can be made only between members of one class defined in the relevant context. We cannot get away with saying that separated ibs cannot be identical if they are observed *simultaneously*, even if we allow ourselves, as we have not done, to speak of "observation" at all; for simultaneity presupposes time, and time, as a property of the physical world, is one of the things we hope to explain, and certainly cannot be used in the beginning without corrupting the whole argument. In fact, the nearest we can get to the idea of simultaneous observation is that it defines a *class* of things so observed — which is where we started from.

It turns out eventually that, starting from relative identity, we can prove that ibs are unorderable, as a theorem; but not vice versa. So we have to incorporate relative identity into our axioms, and immediately encounter another and much bigger difficulty. For as soon as we make the parity-relation of twinship dependent on class membership, the notation in which our theory is expressed becomes context-dependent; for the classes must be defined ***in the relevant context***, which means they are liable to change as the argument proceeds. Normal mathematical notation is context-free, subject to conventionally accepted exceptions such as $\sin^2\theta = (\sin\theta)^2 \neq \sin\sin\theta \neq (\sin^{-1}\theta)^{-2}$, which are already awkward enough. It follows that we shall not be doing "normal" mathematics, but something requiring unusual care and vigilance if proper standards of rigour are to be maintained.

Furthermore, we have said little enough, in saying that the notation is context-dependent. The rules of dependence have to be discovered and precisely formulated. This can be done thanks largely to work which has already been done in mathematical linguistics, which enables us to work out, step by step, the effect of the third parity-relation on the meaning of various possible formulae. What we find takes the form of a ***substitution rule*** to be applied to indistinguishables, corresponding to the rules allowing "free" interchange between equals and no interchange between unequals; the new rule is of course more complex, and refers to the syntax of the formulae. But it is definite and clear, and leads to an "axiom-schema" replacing the much simpler (and usually unstated) one of normal mathematics. The price is paid when we come to scan all our formulae to see whether they mean what we take them to mean in the light of this substitution-rule; the only compensation is that it is usually (but *not* always) possible to express what one wants to in the new formalism. When this cannot be done, it means you are trying to express nonsense. It seems that this mathematics can only be interpreted in formalistic terms — which is more or less what one might expect in treating of entities so elusive as our ibs.

I.4 *Peculiarities of Sort Theory*

In many ways Sort Theory works out differently from Set Theory. One peculiarity is for example that the members of a Sort are always Sorts; there is no analogue of members of Sets which are not themselves Sets. In consequence of this, structures in the Inchoative are not hierarchical in the way of having members which have members ... till eventually we reach a bottom level. In place of this kind of thing however we do have functional hierarchies, the arguments of a "higher" function being themselves functions on "lower" arguments, and here we do eventually reach bottom with arguments which are not functions.

There are may odd things about Sort mappings. Any mapping from a Sort onto a perfect Sort gives identical images for all its arguments, namely a free choice among all the elements of the perfect Sort. But in reverse it is otherwise; each element of a perfect may be mapped onto a distinct element of an ordinable Sort. But of course in neither case can we have an inverse mapping. One effect of these lapses into triviality is that the number of different functions which can be defined over a perfect Sort is very limited. A function of two arguments can have at most three different values, according to what arguments are chosen; it must moreover be commutative and associative. The latter restrictions apply however many arguments are involved, and outweigh the slowly increasing number of values which might appear; there is in fact only one three-argument function (not reducible to combinations of others), and that exists only for the perfect Sort of cardinal 4. No (irreducible) functions of more than three arguments exist at all.

The only non-trivial functions of one argument are called, with a little license, "endomorphisms", because they carry one element of a perfect Sort into another. But there are only two possible results, the argument unchanged, and a free choice among the lot. It follows that two endomorphisms which have the same invariant subdomain are identical, and it can be shown that those with different invariant subdomains are twins; the Sort of endomorphisms over a given perfect Sort is therefore itself a perfect Sort.

There is however one kind of function, of a rather trivial kind, which gives a little extra variety, which I call "multiplets". A multiplet is an ordered or unordered class of multiplets, or an unordered class of members of one perfect Sort. The simplest example is a pair; a more complex one is an ordered quadruplet of a pair, a singleton, another pair, and a triplet, where the first two and the second two are naturally unordered and the whole has cardinal 4, ordinal 3. All pairs taken from a perfect Sort are mutually twin, and form the "pair-Sort"; the pair-Sort of a Sort of n twins has cardinal $\Delta n = n(n+1)/2$.

There are no functions definable over any perfect Sort which are not reducible to some formula containing functions of one two or three arguments and multiplets. Over mixed Sorts of course may more functions can be constructed, but since any mixed Sort can be expressed as an ordinable Sort of perfect, or smaller mixed, Sorts, all they yield is reducible to a mixture of familiar functions over Sets and those over whatever perfect Sorts are involved.

A particular problem is posed by ordinable Sorts, which contain no twins. They are Sorts insofar as they are associated with other Sorts which are not necessarily ordinable; but in the absence of these they are indistinguishable from Sets. It is important, for the exposition of the theory, not to *call* them Sets, provided we remember that for every ordinable Sort there is an isomorphic Set with the same extent (to use the Set-theoretic term). This is called the equivalent Set of the Sort. Mixed Sorts also have equivalent Sets, whose members are Sorts, but it is not usually necessary to remember this, so that Set theory has in this case a very limited application.

I.5 *Rational Sorts*

The definition of Sorts has been so framed, that any class which is directly subject to empirical observation, and so of evidential value in the scientific method, must be a Set. It cannot be a Sort, but it may be the equivalent Set of an ordinable Sort (which hides an exception to the rule under a transparent verbal camouflage). Thus if we are given a Sort S which is not ordinable, the proposition that "S exists" is empirically undecidable.

Now suppose that we can construct, from the members of S and functions definable over S, a Set S' (that is to say, a class all of whose members are either identical or distinct), in which each element of S has a representative (that is, a mapping exists from S to S') such that the twinship or distinction between any pair of elements of S can be determined from their representatives; and in which each function of S is represented by some function defined over S' such that the value derived from given representatives in S' is the representative of the value given by the corresponding members of S under the function of S represented. Then we call S' (with the required functions) an "autogenous representation" of S. In such a case, we can legitimately infer, from the proposition "S exists" alone, that "S' exists", and since S' is a set, it is possible for there to be empirical evidence for S'. If such evidence is forthcoming, we are then justified in saying that this evidence would be explained by the existence of S, since that is a sufficient condition for the existence of S' also.

Clearly, individual instances of this will not be strongly evidential, though the more cases we find, and the fewer failures, the better the matter will stand. Much depends on how many Sorts turn out *not* to have autogenous representations. For if S' were not autogenous, we cannot infer S from S'; additional assumptions will be required beside the mere existence of S, so that the existence of S is no longer a sufficient condition for that of S', and as it is certainly not a necessary condition there is no valid case for S at all.

Any Sort for which one or more autogenous representations can be constructed is called a "rational" Sort, or RS; the above argument shows that there could be positive empirical evidence explainable by a rational Sort, but not for any non-rational Sort.

Now thanks to the relative poverty of functions and/or mappings among Sorts, it is possible without too much trouble to discover whether or not any given perfect Sort is rational, and in some cases to construct mixed Sorts which are so. Mixed Sorts in general can be considered as unions of perfect Sorts, and may be rational though not all the latter are; all ordinable Sorts are of course trivially rational by virtue of their equivalent Sets of autogenous representations. We can thus expect a definitive answer to the question, "Which Sorts are rational?" The answer is, hardly any. Perfect Sorts with 0, 1, 2, or 3 members are RS's, also a mixed Sort with $2^{\aleph_0}$ members, and mixed Sorts including any of these together with the Sorts of endomorphisms up to a certain level, leaving no gaps in the series; all other Sorts are RS's only if they are constructed as unions, intersections, direct products, etc., from among these basic RS's, and so offer no additional information to that deriving from the latter alone.

It may be of interest to explain the nature of failures to find autogenous representations, by considering the perfect Sort of 4 members. This turns out to have a function of three arguments which has no representation, and a symmetry condition among its members which is not satisfied by any representation. The latter failure is reproduced for all larger perfect Sorts. In the case of Sorts of endomorphisms, the only available representations are in terms of structures analogous to matrices, the numbers of which that are available can be shown to be insufficient if we continue the series long enough.

I.6 *The Inchoative Hypothesis*

At the beginning of Section I.5 I proposed that the Inchoative Plane might be characterized as an infinite class of unorderable entities. Even when sharpened up by the replacement of "unorderable" by "indistinguishable", which can be defined (mathematically) by the theory of Sorts this seems rather a bare statement. We can now however use it in a genuinely testable hypothesis, which can be stated thus:

1. There is an infinite class of indistinguishable propertyless entities, call the Inchoative Plane;
2. There is a physical entity manifesting the structure of each biparitous

representation of every rational Sort of indistinguishables in the Inchoative Plane;

3. There are no other physical entities than these, or such as are anylysable in terms of these.

I call this the "Inchoative Hypothesis" in its "strong form"; a weaker form, which alone I claim to give evidence for, is obtained by deleting the clause (3). Even the strong form does not make the Physical Plane correspond to the Inchoative, because the entities it contains may not be (indeed are not) deterministic in their behaviour. If it were true, however, unlikely though that is, it would go a long way towards validating an apriorist philosophy of physics. This is a strong motive for not taking it seriously, though it probably cannot be disproved on present knowledge without disproving (2) also; but I shall mention a few probable counter-examples to (3) in Section I.14.

Clause (1) is a mainly metaphysical support for clause (2); but not wholly so. The term "propertyless" is inserted for the following logical reason: had I said "indistinguishable black entities" this would imply that some more black entities, necessarily not in the Inchoative because they would not be indistinguishable from the black ones, exist; therefore clause (3) could not also be true. The term "infinite" is also probably consequential for the interpretation. If "metaphysical" means "without testable consequences" (as it often does in scientific discourse), then the epithet cannot strictly be applied to (1); neither is (1) incapable of analytical formulation, being embodied as we assume in the axioms of Sort Theory.

Nevertheless, it is clause (2) which has to run the gauntlet of comparison with the known physical world. It comes through, if not scatheless, with no fatal wounds (as presently diagnosed). That it does so is, at first glance, very surprising, and makes it difficult not to take the Inchoative as a serious hypothesis. I shall look into the strategy of testing it in the next section, but meanwhile two points in clause (2) need some comment.

First, the expression "physical entity", occurring in a context where none of the common limitations can be presupposed, has a highly inclusive sense. Microphysical events, space, the uncertainty-principle, protons, gravitation, are among the kinds of things comprehended under this term. Second, note that the term "autogenous" is ***not*** used of the "representations" mentioned; it is of course assumed in the definition of a rational Sort, but if a Sort is known in this way to be rational, there is no logical reason for discounting other representations which satisfy the mapping relation as candidates for empirical interpretation. These non-autogenous representations I call "secondary".

I.7 *A Pattern of Families*

I have mentioned that, for any perfect Sort, there is another perfect Sort whose members are the endomorphisms on the first; and so on of course without limit. In the case of a rational Sort, it can be shown that if an autogenous representation can be found for the Sort or endomorphisms over it the ***union*** of these two Sorts is rational. The means for constructing representations are however limited, and as soon as we reach the point where none can be found, the sequence of rational Sorts terminates. This relation of endomorphism generates Sets of RS's which I have called "families".

All the RS's turn out to belong in one or another of six such families; two of these are intimately interrelated and are best treated as one, and one is trivial. The families contain different numbers of RS's, which form the palindromic series $1\ \infty\ 8\ \infty\ 1$. The first consists of the empty Sort alone. The second has the singular Sort $D1_0$ as its initial member, and contains an ordinable Sort of every succeeding cardinal n. The 8 refers to the combined families $D2$ and $D3$, which contain the perfect Sorts $D2_0$ and $D3_0$ with two and three members, which have respectively five and one descendants, with cardinals 3, 5, 10, 137, and $17_{10}37$ (approx.) in $D2$, and 3, 10 only in $D3$. The second infinite family stems from the initial $D\infty_0$ and the final 1 contains the all-inclusive infinite Sort representing the Inchoative Plane as a whole, which does not call for any specific physical interpretation except presumably the Universe. The presence of this totality-term is an unusual and welcome feature of the theory.

The second infinite family is sensitive in an interesting way to the mathematical philosophy with which we approach it. If, with the strict intuitionists, we will have no truck with "completed infinities" the initial Sort $D\infty_0$ exists and is rational but all the rest and the "total" RS are identical with the first. If we accept completed infinities in the Sorts themselves, as being appropriately beyond the reach of the mind, but reject them in constructing autogenous representations on the grounds that these have a practical role, then the family is indeed infinite, but all have the same cardinality; this is a close analogue of $D1$. Finally, the most indulgent view about infinities allows the Sorts in $D\infty$ to run through all the Carnapian infinities $\aleph_N$. The "total" RS exists (non-identically) only for the last two philosophies, the second making it equal to cardinality to the RS's of $D\infty$, the third giving it an extent beyond any cardinality. The second allows us to see the family $D\infty$ as a picture of strictly objective observations; the first does not allow for any representation of observation as conscious, while the third can accommodate an infinity of subjective states as well: which neatly explains what sort of people prefer each view of infinity, but tells us nothing about physics.

I.8 *Secondary Representations*

It might be feared that the introduction of secondary representations of RS's, combined with the strategy of allowing a separate interpretation to each representation, would widen the field beyond the possibility of definitive testing. It is true that the possibility of finding a secondary representation previously overlooked cannot be ruled out, but to entail a new interpretation it would have to be non-reducible to any previous one, which seems unlikely. All we need to say about this is that the total system is slightly more open-ended than it may at first seem. In fact, there are only two cases where secondary representations are known.

The families $D0$ and $D1$, being inherently ordinable Sorts, have none (or no primary ones, if you prefer). $D2$ does have collectively a secondary representation (which however omits the RS $D2_1^*$ with five members), namely the combinatorial hierarchy of Bastin, Noyes and Kilmister. This can be axiomaticed within the biparitous mathematics of the system, and is thus capable of interpretation in directly physical terms, whereas primary representations can only be interpreted as classes of indistinguishables which have to be correlated with physically observable predicates to become fully empirical. $D3$ also has a secondary representation, but this is contained within that of $D2$. $D\infty_0$ has a secondary representation by non-terminating simply-ordered sequences of digits (virtually = real numbers < 1).

Among secondary representations, the combinatorial hierarchy occupies a unique place, since its biparitous character makes it much more straightforwardly interpretable than its primary rival. I nevertheless do not count the results from that quarter as directly relevant to the success of my hypothesis, since they are logically incapable of providing evidence for it.

I.9 *Varieties of Interpretation*

For primary representations we have to seek interpretations which are faithful to the logical structure of the Sorts involved. In the case of ordinable Sorts, like those in family $D1$, we look for an ordered class of distinct things; in general however our interpretation must be first as some system of indistinguishables, to which distinct, often numerical, predicates can be assigned in the course of "observing" them.

The principal types are "aggregates", "thresholds", and "liberties". An aggregate is a fixed usually small number of indistinguishables, usually thought of in physics as a kind of object but more naturally as so many slots where specific quantities of charge, mass, spin, and so on can be entered. This writing-in of specific quantities is precisely analogous to the writing-in of specific values for the components of a vector, whether they are spatio-temporal coordinates or say angular measures defining the orientation of the spin axis. In each case we are ***predicating*** something observable of something in itself unobservable, but susceptible of ***interpretation*** as having the appropriate role. The temptation to think of the coordinates as a "kind of object" arises when their values are dimensionally congruent with those predicated of actual objects, which are then seen as the sum of a set of smaller constituent parts. Hence the description of baryons, etc., as being made up of quarks.

But, it may be objected, isn't there evidence that those are evidence of discrete centres of scattering within the proton, for example? If what is an aggregate is deemed an object, the location of that object is not precisely defined, and if attempts are made to fix its position various results within an experimentally-determined range will be obtained. If one can actually "see" the object the uncertainty may be ascribed to "vibration" (as with atoms in a crystal lattice) composed of vector components; if this description is inappropriate, what one does observe will be whatever is understood as appropriate. Call them vector components, call them quarks, what's in a name?

The second main type of interpretation of rational Sorts is as "thresholds". When a number of particles all having the same descriptors (quantum numbers) are assembled within so small a space that the spatial location vectors no longer serve to distinguish them, they become truly indistinguishable, and ought then to be observable only in the numbers allowed for RS's. In fact, experimental difficulties make it impossible to assemble in this way more than two or at most three similar particles, so a direct test is (as usual) not feasible. But it is possible to calculate what would happen if one could collect larger numbers; what we find is that in every case there is a threshold number, at which the aggregate becomes unstable. For example, an assembly of (about) $17_{10}37$ nucleons within the Compton radius of one of them produces the smallest possible black hole; and 137 electron-positron pairs so packed initiates a chain-reaction of pair-production and so would "explode". These thresholds correspond to the cardinals of RS's, as the theory says they should.

Last of my three main types of interpretation I have called "liberties", meaning by this that the indistinguishables involved in them are most easily recognised as the degrees of freedom of some system. Of this kind are the three dimensions of space, the ten degrees of freedom specifying the flavour and colour of a quark (three for colour, and still only seven d.f. among the known quantum numbers). This last is one of the few cases where a mixed RS shows its composition from two perfect Sorts of cardinals (three and seven respectively) in the interpretation assigned to it. Not all interpretations belong to these three types, however, as I shall now explain.

I.10 *Space and Time*

Whereas in the strictly biparitous theory of the combinatorial hierarchy it is possible to discern a definite order of appearance of the various structures, no such ordering can be postulated for the theory of indistinguishables. The total repertory of RS's coexist with no time-like ordering; at most within families is there an order of dependence which might be significant (and which in the family $D2$ is in fact the same as the "order" of the combinatorial hierarchy). Thus, while the latter theory must start without any space-time framework the *Theory of Indistinguishables* has one from the first quite independently of the interpretation of the finite families, based on the families $D1$ and $D\infty$.

The members of the simply-ordered denumerably-infinite family $D1$ are the only items in our theory which are all ordinable Sorts, and therefore may correspond to something empirically observed. It is commonly acknowledged that all the empirical data of microphysics come from the observation of "particle interactions" or *events*; we need therefore have no hesitation in saying that family $D1$ correspond to the totality of events (in this sense of the term).

This "totality of events" constitutes a discrete "space" in the topological sense, and it is possible to show that to specify any event we need to give it a position in an infinite succession having a first term, and for any such position can give it a position in up to three independent twin orderings, each of which is finite but unbounded; and that this is the sum total of the information of this kind which is available. Any event therefore can be specified by a set of four coordinates, one of which refers to a structure of a different kind from the other three; any method of specification not reducible to this form is either in general inadequate, or redundant. The space-time structure of the world is thus determined *a priori*, and only the methods by which we can investigate it, and the manner of its subjective apprehension, remain to be considered.

That for everyday purposes space, as thus defined, seems to be "continuous", follows merely from the fact that the discrete events, whose disposition it describes, are far too small and close together to be discerned as discrete by our organs of perception. Physicists however have become accustomed to considering situations where only a few events are relevant, often indeed only one of them. And in this last case at least it is clear that the notion of space as given by the above theory simply has nothing to say. At least two events are needed to provide any standard of measurement; and to provide an event with a position defined as required, at least four other events, making five in all, must be taken into account. Very small regions are thus not catered for by the apparatus provided by family $D1$.

Empty space, if that's what we are talking about, is nothing much to worry about; but the "space" surrounding one event is commonly thought of as containing various fields which can be described, we often think, only by reference to a coordinate system. These coordinates are in fact derived by imaginging the familiar spatial structure interpolated into regions as small as we care to consider. If the nature of space is as I have described, this must be nonsense, and its results must be wrong. As is now well known, they are; quantum theory is provided as a remedy, and for many purposes it works wonderfully well, but few would be prepared to explain *why* it works, except in special cases.

I.11 *Disordinate Space*

Where the space based on $D1$ fails, we turn to its palindromic complement $D\infty$, where we find in $D\infty_0$ an RS which has like $D1$ a secondary representation, already briefly mentioned. This representation has the structure of an infinite Boolean lattice; points are separated not by spatial intervals, as London is from Cambridge, but more like the separation of East from North, by rotations. Every point has an infinite number of neighbours. Moreover, almost all the members of $D\infty_0$ are twins ($D\infty_0$ is "almost perfect"), so that any mapping onto this Sort is (at most) completely unspecified; for this reason, I call the space which we infer from it the "disordinate space". If it is right to assume that this is the kind of space which takes over when the discrete space of $D1$ is no longer applicable, then the basic trouble with working in conventional infinitely-divisible space is that its points are really mapped onto disordinate space, so that literally anything can be any where.

The result is not total chaos. There are few problems that statistics cannot be applied to, and this isn't one of them. "Disordinate statistics" is in principle simple enough, and though its results are sometimes bizarre, that is only to be expected. In selected cases, the technique seems to work well.

Disordinate space has infinite connectivity; it is in fact a realization of a concept which has recently come into prominence in quantum physics, that of "wormholes", according to which the connectivity of space at very small distances increases without limit. But my disordinate space offers no such gradualism. Connectivity *is* infinite albeit in general tempered by a finite probability which offers at least a qualitatively similar smoothing of the transition.

This raises the question, what is the connectivity of ordinary large-scale space? It appears that this is not a question which it is customary to ask, perhaps because it cannot (yet?) be decided by observation. Now the geometrically simplest way of constructing space from the events of $D1$ gives us a hypertoroidal space, whereas cosmologists seem always to assume (so tacitly as to suggest it is not an assumption) that the universe is hyperspherical (or parahyperboloidal). There are enough data supporting the inherently more obvious assumption of hypertoroidal space to persuade me in general to assume that it is true. It may be long before the matter can be settled by observation, but theoretical consistency will probably give the answer much sooner.

I.12 *Particles as Aggregates*

It is a basic point of my theory that indistinguishables as such can never be observed. Yet there are many things which are repeatedly observed (or so they say) which appear to be — apart from accidents of position or momentum — strictly indistinguishables. Electrons with identical spins, or example. Strictly speaking, it is only the strong form of the Inchoative Hypothesis which entails any consequences from such observations — but if the so-called elementary particles are to be themselves counted among the exceptions, we shall have explained very little. Furthermore, it is generally accepted that some of these particles are indeed aggregates of unobservable entities, as the theory predicts. If hadrons, why not leptons?

Because, up to now, there has been no evidence of any kind of compositeness for leptons, and so no motivation for still further conceptual complications. One might add that if quarks are essentially involved in the strong interactions (as are the forelimbs of birds in flight) then particles which don't participate in that force (flightless vertebrates) shouldn't have them. All the same, as many an amateur numerologist must have discovered, it is easy enough to invent a set of lepto-quarks which will "fit" the known leptons in the same manner that classical quarks "explain" the various hadrons. My theory suggests that it may be more profitable to draw inferences from assuming such a structure than to look for direct evidence of their existence, which we have no right to expect anyway.

The theory makes a plain prediction: that, given that there is no essential *epistemological* difference between leptons and hadrons (which has never been suggested) then the former are aggregates of five "partons" of which at any one time three are lepto-quarks ("larks"). These correlate like quarks with the RS $D2_1^*$ with $3 + 2 = 5$ members. As to the partition of the various supposedly quantized attributes among these partons, the requirements of the theory entail that only identical or random dense values can be considered for the partons *in situ* (e.g. *not* $-1/3$, $2/3$, $2/3$), and that identical values would have to be constant and so imply in implausible measure of self-identity for indistinguishables, as well as contradicting the hypothesized quantity of propertylessness. So we end up with a random and perpetually shifting partition of charge (and spin?) subject to all adding up to the electronic charge (or zero for neutrinos) and a second constraint of the same nature — because $D2_1^*$ is a mixed Sort of ordinal 2 — leaving us with a system of three degrees of freedom.

Any such partition of the electronic charge will clearly endow the particle with an intrinsic electrostatic potential, and hence with at least the corresponding mass. The model described enables us, with the help of the aforementioned "disordinate statistics", to calculate the resulting mass. It comes to

$$m = 0.23440233\ \alpha\hbar/d$$

where α is the fine-structure constant, $\hbar$ is Planck's constant divided by 2π, and d

is an assumed minimum distance of approach between the partons (that it is finite is required by the theory, and of course a zero value leads to infinite potential). If we identify this d as the Compton radius of the proton (this being the heaviest stable particle and so giving them smallest d), and α as 1/137 following on the identification of the RS $D2_3$ with the electromagnetic field, and give $\hbar$ its accepted empirical value, m comes out equal to m_e as near as the known errors in $\hbar$ and d allow.

There is nothing here to call in question the assumed parton-structure of the electron, except that the value of α assumed is *not* the empirical value of the fine-structure constant. The latter is defined within the framework of a quite different model of the system, and an attempt to quantify the effects of this difference in the models enables us to account for most of the discrepancy. Overall, this kind of results gives a little support to the model on which it is based, but is by no means conclusive in that regard.

I.13 *Further Results*

A number of other conclusions and predictions can be derived in the course of developing the interpretation of the theory. Most of these are relatively of little weight, and some work against the correctness of the hypothesis. Of a kind too general to carry much weight are for example prediction of conservation laws applying under specified conditions to energy, angular momentum, and linear momentum; and of the irreversability of mass action. Difficult to assess is the conclusion that, if the connectivity of space is hypertoroidal, there is no reason to expect conservation of chirality, which however would characterize hyperspherical connectivity; all one can say is that since (a few) cases of asymmetrical chirality are known, this favours the hypertoroidal theory.

More interestingly, we can predict the existence of an upper limit to the velocity with which a particle may travel, from the minimal topological requirements for the distinctness of time from space; which is by no means original, except for the fact that this distinctness is itself predicted by the theory. Much the same status attaches to the prediction of a finite limit to the accuracy of simultaneous complementary measurements (Heisenberg's principle).

There is a long but straightforward argument which sets an upper limit on the number of distinguishable particles of a given kind which might (ideally) be "observed". This leads, in the case of nucleons, to an estimate of the mean density of matter in the universe many times in excess of current astronomical estimates, but close to the value which, according to relativity theory, would make the overall curvature of space zero. This, if not merely a coincidence — and the uncertainties in the numerical values concerned forbid us to dismiss this possibility — seems to show something, but it is hard to know precisely what. With the same proviso (now almost stultifying) we can derive a tolerable estimate of the gravitational constant G.

It can fairly be claimed that the density of matter is the one apparently counter-factual result the theory has yet come up with; and in view of the uncertainties commonly expressed by astronomers when discussing their evidence, even this may not be so bad as it may seem. So, in the end, the weak form of the Inchoative Hypothesis emerges slightly battered, but surviving. Nothing however can be said in favour of the strong form, which maintains that everything should be in some sense explained either directly or indirectly. The following things for example remain untouched: masses and lifetimes of the unstable particles (though Kari Enqvist has had some success with the masses of hyperons);

disappointingly, perhaps, this theory has nothing original to contribute to the description of the two-slit experiment, only a translation of the "probability-wave" account. There are of course many other gaps of a like kind which it would be tedious to list.

I have deliberately not claimed any support from the combinatorial hierarchy work, since this is conceptually independent, even though logically compatible with the Inchoative Hypothesis. There are many reasons for hoping that some of the gaps left by the latter may eventually be filled by the former; if this happens, we shall approach a little nearer to the essentially implausible "strong form" of the hypothesis.

I.14 *Philosophical Implications*

The Theory of Indistinguishables has no immediately evident practical consequences. The most it might do is to lead eventually to a simpler and perhaps more comprehensible presentation of existing physical theories, whose quantitative results, and in many cases the routes by which they are arrived at, will remain as they are or nearly so. But it may have an effect on the way we look at the world about us, through two factors of which one is new and the other revived from long ago.

The new factor is the concept of "planes". Two of these are generally recognized in one form or another, the physical and the organic. The reductionist view, that all organic phenomena should be ultimately explainable in physico-chemical terms, would in effect abolish the distinction between these, but is becoming ever less tenable. Some would claim the "human plane" as a third, at least partially within the purview of science. I however wish to add one at the other end. If my theory is accepted, there is an Inchoative Plane, of which many aspects of the physical plane are logical consequences. If the strong form of the hypothesis were to be established, this would partially reduce the physical to the inchoative, and so threaten the distinction between these planes. This I do not expect to happen.

From this point of view, the virtue of the present work lies in the extreme simplicity of the Inchoative as described by my theory. Because of this, whatever it can explain is explained in a very strong sense. If sound, therefore, it represents a real advance in our understanding of the nature of the world, however much remains unexplained. It would mean that the physical plane is in part transparent, and in part determined by strictly physical principles for which we must continue to seek physical explanations as we have always done. In this search however it will surely help, if we can go some way towards eliminating certain things from the latter category, as being inevitable according to the kind of principles I have been dealing with here.

But the notion of different planes is not the only characteristic of the theory of indistinguishables. Precisely because of the great simplicity of the initial assumptions, it takes a rather large stride in the direction of a-priorism; and this will be unwelcome in many quarters. It is not, in strict logic, an a-priori theory; it makes a few non-tautologous assumptions (notably of the existence of the Inchoative), from which it draws conclusions which are experimentally falsifyable — and perhaps will be falsified. I believe a ***strictly*** a-priorist theory is logically impossible. But a radical diminution in the number of presuppositions required has much the same psychological effect, in that it suggests that the world is at bottom unexpectedly ***simple***. That is far from the impression given by quantum theory today, and for many may be a welcome change, so far as it goes.

But it cannot but call in question a lot of things in which a huge investment of dedication, and money, has been sunk, perhaps in vain. There will be many deaths before such a conclusion can be admitted. It is still too early to attempt to name such lines of research, and I shall not try; but whoever accepts the new theory must expect to meet some destructive as well as (I hope) constructive effects.

There is one more point to make. It is still part of the conventional wisdom that physical science will never come to an end. This of course is true, obviously so, applied to the investigation of the effects of physical principles in all their manifold interactions. The reduction of chemistry to physics has hardly begun, and might have great consequences if it were to be more nearly achieved. But it is equally obvious that in certain directions my theory implies that we have already reached a non-physical bedrock. In effect the Inchoative Plane is a no-go area for physicists. It fills, much more literally and plausibly, the role which Fridtjof Capra tried to ascribe to sub-atomic physics in his book "The Tao of Physics" — the role of being directly accessible to the mystics. If it does exist, it is a terminus. How much of the present muddle could that explain?

I.15 *Summary*

The foregoing remarks are, of course, in themselves only a summary of the Theory of Indistinguishables. Further to condense the matter is perhaps to seek an excessive shrinkage. The gist of the matter is that, if we assume the existence of an Inchoative Plane (in what sense of "existence" it may not be profitable to enquire) sufficiently described by the unusual mathematical system of Sort in respect of empirical observations. The hypothesis, that all and only these, among the infinite contents of the Inchoative PLane, are reflected in the known physical world, meets with many successes, a few doubtful ones, and no failures that cannot be, at least for the nonce, outfaced. Some spectacular quantitative computations have some relevance to the question. I claim that it is reasonable to conclude that if there were indeed such an Inchoative Plane, a fair scatter of basic physical principles would find therein a common explanation; and some might think the theory worth attention on the grounds of its unusually wide compass alone. For all that, there is plenty of work still for real physicists to do, and the changes are that their work will tend both to expand the scope and erode certain aspects of the theory. If my work gains any attention, it will long be controversial, and in due course superseded. If it has any utility in the meantime, it will be to bring into question some of the meta-scientific attitudes and presuppositions which underlie the present chaotic state of fundamental physical theory.

Appendix II. THE COMBINATORIAL HIERARCHY

by

Clive W. Kilmister

II.1 *Browerian Foundations for the Hierarchy*

[A version of this paper was presented at the second annual meeting of the Alternative Natural Philosophy Association, King's College, Cambridge, 1980.]

The particular algebraic model for which the results of this paper have been found is developed from one described by Bastin, *et.al.* in Ref. 13 of the main text. It is based on three discrete processes and an equivalence relation. A typical functioning of the system consists of discrete steps, in each of which one step of one of the three processes takes place. Which process is involved may be determined by outside considerations or by the state of the machine at the time (just as, in a Turing machine, the next act is determined by the contents of the square being scanned and by the state of the machine). It is important for the particular kind of model we have in mind, however, to realise that the model is not to be thought of as being given in a complete form at the beginning of the investigation but rather as developing in a recursive fashion as the investigation proceeds. A detailed consequence of this is that it is impossible to take the equivalence relation as given in the usual way, and a recursive way of specifying it has to be found.

The constituent parts are:-

(i) a generating process G which yields new elements to adjoin to a set S of previously constructed elements. The actual form of G is not of much importance later, but one suitable form which we have employed is that given, in a completely different context, by Conway (Ref. 17 of the main text). A single operation of G is then of the form:

If L, R are disjoint subsets of S, adjoin $\{L|R\}$ to S.

The great advantage of this form of G is that it is completely recursive in the strong sense that no starting point is needed. The first element generated has to be $\{\emptyset|\emptyset\}$, where $\emptyset$ denotes the empty set; this element plays a special role and will be denoted by 0. The two possible next ones are $\{0|\emptyset\}$ and $\{\emptyset|0\}$, and so on.

(ii) We now introduce an equivalence relation, D, on S; the equivalence classes under D will be called *locations* and any member of a location A (written $a \in A$) will serve as an *address* for A.[Note that the word *address* is used here in a different sense than in the main text.] The relation D is specified recursively as follows: Let $S = \{0, a_1, a_2, ..., a_k\}$, be the set of elements already in play (either as a direct result of processes of G or from other operations to be described below) and a new element b be generated by G. Define by some recursive means a function f of two variables, called a *discrimination function*, so that, if Z is a particular subset of all posssible values of f then the condition

$$Dxy \leftrightarrow f(x, y) \in Z$$

defines a relation which is an equivalence relation. (To put it more directly, f discriminates between pairs (x, y) which are equivalent and those which are not, because its value for an equivalent pair is never the same as for a non-equivalent pair.) The requirement that D should be an equivalence relation imposes obvious restraints on f. Sufficient (but not necessary) conditions to insure that these restraints are satisfied are

$$f(x,x) \in Z \text{ for all } x,$$

$$f(x,y) = f(y,x) \text{ for all } x,y, \qquad (II.1.1)$$

$$f(f(x,y), f(y,z)) = f(x,z) \text{ for all } x,y,z.$$

For simplicity in what follows we assume all of these to hold. If b never turns out to be equivalent to any of the existing members of S, it is assigned to a new location.

(iii) In the course of determining the location of b, a number of values $f(a_i, b)$ will have been determined and (by (II.1.1)) two of these will be equivalent only if the corresponding a_i 's are equivalent. Accordingly for each existing address A_i we generate a new address $F(A_i, B)$ and for each of these we introduce a *minimal address rule* of the form:

Number the addresses 0,1,2,..., where 0 is the address of element 0. Then

$F(A,B) =$ the least address (in the usual order) different from all $F(A,B')$ and all $F(A',B)$ where A' is different from A and B' is different from B.

Successive values of F can now be found recursively, and it is easy to verify that F satisfies the restrictions

$$F(A,A) = 0; F(A,B) = F(B,A); F(A,F(B,C)) = F(F(A,B),C). \qquad (II.1.2)$$

Values for F for small values are given in Table AII.1(a).

The form of the identities (II.1.2) suggest a change in notation, writing $F(A,B) = A + B$. We adopt this in what follows and refer to this process, between addresses, as ***discrimination***. It is now straightforward, if a little tedious, to establish the result:

Theorem 1. Let S be a closed discrimination system. (The table shows how closed systems arise if we stop at 1,3,7, ...) *Then $|S| = 2^n$ for some integral n and there is an isomorphism between $(S,+)$ and $(V_n,+)$, where V_n is the n-dimensional vector space over the field Z_2 with two elements*, the + in the second bracket denoting the usual vector space addition. (Conway (1976)).

(iv) So far the construction of the model has stressed the process aspect, but not the self-organizing one, which involves a hierarchy with interaction between levels. This is introduced by an ***economy process***, in which certain special sets of locations can be given a single address, without disturbing the discrimination in the minimal addressing process. Suppose that T is any set of non-null addresses, and define the ***discriminate closure*** of T, T^* say, recursively as follows:

(a) $T \subseteq T^*$,

(b) If B, C are any two different members of T^*, then $B + C \in T^*$.

(Note that this form of definition makes a closed discrimination system S have the form $S = \{0\} \cup T^*$, for some T^*.)

Consider now a mapping ϕ of a closed discrimination system, S, into itself, which preserves the discrimination:

$$\phi : S \to S, \phi(A+B) = \phi(A) + \phi(B).$$

Then, from theorem 1, there is an evident representation of ϕ as an $n \times n$ matrix over Z_2, and since such matrices constitute a vector space of dimension n^2, the set of all such ϕ corresponds to a new closed discrimination system, S^2 say. (N.B. The vector-space picture suggests that ϕ is an element of a different logical type from the addresses A, B, but this is just an illusion produced by the special representation.) We may use this for the economy process as follows:

Let T^* be any d.c. (discriminately closed) subset. Then it can be proved that there exists one mapping ϕ with the property that

$$\phi(A) = A \leftrightarrow A \in T^*.$$

If also ϕ is chosen (as it can be) so that

$$\phi(A) = 0 \rightarrow A = 0,$$

(ϕ nonsingular), we may use the new address ϕ to represent the old set of addresses T^*, so that the information contained in T^* is represented in a more economical way at a higher level.

If, moreover, we choose the ϕ's for different d.c. subsets to be independent (that is, so that any k such themselves generate a d.c.subset of $2^k - 1$ members, and no fewer), these ϕ's will serve to allow the whole process to be repeated, so that we have a hierarchical structure as required above.

Let us call a generation of such a heirarchy *complete* when it so happens that the creation and discrimination operations have been carried out in such an order as to maximize the information-carrying of the structure. The complete hierarchy serves to define bounds on the amount of information that can be dealt with. We can then prove

Theorem 2. There is a unique complete hierarchy with more than two levels;it has successively completed levels of $3, 10, 137, 2^{127}-1+137 \simeq 1.7 \times 10^{38}$ *elements, beyond which further extension is impossible.*

The proof of this result is lengthy and at present clumsy. Instead of describing it, I prefer to indicate how the construction can proceed at lower levels. It will be convenient to have an abbreviated notation for vectors and operators. Write, for any vector v, $v = i + j + ... + k$, where the only rows occupied by 1's are the $i^{th}, j^{th}, ...$ (If we are in a low dimension so that none of $i.j, ...$ exceed 9, we can simply write $ij...k$.) An operator can then be written as an assemblage of column vectors, and this allows calculations to be carried our quickly, since $(p, q, r)1 = p$, $(p, q, r)2 = q$. To begin the construction choose two basis vectors in two dimensions, 1 and 2. Then we have to find operators with the invariant spaces:

(a) $\{1\}$, and this is evidently the operator $(1, 12)$.

(b) $\{2\}$, and this is $(12, 2)$.

(c) $\{1, 2, 12\}$, and this the unit matrix $(1, 2)$.

It should be noted that there is no choice at this stage. These operators may be rewritten at the next level as vectors 134, 124, 14. For some purposes it is necessary to keep them in this form but for the mere existence theorem it is possible to simplify by taking these as a new basis 1,2,3. There are now 7 invariant subspaces, and it is possible to find 7 corresponding operators, in a number of ways, which are linearly independent. For example, in the three-dimensional subspace, the operator $(1, 3, 23)$ has unique eigenvector 1, and so, interchanging the first and second directions $(3, 2, 13)$ has 2, and $(12, 1, 3)$ has 3. In much the same way the three $(13, 2, 3), (1, 23, 3), (1, 2, 13)$ serve for the three element spaces, and $(1, 2, 3)$, the unit matrix for all 7 vectors. It is not hard to verify that these are all linearly independent. It is harder to establish the existence of the 127 operators at the next level, but several different versions of this have now been carried out. The termination of the process arises because the dimensionality of the spaces does not increase fast enough to accommodate the number of linearly independent vectors, as is demonstrated in Table AII.1(b).

We need to understand something of the "geography" of the higher levels. Some of this information is really about the structure of the finite group $GL(n,2)$ where $n = 256$ (or perhaps 16). Such a project will evidently require computation, but it is important first to determine what should be computed. The "model" cases of $GL(3,2)$ and $GL(4,2)$ will be presented at ANPA 83. A certain amount of assistance comes from the fact that, if $n \geq 2, GL(n,2)$ is simple and therefore has been an object of study by simple group theorists, but to a large extent the information they derive is not sufficiently specific for our purposes.

The number of elements of $GL(n,2)$ is easily seen to be $(2^n-1)(2^n-2)(2^n-2^2)(2^n-2^{n-1})$. It is easier to express it as $n^* \cdot 2(n-1)^* \cdot 2^2(n-2)^* ... = 2^{n(n-1)/2}\phi(n^*)$ where $r^* = 2^r - 1, \phi(r^*) = r^*(r-1)^*...2^*1^*$. Thus $GL(3,2)$ has order $8 \cdot 7 \cdot 3 \cdot = 168$. Its structure was further anylysed by Steinberg (R.Steinberg (1951), *Canadian Jour. of Math. 3*, 225.) and he divided it into classes containing 1, 21, 42, 56, 24, 24 elements. If we compare these figures to some due to Amson (private communication), who finds one member with 7 eigenvectors, 21 with 3, 98 with 1 and 48 which unfix every vector, there is obviously some connection. In fact Amson's 98 with one eigenvector split into two subclasses, one of 42 members which permute the remaining 6 vectors in a 2-cycle and a 4-cycle, and one with 56 which produce two 3-cycles. This points to the need to find all possible combinations of cycle-lengths, and this is a project on which some members [of ANPA] did some calculations (up to the n=16 case) some years ago. The division of the unfixers is more obvious. If A is an unfixer, then for any vector v, Av is neither v nor zero; Hence also $(A+I)v$ is neither 0 nor v, and so $(A+I)$ is an unfixer. Though it is possible to carry the analysis of $GL(3,2)$ much further, it should be noted that the order of $GL(4,2)$ is 20160, so that already (at the first stage where the results would be of interest to the study of the hierarchy) the numbers are beginning to become unmanageable.

However, matters are not quite so bad; in the first place, it is not actually the whole group with which we are concerned. We have to deal with a construction which considers just those non-singular operators for which none of the vectors not in a preferred subspace can be eigenvectors. This lowers the number of operators to be considered by only a trivial amount (for example, when n=4 from 20160 to 18816) but the new set of operators is no longer a subgroup, and this suggests that the group theoretic analysis cannot give the fine detail needed. This work is continuing.

Appendix II.2. *On Generation & Discrimination*

[This paper was presented at the fourth annual meeting of the Alternative Natural Philosophy Association, King's College, Cambridge, 1982.]

The algebraic model is based on three discrete processes, and a typical functioning of the model consists of discrete steps in each of which one step of one of the three processes takes place. Which process is involved may be determined by:-

(i) outside considerations

(ii) internal ones, e.g. state of the system at the time, or the constraint that the third process cannot be carried out in th early stages.

What is important here is the requirement that the model is not given in a complete form at the beginning but develops as the investigation proceeds.

The first process is a ***generating operation G*** which adjoins elements to the (finite) set S of elements which have already arisen. [I do not know that the details of Conway's $\{L|R\}$ construction are important, but G should have in common with Conway's construction the ability to start with nothing. Since the only "nothing" which I know of as freely available is the null set, this suggests that G must be some sort of construction in terms of sets.]

The second process is needed to check whether elements generated are really new ones or not. A formalist way of putting this would be to say that there was an equivalence relation D, and the question, when x is produced, is whether Dxy holds for any y in S. We cannot use this way of putting it because, both in the specification of D and in checking Dxy for any y in S we are supposing S completely given, contrary both to the original requirement and to the fact that S is growing with the development of the model. None the less this formalist approach gives two insights: firstly that *some* form of memory is essential. I shall assume that I can call elements of S but only at random. Secondly it suggests a recursive specification, instead of D, of a function $f : S \times S \rightarrow T$ (here T is some set, which includes S) with the property that there is a fixed subset Z of S and $fxy \in Z$ if and only if x and y are the same element.

Since f is our recursive substitute for D, we must require

(i)$fxx \in Z$

(ii)$fxx \in Z \rightarrow fxy \in Z$

(iii) $fxy \in Z$ and $fyz \in Z$ if and only if $fxz \in Z$

Any such ***recursive*** f will be called a ***prediscrimination.*** Two prediscriminations $f, \bar{f}$ will be called equivalent, $f \equiv \bar{f}$, if $fxy \in Z$ if and only if $\bar{f}xy \in \bar{Z}$ where $f : S \times S \rightarrow \bar{T}$ and $\bar{Z}$ is some subset of S. (Of course it is not ruled out that $T = \bar{T}$ and $Z = \bar{Z}$.)

Theorem 1. Every f is equivalent to some g for which

(i) $gxx = 0$

(ii) $gxy = gxy$

(iii)$gx(gyz) = g(gxy)z$

(where 0 is written for the unique element of a one element Z)

Proof

(a) Define $\bar{f} = 0$ if $x = y$, $= fxy$ otherwise.

Evidently $\bar{f} \equiv f$: and $\bar{f}\, xx = 0$.

(b) Suppose the elements of S to have been numbered by any recursive process; and define

$$\bar{F}\, xy = min(\bar{f}\, xy, \bar{f}\, yx)$$

(where 0 is counted as the least element and the others 1,2,3,.. are regarded as ordered in the usual way). Easily $\bar{F} \equiv \bar{f} \equiv f$ and

$$\bar{F}\, xx = 0; \bar{F}\, xy = \bar{F}\, yx$$

(c) [Conway's trick] Define

$$gxy = \textit{the least element } z \textit{ such that } \bar{F}\, z(g\,\bar{x}\,y) \neq 0, \bar{F}\, z(gx\,\bar{y}) \neq 0,$$

for all $\bar{x} < x, \bar{y} < y$. Then, easily, $g \equiv \bar{F} \equiv f$, and, since $gxy = 0$ if and only if x and y are the same element, g satisfies

$$gxx = 0$$

$$gxy = gyx$$

$$gxy = 0 \textit{ and } gyz = 0 \rightarrow gxz = 0.$$

It remains to prove that

$$gx(gyz) = g(gxy)z.$$

It is simplest to verify this by explicit construction as in Conway's book. From the definition $g00 = 0$, $g01 = 1$, so $g11$ cannot be 1 but can be 0: $g11 = 0$. $g02$ cannot be 1 or 0 but can be 2. Then $g12$ cannot be 2,1,0 so must be called 3. Next $g22 = 0$ and so on.

The requirements on g are exactly those of a discrimination function. It is straightforward (if a little tedious) to prove

Theorem 2. If S is a closed system then $|S| = 2^n$ for integral n and there is an isomorphism

$$(S, g) \simeq (V_n, +_2)$$

where V_n is the vector space of n dimensions over Z_2.

The third process is an economy process for labeling sets of elements with a single element.[Here follows the usual eigenvalue construction given above in AII.1.]

The three processes are carried out in various orders and in this way generate a hierarchical structure. Call this hierarchy *complete* [perhaps some other word e.g. maximal would be better] when it so happens that creation, discrimination & economy processes have been carried out in such an order as to maximize the structure. A complete hierarchy then serves to define bounds on the amount of information that can be dealt with. Then

Theorem 3. There is a unique complete hierarchy with more than two levels having successfully completed levels of $3, 10, 137, 2^{127} - 1 + 136$ *elements, beyond which further extension is impossible.*

However, after the complete hierarchy has arisen (so that the third process can no longer intervene) the generation process and discrimination can still proceed. One can picture this best in the vector space picture. The elements of the vector space have a 256 - bit segment to which further strings are affixed. We can call the 256- bit segment the *label* and the remainder the *address*.

Appendix II.3 HIERARCHY CONSTRUCTION (second version)

[This is a new version for work done about three years ago rewritten for this paper 10 April 1983]

By a *hierarchy* is meant a collection of *levels* related as follows:

a) The elements at one level are a basis of a vector space V/Z_2, a subspace of V_n/Z_2.

b) The elements at the next (higher) level are non-singular linear operators: $V_n \to V_n$ (again, of course $/Z_2$).

c) Each element A at the higher level corresponds to a subset S of the elements at the lower level by the correspondence: the proper eigenvectors of A are exactly the linear subspace generated by S. (NOTE: *Proper* eigenvector means $Au = u$.)

d) The operators $\{A\}$ are then vectors in V_{n^2}. In order to repeat the operation, they must be chosen linearly independent at the higher level.

Evidently if there are r elements at one level, the next one must contain $2^r - 1 = r^*$ (say) elements.

Theorem 1. (The Parker-Rhodes theorem) *There is only one candidate for a hierarchy with more than two stages (3 levels) and that has successive numbers of elements.*

$$2, 2^* = 3, 3^* = 7, 7^* = 127, 127^*$$

Proof: The proof is easy, since impossibility results from the fact that the operators have n^2 elements and 2^n increases too fast. In the case given above (Table II.1(b)), however, it may be possible to find a candidate, though at the last stage the operators cannot be linearly independent, so the construction terminates. The point of this paper is to show that it is indeed possible.

Notation. Write for the vector having 1 in its k^{th} place and zero elsewhere simply k. For $k + l$, $/Z_2$,write kl. For an $n \times n$ operator form an ordered set of vectors (its columns) in the form $(A_1, A_2, ..., A_n)$. Then

$$(A_1, A_2, ..., A_n)i = A_i$$

At the first level of the hierarchy assume that the two vectors chosen initially are 1,2. (The other choices, involving 12, give very similar algebra.) The operators with given invariant subspaces are then, uniquely

Subspace	*Operator*
$\{1\}$	$(1, 12)$
$\{2\}$	$(12, 2)$
$\{1, 2, 12\}$	$(1, 2)$

Writing the operators as vectors in 4 dimensions they are 134,124,14 respectively. One can perform a basis transformation to turn these into 1,2,3. Before we go on to finds the appropriate 7 operators, it is useful to look at the remaining non-singular operators at the first level. These are, in turn:

(2,1) with proper eigenvector 12 and

(2,12), (12,1) which unfix every vector.

So there are, at this level, exactly two unfixers.

As a next exercise, consider the 3 vectors 1,2,3 in a *three*-dimensional space. We discuss in some detail how to construct operators having the appropriate eigenvectors, as the same methods will be needed again.

(a) Firstly, for all three vectors $\{1,2,3\}$, only the identity operator (1,2,3) will serve.

(b) For the subspace generated by $\{2,3\}$, which we denote by $D(2,3)$ we use the ***Noyes trick*** of getting the operator (13,2,3).

(c) For the subspace $\{3\}$, we use a direct sum representation, and an unfixer on the 1,2 columns, so, for example (12,1,3).

Consider now the operation of interchange two basis vectors, say 2 and 3. This is a linear operation, L, on vectors; in fact $L = (1,3,2)$. But if, say, $Au = v$, then $Lv = (LAL^{-1})Lu$, so that the result of such a change of basis is to make A become LAL^{-1}. Now $L^{-1} = L$, so if $A = (A_1, A_2, A_3)$ we have

$$A' = LAL^{-1} = LAL = (1,3,2)(A_1, A_2, A_3)(1,3,2) =$$

$$= (1,3,2)(A_1, A_3, A_2),$$

and to evaluate this product notice that if a column contains a 2, it becomes a 3, and ***visa versa***. So $A' = (\bar{A}_1, \bar{A}_3, \bar{A}_2)$ where bars denote the operation of interchanging 2 and 3. We can now use these to tabulate the seven operators in the following way:

Subspace		*Operator*
ALL		(1,2,3) α
$D(2,3)$		(13,2,3) β
$D(1,3)$	$1 \leftrightarrow 2$	(1,23,3) γ

$D(1,2)$ $1 \leftrightarrow 3$ (1,2,13) δ

$\{3\}$ (12,1,3) ϵ

$\{1\}$ $1 \leftrightarrow 3$ (1,3,23) η

$\{2\}$ $2 \leftrightarrow 3$ (13,2,1) ς The seven operators so found are, in fact, linearly independent. This can be seen in the following way (for which I am indebted to Dr. Mary Warner):

A basis for 3×3 matrices is obviously provided by (1,A,B), (2,C,D), (3,E,F), (0,1,G), (0,2,H), (0,3,K), (0,0,1), (0,0,2), (0,0,3). One simply works systematically to put the given set in this form.

$\alpha = (1,2,3)^*$ First step:

$\epsilon + \alpha = (2,12,0)^*$

$\beta + \alpha = (3,0,0)^*$ Second step:

$\gamma + \alpha = (0,3,0)^*$

$\delta + \alpha = (0,0,1)^*$

$\eta + \alpha = (0,23,2)$ $\eta + \gamma = (0,2,2)^*$

$\beta + \varsigma = (0,0,13)$ $\beta + \varsigma + \delta + \alpha = (0,0,3)^*$

The final starred elements are obviously 7 of the basis mentioned and so are linearly independent.

The above argument is for the set of vectors $\{1,2,3\}$ in three dimensions. The hierarchy construction has them in four dimensions. However it is easy to derive a corresponding solution. Let $A_i (i = 1, ..., 7)$ be the seven operators starred, and adjoin one more column. Then the operators

$$\bar{A}_i = (A_i, 14)(i = 1, ..., 7)$$

will be linearly independent, and have the same eigenvectors.

This has established, then, the first two stages of the hierarchy construction. The next step is the core of the proof. It is equivalent to proving:

Theorem 2. Given any 7 linearly independent vectors in Z_2^{16}*, there exist 127 linearly independent* 16×16 *matrices* $/Z_2$ *such that*

(a) *each is non-singular,*

(b) *each corresponds to a unique linear space generated by a subset of the 7 vectors by the proper eigenvector correspondence.* In order to prove this we first make a basis transformation so that the seven vectors are respectively 1,2,3,4,5,6,7. The proof relies on two simpler theorems:-

Theorem 3.

(a) There exist (many sets of) 127 non-singular operators $Z_2^7 \to Z_2^7$*, one for each subset of* $\{1, 2, 3..., 7\}$ *in the proper eigenvector correspondence.*

(b) These 127 operators can be chosen to span $Z_2^7 \times Z_2^7$*.*

Theorem 4. For n>2, unfixers span $Z_2^n \times Z_2^n$*.*

The use of the theorems to prove theorem 2 is as follows:

From theorem 3(b)we can select 49 linearly independent operators, say $A_i(i = 1, 2, ..., 49)$. Denote the remaining operators by $B_j(j = 1, 2, ..., 78)$. By Theorem 4, for $n = 9$, there are 81 linearly independent unfixers on $Z_2^9 \times Z_2^9$. Choose any 79 of them, $\phi_k(k = 1, 2, ..., 79)$. Since 7+9 = 16 one can construct operators of the direct sum form: $(A_i, \phi_79), (B_j, \phi_j)(i = 1, 2, ..., 49, j = 1, 2, ..., 78)$ which are 127 in number corresponding correctly and are linearly independent. This proves Theorem 2.

It remains to prove the subsidiary results, Theorems 3 and 4. We begin with Theorem 4, that unfixers span (if $n \geq 2$, since, if $n = 2$, there are only two unfixers).

In n dimensions an obvious unfixer is $A = (2,3,4,...,n,U)$ where (from non-singularity)U cannot belong to $D(2,3,4,...,n)$ and so must be 1 or $1v$, where $v \in D(2,3,4,...,n)$. But $U = 1$ will not serve since 123...n is then an eigenvector, so $U = 1v$.

So long as $n \geq 3$, two unfixers are (2,3,...,n,12) and (2,3,...,13) and their sum is (0,0,...,0,23). Interchanging 1 and 2 will similarly produce (0,0,...,0,13) and adding results will give (0,0,...,0,12).

Now perform the automorphism $L()L^{-1}$, where $L = L^{-1} = (12,2,3,...,n)$. Then

(12,2,3,...,n)(0,0,...,0,12)(12,2,3,...,n)= (12,2,3,...,n)(0,0,...,0,12)=(0,0,...,0,1).

Since this can be produced, we can, by interchanging 1 and 2, 1 and 3,....., 1 and (n-1), produce in turn (0,0,...,0,2), (0,0,...,0,3),...,(0,0,...0,n-1). In order to produce (0,0,...,0,n) begin with the two unfixers (2,3,...,n,12) and (2,3,...,n,1n) with sum (0,0,...,0,2n) which, by adding (0,0,...0,2) gives (0,0,...,0,n).

Since there is evidently nothing special about the last column, it is clear that unfixers may be found to give a 1 in any place at all, i.e. unfixers span.

Now to prove Theorem 3 one uses the same procedure in $Z_2^7 \times Z_2^7$ as has been used above in $Z_2^3 \times Z_2^3$. It is, in fact, sufficient to consider the cases:

Eigenvectors	*Operator*
ALL	(1,2,...,7)
$D(2,3,4,5,6,7)$	$(17,2,...,)7$ (The Noyes trick),
$D(3,4,5,6,7)$	$(12,1,3,...,7)$ (unfixer in [2]),
$D(4,5,6,7)$	$(2,3,13,4,...,7)$ (unfixer in [3]).

When all possible interchanges between 1,2,...,7 are performed so as to list the 6,5,4 dimensional subspaces and their operators, the number listed will be

$1+7+21+35=64$. Applying the Warner technique, it will be found that these 64 operators include 49 linearly independent ones. This completes the proof.

Table AII.1

(a) Values for F found recursively.

(b) Dimensionality of the spaces vs. no. of linearly independent vectors.

a) $F(A,B)$

$B=$ \ $A=$	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	0	3	2	5	4	7	6	9	8
2	2	3	0	1	6	7	4	5	10	11
3	3	2	1	0	7	6	5	4	11	10
4	4	5	6	7	0	1	2	3	12	13
5	5	4	7	6	1	0	3	2	13	12
6	6	7	4	5	2	3	0	1	14	15
7	7	6	5	4	3	2	1	0	15	14
8	8	9	10	11	12	13	14	15	0	1
9	9	8	11	10	13	12	15	14	1	0

b) vectors and dimensions

No. vectors:	2	3	7	127	$2^{127}-1$
Dimension:	2	4	16	256	65536

(terminates: dimensionality of the spaces falls behind no. linearly independent vectors)

Appendix III. COMPLEMENTARITY AND ALL THAT

by

Ted Bastin

[This is Chapter 3 from Ted Bastin's unpublished book *The Combinatorial Basis of the Physics of the Quantum*]

Bohr is credited with the remark that "truth and clarity are complementary" and Peierls to whom I am indebted for the quotation, adds that Bohr leaned heavily to the side of truth. Attempts at clarity about observation, in the sense of a brief and definite statement within the intellectual structure that we call the quantum theory, tend to run into the difficulties that have occupied us at some length already. For example we find the kind of clarity that the physicist is used to expect admirably provided in the statements on the subject by Dirac that were extensively used in the arguments of the last chapter. However, the more satisfying the clarity, the more we find the difficulties thrown into sharper relief, and we may set our desired succinctness and clarity only at the expense of our being prepared to live with an underlying muddle. From this point of view the surprise felt by many physicists at the prolixity of Bohr's discussions of complementarity is misplaced; it is, to say the least, a moot point whether or not Bohr should be taken to task for failing to be clear in his presentation of a muddle. In any case I shall be presenting the very different judgment that Bohr differed from his contemporaries in the mainstream of quantum physics in being not prepared to temporize with an incomplete understanding of the basic quantum principles, and that the difficulties being as inveterate as I claim, the endlessness of Bohr's search was an inevitable consequence.

In this chapter I shall be concerned with one question: does Bohr's complementarity principle enable us to deduce the differences between quantum physics and classical physics that appear – in particular – in the uncertainty principle. I shall conclude that they do not. It will also follow that – Bohr's profound critique not having issued in an explanation – no understanding of these differences exists at present.

The doctrine called "complementarity" is one of the principles which guide the treatment of observation in mainstream quantum theory. There, it usually refers either to the relationship of the particle picture and the wave-picture, or to a more technically articulated relationship that exists between certain *pairs* of dynamical variables that appear in the specification and solution of a single dynamical problem. In both cases there is an idea of exclusivity in the application of two analytical techniques or concepts at a given time, even though both are required for the full understanding of the problem. I shall criticize the use that is made of the idea of complementarity in mainstream quantum theory severely, and it therefore matters what form of presentation of it one takes a representative. One could scarcely hope for a more insightful brief account of it as an element in that corpus of thinking and knowledge than the following from Born ("Atomic Physics", Blackie, IIIrd Ed. 1944, p. 144).

"The true philosophical import of the statistical interpretation ... consists in the recognition that the wave picture and the corpuscle picture are not mutually exclusive but are two complementary ways of considering the same process – a process whose accessibility to intuitive apprehension is never complete, but always subject to certain limitations given by the principle of uncertainty. ... The uncertainty relations, which we have obtained simply by contrasting with one another the descriptions of a process in the language of waves and in that of corpuscles, may also be rigorously deduced from the formalism of quantum mechanics – as inexact inequalities, indeed: for instance between the coordinate Q and momentum P we have the relation

$$\delta Q \delta P > h/4\pi ,$$

if δQ and δP are defined as root squares ...."

In this account, Born is unusually definite among expositors in making the Heisenberg uncertainty relation depend upon the complementarity of the wave and particle pictures ("corpuscle" picture, as Born calls it). However even he leaves the deductive situation ambiguous, he suggests that the strong expectation that the complementarity principle gives us that there must arise an uncertainty relation, will then be happily confirmed by the more rigorous treatment. Of course this would be fine if the more rigorous treatment included a more rigorous formulation of the wave/particle duality, but the actual situation is that the treatment of that topic that appears in the above quotation is all the justification of it that he provides. As a result, his readers are left chasing round and round, and never sure at what point they are meant to break into the argument. The evident – though perhaps never consciously expressed – invitation is that one should build up support for the quantum-mechanical approach as a whole by deriving a little from each of an array of principles of which the complementarity of the wave- and particle- pictures is one.

We might argue that this is what happens in classical mechanics. There, if we ask for a definition of mass, we are referred to statements which presuppose that we already know what force and acceleration mean; and vice versa. And so on round and round in circles. The closure of the system of definitions works, moreover. Everyone who is trained in physics knows exactly how to apply the classical theory and what constitutes a proper argument within it. The miraculous-seeming quality of the coherence is what I tried to draw attention to in chapter 1 by my introduction of the term "theory-language". In the classical case we indeed have a closure of the definition system that justifies us in starting from any of many equivalent points in our deductive treatment of any problem. Moreover, as happens with a language, every piece legitimately contributes to the meaningfulness of the whole. The vital point, in the case of the classical theory-language however, is that the principles that would be invoked in justifying any one piece would be consistent with those for all the rest, whereas in the case of the quantum theory, this consistency is just what is being called in question.

In any case, Bohr did not regard the complementarity principle as being at the same level as the technical constructions of the quantum theory. He regarded it single-mindedly as an autonomous principle which required no justification backwards from the success of the quantum theory. On the contrary it was this principle which should carry the weight of the quantum-theoretical vision of the world.

To pursue this programme, Bohr's first effort has to be to provide a conceptual framework within which the complementarity of pairs of dynamical quantities was natural and predicable; subsequently he has to show that the rest of quantum physics could reasonably be seen in this setting. It was this second effort that made his writings so voluminous, and so involved as to give him his reputation for obscurity. It is, as a matter of the general way theories evolve, likely, and in my own opinion demonstrably the case, that the difficulties of exposition that

Bohr found, and the way in which he found his arguments getting ever more complex, indicated that he had not assembled the essential ingredients for the solution of the task he had set himself. Whether or not this is true, there is no one writing on the foundational aspects of the quantum theory at the present time who will accept, or even attempt to argue the case for the correctness of the conceptual framework supplied by Bohr in the detailed context of the mutual exclusion of the pairs of dynamical variables.

The position of von Weizsaecker on complementarity will be discussed in chapter 11. It is worth putting on record an opinion expressed by Heisenberg a few months before his death. Asked[1] what he now felt about complementarity, and whether in particular he would support the possibility of there being simultaneously incompatible dynamical variables, as a matter of general principle, Heiseberg said that he thought that science does indeed throw up situations of this sort from time to time where there seems to be a conflict of principles operating in such a way as to close investigation. He said he thought that such situations were an augury of major simplifications at the most profound level about to emerge, but as yet only to be guessed at. Heisenberg was not to be drawn by the further question whether this point of view did not strike at the heart of the Copenhagen philosophy, and drew the conservation to an end by remarking that it must be time for lunch.

In spite of this absence of currency for the complementarity idea as a piece of organized thinking at the technical (as distinct from the general philosophical) level it continues to be presented as an element of the mainstream position on quantum theory. If one enquires about the relationship between canonically

[1]Conversation with the author.

related dynamical quantities, as the accepted way of reaching an understanding of the quantum-mechanical doctrine on observation, one is likely to find oneself referred to Bohr's discussion of the general concept of complementarity. It is for all the would as though that discussion could give the technical form of the dual relations the validity of a sort of counter which could be played at will in some sort of game in complete disregard of the circumstances under which the exclusion prescribed by the principle would be expected to manifest itself.

It is not impossible to obtain a short statement by Bohr himself of the complementarity concept in its general form. The following appears in an essay by Bohr entitled "Natural Philosophy and Human Cultures" (Address at the International Congress of Anthropological and Ethnological Sciences in Copenhagen, delivered at a meeting in Kronberg Castle, Elsinore, August 1938. This essay appeared in *Nature, 143*,268, (1939), and was reprinted in Bohr's book *Atomic Physics and Human Knowledge*, Wiley, 1957.)

"Information regarding the behavior of an atomic object obtained under definite experimental conditions may, however, according to a terminology often used in atomic physics, be adequately characterized as *complementary* to any information about the same object obtained by some other experimental arrangement excluding the fulfillment of the first conditions. Although such kinds of information cannot be combined into a single picture by means of ordinary concepts, they represent indeed equally essentially aspects of any knowledge of the object in question which can be obtained in this domain."

This definition makes use of several principles which Bohr considered established in current theory, or which he considered he had himself established, and which can be though about separately. Firstly there is the statement about the obtaining of information by an experimental arrangement. This refers to Bohr's position that the units into which it was alone legitimate to analyze knowledge about the world of quantum objects was the whole experimental procedure together with whatever logical relationships are necessary to demarcate the arrangement that we have in mind off from the rest of the physical surroundings of the experiment.

This position sounds arbitrary until we see it against the special operational circumstances of the quantum objects. It is part of what we mean by the term "particle" in the classical way of thinking that there should automatically be a possibility of defining other particles in the neighbourhood of the first without making any special theoretical provision for their intrusion. If we could not assume this without question we should not be able to use the dynamical variables with their usual meaning. Now in the quantum domain this assumption is consistently and as a matter of fundamental principle invalid. If we wish to refer to a new particle then we must specify a new, and usually much more complex, theoretical background capable of describing the combined system. For Bohr, the right way to express this specifically quantum view was to stress the unity of observed entity and observing system, and indeed to insist that neither should be ascribed reality independently. That it is a correct understanding of Bohr to interpret his assimilation of the atomic object (I use Bohr's phrase) itself, to the circumstances of its measurement is further borne out by his giving central importance to what he called the "quantum postulate". This, he says (*Atomic Theory and the Description of Nature*, Cambridge, 1934, p. 52) attributes to any atomic process an essential discontinuity or rather individuality, completely foreign to the classical theories, and symbolized by Planck's quantum of action."

One sees from this quotation that Bohr saw the very discreteness or particularity of the quantum particle as something to be imagined in quite a different way from the way we imagine a classical particle (and the matter of the existence of a background of related particles would certainly be an important part of the imaginative apparatus that Bohr would require us to renounce). To set a right view of the atomic object we had to preserve a lively consciousness of the unity of object-system and experimental milieu.

Bohr's insistence on the quantum postulate was needed as a protection against the sort of crudity in thinking about quantum particles which retains elements of the "atoms are bits of matter cut up small" variety. The combinatorial principles to be developed in this book are easily misinterpreted as putting the theory into the class of statistical theories which use particles whose discreteness is of the classical kind, and it will be useful to bear in mind how important it was to Bohr to avoid this misconception. This part of his doctrine is absolutely integral to our way of thinking.[2]

The second of the principles which contribute to Bohr's idea of complementarity concerns the inevitability of the classical description using the classical dynamical concepts as the only possible way of talking about the physical world in any of its aspects, and in particular in the quantum aspect. This principle is obviously connected with the first; however they are not equivalent. The second goes much further than first in the way that it asserts that change from the classical language is for ever ruled out. Bohr was insistent on this strong prohibition.

[2]In chapter 1 we discussed how far the classical concept of the particle could be laid at Newton's door and how far he was careful not to commit himself in this way. A later discussion in chapter 11 of the ideas of Bohr on what he called the "mechanistic concept of the particle" is also relevant.

Any suggestion that one should be open to the possiblity of change in the way we imagine the physical world at the basic level of the intuition of spatial events, seemed to him entirely fanciful. To the criticism that such self-assurance could hardly be reconciled with a modest awareness of the infinite corrigibility of science, Bohr would simply be incredulous. He evidently thought that anyone who made proposals of the sort that he was brushing aside had failed to consider the vastness of the task they were proposing, or indeed to set its real nature clearly into focus. On Bohr's side one does have certainly to recognized that most of the *soi disant* exercises in the invention of original conceptual frameworks for physical thinking which are intended to handle the unfamiliarity of the quantum world, do fall back for their very expression at a very early stage on the familiar imaginative pictures that they were meant to replace.

The conclusive failure of attempts like these lend colour to the working physicist's belief that the familiar approach is simply commonsense about the reality of the world. Bohr's position, however, was poles removed from that of the naive realist. In his positivistic attitude to the language of physics Bohr was, in a way, being explicit about the dominance of what I have called the classical theory-language. He was rejecting the view that the dominant language was an expression of commonsense about the reality of the world for, of course, sophistication about the part played by language in affecting what we say we observe is at the opposite pole from simple realism. Yet to a great extent the effect of his argument was to make him an ally of those who never questioned the inevitability of the classical language because it never occurred to them to do so.

Among the quantum physicists there is a further class of those who have given thought to the possiblity of profound conceptual change, and who perhaps would welcome it in principle but who cannot see any likelihood of its coming about. Bohr was not in this class either, for he made a very positive virtue of the necessity of the classical language.

A good deal has been written about the influence of idealist ways of thinking that may have made Bohr feel that he was on the right track in insisting on the classical language as a necessary form of thought, or at least as a precondition for all thinking that could be labelled "physics". In particular, Bohr may have seen an analogy between the part played by the classical language and the ***synthetic a priori*** place of space and time in the Kantian philosophy. However philosophical tenets which do not play a part directly in scientific argument are beyond the scope of my discussion. Reference may be made on this, and similar points to it Quantum Physics and the Philosophical tradition by Aage Petersen (MIT, 1966).

The last component that we always find in Bohr's statements of the complementarity principle, such as the one quoted above, is that of ***incompatibility***. We already have the unity of the operations and language that go to make up a measurement; we have the restriction on the scope of that language to that which is current in the classical understanding of the world; now we are to understand that there will typically be more than one such description required to present the essentials of any given quantal situation, and that these will consistently so appear that the provision of one will prevent the provision of the rest. As Petersen puts it:

"... the experimental arrangements that define elementary physical concepts are the same in quantum as in classical physics. For example, in both cases, the concept of position refers to a coordinate system of rigid rulers and the momentum concept refers to a system of freely moving test-bodies. In classical physics, these instruments can be used jointly to provide information about the object. In the quantum domain, however, the two types of instrument are mutually exclusive; one may use either a position instrument or a momentum instrument, but one cannot use both instruments together to study the object." (Petersen, Aage, *Quantum Physics and the Philosophical Tradition*, MIT, 1966).

Why not? It is very difficult even to imagine what it would be like to argue in favour of Petersen's assertion, let alone actually to produce the argument. (Let me again remind the reader that my purpose is not to argue that it is impossible to *postulate* an exclusivity; it is only to show that there can be no case internal to the physical argument in favour of it.) What sort of thing could it be that would prevent one kind of instrument being used because of the presence of the other? Or would the argument be that it was the successful *operation* of the one instrument that must inhibit the operation of the other? In the latter case, what would the mechanism of the interaction between the two be? It is obvious that if one restricts oneself to classical argument then there is no reason why one should not, for example, construct measuring techniques which measure momentum and position and other dynamical variables as well in indefinitely complex relationship. Indeed it is notorious that, far from it being the case that simple dynamical variables force themselves on the attention of the experimenter, his ingenuity is always stretched by the need to provide experimental techniques that exhibit those conceptually simple properties of a system that theory demands.

Commentators usually continue the argument at this point by appeal to the uncertainty principle. Thus they may argue as follows: "Suppose we measure position. Then if we measure momentum this must by the use of scattering with some sort of "test-particle"; however we don't have test-particles which are small compared with the particle being observed (as we always do classically). Hence the momentum measurement must disturb the position, and this is so merely as a fact about measurement." In fact we cannot permit recourse to this argument. The finiteness of test- and all other- particles is supposed to be a consequence of whatever quantized theory we come up with, and to use it in the argument about the most fundamental step in establishing discreteness is to beg the whole question we are trying to answer. One is inclined to say that if one is allowed to assume the uncertainty principle then one has already got quantum theory, and has no need of complementarity. Such a claim may be too strong (though Noyes has argued that one may build a quantum theory upon an operational basis of counts of particles in detectors with an assumption of an irreducible statistical fluctuation in the counts that has the uncertainty relation as a special case, [see main text and references therein]). The correct relationship in Bohr's eyes was probably more that the complementarity philosophy was needed to make the uncertainty principle a comprehensible assumption. The fact would remain, in that case, that the former had to stand in its own right.

Bohr's position on the incompatibility of simultaneous descriptions is reminiscent of an argument that I have already maintained to be the best way of summarizing the difference between the quantum and the classical views of measurement. A quantum measurement does not presuppose the potential existence of a background of related experimental results in the way that classical measurement does. Even for such a simple case as the measurement of two momenta of a particles at contiguous points in space at high energies, we require a quite different experimental arrangement from what is needed for the single measurement, and one that is usually of a different order of complexity. There is a complexity in the experimental step-up that provides the information about the spatial relationships of the separate components of the complex measurement, and to assert this seems to come near to saying, as Bohr does, that different ways of measuring imply that different things are measured. Indeed it would only be a small step to suggest that the whole quantum theoretical concept of measurement could be built round an application of this idea to the central concepts of momentum and spatial position. Then one would have, in all essentials, the uncertainty principle.

There seems nothing wrong with this way of looking at the complementarity principle; the error is to try to make it follow from the *classical* concept of observation. Bohr had stressed that his quantum postulate was something quite new on the horizon of physics, and it is ironical that it was also his embargo on attempts to transcend the classical description of of experiments that made him locate the characteristics of that postulate in a place which could not have the right kind of room for them.

In the discussion of the Einstein-Podolsky-Rosen paradox in the previous chapter the center point of principle in the controversy was shown to be over the proper requirements that scientific enquiry in its most general aspect ought to impose on measurement. The "reality principle" that was set up by the critics of mainstream quantum theory was an attempt to separate the result of a measurement from any essential dependence on the techniques that are involved in making it, and in that way to ensure that the results of measurements have an objectivity of the familiar sort. It was as though this kind of objectivity has been presupposed by everyone so implicitly that no one had noticed that the quantum theory had done away with it; at any rate that is one way of putting the critics' case.

However it was not to be the critics who were generally held to have won the day, for Bohr's arguments were generally assumed to have answered the opposition, however carefully or casually they were, considered. Summarizing the outcome, Jammer (*The Conceptual Development of Quantum Mechanics*, McGraw-Hill, 1966, p. 382) has this to say:

"The challenge was soon answered, at least from the viewpoint of the complementarity interpretation of the theory, by Bohr's insistence on the essential influence of the procedure of measurement on the conditions underlying the very definition of physical quantities, considering these conditions as an inherent element of any phenomenon to which physical reality can be attributed. Bohr pointed out that a mechanical system, even though having ceased to interact dynamically with any other system, does not contribute an independent set of 'real' attributes. Bohr's rejection of the possibility of associating quantities with physical systems in a possessive manner, which rejection invalidated the epistemological premise of the paradox, was clearly but an expression of the fact that, within the framework of the Bohr-Heisenberg interpretation, quantum mechanics is ultimately a physics of processes and not of properties, a physics of interactions and not of attributes, even out of primary quantities of matter.

"From this point of view quantum mechanics may rightfully be regarded as falling in line with the general development of theoretical physics."

This passage puts the contrast between the opposing views in such a way as to make very clear the importance of the issue as a turning point in physics, but arguments of this kind which point out the desirability of change cannot be said to have justified the turn (if, indeed, the turn was to be as irrevocable as the victors at that time supposed). For what was at issue was whether the "essential influence of the procedure of measurement" could really be shown to replace the reality condition of Einstein and to put something in its place which should be as satisfying as the old, though in the new framework. According to the discussion of this chapter Bohr has not supplied this demonstration, and therefore the challenge of Einstein, Podolsky and Rosen had not been met.

Appendix IV. PROGRAM UNIVERSE

A Constructive Bit-String Model of the Early Universe

by

Michael J. Manthey

```
const doomsday = false;
type  onebit = (0,1);
    Uptr: [1..Usize];   {index of a string in U.}
    ensemble = record
      last: [1..*];   {index of current last element of E.}
      E   : array[1..*] of Uptr;
    end;
    string = record
      bits: array[1..*] of onebit;
      try : boolean
    end;
    semaphore = (avail,busy); {used to guarantee mutual exclusion
              on updates to U}

var U: array[1..*] of string;
   Usize: integer;    {initially zero = no strings in universe}
   Umutex: semaphore; {initially = avail}

{level  I       II           III              IV

    1 2 (3) 4..6 (7..10) 11..17 (18..137) 138..255 (256..2^127-1)

basis .2. |  .3.  |      .7.      |      .127.      |
size      |       |               |                 |
          |.......|.......strings in closures.......|

    Levels: array[1..4] of {indices into U}
          record  {basis slots}
            LB,        {1,4,11,138}
            Cur,
            UB         {2,6,18,255}
              :[1..*]
          end; {nb:closure slots from [i][UB+ 1]-[i+ 1][LB-1]}
    Labels: record
            last: [1..*]; {index of current last element of L.}
            L   : array[1..*] of ^ensemble
          end;
   empty: string;  {an empty string, i.e. one whose length is zero.}

   slength: record        {current length of strings in U}
       sem: semaphore;
       len: integer
     end;

   Bit: onebit;    {one random bit...see function Random below}

   CurLvl: 1..4   {the level currently being "constructed"}
   BasesComplete,  {all four basis vector sets formed yet?}
   HierarchyComplete {all four bases and closures formed yet?}
       :boolean;
```

```
{------- Synchronization -------------}

procedure wait(var s:semaphore); {poll s until it's avail}
  var t: semaphore;
begin  {presumes mutual exclusion on procedure swap, which
    is formally undefined (universal primitive) and
    which interchanges the values of two variables.}

  t:=busy;
  repeat  swap(s,t)  until t=avail;

end; {wait}

procedure signal(var s:semaphore);  {signal that s is available again}
begin
  swap(s,avail)
end; {signal}

{------- Random 1/0 Generation ----------}

procedure RandomBit;  {Actual random bit generation...a function of}
  var i,j: integer;  {the strings in U. An independent process.  }
begin
  repeat     {flip Bit forever}

    Bit := 1;        {important when U is small}

    for i:=1 to slength.len do  {set Bit as a fcn of U}
      for j:=1 to Usize do
        Bit := (Bit + U[i][j]) mod 2

    Bit := 0;        {important when U is small}

  until doomsday
end; {RandomBit}

{The randomness of the value returned by function Random below
  depends on the fact that procedure RandomBit runs as an independent
  asynchronous process to everything, constantly scanning U's strings
  and updating the value of Bit appropriately. This occurs even as
  U is locked during discrimination and pre-scattering calculations.}

function Random:onebit; {Called whenever a random bit is needed.}
begin
  Random := Bit
end; {Random}
```

```
{----------Managing the Universe------}

function Generate:string; {generates the first two strings in U.}
  var g: string;
begin
  if Usize=0 then Generate:=Random
  else {Usize can only be 1}
  begin
      repeat g:=Random until g<>U[1];
      Generate := g
  end
end; {Generate}

{-------------------------------}

procedure LockUniverse;
begin
  wait(slength.sem);
  wait(Umutex)
end;

procedure UnlockUniverse;
begin
  signal(Umutex);
  signal(slength.sem)
end;

{-------------------------------}

procedure Tick; {increments the universal string length by one bit.
      This is done under mutual exclusion, so U grows, but no one
      ever sees it, and all bit strings are (for all practical
      purposes) always of equal length.}

  var i:integer;
begin
  LockUniverse; {stop the world while we change it}

  slength.len := slength.len + 1;

  if Usize=0 then
  begin
      U[1] := Generate;
      Usize:=Usize+ 1
  end
  else {increase the length of every string in U by 1 bit.}
    for i:=1 to Usize do U[i][slength.len] := Random;

  if BasesComplete and not HierarchyComplete then
  begin
    for i:=1 to Usize do
      if U[i] not in Labels then {U[i] not yet in a closure}
      begin
        for j:=1 to 4 do
```

```
          if not ClosureFull(j) then
          begin
            if Levels[j].closure=nil then
              Level[j].closure := genclosure(j);
            if U[i] in Levels[j].closure then PutClosure(i,j)
          end; (j-loop)
      end; {i-loop}
    if AllClosuresFull then
    begin
      HierarchyComplete := true;
      N := slength.len
    end
    else DiscardIncompleteClosures
  end;

  UnlockUniverse; {let the world breathe again}

end; {Tick}
```

```
{- - - - - - - - - - - Bit-Picking Routines - - - - - - - - - -}

function ones(s:string):integer; {counts # of ones in s}
  var i,c: integer;
begin
  c:=0;
  for i:=1 to slength.len do
      if s.bits[i]=1 then c := c + 1;
  ones := c
end; {fcn ones}

function zeroes(s:string):integer; {counts # of zeroes in s}
begin
  zeroes := slength.len - ones(s)
end; {fcn zeroes}

function complement(s:string):string; { complement s }
    var i:integer;
  begin
   for i:=1 to slength.len do complement.bits[i]:=(s.bits[i]+ 1) mod 2;
   complement.len := slength
  end;

{- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -}

function discrim(s,t:string):string; {exclusize-or of s and t}
begin
  for i:= 1 to slength.len do
      if s[i] = t[i] then discrim[i] := 0
            else  discrim[i] := 1;
end; {fcn discrim}

{- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -}

function Pick:string {picks a string at random from U}
  var i,index: integer; {index will be random in 1..Usize}
begin
  index := 0;
  repeat
     for i:=0 to ceiling(lg(Usize)) do index := 2*index + Random
  until index in [1..Usize];

  Pick := U[index]  {assign random string to Pick}

end;  {function Pick}
```

```
{- - - - - - - - - - Hierarchy Construction- - - - - - - - - -}

function InU(s:string):boolean; {true if s in U else false}
   var i,j: integer; found: boolean;
begin

  for i := 1 to Usize do {search all of U}
  begin
     found := true;
     for j := 1 to Slength.len do found := found and (s[j]=U[i][j]);
     if found then goto 1;
  end;

1: InU := found
end; {procedure InU}

function LindepL(S:string; lvl:[1..4]):boolean;
   {true if S is linearly independent of the strings in level lvl
   only.}
begin
   --mucho recursive-- generates n(b)= B(l)!/{b![B(l)-b]!}
     discriminations with S.
end; {fcn LindepL}


function Lindep(S:string; lvl:[1..4]):boolean;
   {true if S is linearly independent of all levels 1 to lvl.
    NB: Assumes (correctly) that it is not called if there is
    no room in basis[lvl]...because of the value of Curlvl.}

begin
   Lindep := false; {default value}
   if lvl<1 then Lindep := true  {base case}
   else {check previous levels, then current level.}
     if Lindep(S,lvl-1) then Lindep := LindepL(S,lvl)
end; {fcn Lindep}

procedure PutBasis(Si: Uptr); {inserts U[Si] into basis of CurLvl}
   {if the current level is full, increments CurLvl.}
begin

   index := Levels[CurLvl].Cur; {find out where we are}
   new(Labels[index]);          {make an ensemble}
     Labels[index]^.E[1] := Si; {point 1st ensemble element to its string}
     Labels[index]^.last := Labels[index]^.last+ 1 {point to next open
                                          slot in ensemble}
   Levels[CurLvl].Cur := Levels[CurLvl].Cur +  1;
   if Levels[CurLvl].Cur > Levels[CurLvl].UP then CurLvl:=CurLvl+ 1;
     {Current basis is complete..start basis of next level in hierarchy}
   if CurLvl>4 then BasesComplete := true

end; {procedure PutBasis}
```

```
procedure PutClosure(Si: Uptr; lvl:[1..4]);
  {inserts U[Si] into the closure of lvl}
begin
 ---------similar to the above---------
end; {PutClosure}

procedure Label(Si: Uptr); {categorize U[Si] in terms of the hierarchy}
begin
  if not BasesComplete then {try to put S into Labels array}
  begin
    if Lindep(S, CurLvl) then PutBasis(Si)
  end
  else
  begin
    if Lsize<2^N then  {there are fewer than 2^N labels currently.}
    begin
      For all S in U such that S not in any basis or closure YET
      do
        for all lvls in Levels do
          if S in closure(basis(lvl)) then put S into that closure
      if all closures are full then define N else scrub all
      incomplete closures; let Tick go; put any remaining strings
      in U into ensembles.
    end;

    add S to ens(S) {ens=given S, returns ptr to S's ensemble}
  end

end; {procedure Label}
```

```
{---------- The Life of a Bit String---------}

procedure stringevolution(var l:string; me:Uptr);
  {every string (except empty) becomes a separate incarnation of this
  procedure, i.e. a separate, independent asynchronous process.}

  var d,m:string;  {local working variables}
      home: boolean; {true => I am a member of a basis, closure, or
                      ensemble.}

begin
  repeat
     if Usize=0 then     {we need two strings to get started}
     begin
        Tick; {go from no strings in U to one.}
        l := Pick;            {we become this first string i.e.
                         the original empty-process becomes
                         the U[1] process herewith.}

      m := Generate;        {generate a second string}
      Usize := Usize + 1;    {Universe now has two strings}
      U[2] := m;
        spawn stringevolution(U[2]); {give U[2] life.}
     end
     else {universe is already rolling, so just scatter w/someone}
     begin
        Label(me);
        if HierarchyComplete and not home then
             {Pierre - is this the right place for this insertion?}
             begin {insert}
               i:=1;
               while not home and i<=Labels.last do
               begin
                 j:=1;
                 while not home and j<=Labels[i].L^.last do
                  with Labels[i].L^ do
                  begin
                     home := Si=E[j];
                     j:=j+1
                  end;
                  if not home
                  and LabelPart(Labels[i].L^.E[1]=LabelPart(S) then
                  begin {I belong in this ensemble}
                     PutEnsemble(Si, Labels[i].L^);
                     home := true
                  end
               end;
            — if not home then
               begin
                  Labels.last := Labels.last + 1;
                  new(Labels.L[Labels.last];
                  with Labels.L[Labels.last]^ do
                  begin
                     E[1] := Si;
```

```
                last := 2
              end
            end
          end {insert}
        UnlockUniverse {spawn unlocks what spawner locked...
                    extra Unlock okay...}
          LockUniverse;
          m := Pick;
          d:= discrim(l,m);
          if d<>zerostring then
          begin
            s := zerostring;  {"flag" S for later}
            if not InU(d) then s:=d
            else
              if not InU(complement(l)) then s:=complement(l)
              else if not InU(complement(m)) then s:=complement(m)
                else if ones(d)<>zeroes(d) then Tick;
                {else we have an elementary scattering event.}
            if s<>zerostring then {put s into U (novelty)}
            begin
              Usize := Usize+ 1;
              U[Usize] := s;
              spawn stringevolution(U[Usize],Usize); {give S life}
              {hierarchy construction inserted here}
            end
            else UnlockUniverse
        end
      end
    until doomsday {strings never die}
  end; {string evolution.}


begin {----------------Universe starts here----------------}
  {Initialization}
     BasesComplete := false;
     CurLvl := 1;
     HierarchyComplete := false;
     .
     .
     .
     Levels array...
   {end of initialization}

  spawn RandomBit;     {start random number generator going}

  BigBang: stringevolution(emptyset);

end. {Universe  (we never get here) }
```

Construction of a Random Walk Ensemble $\mathcal{E}(\nu, b)$

$\{\mathcal{U}(N + B + b, SU)$ exists$\}$

{PICK (L,b) := From the ensemble in $\mathcal{U}$ labeled by L pick at random one string and delete all but the last b bits. For this string $v = (N^1 - N^0)/b$}

```
Begin
        E(1) := PICK (L,b)
        input values ν, δ
        v_av := v(1)
        v_Σ  := v(1)
        SE   := 1
        k    := 2
        While |v − v_av| > δ do

                Begin
                        if v < v_av then
                            repeat E(k) := PICK (L,b)
                            until v(k) < ν
                        else
                            repeat E(k) := PICK (L,b)
                            until v(k) ≥ ν
                        ℰ := ℰ𝒰 E(k)
                        v_Σ := v_Σ + v(k)
                        SE := SE + 1
                        v_av := v_Σ/SE
                End

End
```

FIGURE CAPTIONS

1. Definitions and flow chart for constructing a growing universe $\mathcal{U}(N, SU)$ containing SU distinct bit strings each containing N bits.
2. Basic flow chart for constructing the four levels of the combinatorial hierarchy, and using them to construct ensembles of labeled addresses.
3. The random walk paradigm.
4. The paradigm for constructing space time from three events.
5. Paradigmatic configurations for the construction of 2+1 Minkowski space.
6. The geometric paradigm for constructing 3+l space.
7. The double slit paradigm.
8. The six basic processes in $\mathcal{U}$ for labels of bit length 2.
9. The basic elastic scattering paradigm.
10. Driving terms for the integral equations of the MUST theory: (a) particle-particle scattering via single quantum exchange; and (b) quantum-particle scattering via single particle exchange.
11. Mapping matrices for the level 2 to level 3 connection.

FLOW CHART FOR PROGRAM UNIVERSE

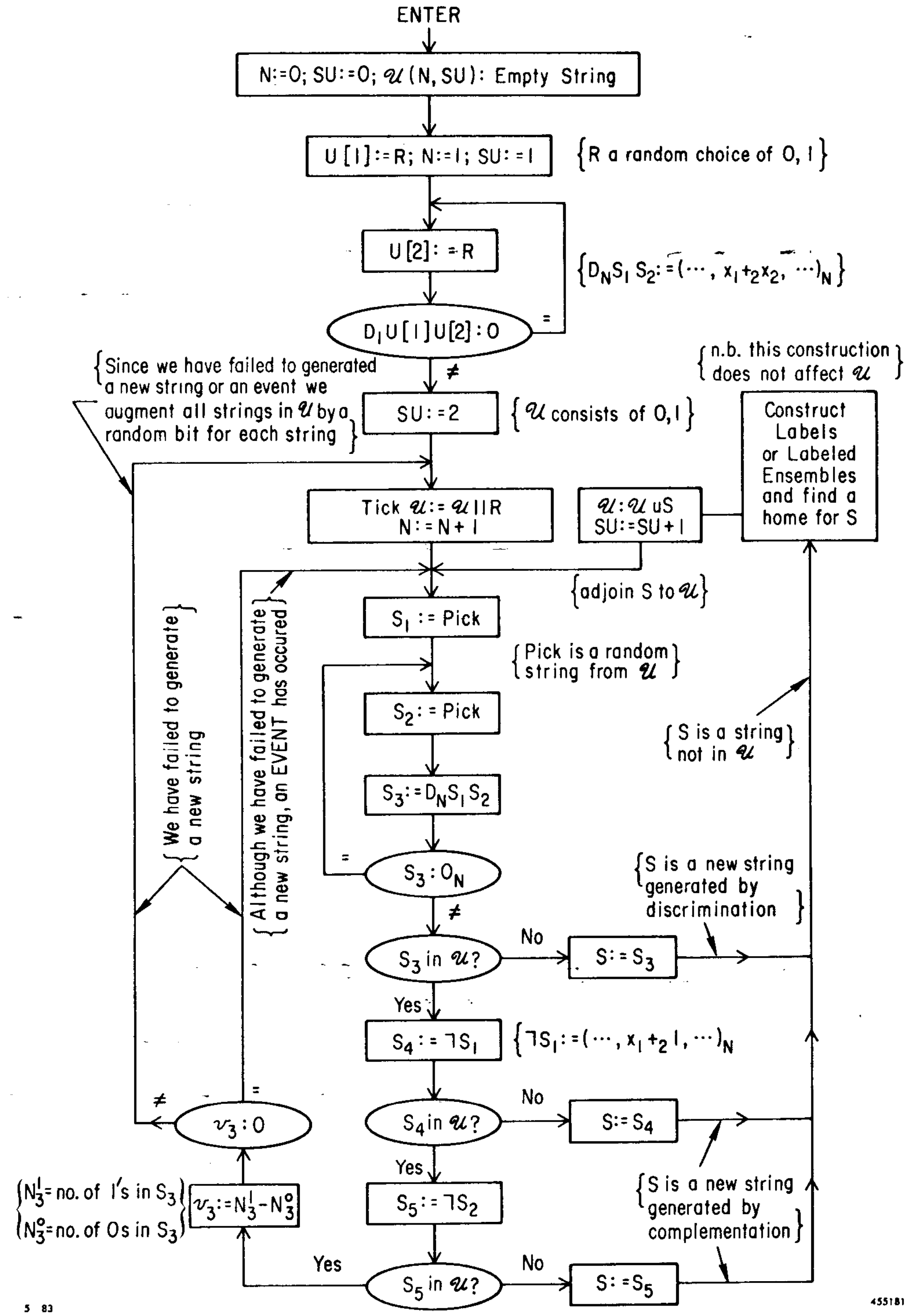


Fig. 1

CONSTRUCT LABELS FLOW CHART

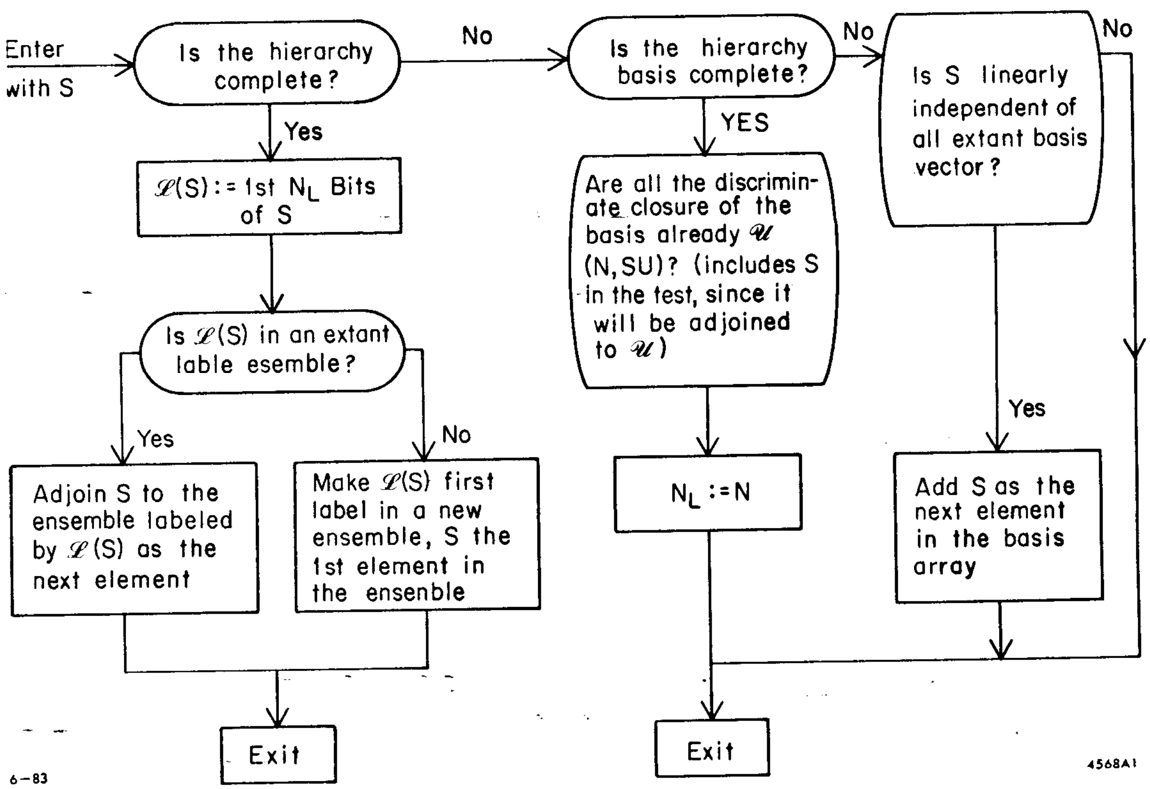
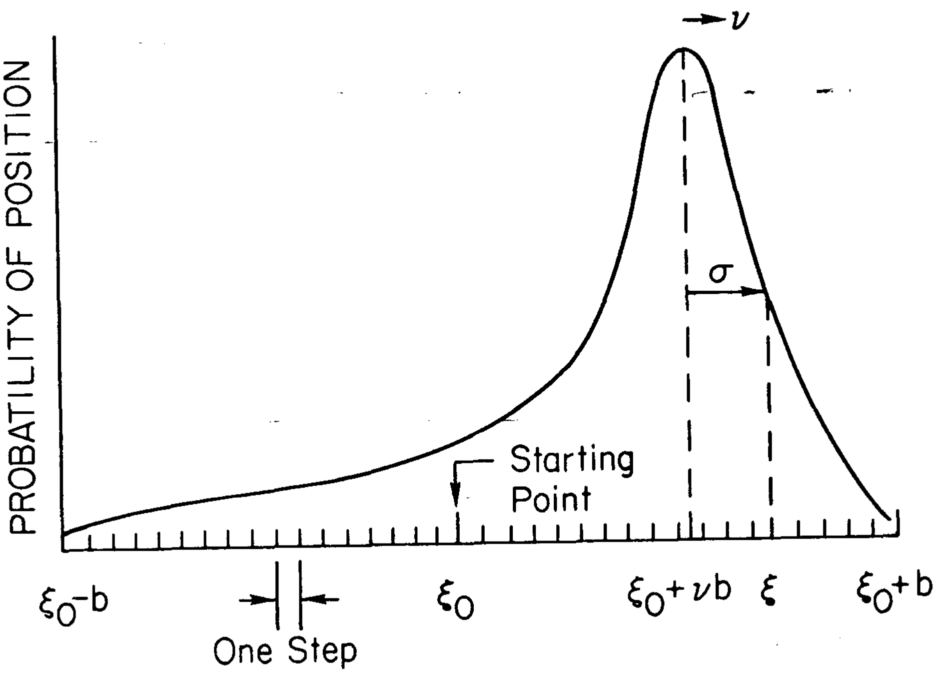


Fig. 2



$$\xi = \xi_0 + \nu b + \sigma(\nu, b)$$
$$\sigma(\nu, b) = (b/4)^{1/2}\sqrt{1-\nu^2}$$
$$p = \frac{1}{2}(1+\nu)$$

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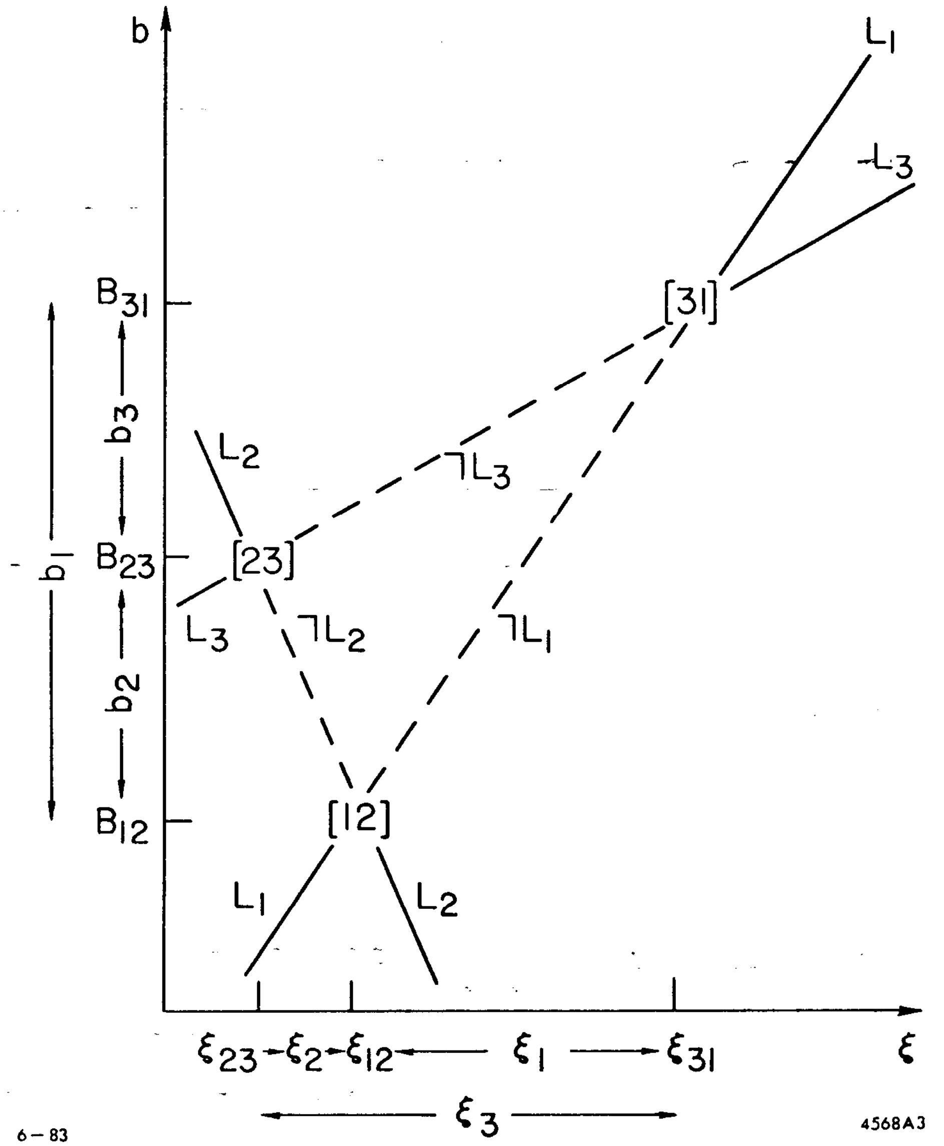
Fig. 3


b
L1
L3
B31
[31]
b3
L2
¬L3
b1
B23
[23]
L3
¬L2
¬L1
b2
B12
[12]
L1
L2
ξ23
ξ2
ξ12
ξ1
ξ31
ξ
ξ3
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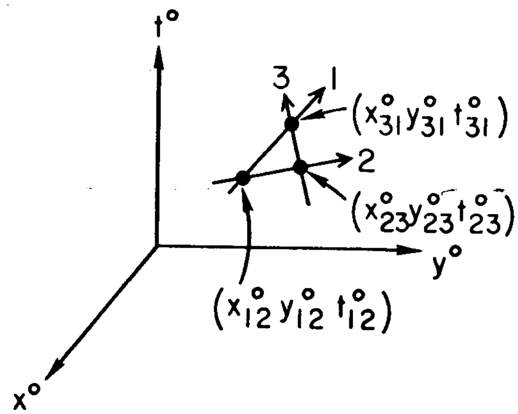
4568A3

Fig. 4



$$r_i^{\circ} = \sqrt{(x_{ij}^{\circ} - x_{ik}^{\circ})^2 + (y_{ij}^{\circ} - y_{ik}^{\circ})^2}$$

$$v_i = r_i^{\circ} / (t_{ij} - t_{ik})$$

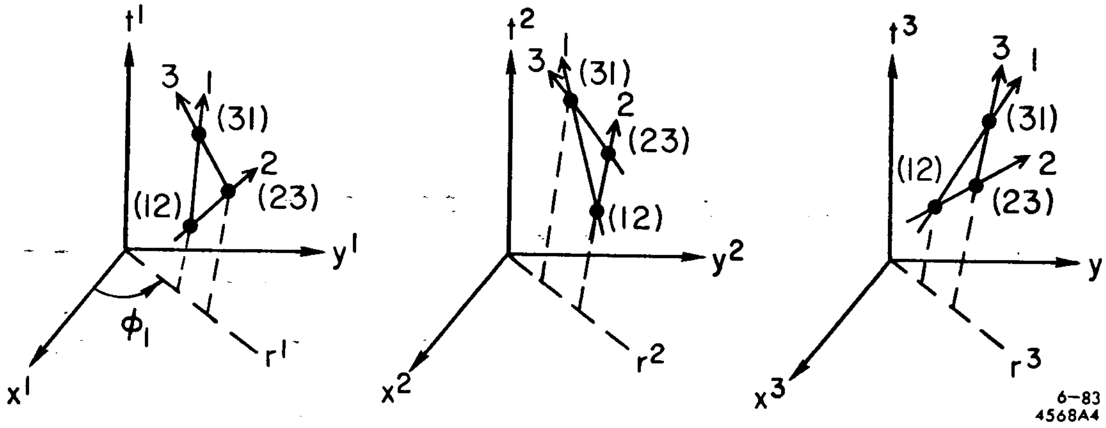
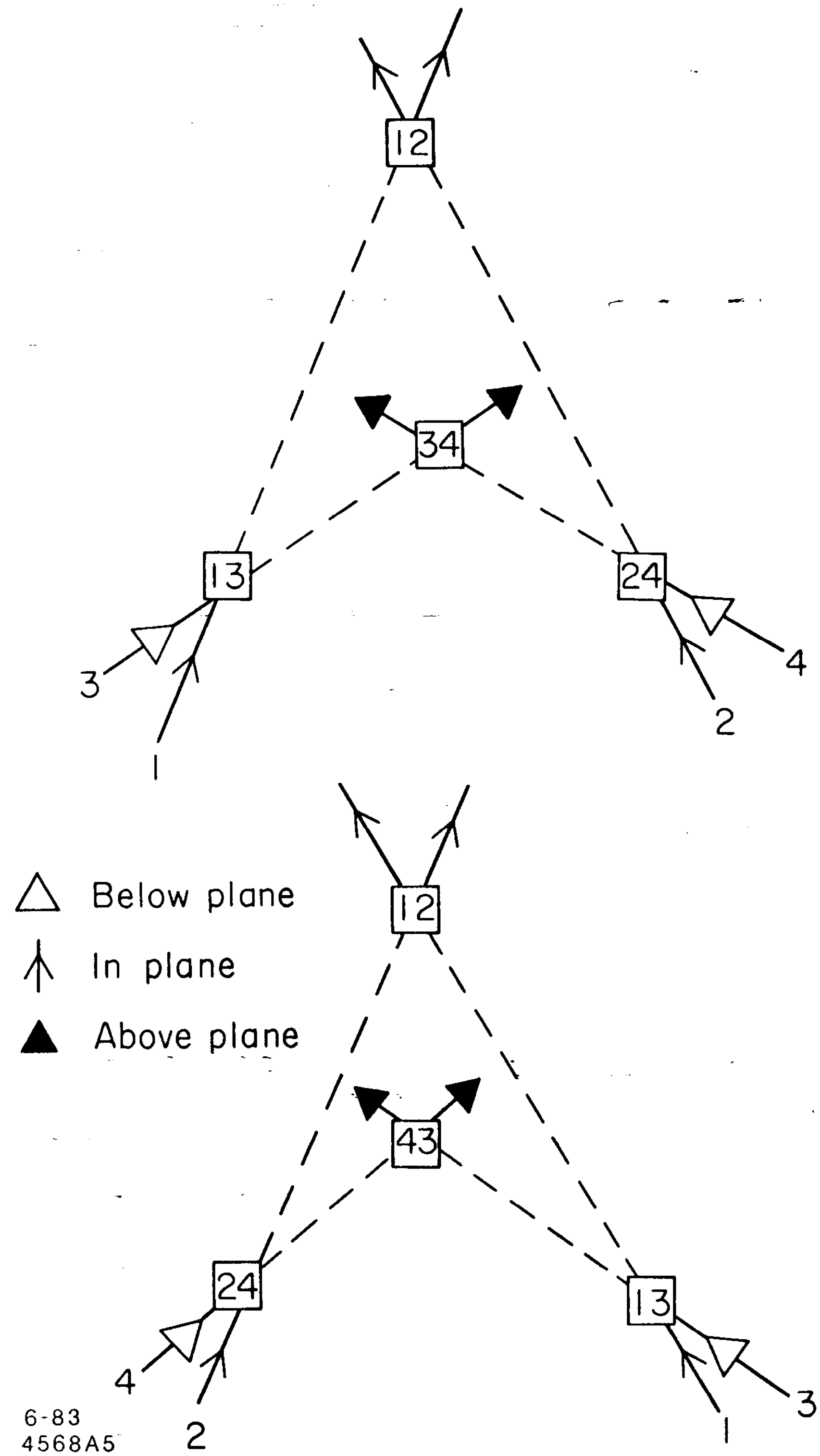
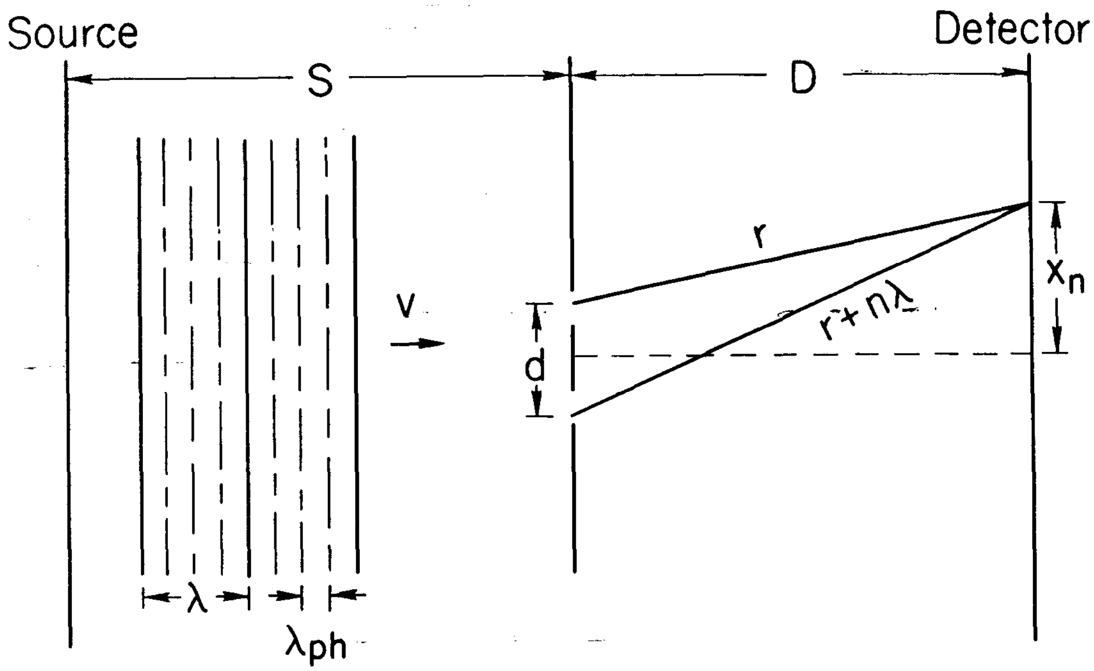


Fig. 5



12
34
13
24
3
1
2
4
Below plane
In plane
Above plane
12
43
24
13
4
2
1
3
6-83
4568A5

Fig. 6



$$v = (S+D)/T$$

$$p = \frac{mv}{\sqrt{1-v^2/c^2}} \qquad E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$$

$$\lambda = \frac{h}{p} \qquad \lambda_{ph} = \frac{hc}{E}$$

$$D^2 + \left(x_n - \frac{d}{2}\right)^2 = r^2$$

$$D^2 + \left(x_n + \frac{d}{2}\right)^2 = (r+n\lambda)^2$$

$$2dx_n = 2n\lambda r + n^2\lambda^2 \approx 2n\lambda D$$

$$x_n = \frac{n\lambda D}{d} \qquad \lambda = \frac{dx_n}{nD}$$

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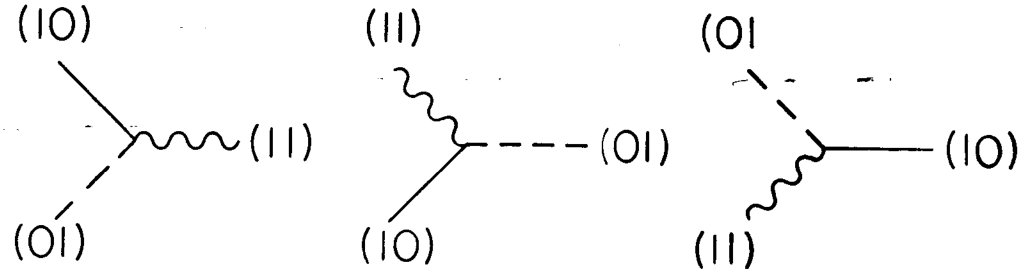
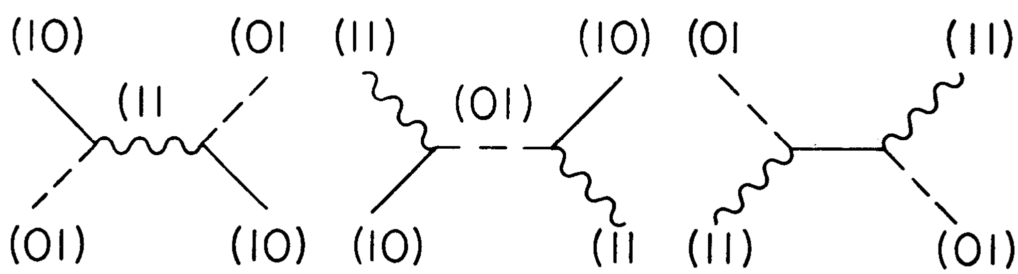
Fig. 7



(a) Vertices generated by discrimination or complementation.



(b) Basic scattering events generated by the main program.

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Fig. 8

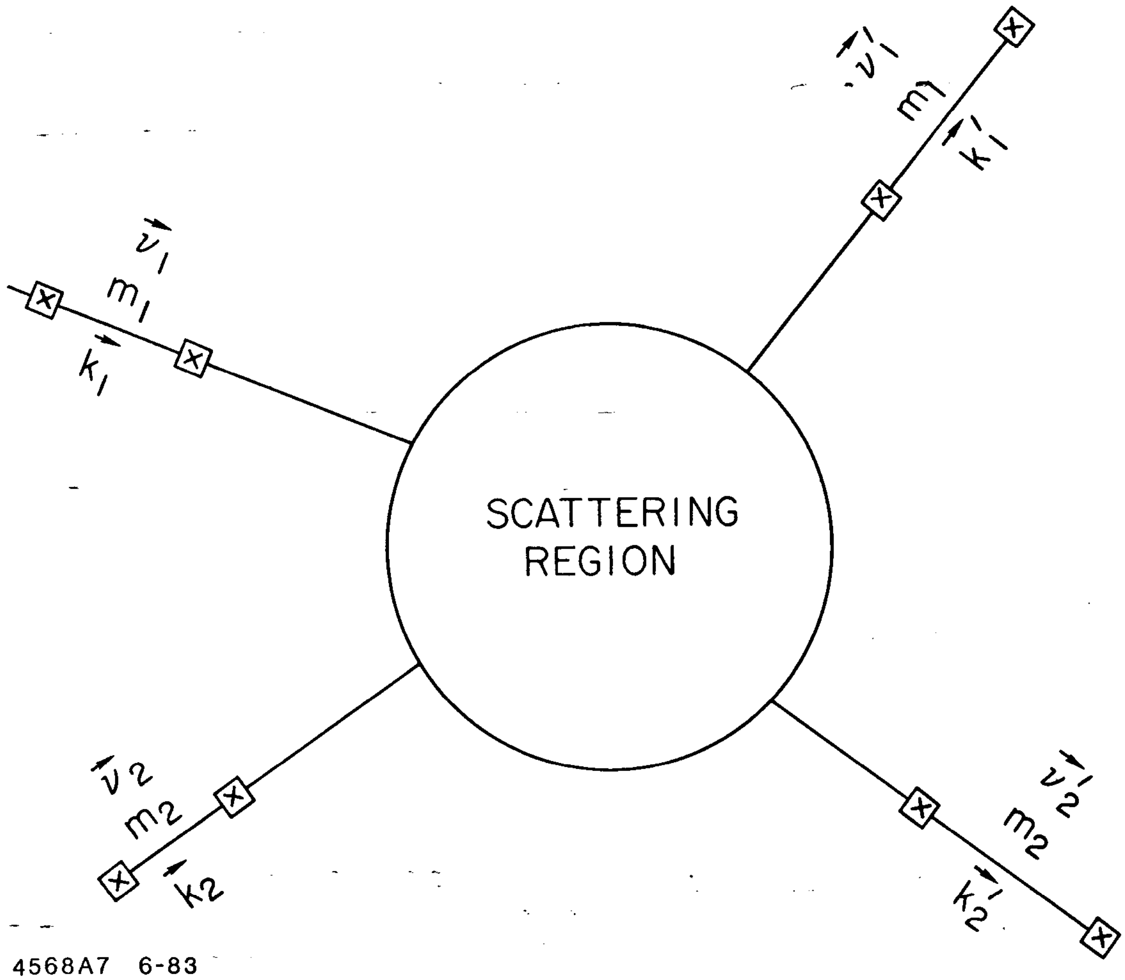
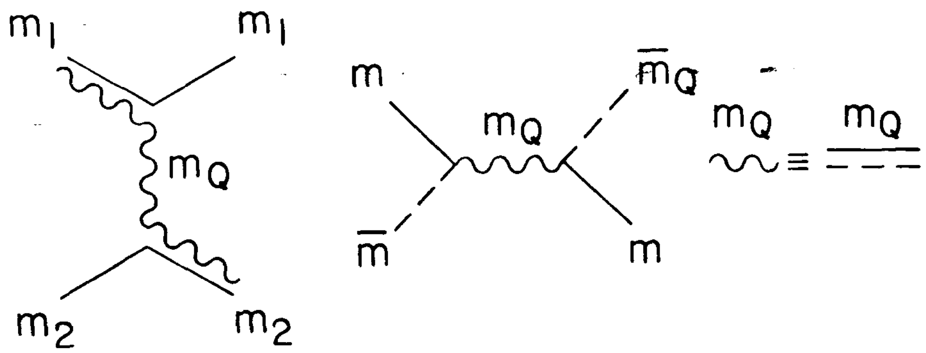
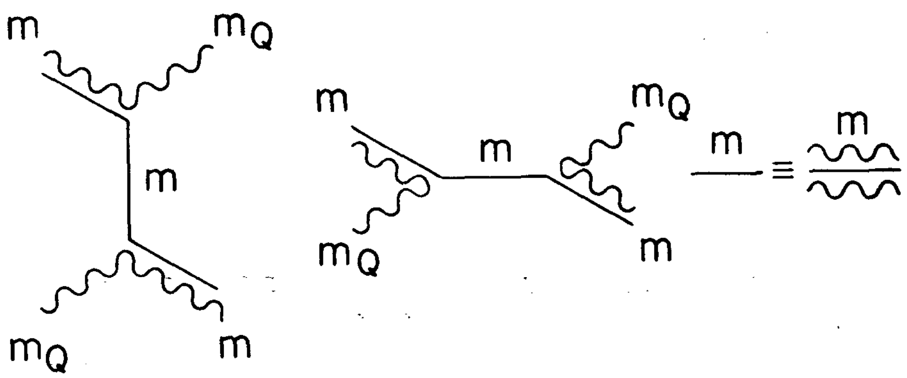


Fig. 9



(a) Particle-particle and particle-antiparticle scattering in the MUST theory.



(b) Quantum particle scattering in the MUST theory.

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Fig. 10

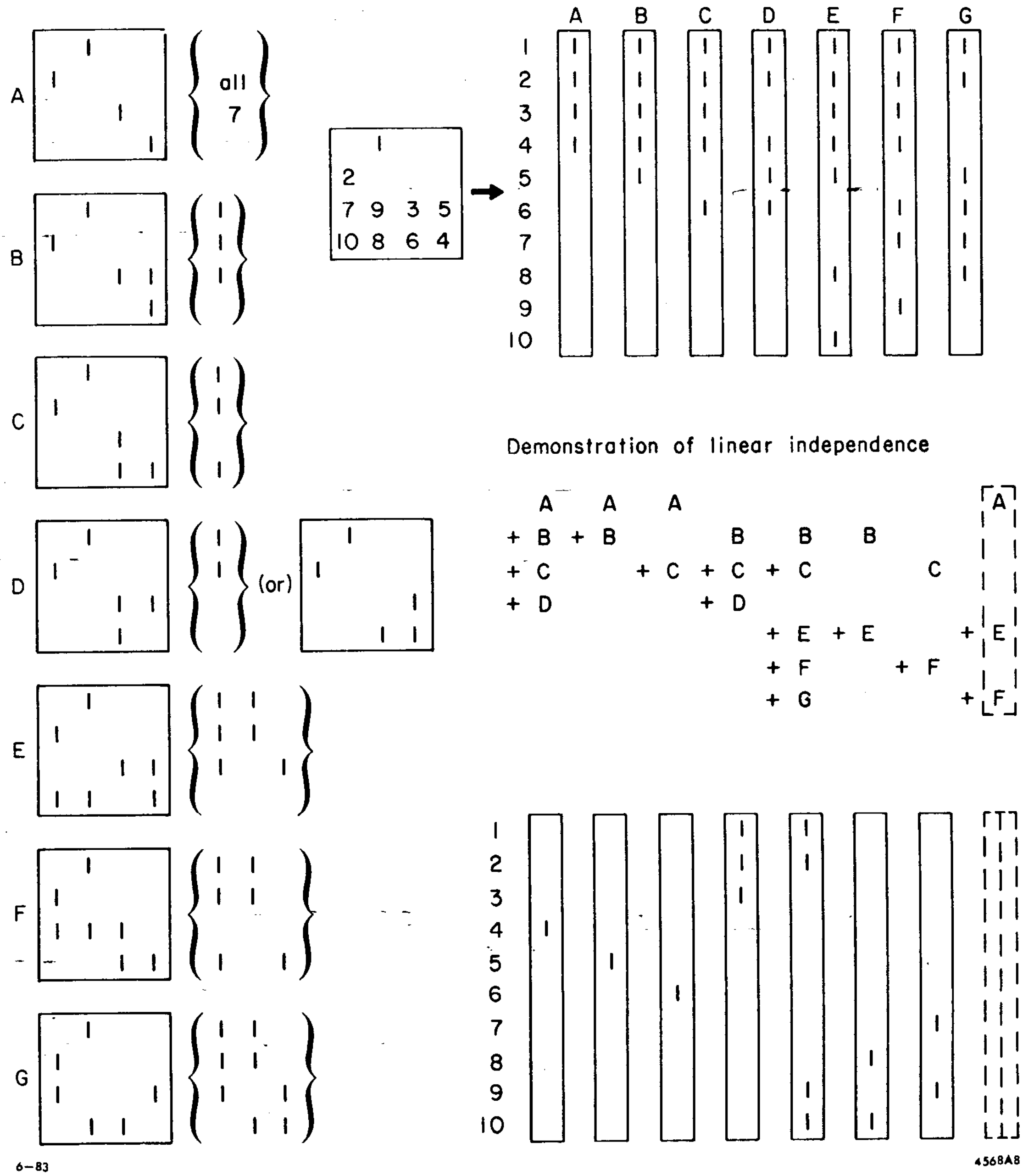
A
B
C
D
E
F
G
all 7
(or)
2
7 9 3 5
10 8 6 4
1 2 3 4 5 6 7 8 9 10
Demonstration of linear independence
A + B + C + D
A + B
A + C
B + C + D
B + C + E + F + G
B + E
C + F
A + E + F
6–83
4568A8

Fig. 11