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ERRATA

The values quoted for s_2 and s_3 in Eqs. (3.11)-(3.13) should be divided by 4. For these copying errors, the authors humbly apologize.

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ABSTRACT

One of the most interesting features of the $\Upsilon(4s)$ (one of the resonances in the Υ family, which provided clear evidence of the existence of the "beauty" flavour) is the enhanced production of kaons from its decay. We calculate the number of kaons yielded by the $\Upsilon(4s)$ decay, making use of the analytic formulas prescribed by the fragmentation model of Field-Feynman. We use the calculation as a test which compares the "Standard Model" (with top quark) with a topless model, based on the unified group E_6. It turns out that the topless model yields the higher number of kaons and is in agreement with the experimental data. Thus, the main problem for topless models of the E_6-type is the strength of their flavour changing neutral currents. Interesting indications come also from the study of different mixings in the two models. For instance, the $b \to u$ channels are not as disfavored as they look.

1. INTRODUCTION

The discovery, two years ago, of the resonance Υ''' [$\Upsilon(4s)$ of the quarkonium model] in e^+e^- annihilation [1,2] has thrown new light on the world of elementary particle physics. Besides yielding clear evidence of the existence of a new flavour (the "beauty" of the bottom quark), it gave important indications on the weak interaction properties of the b-quark itself.

If we knew how the b-quark decays, we could find out whether the existence of the top quark (for which there is yet no direct evidence) is needed, and thus rule out certain classes of unified models. Unfortunately, this is true only in principle. In the present situation, the interpretation itself of the experimental data is so strongly model dependent as to make it impossible either to validate, or to rule out, with one hundred percent confidence, models involving quark-lepton symmetry (except for the very "exotic" ones). The search for indications and the attempts at interpreting them by use of theoretical models can test the validity only in terms of the likelihood of the models themselves.

The fact that the top quark has not been discovered, in a mass range up to about 20 GeV, keeps alive the interest in the otherwise unlikely topless models for Grand Unification (the main argument against these models being the limit, which is getting more and more narrow, on the flavour changing neutral currents, which a topless model is forced to have).

The object of this paper is the critical examination of one of the important characteristics demonstrated by the Υ''' resonance: the enhanced production of kaons. We interpret it in terms of two distinct models: a topless model (with E_6 as gauge group) and the standard SU(2) $\times$ U(1) model with the top quark, with the aim of testing these two models.

The Υ''' resonance is the broadest of a series of four resonances (Υ, Υ', Υ'' are the others [3,4]) observed in e^+e^- annihilation at the Cornell Electron Storage Ring (CESR), in the energy range 9.4 to 10.6 GeV. The family of Υ resonances is interpreted as a $b\bar{b}$ bound system ($1s$, $2s$, $3s$, $4s$, respectively). The Υ''' resonance, which is observed at 10.55 GeV, is supposed to decay rapidly into a $B\bar{B}$ meson pair, in analogy with the decay of $\psi(3770)$ into a $D\bar{D}$ pair. The B meson, which is supposed to be a bound state of a b-quark and a light antiquark, then decays weakly. Evidence of semileptonic decays [5,6] confirms this picture: there is an enhanced production of electrons and muons from the Υ'''.

The most striking data on Υ''', obtained with the CLEO detector at CESR, is the enhanced production of kaons [7,8]. In the picture described above, this has direct implications for the weak interactions of the bottom quark. In particular, it seems to suggest that b decays more into c than into u, since c decays mainly into s.

We examine this point critically in the present paper, which consists of a theoretical calculation of the kaon yield per B decay, assuming the weak decay properties of the b-quark as they stem from the standard model (with top quark) and from an E_6 topless model. We assume the validity of the "spectator model" which is supported by recent experimental data [9]. The B decay properties are then reduced to the b-decay properties, whereas the other light antiquark in B is just a spectator bearing only the duty of recombining with the products of the b decay to form new hadrons. The most important ingredient in our calculation is the use of the Feynman-Field (FF) prescription [10] to compute the number of hadrons produced by the fragmentation of a quark. We want to stress that the FF formulae apply with good agreement in the case of jets of mesons produced by fast outgoing quarks [11]. That might seem, at first glance, not to be suitable to the case of a decay. On the other hand, the energy of the b-quark (about 5 GeV) justifies, in our opinion, our use of the FF formulae. In any case we demonstrate a nontrivial method to calculate the number of kaons in the B-decay; one might search in the future for fragmentation functions considered more suitable to the case for B decay, but the required changes would not be difficult. We believe, though, that such modifications would not seriously affect the main result of this paper, which is a direct comparison between a topless model and a model with top quark in effecting weak B-meson decays.

Only part of the calculation can be done analytically. The final integrations are numerical integrations, performed on a computer with the aid of a Monte Carlo routine.

Our paper is organized as follows. In section 2 we describe the method used for calculating the number of kaons produced by a decaying particle. The FF fragmentation model is outlined first (2.1); then its application to the case of a decay is given (2.2) and the respective differential decay rate is calculated from a general effective Lagrangian for the weak interaction (2.3). The limits of the integration to be performed are then evaluated (2.4) and the way we average on the transverse momentum of the kaon (transverse with respect to the direction of the quark which fragments) is shown (2.5). Section 2 ends with a summary of the formulae to be used. In section 3 we apply the procedure just described to the E_6 topless model and to the "standard model", specifying our choice of the mixing angles, among those which phenomenology still allows. The results obtained by our calculation are presented, together with our analysis and comments on these results. Section 4 contains some concluding remarks.

2. THE PROCEDURE

We indicate in this section the procedure that we have followed in order to calculate the number of kaons produced in the weak decay of the b-quark. Throughout this section we keep to a general point of view, independent of any specific model for grand unification. (Applications to the E_6 topless model and the standard model with top quark will follow in the next section.) This general framework is based on the fragmentation picture to which the Feynman-Field formulae apply. We refer to the original paper [10] and to the references therein for details, while we briefly outline in subsection 2.1 the main features of the FF fragmentation model.

2.1 The Fragmentation Model

The basic assumption is that a quark of flavour "a", having a certain momentum w_a, creates a colour field (by soft gluon effects) in which new quark-antiquark pairs are produced. The quark "a" combines with an antiquark "$\bar{q}$" of the created $q\bar{q}$ pair to form a meson "$a\bar{q}$", while "q" combines with another antiquark "$\bar{r}$" of another created pair, ... The mesons thus created are called "primary mesons" (they may undergo a decay process later, but in the applications we shall make, we shall not consider this). A hierarchy of primary mesons is thus created, starting from the meson "$a\bar{q}$", called the "rank-one primary meson", going on to the meson "$q\bar{r}$", called the "rank-two primary meson", et cetera. This process is called "fragmentation" and it is said that the quark "a" fragments into the "cascade" of hadrons "$a\bar{q}$, $q\bar{r}$", et cetera. The hadrons in the cascade move roughly in the same direction as the fragmenting quark "a".

The next assumption of the model is the validity of the following recursive principle (called the "chain decay" ansatz): if the rank-one primary meson carries away a momentum w_1 from the quark "a" of momentum w_a, then the remaining cascade is distributed in the same way as the cascade created by a quark "q" of momentum $w_q = w_a - w_1$. It is moreover assumed that all distributions, for large momenta, scale so that they depend only on the ratio of the hadron momenta to the quark momenta (the problem arising when dealing with small momenta will be solved shortly).

With these hypotheses, the properties of the fragmentation can essentially be deduced from one function $f(\eta)$, determining the probability that the rank-one primary meson leaves the fraction of momentum η to the remaining cascade. Aside from this function there is a dependency only on three parameters: the flavour, the spin of the primary meson, and the transverse momentum (transverse with respect to the direction of the original fragmenting quark) of the primary meson.

For what concerns the flavour it is assumed that the probability of creating an $s\bar{s}$ pair is half the probability of creating a $u\bar{u}$ or $d\bar{d}$ pair. Therefore, if we denote by γ the probability of creating a $u\bar{u}$ or $d\bar{d}$ pair, the probability of creating an $s\bar{s}$ pair is $1 - 2\gamma$. Hence $\gamma = 0.4$. We assume moreover that all the primary mesons are pseudoscalar, thus eliminating the question of the spin of the primary mesons. The problem of the inclusion of the transverse momentum of the primary mesons can be solved only be comparison with the experimental data and will be discussed in subsection 2.5.

It is possible to show [10] that, given the function $f(\eta)$ and fixing the parameters relative to flavour and spin of the primary mesons, one can determine the distribution in the fraction of momentum z of the primary meson h from the fragmentation of the quark q. This distribution is denoted by $D_q^h(z)$. The function $f(\eta)$ is then determined by choosing for it the simple form of a parabola, with parameters fixed by making them fit the experimental data. Before giving the explicit form of the distribution $D_q^k(z)$ [where k stands for kaon] we want to give the resolution of the only problem we left behind. The model works well for very high momenta, but for low momenta, or even for fairly large momenta but small z, it presents ambiguities. These can be solved [10], by replacing the momentum variables with the light cone momentum variables [12,13], which is like saying that the whole procedure is valid when interpreted as viewed from the "infinite momentum frame", a system of reference moving at the velocity of light with respect to the cascade in the direction of the fragmenting quark q. If we fix this direction to be the z-direction, then the light cone momentum of the particle r is $E_r + p_r^z$. The variable z of the distribution $D_q^k(z)$ is then:

$$z = \frac{E_k + p_k^z}{E_q + p_q^z} \quad . \tag{2.1}$$

We give the explicit form of $D_q^k(z)$, for $q = u, d, s$, as they are deduced from the FF formulae in the case of production of pseudoscalar mesons only:

$$\begin{aligned} D_u^k(z) &= D_{\bar{u}}^k(z) = 0.32\,F(z) - 0.12\,f(1-z) \\ D_d^k(z) &= D_{\bar{d}}^k(z) = D_u^k(z) \\ D_s^k(z) &= D_{\bar{s}}^k(z) = 0.32\,F(z) + 0.48\,f(1-z) \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} F(z) &= \frac{2.42}{z} + 3.47\,z - 5.77\,z^{0.24} \\ f(1-z) &= 0.12 + 2.64\,(1-z)^2 \quad . \end{aligned} \tag{2.3}$$

2.2 The Main Formula

Now that we have the possibility, using the formulae (2.2), of calculating the number of kaons produced by a light quark q fragmenting into hadrons, we can solve the original problem of determining the kaon yield per B decay, by letting b decay into q [14]. We recall, in fact, that, having assumed the validity of the spectator model, the antiquark combined with b to form the B-meson does not play any role in the calculation.

In the decay of b into light quarks, we must distinguish two cases:

(1) b decays directly into the light quark q

(2) b decays into a light quark q via a decay into a c-quark or τ-lepton.

In the first case we simply "multiply" the probability of getting q from the b decay times the appropriate distribution in (2.2). In the second case we make use of an obvious recursive formula.

We use the following notation:

T_r	denotes the lifetime of the particle r;
b	stands for the b-quark;
$\Gamma_r^s(y)$	denotes the differential decay rate for having the particle s, in a particular decay channel of the particle r, with a light cone momentum fraction y with respect to r;
$\sum_{decay}^r$	denotes the sum on every possible decay channel of the particle r;
$\sum_s^r$	denotes the sum over the three particles produced by a particular decay of the particle r.

The number of kaons per B decay, denoted by $\langle n_k \rangle$, is then given by:

$$\begin{aligned}\langle n_k \rangle &= \frac{\sum_{decay}^b \sum_a^b \int \left[\int \Gamma_b^a(y)\, D_a^k\left(\frac{z}{y}\right) dy\right] \frac{dz}{y}}{\sum_{decay}^b \sum_a^b \int \Gamma_b^a(y)\, dy} \\ &= T_b \sum_{decay}^b \sum_a^b \int \left[\int \Gamma_b^a(y)\, D_a^k\left(\frac{z}{y}\right) dy\right] \frac{dz}{y}\end{aligned} \tag{2.4}$$

where

$$y = \frac{E_a + p_a^z}{E_b + p_b^z} \tag{2.5}$$

$$z = \frac{E_k + p_k^z}{E_b + p_b^z} \tag{2.6}$$

where E_k and p_k denote the energy and momentum of the kaon and:

(a) for $a = u,\, d,\, s,\, \bar{u},\, \bar{d},\, \bar{s}$,

$$D_a^k \quad \text{is given by the formulae} \quad (2.2) \tag{2.7}$$

(b) for $a = c,\, \tau,\, \bar{c},\, \bar{\tau}$,

$$D_a^k\left(\frac{z}{y}\right) = T_a \sum_{decay}^{a} \sum_{q}^{a} \int \Gamma_a^q(y')\, D_q^k\left(\frac{z}{yy'}\right) \frac{dy'}{y'} \tag{2.8}$$

with $q = u,\, d,\, s,\, \bar{u},\, \bar{d},\, \bar{s}$, and

$$y' = \frac{E_q + p_q^z}{E_a + p_a^z} \tag{2.9}$$

(c) if $a =$ any lepton, except $\tau,\, \bar{\tau}$,

$$D_a^k\left(\frac{z}{y}\right) = 0 \quad . \tag{2.10}$$

For example, consider the decay channel $b \to c\bar{u}d$. In this case $\sum_a^b$ denotes in formula (2.4) the sum over the contributions by $c, \bar{u}, d$ in this decay. The multiplicity of kaons, produced by d (and analogously for $\bar{u}$) in the considered decay channel, with light cone momentum fraction z, with respect to the b-quark,* hence with a fraction z/y with respect to d, is:

$$\int \Gamma_b^d(y)\, D_d^k\left(\frac{z}{y}\right) \frac{dy}{y} \tag{2.11}$$

where y must be greater than $z(z/y \leq 1)$ and D_d^k is given by formulae (2.2),(2.3). The total number of kaons, produced by d in the decay channel under consideration, is then obtained by integrating formula (2.11) over the range of z one is interested in.

$\star$ We should say: the multiplicity of kaons with light cone momentum fraction in the interval dz around z.

For c the formula is more complicated. To apply the fragmentation picture, we let c decay into light quarks. We must then sum over every possible decay mode of the c-quark, according to the particular model under consideration. This is the sum denoted by $\sum_{decay}^{a}$ in formula (2.8), with $c = a$ in the present case. In any particular decay channel of c, we shall only have light quarks and light leptons. The former yield kaons [formula (2.8)], whereas light leptons, of course, do not produce any kaon [formula (2.10)].

This example explains the formula (2.4), in which the contributions of every decay mode of b are summed up $\left(\sum_{decay}^{b}\right)$.

2.3 THE DIFFERENTIAL DECAY RATE

Consider the weak decay of a particle r, of four momentum p_r, into the (spin 1/2)-particles α, $\bar{\beta}$, ξ, of four momenta p_1, p_2 and p_3, respectively, according to fig. 1.

We write the effective Lagrangian for this decay in the general form:

$$\mathcal{L} = \left(\frac{G_F}{\sqrt{2}}\right) \left(G_L \, \bar{\xi}_L \, \gamma_\mu \, r_L\right) \left(g_L \, \bar{\alpha}_L \, \gamma^\mu \, \beta_L + g_R \, \bar{\alpha}_R \, \gamma^\mu \, \beta_R\right) \tag{2.12}$$

where G_F is the Fermi constant; G_L, g_L and g_R are coupling constants (including mixing angles) due to the left-handed (L) and right-handed (R) current contributions, and depend on the specific unified model.

From the Lagrangian (2.12) one obtains the following differential decay rate, $d\Gamma$,

$$\begin{aligned} d\Gamma = &\left(\frac{G_F^2 \, G_L^2}{4 \, E_r \, E_1 \, E_2 \, E_3}\right) \\ &\times \left[\, g_L^2 \, (p_2 \cdot p_r) \, (p_1 \cdot p_3) + g_R^2 \, (p_r \cdot p_1) \, (p_2 \cdot p_3) + g_L \, g_R \, m_1 \, m_2 \, (p_r \cdot p_3) \,\right] \\ &\times (2\pi)^4 \, \delta \, (p_r - p_1 - p_2 - p_3) \; d^3p_1 \, d^3p_2 \, d^3p_3 \, (2\pi)^{-9} \end{aligned} \tag{2.13}$$

where E_i (E_r), m_i (m_r) are the energy and the mass of the particle with momentum p_i (p_r), $i = 1, 2, 3$.

Since in formula (2.13) there is no symmetry in the indices 1, 2, 3, we must distinguish the three different cases in which we consider the probability distribution of b to decay into ξ or α or $\bar{\beta}$.

Let us focus our attention first on the distribution of the particle ξ. We first integrate in $d^3p_1\, d^3p_2$. Using the notation:

$$s' = |p_r - p_3|^2 \tag{2.14}$$

we get [15];

$$\begin{aligned}
\frac{d^3\,\Gamma_r^\xi}{d^3p_3} = \frac{\pi}{12}\,\frac{G_F^2\,G_L^2}{(2\pi)^5\,E_r\,E_3}\, &\left[\left(g_L^2 + g_R^2\right)\,\frac{1}{s'}\,W\left(s',\,m_1^2,\,m_2^2\right)\right. \\
&\times \left\{\frac{1}{4}\left(m_r^2 + m_3^2 - s'\right)\left[\,W\left(s', m_1^2, m_2^2\right)\,\right]^2 + \frac{1}{4s'}\,\left[\left(m_r^2 - m_3^2\right)^2 - s'^2\right]\right. \\
&\times \left.\left[s'^2 + s'\left(m_1^2 + m_2^2\right) - 2\left(m_1^2 - m_2^2\right)^2\right]\right\} + 3\,g_L\,g_R\,m_1\,m_2 \\
&\left.\times \left(m_r^2 + m_3^2 - s'\right) W\left(s',\,m_1^2,\,m_2^2\right)\right]
\end{aligned} \tag{2.15}$$

where the function W is defined by:

$$\begin{aligned}
W(a,b,c) &= \left(a^2 + b^2 + c^2 - 2ab - 2bc - 2ac\right)^{1/2} \\
&= \left\{\left[a - \left(\sqrt{b} + \sqrt{c}\right)^2\right]\left[a - \left(\sqrt{b} - \sqrt{c}\right)^2\right]\right\}^{1/2} \quad .
\end{aligned} \tag{2.16}$$

In order to find the differential decay rate expressed in terms of the light cone momentum fraction, we distinguish the two cases:

(1) b decays directly into a light quark $(l.q.)$, which fragments: $b \to l.q.$

(2) b decays into $l.q.$ via a decay into $a = c,\, \tau :\; b \to a \to l.q.$

Case (1): $b \to l.q.$

Since the energy at which Υ''' is observed is very close to twice the expected mass of the b-quark, we shall assume, from now on, that the b-quark is at rest (a good approximation for our purposes) in the e^+e^- center of mass frame (hence in the laboratory frame).

Suppose that the light quark under consideration is the particle ξ in fig. 1 (in the present case the particle r in fig. 1 is the bottom quark). Then, assuming that the direction z is the direction of the momentum of ξ, we get:

$$y = \frac{E_3 + p_3^z}{E_b + p_b^z} = \frac{E_3 + |\vec{p}_3|}{m_b} \tag{2.17}$$

where $|\vec{p}_3|$ denotes the modulus of the three momentum of ξ and the index b refers to the b-quark.

Defining

$$s = \frac{s'}{m_b^2} = \frac{|p_r - p_3|^2}{m_b^2} \tag{2.18}$$

and denoting $x_i = (m_i/m_b)$, i = 1,2,3, we get from formulae (2.17),(2.18):

$$s = 1 + x_3^2 - \frac{x_3^2}{y} - y \tag{2.19}$$

$$\frac{d^3p_3}{E_3} = |\vec{p}_3|^2 \, \frac{d|\vec{p}_3| \, d^2\Omega}{E_3} = |\vec{p}_3|^2 \, d^2\Omega \, \frac{dy}{y} \tag{2.20}$$

$$|\vec{p}_3|^2 = \frac{m_b^2 \left(y^2 - x_3^2\right)^2}{4y^2} \quad . \tag{2.21}$$

Since $d^3\Gamma_r^\xi/d^3p_3$ in formula (2.15) depends only on s, which in turn is just a function of y, by formula (2.19), we can trivially integrate in $d^2\Omega$, getting a factor 4π. We can now write down the explicit form of the distribution $\Gamma_b^\xi(y)$, in the notation of the formula (2.4), in the case in which ξ is a light quark which fragments. Before doing that, we introduce [16,17] a multiplicative parameter in the function Γ_r^a which takes QCD into account. It is the first order QCD corrected colour factor N, whose values [18] are: $N = 1$ when the particles α and β, in fig. 1, are leptons; $N = 3.9$ when α and β are quarks.

Writing for convenience $a = \xi$, we get:

$$\Gamma_b^a(y) = \left(\frac{G_F^2\, m_b^5}{192\pi^3}\right)\, G_L^2\, N\, \mathcal{A}^a[s(y)]\, Y^a(y) \tag{2.22}$$

where, for W as in formula (2.16), and $s = s(y)$ as in formula (2.19):

$$\begin{aligned} \mathcal{A}^{\xi}(s) = \Bigg[& \left(g_L^2 + g_R^2\right) \frac{1}{4s^2}\, W\left(s, x_1^2, x_2^2\right) \Big\{ \left(1 + x_3^2 - s\right) \left[W\left(s,\, x_1^2,\, x_2^2\right) \right]^2 \\ & + \frac{1}{s} \left[\left(1 - x_3^2\right)^2 - s^2 \right] \left[s^2 + s\left(x_1^2 + x_2^2\right) - 2\left(x_1^2 - x_2^2\right)^2 \right] \Big\} \\ & + 3\, g_L\, g_R\, x_1\, x_2\, \frac{1 + x_3^2 - s}{s}\, W\left(s, x_1^2, x_2^2\right) \Bigg] \end{aligned} \tag{2.23}$$

$$Y^{\xi}(y) = \frac{\left(y^2 - x_3^2\right)^2}{2y^3} \quad . \tag{2.24}$$

Applying the same procedure described above, we obtain the form of $\Gamma_b^a(y)$ in the cases in which $a = \bar{\beta}$ or $a = \alpha$, when $\bar{\beta}$ and α (fig. 1) are light quarks. We get the same expression, (2.22), with

$$\begin{aligned} \mathcal{A}^{\bar{\beta}}(s) = \frac{1}{2} \Big\{ & 3 g_L^2 \left(\frac{1 + x_2^2 - s}{s} \right) W\left(s,\, x_1^2,\, x_3^2\right) \left(s - x_1^2 - x_3^2\right) \\ & + \frac{1}{2}\, g_R^2\, \frac{1}{s^2}\, W\left(s, x_1^2, x_3^2\right) \Big\{ \left(1 + x_2^2 - s\right) \left[W\left(s,\, x_1^2,\, x_3^2\right) \right]^2 \\ & + \frac{1}{s} \left[\left(1 - x_2^2\right)^2 - s^2 \right] \left[s^2 + s\left(x_1^2 + x_3^2\right) - 2\left(x_1^2 - x_3^2\right)^2 \right] \Big\} \end{aligned}$$

$$+\,3\,g_L\,g_R\,x_1\,x_2\,\frac{1}{s^2}\,W\left(s\,,\,x_1^2\,,\,x_3^2\right)\left(s+x_3^2-x_1^2\right)\left(1-x_2^2+s\right)\Big\}\tag{2.25}$$

$$Y^{\bar{\beta}}(y)=\frac{\left(y^2-x_2^2\right)^2}{2y^3}\qquad .\tag{2.26}$$

$$\begin{aligned}\mathcal{A}^{\alpha}(s)=\frac{1}{2}\,&\Big[g_L^2\,\frac{1}{2s^2}\,W\left(s,x_2^2,x_3^2\right)\Big\{\left(1+x_1^2-s\right)\left[\,W\left(s,x_2^2,x_3^2\right)\right]^2\\&+\frac{1}{s}\,\left[\left(1-x_1^2\right)^2-s^2\right]\left[\,s^2+s\left(x_2^2+x_3^2\right)-2\left(x_2^2-x_3^2\right)^2\right]\Big\}\\&+3\,g_R^2\,\left(\frac{1+x_1^2-s}{s}\right)\,W\left(s,x_2^2,x_3^2\right)\,\left(s-x_2^2-x_3^2\right)\\&+3\,g_L\,g_R\,x_1\,x_2\,\frac{1}{s^2}\,W\left(s\,,\,x_2^2\,,\,x_3^2\right)\left(s+x_3^2-x_2^2\right)\left(1-x_1^2+s\right)\Big]\end{aligned}\tag{2.27}$$

$$Y^{\alpha}(y)=\frac{\left(y^2-x_1^2\right)^2}{2y^3}\qquad .\tag{2.28}$$

Case (2): $b\to\ a\to\ l.q.$ (where $a=c,\bar{c},\tau,\bar{\tau}$)

In this case the whole process becomes much more complicated. We make a choice, therefore, which enables us to simplify the calculations without sensibly affecting the result. We choose the z-direction of the light cone variable as the direction of the three-momentum $\vec{p}_a$ of a. This allows, as we show next, to exploit the formulae already obtained in Case (1), using the property of the light cone variable of being invariant under finite boosts in the z-direction.

In a decay in which $b\to a$, which in turn decays into a l.q. q, the number of kaons produced by the fragmentation of q is, by formula (2.4):

$$\langle n_k\rangle=T_b\,T_a\int\Gamma_b^a(y)\,\Gamma_a^q(y')\,D_q^k\left(\frac{z}{yy'}\right)\,\frac{dy'}{y'}\,\frac{dydz}{y}\qquad .\tag{2.29}$$

Having chosen the z-direction as the direction of $\vec{p}_a$, we apply the same arguments as in Case (1) to determine the same form of the distribution Γ_b^a. We have also in this case the expression (2.22) for Γ_b^a with A^a, Y^a defined by the formulae (2.23) to (2.28).

Because of the invariance of the light cone variable with respect to finite boosts in the z-direction, we can evaluate also Γ_a^q in the rest frame of a. Let us define:

$$v = \frac{|p_a - p_q|^2}{m_a^2} \quad . \tag{2.30}$$

In the rest frame of a we have:

$$y' = \frac{E_q + p_q^z}{m_a} = \frac{E_q \pm \sqrt{E_q^2 - m_q^2 - p_q^{\perp 2}}}{m_a} \tag{2.31}$$

(the $+$ or $-$ sign in formula (2.31) depends upon the sign of p_q^z),

$$v = 1 + x_q^2 - \frac{2E_q}{m_a} \quad . \tag{2.32}$$

In formulae (2.30)-(2.32) we have denoted by $p_a, p_q, m_a, m_q, E_a, E_q$ the four-momenta, the masses and the energies of a and q, respectively, by $p_q^\perp$ the modulus of the component of the three-momentum of q perpendicular to the z-direction and by x_q the ratio of the mass of q to the mass of a.

From formulae (2.31), (2.32), we obtain

$$v = 1 + x_q^2 - \frac{x_q^2}{y'} - \frac{p_q^{\perp 2}}{m_a^2 y'} - y' \quad . \tag{2.33}$$

Using the identity:

$$\frac{dp_q^z}{E_q} = \frac{dy'}{y'} \tag{2.34}$$

we get:

$$\frac{d^3 p_q}{E_q} = \frac{dp_q^z}{E_q}\, d^2\, p^\perp = \frac{1}{2}\, d\theta\, d\left(p_q^{\perp 2}\right)\, \frac{dy'}{y'} \tag{2.35}$$

where $d^2 p_q^\perp$ is the differential in the two components perpendicular to p_q^z and θ is the angle in the plane perpendicular to the z-direction.

From formulae (2.33), (2.35) we derive:

$$\frac{d^3p_q}{E_q} = -\frac{1}{2}\, d\theta\, m_a^2\, dv\, dy' \qquad . \tag{2.36}$$

The minus sign in formula (2.36), due to the fact that v is a decreasing function of $p_q^{\perp 2}$, gets cancelled by ordering the limits of integration that we are going to determine in the next section.

Since $d^3\Gamma_a^q/d^3p_q$ depends only on v, and the limits of integration in dv depend only on y', as we shall see, we can trivially integrate in $d\theta$, getting a factor 2π. This factor 2π and the factor m_a^2 in formula (2.36) give the usual coefficient m_a^5/π^3 in the weak decay rate, and enable us to write:

$$\Gamma_a^q(y') = \left(\frac{G_F^2\, m_a^5}{192\,\pi^3}\right)\, \frac{G_L^2\, N}{2} \int \mathcal{A}^q(v)\, dv \qquad , \tag{2.37}$$

where $\mathcal{A}^q$ is the function defined in the equations (2.23), (2.25), (2.27), depending on whether q is the particle ξ, α, or $\bar{\beta}$ in fig. 1.

We stress again that, having chosen v as the variable of integration, the dependency of Γ_a^q on y' is in the domain of integration, which we are going to determine in the next subsection.

2.4 The Domain of Integration

The calculation we have to carry out is indicated in the formula (2.4). We calculate first the number of kaons produced with a certain light cone momentum fraction in dz around z and then we integrate on the range of z that we shall specify later on. We must find therefore what is the range of y, for a given z.

We recall that, in the rest frame of b, having chosen the z-axis along the three-momentum $\vec{p}_a$ of a, we have

$$y = \frac{E_a + p_a^z}{m_b} = \frac{E_a + |\vec{p}_a|}{m_b} \qquad . \tag{2.38}$$

The limits on y are thus determined by the extreme values of E_a. The lower limit for E_a is obviously the value of the mass m_a of a. Corresponding to this value of E_a, $|\vec{p}_a| = 0$, hence the lower limit for y is:

$$y_1 = \frac{m_a}{m_b} = x_a \quad . \tag{2.39}$$

The upper limit of E_a, in the rest frame of b, is given by

$$E_{a_{max}} = \frac{m_b^2 + m_a^2 - m_{12}^2}{2m_b} \tag{2.40}$$

where m_{12} denotes the sum of the masses of the other two particles occurring in the decay.

The value $|\vec{p}_a|_{max}$ of $|\vec{p}_a|$ corresponding to $E_{a_{max}}$ is:

$$|\vec{p}_a|_{max} = \left[\frac{1}{4m_b^2}\,\left(m_b^2 + m_a^2 - m_{12}^2\right)^2 - m_a^2\right]^{1/2} = \frac{1}{2m_b}\,W\left(m_b^2,\, m_a^2,\, m_{12}^2\right) \tag{2.41}$$

for W as defined by the equation (2.16).

From the equations (2.40) and (2.41), we get the maximum value y_2 of y:

$$y_2 = \frac{1}{2}\,\left[\,1 + x_a^2 - x_{12}^2 + W\left(1\,,\,x_a^2\,,\,x_{12}^2\right)\right] \tag{2.42}$$

where $x_{12} = m_{12}/m_b$.

Hence the range of y is:

$$y_1 \leq y \leq y_2 \quad . \tag{2.43}$$

On this domain we have, though, the restriction:

$$y \geq z \tag{2.44}$$

due to the fact that z/y, which represents the light cone momentum fraction of the kaon with respect to a, must obviously be less than or equal to one.

In the case in which a is a c-quark or a τ lepton, it follows from equations (2.8) and (2.37) that we must integrate in the variables y' and v, defined in the equations (2.31), (2.32).

Let us first consider the limits on y'. The condition analogous to equation (2.44) is:

$$y' \geq \frac{z}{y} \quad . \tag{2.45}$$

From equations (2.31), (2.40), and (2.41), we get:

$$y' \leq \frac{\left(E_q + \sqrt{E_p^2 - p_q^2 - m_q^2}\right)}{m_a} \leq \frac{\left(E_q + \sqrt{E_q^2 - m_q^2}\right)}{m_a} \leq \frac{\left(E_{q_{max}} + \sqrt{E_{q_{max}}^2 - m_q^2}\right)}{m_q}$$
$$= \frac{1}{2}\left[1 + x_q^2 - x_{12}'^2 + W\left(1, x_q^2, x_{12}'^2\right)\right] = y_2' \tag{2.46}$$

where $x_q = m_q/m_a$, $x_{12}' = m_{12}'/m_a$ and m_{12}' is the sum of the masses of the other two particles, besides q, occurring in the decay of a.

On the other hand we have:

$$y' \geq \frac{\left(E_q - \sqrt{E_q^2 - p_q^{\perp 2} - m_q^2}\right)}{m_a} \geq \frac{\left(E_q - \sqrt{E_q^2 - m_q^2}\right)}{m_a} \geq \frac{\left(E_{q_{max}} - \sqrt{E_{q_{max}}^2 - m_q^2}\right)}{m_a}$$
$$= \frac{1}{2}\left[1 + x_q^2 - x_{12}'^2 - W\left(1, x_q^2, x_{12}'^2\right)\right] = y_1' \quad . \tag{2.47}$$

In equation (2.47) we have used the fact that the function $x - \left(x^2 - a^2\right)^{1/2}$ is a decreasing function of x.

From equations (2.45), (2.46), and (2.47), we conclude that the domain of integration in y' is:

$$y_1' \leq y' \leq y_2' \tag{2.48}$$

for y_1' and y_2' as defined in equations (2.47), (2.46), with the overall condition (2.45).

Consider now a certain value of y' in the range (2.48). We must find the possible values of v for y' fixed. The definition (2.32) of v gives the general condition:

$$x_{12}'^2 \leq v \leq (1 - x_q)^2 \qquad . \tag{2.49}$$

From equation (2.33) we see that v, for a fixed y', is a decreasing function of $p_q^{\perp 2}$. The extreme values of $p_q^{\perp 2}$ are:

$$0 \leq p_q^2 \leq |\vec{p}_q|^2_{max} \qquad . \tag{2.50}$$

For $p_q^{\perp 2} = 0$ we get from equation (2.33):

$$v \leq 1 + x_q^2 - \frac{x_q^2}{y'} - y' = v_2 \qquad . \tag{2.51}$$

The maximum value of v_2 is reached for $y' = x_q$, for which $v = (1 - x_q)^2$. Hence the condition (2.51) respects the general condition (2.49), and we can take v_2 as the upper limit for v at y' fixed.

For $p_q^{\perp 2} = |\vec{p}_q|^2_{max}$ we get, again from equation (2.33):

$$v \geq 1 + x_q^2 - \frac{1}{y'}\frac{E^2_{q_{max}}}{m_a^2} - y' = v_1(y') \qquad . \tag{2.52}$$

The function of $v_1(y')$ gets its maximum value for $y' = E_{q_{max}}/m_a$, for which its value is $x_{12}'^2$:

$$v_1(y') \leq x_{12}'^2 \qquad . \tag{2.53}$$

From equations (2.52), (2.53) and (2.49) it follows that the lower limit v_1 for v is:

$$v_1 = x_{12}'^2 \tag{2.54}$$

independently of y'.

REMARK 2.1

It is intuitively clear that $y' = x_q$ and $y' = E_{q_{max}}/m_a$ are critical values in determining the extrema of v. In fact, it follows from the definition of y' that for both $y' < x_q$ and $y' > E_{q_{max}}/m_a$, p_q^z cannot be zero. In particular, for $y' < x_q$, p_q^z must be negative and for $y' > E_{q_{max}}/m_a$, p_q^z must be positive. This restricts the range of $p_q^{\perp 2}$ on which v depends.

REMARK 2.2

We must check that $v_2 \geq v_1$ for every y', so as to cancel the minus sign in equation (2.36). Since v_2 is a decreasing function of y' for $y' > x_q$, and is increasing for $y' < x_q$, we get the minimum values of v_2 for the extreme values of y', that is, for:

$$y' = \frac{E_{q_{max}} \pm |\vec{p}_q|_{max}}{m_a} \quad . \tag{2.55a}$$

For these values of y' we get:

$$\begin{aligned} v_{2min} &= 1 + x_q^2 - \frac{x_q^2}{y'} - y' - x_{12}'^2 + x_{12}'^2 \\ &= 2\,\frac{E_{q_{max}}}{m_a} - \frac{m_q^2}{m_a\left(E_{q_{max}} \pm |\vec{p}_q|_{max}\right)} - \frac{E_{q_{max}} \pm |\vec{p}_q|_{max}}{m_a} + x_{12}'^2 \qquad (2.55b) \\ &= x_{12}'^2 \quad . \end{aligned}$$

We have thus proven that $v_2 \geq x_{12}'^2 = v_1$.

Now that we have fixed the domain of integration in the variables y, y', v, we are left only with the determination of the range of z, the light quark momentum fraction of the kaon with respect to the b quark. To do this we start by choosing a range for the momentum of the kaon. This range could be, for instance, the one in which the momentum of the kaons is experimentally observed. Denoting by $|\vec{p}_k|$ the modulus of the three-momentum of the kaon, we suppose:

$$p_1 \leq |\vec{p}_k| \leq p_2 \quad . \tag{2.56}$$

How do we relate equation (2.56) to the range of z? We can determine the range of z from that of $|\vec{p}_k|$ if we get rid somehow of the transverse momentum of the kaon. The only way we can do it is to average over $p_k^{\perp 2}$ with weights obtained from experimental data.

2.5 The Transverse Momentum of the Kaon

The distribution of $p_k^{\perp}$ is assumed to be Gaussian:

$$c\, e^{-Bp_k^{\perp 2}} \tag{2.57}$$

where c is a normalization constant and B is obtained by the experimental data (we shall fix its value in Section 3.3).

We want to average over $p_k^{\perp 2}$ before integrating in dz, hence we must determine the domain of integration in $dp_k^{\perp 2}$ at z fixed.

Since we limit the momentum of the kaon in the range from p_1 to p_2, it follows that $p_k^{\perp 2}$ can vary within the range:

$$0 \leq p_k^{\perp 2} \leq p_2^2 \quad . \tag{2.58}$$

However, having fixed z, $p_k^{\perp 2}$ cannot reach, in general, every possible value in the range (2.58). For instance, for z $= m_k/m_b$, it cannot be null, because otherwise also p_k^z would be null, contrary to the condition (2.56) (if $p_1 > 0$). On the other hand, for $z > E_2/m_b$, where $E_2 = \left(p_2^2 + m_k^2\right)^{1/2}$, $p_k^{\perp 2}$ must be less than p_2^2, because p_k^z must be greater than zero.

To find out how to limit the range of $p_k^{\perp 2}$ according to the values of z, we argue in the following way. From the definition of z, we derive:

$$p_k^{\perp 2} = 2m_b\, E_k\, z - m_b^2\, z^2 - m_k^2 \quad . \tag{2.59}$$

Thus, for z fixed, $p_k^{\perp 2}$ can range within:

$$2\, m_b\, E_1\, z - m_b^2\, z^2 - m_k^2 < p_k^{\perp^2} < 2m_b\, E_2\, z - m_b^2\, z^2 - m_k^2 \tag{2.60}$$

where

$$E_i = \left(p_i^2 + m_k^2\right)^{1/2} \;, \qquad\qquad i = 1,\, 2 \quad .$$

Since $p_k^{\perp 2}$, for a fixed z, must satisfy both equations (2.58) and (2.60), its range is the intersection of the sets defined in equations (2.58) and (2.60). Let us consider the lower bound. We have:

$$2\, m_b\, E_1\, z - m_b^2\, z^2 - m_k^2 = 0 \qquad \text{for} \qquad z = z_{\pm} = \frac{E_1 \pm p_1}{m_b} \quad . \tag{2.61}$$

Hence $2m_b E_1 z - m_b^2 z^2 - m_k^2 < 0$ for $z < z_-$ and $z > z_+$, in which cases we must take zero for the lower limit of $p_k^{\perp 2}$. For $z_- < z < z_+$, on the contrary, equation (2.60) holds as for the lower limit.

Let us now examine the upper bound. We have:

$$2\, m_b\, E_2\, z - m_b^2\, z^2 - m_k^2 \le p_2^2 \tag{2.62}$$

which implies:

$$2\, m_b\, E_2\, z - m_b^2\, z^2 - E_2^2 \le 0 \tag{2.63}$$

which is equivalent to:

$$-\left(m_b\, z - E_2\right)^2 \le 0 \quad . \tag{2.64}$$

Since equation (2.64) is always satisfied, so is equation (2.62); hence equation (2.60) holds as for the upper limit of $p_k^{\perp 2}$.

REMARK 2.3

Notice, indeed, that for $p_k^{\perp 2} = p_2^2$, it must be true that $p_k^z = 0$, thus $z = E_2/m_b$; hence for this and only this value of z, $p_k^{\perp 2}$ can reach the value p_2^2 (as it is shown in equation (2.64)).

To summarize, if we denote by $p_1^{\perp 2}$ and $p_2^{\perp 2}$ the lower and upper limit of $p_k^{\perp 2}$ respectively, we have:

$$p_1^{\perp 2} = \begin{cases} 2m_b\, E_1\, z - m_b^2\, z^2 - m_k^2 & \text{for } \dfrac{E_1 - p_1}{m_b} < z < \dfrac{E_1 + p_1}{m_b} \\ 0 & \text{for } z < \dfrac{E_1 - p_1}{m_b} \text{ or } z > \dfrac{E_1 + p_1}{m_b} \end{cases} \tag{2.65}$$

$$p_2^{\perp 2} = 2m_b\, E_2\, z - m_b^2\, z^2 - m_k^2 \quad \text{for every } z \quad . \tag{2.66}$$

Consider now the normalization factor c in equation (2.57). It is defined by the condition:

$$\int_0^{p^2} c\, e^{-Bp^{\perp 2}}\, d\, p^{\perp 2} = 1 \quad . \tag{2.67}$$

Thus:

$$c = \frac{B}{1 - e^{-Bp^2}} \quad . \tag{2.68}$$

The meaning of equation (2.67) is obvious: the integral of the distribution of $p_k^{\perp 2}$ over every possible value, for a given momentum of modulus p, represents the probability of getting any possible $p^{\perp}$ for the given p, hence it must be equal to unity.

In the integration we are making, though, we are not fixing p but z. It follows from the definition of z that:

$$|\vec{p}_k|^2 = \left(\frac{m_b^2\, z^2 + m_k^2 + p_k^{\perp 2}}{2\, m_b\, z}\right)^2 - m_k^2 \qquad . \tag{2.69}$$

Therefore, for a given z, to any value of $p_k^{\perp 2}$ corresponds a value of $|\vec{p}_k|^2$. Hence, the probability of having a kaon, for a given z, with momentum in the interval $d\left(p_k^{\perp 2}\right)$ around $p_k^{\perp 2}$ is, from equations (2.57) and (2.68),

$$\frac{B}{1 - e^{-B|\vec{p}_k|^2}}\, e^{-B p_k^{\perp 2}}\; dp_k^{\perp 2} \tag{2.70}$$

with $|\vec{p}_k|^2$ given by equation (2.69). The function (2.70) can now be integrated in the variable z.

We finally calculate the range of z. Since the momentum of the kaon can range within the values p_1 and p_2 [see equation (2.56)], the extreme values for z are:

$$z = \frac{E_2 + p_2}{m_b} \tag{2.71}$$

$$z = \frac{E_2 - p_2}{m_b} \qquad . \tag{2.72}$$

We shall however exclude negative values of the longitudinal component of the kaons, since they go roughly in the same direction of the fragmenting quark. Hence we shall assume:

$$z_1 = \frac{E_1}{m_b} \leq z \leq \frac{E_2 + p_2}{m_b} = z_2 \qquad . \tag{2.73}$$

2.6 Summary of the Section

We summarize in a few formulae what we have discussed in section 2. The number of kaons produced in the decay of the B meson is given by the following formula:

$$\langle n_k \rangle = T_b \sum_{decay}^{b} \sum_{a}^{b} \int_{z_1}^{z_2} \left[\int_{p_1^{\perp 2}}^{p_2^{\perp 2}} \frac{B\, e^{-Bp_k^{\perp 2}}}{1 - e^{-B|\vec{p}_k|^2}} \right.$$

$$\left. \times \left\{ \int_{y_1}^{y_2} \left(\frac{G_F^2\, m_b^5}{192\, \pi^3} \right) G_L^2\, N\, \mathcal{A}^a\, [s(y)]\;\; Y^a(y)\; D_a^k\, \left(\frac{z}{y} \right)\;\; \Theta(y - z)\, \frac{dy}{y} \right\}\; dp_k^{\perp^2} \right] dz$$

where:

1) $\mathcal{A}^a$ is given by the formulae (2.23), (2.25) and (2.27);

2) Y^a is given by the formulae (2.24), (2.26), and (2.28);

3) i) for $a = u,\, d,\, s,\, \bar{u},\, \bar{d},\, \bar{s}$:

D_a^k is given by the formulae (2.2) ;

ii) for $a =$ any lepton but $\tau,\, \bar{\tau}$:

$$D_a^k \left(\frac{z}{y} \right) = 0$$

iii) for $a = c,\, \tau,\, \bar{c},\, \bar{\tau}$:

$$D_a^k \left(\frac{z}{y} \right) = T_a \sum_{decay}^{a} \sum_{q}^{a} \int_{y_1'}^{y_2'} \left[\int_{v_1}^{v_2} \left(\frac{G_F^2\, m_a^5}{192\, \pi^3} \right) G_L^2\, N\, \frac{1}{2}\, \mathcal{A}^q(v)\, dv \right]$$

$$\times D_q^k \left(\frac{z}{yy'} \right)\, \Theta \left(y' - \frac{z}{y} \right)\, \frac{dy'}{y'}$$

where $q = u,\, d,\, s,\, \bar{u},\, \bar{d},\, \bar{s}$;

4) $\Theta(x) = 1$ for $x > 0$, $\Theta(x) = 0$ for $x < 0$, [$\Theta(y - z)$ and $\Theta(y' - z/y)$ have been introduced because of the conditions (2.44), (2.45)];

5) T_r denotes the lifetime of the particle r ;

6) $\sum_{decay}^{r}$ denotes the sum over the decay modes of the particle r ;

7) $\sum_{s}^{r}$ denotes the sum over the three different particles in a decay channel of r ;

8) B determines the Gaussian distribution of the transverse momentum of the kaon, $p_k^{\perp}$, and is taken from the experimental data (see subsection 3.3) ;

9) $|\vec{p}_k|^2$ is given by the formula (2.69) ;

10) G_F is the Fermi constant ;

11) m_b (m_a) is the mass of the b-quark (of the particle a) ;

12) G_L and N are defined in subsection 2.3 ;

13) i) z_1 and z_2 are defined by formula (2.73) ;

ii) $p_1^{\perp 2}$ and $p_2^{\perp 2}$ are defined by formulae (2.65) and (2.66), respectively ;

iii) y_1 and y_2 are defined by formula (2.43) ;

iv) y_1' and y_2' are defined by formulae (2.47) and (2.46), respectively ;

v) v_1 and v_2 are defined by formulae (2.53) and (2.51), respectively .

The procedure outlined here is applicable to any unified model. The specification of the model, in fact, determines the decay modes of b, c and τ, fixing the parameters G_L, g_L, g_R and N and the values of the masses. As stated in the introduction, we are interested in the comparison of two kinds of models: the topless models and the models with the top quark. For the first class we examine an E_6 model and, for the second one, the "standard" SU(2) $\times$ U(1) model.

3. APPLICATION

We now apply the procedure described in the previous section to the unified models we want to compare: the E_6 topless model and the standard SU(2) $\times$ U(1) model (with top quark).

3.1 THE E_6 TOPLESS MODEL

There is an extensive literature on the use of the exceptional Lie group E_6 as a gauge group for grand unified theories (see references [19]-[22], to cite a few). A characteristic feature of E_6 is that the lowest dimensional irreducible representation of its flavour subgroup $SU(3)_L \times SU(3)_R$ [23] contains only an $SU(2)_L$ singlet b_L. Hence, in a two family model based on E_6 [24], there is no room for the top quark. One is forced, though, to insert another new quark h, of charge -1/3, which is supposed to be heavier than b. The six quarks are then accommodated in two $SU(3)_L$ triplets [one for each family]: $(u', d', b',)_L$ and $(c', s', h',)_L$, where the primes denote suitable mixtures of the quarks to fit the experimental data.

Since b [being an $SU(2)_L$ singlet] can only decay via mixtures, flavour changing neutral currents are introduced. Some of these currents are strictly forbidden; the others have strong constraints [7], but their existence is not (yet) ruled out. For the determination of the mixings which fit the phenomenological constraints, we shall follow the prescription of Achiman, whose paper [17] we refer to.

Let us define:

$$B_1 = cos\alpha \; b + sin\alpha \; h \qquad , \qquad B_2 = -sin\alpha \; b + cos\alpha \; h \qquad . \tag{3.1}$$

Then the $SU(2)_L$ doublets are:

$$\left(u' , d'\right)_L = (cos\theta \; u - sin\theta \; c \; , \; cos\beta \; d + sin\beta \; B_1)_L \tag{3.2}$$

$$(c', s')_L = (sin\theta\ u + cos\theta\ c\ ,\ cos\beta\ s + sin\beta\ B_2)_L \tag{3.3}$$

and the $SU(2)_L$ singlets are:

$$b'_L = (-sin\beta\ d + cos\beta\ B_1)_L \tag{3.4}$$

$$h'_L = (-sin\beta\ s + cos\beta\ B_2)_L \tag{3.5}$$

where θ is the Cabibbo angle, α is an angle which is left arbitrary for the moment, β is constrained by the limit [25]:

$$\begin{aligned} |b \rightarrow u| &= A \\ &= sin\beta\,(cos\theta cos\alpha - sin\theta sin\alpha) \\ &= 0.06 \pm 0.06 \end{aligned} \tag{3.6}$$

The part concerning the quarks of the usual current $\times$ current Lagrangian follows from equations (3.1)-(3.5), and from it the constants G_L, g_L, g_R in equation (2.12).

The leptonic currents are taken from the standard model, because they fit the experimental data very well. The only difference from the standard model is the introduction of a new doublet [24] composed of a heavy lepton, τ', and the corresponding neutrino $\nu_{\tau'}$, which is supposed to be light. In the Lagrangian of the b decay, we suppose it to be below the h and τ' threshold, but we must take into account the existence of the fourth neutrino $\nu_{\tau'}$, when considering the neutral leptonic currents.

In the calculation we shall consider one value of β, corresponding to $A = 0.06$ in equation (3.6), and five values of α: $0, \pi/6, \pi/4, \pi/3, \pi/2$.

The values of the masses will be given in section 3.3.

3.2 THE STANDARD MODEL WITH THE TOP QUARK

In the standard SU(2) $\times$ U(1) model [26]-[28], with top quark [29], there are three left-handed doublets:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad , \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad , \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \tag{3.7}$$

where d', s', b' are orthogonal mixtures of d, s, b, parametrized by the Kobayashi-Maskawa matrix [29]. The decay b occurs via charged currents only, through the mixtures d' and s'. In particular, then, the constant g_R defined in the Lagrangian (2.12) is equal to zero, since the right-handed currents are always neutral.

The mixtures d' and s' we are interested in are defined by:

$$d' = (c_1)\, d + (s_1\, c_3)\, s + (s_1\, s_3)\, b \tag{3.8}$$

$$s' = (-s_1\, c_2)\, d + \left(c_1\, c_2\, c_3 + s_2\, s_3\, e^{i\delta}\right) s + \left(c_1\, c_2\, s_3 - s_2\, c_3\, e^{i\delta}\right) b \tag{3.9}$$

where c_i and s_i are cosines and sines of angles $[c_i = (1 - s_i^2)^{1/2}, \;\; i = 1,\, 2,\, 3]$. Only s_1, the sine of the Cabibbo angle, is determined with good approximation:

$$s_1 = 0.23 \pm 0.01 \qquad . \tag{3.10}$$

On the others there are only constraints [30]. We have chosen the following three sets of values [31] for s_2, s_3, and δ:

$$1) \quad s_2 = 0.11 \quad , \quad s_3 = 0.42 \quad , \quad \delta = \pi \text{ - } 0.01 \tag{3.11}$$

$$2) \quad s_2 = 0.6 \quad , \quad s_3 = 0.5 \quad , \quad \delta = 0.001 \tag{3.12}$$

$$3) \quad s_2 = 0.24 \quad , \quad s_3 = 0.02 \quad , \quad \delta = 0.07 \quad . \tag{3.13}$$

The first choice corresponds to near maximal $|b \to c|$ coupling strength, for δ in the second quadrant; the second and third correspond to near maximal and minimal ratios (respectively) $|b \to u|/|b \to c|$ for δ in the first quadrant.

3.3 RESULTS

We fix the following values of the parameters:

$$sin^2\theta_w = 0.23\,, \qquad sin\theta_c = 0.23\,, \qquad m_k = 0.494\,, \qquad m_b = 5.1$$

$$m_e \simeq m_\mu \simeq m_{\nu_e} \simeq m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq m_{\nu_{\tau'}} = 0$$

$$m_u \simeq m_d = 0.3\,, \qquad m_s = 0.5\,, \qquad m_c = 1.75\,, \qquad m_\tau = 1.8$$

where θ_w is the Weinberg angle, θ_c is the Cabibbo angle and m_n denotes the mass in GeV of the particle n.

The parameter B, which determines the distribution (2.57) of transverse momentum of the kaon, is taken from experimental data [32] (we took the value corresponding to the lowest energy region in reference [32]). We have:

$$B = 8.35\ (\mathrm{GeV})^{-2} \quad . \tag{3.14}$$

The results are given in tables 1 - 9. On top of each table we specify the model which the results refer to and the different choices of mixing in that model. For the E_6 model we consider different values of the angle α defined in equation (3.1), and for the standard model we examine the cases 1, 2, 3 of section 3.2 [see equations (3.11) - (3.13)].

In tables 1 and 5 the percentage rate for each decay channel of b is given for the E_6 and standard model, respectively. Beside each rate we give the number of kaons (NK) which one would obtain by counting one for each c- or s-quark in a decay channel times the probability for b to decay in that channel. This is a very naive computation, sometimes used in order to get a rough approximation of NK, with which our results strongly disagree. The total number of kaons, computed in this way, is given at the bottom of tables 1 and 5, together with the lifetimes τ_b of b (in units of 10^{-13} sec).

In tables 2, 3 and 4 (for the E_6 model) and tables 6, 7 and 8 (for the standard model) we report the number of kaons, for each decay channel and in total, that we have obtained with our calculation, in different ranges of the momentum p (in GeV) of the kaon.

In table 9 we give the number of charged kaons (NCK) in the range $.5 < p < 1$ GeV and the number of neutral kaons (NNK) in the range $.3 < p < 3$ GeV.

The results reported in table 9 concern only the choices $\alpha = 0,\ \pi/2$ of the E_6 model and cases 2) and 3) (see section 3.2) of the standard model.

We proceed now to the analysis of the results reported in the tables and to their comparison with the experimental data.

LIFETIME

The calculated lifetime of the b-quark (tables 1 and 5) both in the E_6 model and in the standard model, for any mixing we consider, agrees with the experimental indication $\tau_B < 10^{-10}$ sec [33],[8]. We also had to calculate the lifetimes of the c-quark and τ-lepton. We got $\tau_\tau = 3.5$ to 3.6×10^{-13} sec and $\tau_c = 8.2$ to 10.8×10^{-13} sec. These results also agree with the experimental data [34],[9].

DECAY RATES

As one can see by examining table 1, in the E_6 model, the rates for $b \to u$ and for $b \to c$ are very sensitive to the choice of the angle α. The branching ratio $[Br(b \to u)/Br(b \to c)]$ decreases for increasing α $(0 \leq \alpha \leq \pi/2)$. The extreme case $\alpha = 0$ (respectively $\alpha = \pi/2$) corresponds to b being only in the $SU(2)_L$ doublet (3.2) [respectively (3.3)], hence it corresponds to minimal (respectively maximal) branching fractions $Br(b \to c)$ and $Br(b \to s)$.

In the standard model, cases 1) and 3) of subsection 3.2 correspond to nearly maximal $|b \to c|$ coupling strength while case 2) corresponds to nearly maximal $|b \to u|$ coupling strength.

Therefore, guessing that c and s should yield most of the kaons, one would expect to have case $\alpha = \pi/2$, for the E_6 model, and cases 1) and 3), for the standard model, clearly put in evidence by an enhanced yield of kaons. As we shall see next, this is not quite so.

NEUTRAL + CHARGED KAONS

We have considered three intervals for the momentum p of the outcoming kaon:

$$\begin{aligned} &\text{i)} \quad 0.3 < p < 3 \text{ GeV} ; \\ &\text{ii)} \quad 0.5 < p < 1 \text{ GeV} ; \\ &\text{iii)} \quad 0.3 < p < 1 \text{ GeV} . \end{aligned} \tag{3.15}$$

The first range is close to the widest possible range for p. The momentum of the kaon can never reach 3 GeV, indeed. Since our procedure has ambiguities for p close to zero, we also had to cut p from below. The first interval is also the range in which neutral kaons are detected, while the second one is the range in which charged kaons are detected. The reason for also examining the third interval for p is that it gives us an estimate of the behaviour of $\langle n_k \rangle$ at low momenta.

Let us consider first the E_6 case (tables 2, 3, and 4). The first, rather striking, consideration is that there is not, as anticipated, a strong dependency of the results on the value of α. For instance, we get 1.11* (for $\alpha = 0$) versus 1.17 (for $\alpha = \pi/2$) kaons in the interval $0.3 < p < 1$ GeV. This means 2.22 versus 2.34 kaons per $\Upsilon(4s)$ decay. We also see that for small $\alpha (\alpha < \pi/4)$ sizeable contributions come from decay channels like $u\bar{u}d$, $d\bar{d}d$ where no c or s appears.

Actually, by looking closely at the fragmentation model, we should not be much surprised by these results. The fact that the fragmenting quark is an s versus a u or a d, only increases the probability that the *first* rank primary meson is a kaon; but the flavor of the fragmenting quark, by itself, does not influence the probability of getting kaons of rank higher than one.

⋆ The results we give are affected by an estimated error of twenty percent due to the Monte Carlo calculation.

On the other hand, the mass and the momentum of the fragmenting quark are important in determining the phase space available to the remaining cascade, and hence, how many mesons with certain momenta can be produced.

Our calculation shows that these two aspects contribute so as to almost balance each other. In the standard model (tables 6, 7 and 8) we even see that the highest number of kaons is obtained just in the case in which the $|b \to u|$ coupling is stronger than the $|b \to c|$ coupling [case 2), in equation (3.12)]. This otherwise surprising result becomes understandable in the light of the previous consideration.

Hence the first result of our work: contrary to what is usually argued, the enhanced production of kaons in the Υ(4s) decay does not, *a priori*, imply that b decays mainly into c. We believe our calculation stands as a reasonable counterexample.

The second result is that the topless models are not at all ruled out by our test on the kaon yield: we get more kaons in the E_6 topless model than in the standard model with top quark. The flavour changing neutral currents are definitely responsible for this fact. (We shall comment further about this below).

Let us now compare our results with the experimental data.

In the CLEO experiments [7],[8], charged kaons are identified by time of flight, in the momentum range $0.5 < p < 1.0$ GeV; whereas neutral kaons are identified through their decay in $\pi^+\pi^-$ with $p > 0.3$ GeV. The results are [7],[8]:

$$0.82 \pm 0.10 \text{ charged kaons per } \Upsilon(4s) \text{ decay with } 0.5 < p < 1.0 \text{ GeV} \;, \tag{3.16a}$$

$$1.13 \pm 0.20 \text{ neutral kaons per } \Upsilon(4s) \text{ decay with } 0.3 < p < 3.0 \text{ GeV} \;. \tag{3.16b}$$

An estimate of the total number of kaons, for all p, is obtained [7],[8] by Monte Carlo simulation, and yields:

$$3.45 \pm 0.49 \text{ kaons per } \Upsilon(4s) \text{ decay} \quad (\text{all } p) \quad . \tag{3.17}$$

In order to compare the data (3.17) to our results, let us consider tables 2 and 6. Notice, once again, that the figures in the tables concern the B decay; hence, we must multiply by two to get them relative to the Υ(4s) decay (since there is an equal contribution by $\bar{B}$. Considering that we have a lower cut at 0.3 GeV and an error of twenty percent, we conclude that our results fit the data quite well (especially in the E_6 case).

CHARGED KAONS

In table 9 we report the results of the calculation of the number of charged kaons in the momentum interval $0.5 < p < 1.0$ GeV. The figures do not agree well with the experimental data (3.16a) in this case. We wish to stress that the extrapolation of the experimental data to the same momentum interval, for both charged and neutral kaons, shows an enhanced production of charged kaons with respect to neutral ones (2.02 ± 0.24 versus 1.43 ± 0.25, for all p) [7],[8]. This is an interesting feature our calculation does not yield.

NEUTRAL KAONS

The calculation of the number of neutral kaons in the momentum range $0.3 < p < 3.0$ GeV (table 9) is in very good agreement with the corresponding experimental data (3.16b).

SEMILEPTONIC DECAY

Kaon-lepton events have also been detected experimentally. These events correspond to the case in which either one or both of B and $\bar{B}$ decay semileptonically. The experimental data is [7]: $2.5 \pm 0.5 \pm 0.5$ kaons per event. Also in this case our calculation fits the data, as it can be derived from tables 2 and 6. In the E_6 case we get between 1.3 and 1.4 kaons in the momentum range $0.3 < p < 3.0$ in the kaon-lepton events. Extrapolating to all p we can get reasonably close to the experimental data.

We end this section with two remarks:

REMARK 3.1

To our knowledge, there is just one other experimental result–besides the enhanced production of kaons–found in the $\Upsilon(4s)$ decay, which argues for a strong $|b \to c|$ coupling: the observed endpoint energy spectrum of the electrons, which indicates the production of a recoil mass ~ 1.8 GeV, in reasonable agreement with the D and D^* masses [6]. This stands, in our opinion, as the strongest support for saying that b decays mainly in c.

REMARK 3.2

Our analysis shows that the topless model can fit very well the experimental data on kaon yield per B decay, even better than the standard model with top quark. Nevertheless, we must recall that topless models are in serious trouble for another reason: the existence of strong flavour changing weak neutral currents. Experimentalists set the following limits with ninety percent confidence [7]:

$$r = \frac{Br(B \to X\,\ell^+\ell^-)}{Br(B \to X\,\ell\,\nu)} \leq 10\,\%$$

$$\ell = \mu\,, e \tag{3.18}$$

$$s = Br\left(B \to X\,\ell^+\ell^-\right) \leq 0.74\,\%$$

By looking at table 1, we get the following values of r and s:

$\alpha = 0$	r = 13 %	s = 2.42 %
$\alpha = \pi\ /\ 6$	r = 17 %	s = 2.98 %
$\alpha = \pi\ /\ 4$	r = 21 %	s = 3.44 %
$\alpha = \pi\ /\ 3$	r = 24 %	s = 3.94 %
$\alpha = \pi\ /\ 2$	r = 24 %	s = 4.04 % .

The case $\alpha = 0$ is, therefore, favoured under this aspect.

Clearly the indication provided by the limits (3.18) is very strong, so that the primary result of our work from the standpoint of model building is to leave equations (3.18) as the main argument against topless models.

4. CONCLUSIONS

There is a substantial agreement between our results and the experimental data. The fit to the kaon yield data is better in the E_6 topless model than in the standard model with top quark. The enhanced production of kaons does not indicate, as supposed, that b decays mainly into c. In fact in the standard model, the case in which more kaons occur is the one in which there is a strong coupling between b and u.

Less agreement with the experiments is found when considering the number of charged kaons produced in the momentum range $0.5 < p < 1.0$ GeV. We have observed that the experiments show an enhanced production of charged versus neutral kaons; the model we have used does not show this effect.

Turning to the question about the need for the top quark, we have recalled that the existence of strong flavour changing weak neutral currents is the most serious indication against topless models. In the E_6 case, this indication favours the case $\alpha = 0$, in which the coupling of b to u is stronger (see *REMARK 3.2*). This case is compatible with an enhanced production of kaons, as we have shown, but it is in strong disagreement with the indication coming from the observed electron energy spectrum in the $\Upsilon(4s)$ decay (see *REMARK 3.1*). On the other hand, the case which fits both the data on the enhanced production of kaons and the electron energy spectrum (the case $\alpha = \pi/2$) has flavour changing neutral currents which are quite beyond the experimentally indicated limit.

These facts confirm that the topless models have a rather limited possibility of survival. However, our work leaves flavour changing neutral currents as the primary argument against such models.

Finally, we emphasize that what we have presented here are illustrations of a prescription for calculating the kaon yield in b-decay in a given model. Such illustrations have not appeared elsewhere.

Note Added:

One may wonder whether the kaon momentum distributions which would follow from our B-meson decay scenarios are consistent with the available data on such distributions. We have addressed this issue by comparing this data to our predicted kaon distributions for the following cases: (a) the standard model with parameters $s_2 = .6$, $s_3 = .5$, $\delta = .001$; (b) the standard model with parameters $s_2 = .06$, $s_3 = .005$, $\delta = .07$; and (c) the E_6 model with the parameter $\alpha = \pi/2$ (which we find to be very close to the case $\alpha = 0$ insofar as these particular distributions in momentum are concerned). These comparisons are shown in figs. 2-4. What we conclude from this is that, in view of the errors on the data, neither of our theoretical models is obviously inconsistent with observation. This gives us additional confidence in the conclusions which we have drawn in our analysis. As a further check on the general applicability of our method of analysis of B-decay, we have also computed the total charged multiplicity in B-decay in the three cases (a)-(c). We find $n_{ch} = 7.3$, 7.5 and 6.3 for cases (a), (b) and (c) respectively. Experimentally, the CLEO group [A. Silverman, *Proc. 1981 Lepton-Photon Conference*, ed. W. Pfeil (Universität Bonn, Bonn, 1981)] has reported ~ 5.4 charged hadrons per B-decay. Again, our model predictions and the data agree within the various theoretical and experimental uncertainties. Hence, we conclude that there are no obvious flaws in the methods which we have used to compute the kaon yield per decay in the present communication.

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Table 1

E_6 Topless Model: Naive Estimate										
Decay Mode	$\alpha = 0$ RATE	NK	$\alpha = \pi/6$ RATE	NK	$\alpha = \pi/4$ RATE	NK	$\alpha = \pi/3$ RATE	NK	$\alpha = \pi/2$ RATE	NK
$u\bar{\nu}_e e$	9.10	0	6.31	0	3.88	0	1.39	0	0.95	0
$u\bar{\nu}_\mu \mu$	9.10	0	6.31	0	3.88	0	1.39	0	0.95	0
$u\bar{\nu}_\tau \tau$	3.64	0	2.52	0	1.55	0	0.55	0	0.38	0
$c\bar{\nu}_e e$	0.22	.002	2.46	.025	4.47	.045	6.79	.068	7.44	.074
$c\bar{\nu}_\mu \mu$	0.22	.002	2.46	.025	4.47	.045	6.79	.068	7.44	.074
$c\bar{\nu}_\tau \tau$	0.05	.001	0.53	.005	0.95	.010	1.45	.015	1.59	.016
$d\bar{\nu}\nu$	9.64	0	8.94	0	6.99	0	4.06	0	0	0
$d\bar{e}e$	1.21	0	1.13	0	0.88	0	0.51	0	0	0
$d\bar{\mu}\mu$	1.21	0	1.13	0	0.88	0	0.51	0	0	0
$d\bar{\tau}\tau$	0.12	0	0.11	0	0.09	0	0.05	0	0	0
$s\bar{\nu}\nu$	0	0	2.84	.028	6.66	0.67	11.60	.116	16.02	.160
$s\bar{e}e$	0	0	0.36	.004	0.84	.008	1.46	.015	2.02	.020
$s\bar{\mu}\mu$	0	0	0.36	.004	0.84	.008	1.46	.015	2.02	.020
$s\bar{\tau}\tau$	0	0	0.03	0	0.07	.001	0.12	.001	0.17	.002
$u\bar{u}d$	31.46	0	21.75	0	13.27	0	4.60	0	3.08	0
$u\bar{u}s$	1.69	.002	1.16	.012	0.71	.007	0.25	.003	0.17	.002
$u\bar{c}d$	0.75	.008	0.52	.005	0.32	.003	0.11	.001	0.07	.001
$u\bar{c}s$	12.35	.247	8.54	.171	5.21	.104	1.81	.036	1.21	.024
$c\bar{u}d$	0.75	.008	8.22	.082	14.81	.148	21.79	.218	23.27	.233
$c\bar{u}s$	0.04	.001	0.43	.009	0.77	.015	1.13	.023	1.21	.024
$c\bar{c}d$	0.01	0	0.11	.002	0.19	.004	0.28	.006	0.30	.006
$c\bar{c}s$	0.14	.004	1.57	0.47	2.83	.085	4.16	.125	4.44	.133
$d\bar{u}u$	5.16	0	4.78	0	3.74	0	2.17	0	0	0
$d\bar{d}d$	6.55	0	6.04	0	4.65	0	2.51	0	0	0
$d\bar{s}s$	5.98	.120	5.51	.110	4.24	.085	2.29	.046	0	0
$d\bar{c}c$	0.61	.012	0.57	.011	0.44	.009	0.26	.005	0	0
$s\bar{u}u$	0	0	1.52	.015	3.56	.036	6.20	.062	8.56	.086
$s\bar{d}d$	0	0	1.92	.019	4.43	.044	7.17	0.72	9.33	.093
$s\bar{s}s$	0	0	1.74	.052	4.03	.121	6.53	.196	8.50	.255
$s\bar{c}c$	0	0	0.16	.005	0.37	.011	0.64	.019	0.89	.027
τ_b *	2.2		1.5		.95		.34		.23	
Total NK		.407		.626		.856		1.11		1.25

*10E − 13 SEC

Table 2

Decay Mode	$\alpha = 0$ NK	$\alpha = \pi/6$ NK	$\alpha = \pi/4$ NK	$\alpha = \pi/3$ NK	$\alpha = \pi/2$ NK
E_6 Topless Model $.3 < p < 3$ GeV *					
$u\bar{\nu}_e e$	.039	.029	.021	.008	.005
$u\bar{\nu}_\mu \mu$	.039	.029	.021	.008	.005
$u\bar{\nu}_\tau \tau$	.021	.011	.008	.003	.002
$c\bar{\nu}_e e$	.001	.010	.017	.026	.028
$c\bar{\nu}_\mu \mu$	.001	.010	.017	.026	.028
$c\bar{\nu}_\tau \tau$	0	.002	.003	.006	.006
$d\bar{\nu}\nu$	.040	.043	.033	.021	0
$d\bar{e}e$	.004	.006	.005	.003	0
$d\bar{\mu}\mu$	.004	.006	.005	.003	0
$d\bar{\tau}\tau$	0	0	0	0	0
$s\bar{\nu}\nu$	0	.026	.055	.104	.137
$s\bar{e}e$	0	.003	.008	.013	.019
$s\bar{\mu}\mu$	0	.003	.008	.013	.019
$s\bar{\tau}\tau$	0	0	0	0	0
$u\bar{u}d$	.420	.304	.181	.061	.044
$u\bar{u}s$	.031	.021	.013	.005	.003
$u\bar{c}d$	.008	.006	.004	.001	.001
$u\bar{c}s$	.198	.120	.069	.028	.017
$c\bar{u}d$	.008	.096	.163	.227	.258
$c\bar{u}s$	.001	.006	.011	.016	.018
$c\bar{c}d$	0	.001	.001	.002	.002
$c\bar{c}s$	.001	.015	.025	.038	.044
$d\bar{u}u$	.074	.066	.050	.031	0
$d\bar{d}d$	.102	.082	.063	.035	0
$d\bar{s}s$	.120	.120	.083	.045	0
$d\bar{c}c$	.003	.003	.002	.001	0
$s\bar{u}u$	0	.027	.060	.106	.150
$s\bar{d}d$	0	.035	.078	.121	.161
$s\bar{s}s$	0	.039	.094	.150	.215
$s\bar{c}c$	0	.001	.002	.004	.005
Total NK	1.11	1.12	1.10	1.11	1.17

*Error Estimate: 20%

Table 3

E_6 Topless Model $.3 < p < 1$ GeV *					
Decay Mode	$\alpha = 0$ NK	$\alpha = \pi/6$ NK	$\alpha = \pi/4$ NK	$\alpha = \pi/3$ NK	$\alpha = \pi/2$ NK
$u\bar{\nu}_e e$	.041	.033	.018	.006	.004
$u\bar{\nu}_\mu \mu$	.041	.033	.018	.006	.004
$u\bar{\nu}_\tau \tau$	.019	.012	.009	.003	.002
$c\bar{\nu}_e e$	.001	.010	.018	.027	.031
$c\bar{\nu}_\mu \mu$	.001	.010	.018	.027	.031
$c\bar{\nu}_\tau \tau$	0	.002	.004	.006	.006
$d\bar{\nu}\nu$	.045	.054	.032	.019	0
$d\bar{e}e$	.007	.005	.004	.002	0
$d\bar{\mu}\mu$	.007	.005	.004	.002	0
$d\bar{\tau}\tau$	0	0	0	0	0
$s\bar{\nu}\nu$	0	.018	.049	.075	.119
$s\bar{e}e$	0	.002	.006	.011	.018
$s\bar{\mu}\mu$	0	.002	.006	.011	.018
$s\bar{\tau}\tau$	0	0	0	0	0
$u\bar{u}d$	.429	.292	.179	.061	.040
$u\bar{u}s$	.025	.018	.012	.004	.003
$u\bar{c}d$	.008	.006	.003	.001	.001
$u\bar{c}s$	.161	.109	.064	.024	.016
$c\bar{u}d$	.008	.086	.159	.255	.257
$c\bar{u}s$	.001	.006	.010	.015	.016
$c\bar{c}d$	0	.001	.001	.002	.002
$c\bar{c}s$	.001	.015	.026	.038	.040
$d\bar{u}u$	.065	.054	.046	.028	0
$d\bar{d}d$	.086	.079	.056	.033	0
$d\bar{s}s$	.101	.096	.078	.044	0
$d\bar{c}c$	.003	.003	.002	.001	0
$s\bar{u}u$	0	.023	.054	.096	.134
$s\bar{d}d$	0	.029	.066	.110	.150
$s\bar{s}s$	0	.037	.088	.137	.181
$s\bar{c}c$	0	.001	.002	.004	.005
Total NK	1.05	1.04	1.04	1.05	1.08

*Error Estimate: 20%

Table 4

E_6 Topless Model $.5 < p < 1$ GeV *					
Decay Mode	$\alpha = 0$ NK	$\alpha = \pi/6$ NK	$\alpha = \pi/4$ NK	$\alpha = \pi/3$ NK	$\alpha = \pi/2$ NK
$u\bar{\nu}_e e$	.018	.012	.006	.003	.002
$u\bar{\nu}_\mu \mu$	.018	.012	.006	.003	.002
$u\bar{\nu}_\tau \tau$	.007	.005	.003	.001	.001
$c\bar{\nu}_e e$	0	.004	.006	.010	.010
$c\bar{\nu}_\mu \mu$	0	.004	.006	.010	.010
$c\bar{\nu}_\tau \tau$	0	.001	.001	.002	.002
$d\bar{\nu}\nu$	.024	.018	.014	.008	0
$d\bar{e}e$	.002	.002	.002	.001	0
$d\bar{\mu}\mu$	.002	.002	.002	.001	0
$d\bar{\tau}\tau$	0	0	0	0	0
$s\bar{\nu}\nu$	0	.010	.020	.042	.058
$s\bar{e}e$	0	.001	.003	.005	.007
$s\bar{\mu}\mu$	0	.001	.003	.005	.007
$s\bar{\tau}\tau$	0	0	0	0	0
$u\bar{u}d$	.174	.121	.077	.025	.018
$u\bar{u}s$	.012	.008	.005	.002	.001
$u\bar{c}d$	.003	.002	.001	0	0
$u\bar{c}s$	.068	.043	.029	.009	.006
$c\bar{u}d$	.003	.032	.056	.085	.096
$c\bar{u}s$	0	.002	.004	.006	.006
$c\bar{c}d$	0	0	0	.001	.001
$c\bar{c}s$	0	.004	.008	.012	.013
$d\bar{u}u$	.028	.028	.020	.012	0
$d\bar{d}d$	.035	.033	.023	.014	0
$d\bar{s}s$	.047	.045	.035	.017	0
$d\bar{c}c$	.001	.001	.001	0	0
$s\bar{u}u$	0	.011	.024	.045	.060
$s\bar{d}d$	0	.014	.031	.051	.065
$s\bar{s}s$	0	.017	.038	.062	.087
$s\bar{c}c$	0	0	.001	.001	.002
Total NK	.444	.434	.423	.430	.452

*Error Estimate: 20%

Table 5

Standard Model: Naive Estimate

DECAY MODE	1 RATE	1 NK	2 RATE	2 NK	3 RATE	3 NK
$u\bar{\nu}_e e + u\bar{\nu}_\mu \mu$	2.39	0	17.99	0	0.03	0
$u\bar{\nu}_\tau \tau$	0.48	0	3.59	0	0.01	0
$u\bar{u}d$	4.18	0	31.45	0	0.05	0
$u\bar{u}s$	0.18	.002	1.23	.012	0	0
$u\bar{c}d$	0.10	.001	0.47	.005	0	0
$u\bar{c}s$	1.20	.024	12.37	.247	0.02	0
$c\bar{\nu}_e e + c\bar{\nu}_\mu \mu$	29.27	.293	10.28	.103	31.11	.311
$c\bar{\nu}_\tau \tau$	3.12	.031	1.10	.011	3.32	.033
$c\bar{u}d$	49.52	.495	17.39	.174	52.62	.526
$c\bar{u}s$	2.07	.041	0.66	.013	2.67	.053
$c\bar{c}d$	0.62	.012	0.14	.003	0.63	.013
$c\bar{c}s$	6.90	.207	3.32	.100	9.55	.286
τ_b *	0.1		0.6		0.6	
TOTAL NK		1.11		.668		1.22

*10E − 13 SEC

Table 6

Standard Model $.3 < p < 3$ GeV *			
Decay Mode	1 NK	2 NK	3 NK
$u\bar{\nu}_e e + u\bar{\nu}_\mu \mu$	.013	.099	0
$u\bar{\nu}_\tau \tau$	.003	.020	0
$u\bar{u}d$	.056	.419	.001
$u\bar{u}s$	.003	.022	0
$u\bar{c}d$	.001	.005	0
$u\bar{c}s$	.018	.184	0
$c\bar{\nu}_e e + c\bar{\nu}_\mu \mu$	.118	.043	.134
$c\bar{\nu}_\tau \tau$	.014	.005	.012
$c\bar{u}d$	.524	.196	.639
$c\bar{u}s$	.026	.009	.037
$c\bar{c}d$	.004	.001	.005
$c\bar{c}s$	.074	.033	.086
Total NK	.852	1.04	.913

*Error Estimate: 20%

Table 7

Standard Model $.3 < p < 1$ GeV *			
Decay Mode	1 NK	2 NK	3 NK
$u\bar{\nu}_e e + u\bar{\nu}_\mu \mu$	.010	.081	0
$u\bar{\nu}_\tau \tau$	.002	.018	0
$u\bar{u}d$	.049	.426	.001
$u\bar{u}s$	.003	.019	0
$u\bar{c}d$	.001	.005	0
$u\bar{c}s$	.015	.181	0
$c\bar{\nu}_e e + c\bar{\nu}_\mu \mu$	.118	.043	.127
$c\bar{\nu}_\tau \tau$	.012	.004	.013
$c\bar{u}d$	.533	.180	.561
$c\bar{u}s$	.028	.009	.036
$c\bar{c}d$	.005	.001	.005
$c\bar{c}s$	.064	.027	.085
Total NK	.839	.995	.828

*Error Estimate: 20%

Table 8

Standard Model $.5 < p < 1$ GeV *			
Decay Mode	1 NK	2 NK	3 NK
$u\bar{\nu}_e e + u\bar{\nu}_\mu \mu$	.006	.033	0
$u\bar{\nu}_\tau \tau$	.001	.007	0
$u\bar{u}d$	.023	.178	0
$u\bar{u}s$	.001	.009	0
$u\bar{c}d$	0	.002	0
$u\bar{c}s$	.006	.065	0
$c\bar{\nu}_e e + c\bar{\nu}_\mu \mu$	.041	.015	.043
$c\bar{\nu}_\tau \tau$	.003	.001	.004
$c\bar{u}d$	.199	.072	.215
$c\bar{u}s$	.011	.004	.014
$c\bar{c}d$	.001	0	.001
$c\bar{c}s$	.020	.010	.028
Total NK	.313	.395	.307

*Error Estimate: 20%

Table 9

	E_6 Model		Standard Model	
	$\alpha = 0$	$\alpha = \pi/2$	2	3
$K^{\pm}$ per Υ(4s) decay $0.5 < p < 1.0$ GeV	0.448	0.452	0.410	0.304
K° per Υ(4s) decay $0.3 < p < 3.0$ GeV	1.08	1.18	0.96	0.99

FIGURE CAPTIONS

Fig. 1. Weak decay of a fermion r into $\xi + \alpha + \bar{\beta}$.

Fig. 2. Charged and neutral kaon momentum distributions for $\Upsilon(4S)$ events – our results in the case of the standard model with parameters: $s_2 = 0.6$, $s_3 = 0.5$, $\delta = 0.001$. The triangular points refer to the data of Brody *et al.* [Phys. Rev. Lett. 48 (1982) 1070].

Fig. 3. Charged and neutral kaon momentum distributions for $\Upsilon(4S)$ events – our results in the case of the standard model with parameters: $s_2 = 0.06$, $s_3 = 0.005$, $\delta = 0.07$. The triangular points refer to the data of Brody *et al.* [Phys. Rev. Lett. 48 (1982) 1070].

Fig. 4. Charged and neutral kaon momentum distributions for $\Upsilon(4S)$ events – our results in the case of the E_6 model with the parameter $\alpha = \pi/2$. The same distribution is followed in the case $\alpha = 0$. The triangular points refer to the data of Brody *et al.* [Phys. Rev. Lett. 48 (1982) 1070].

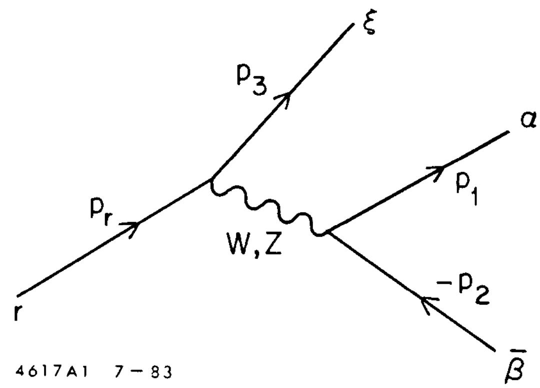

Fig. 1

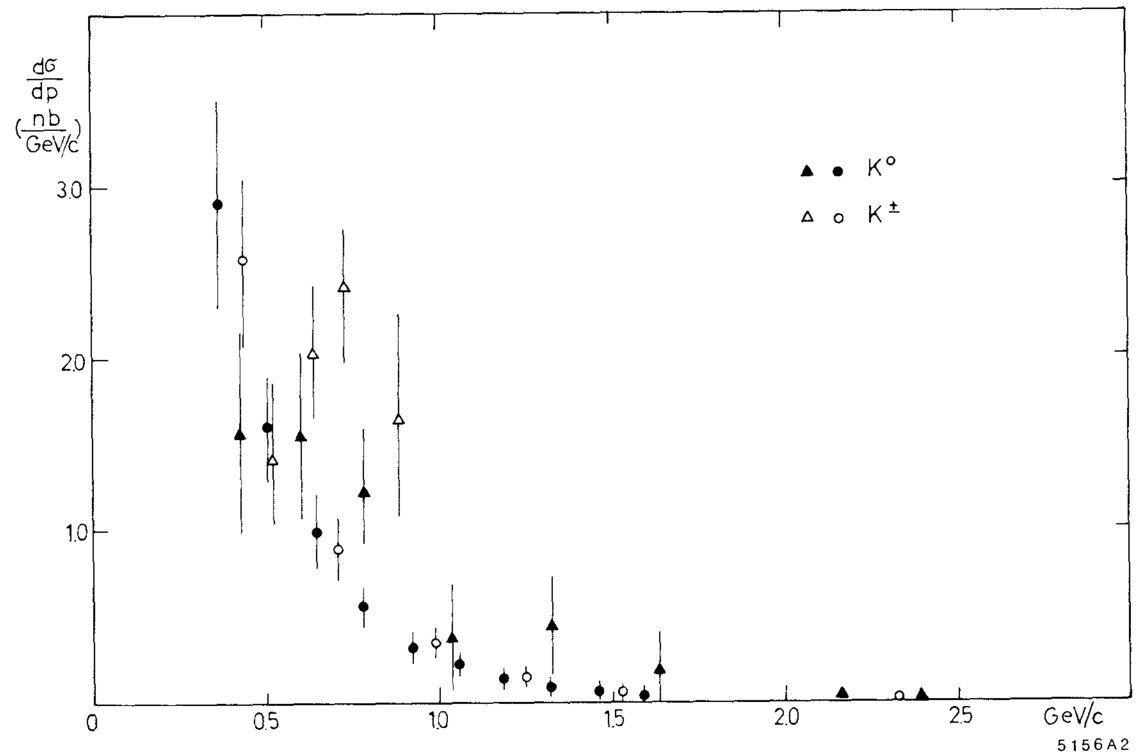

Fig. 2

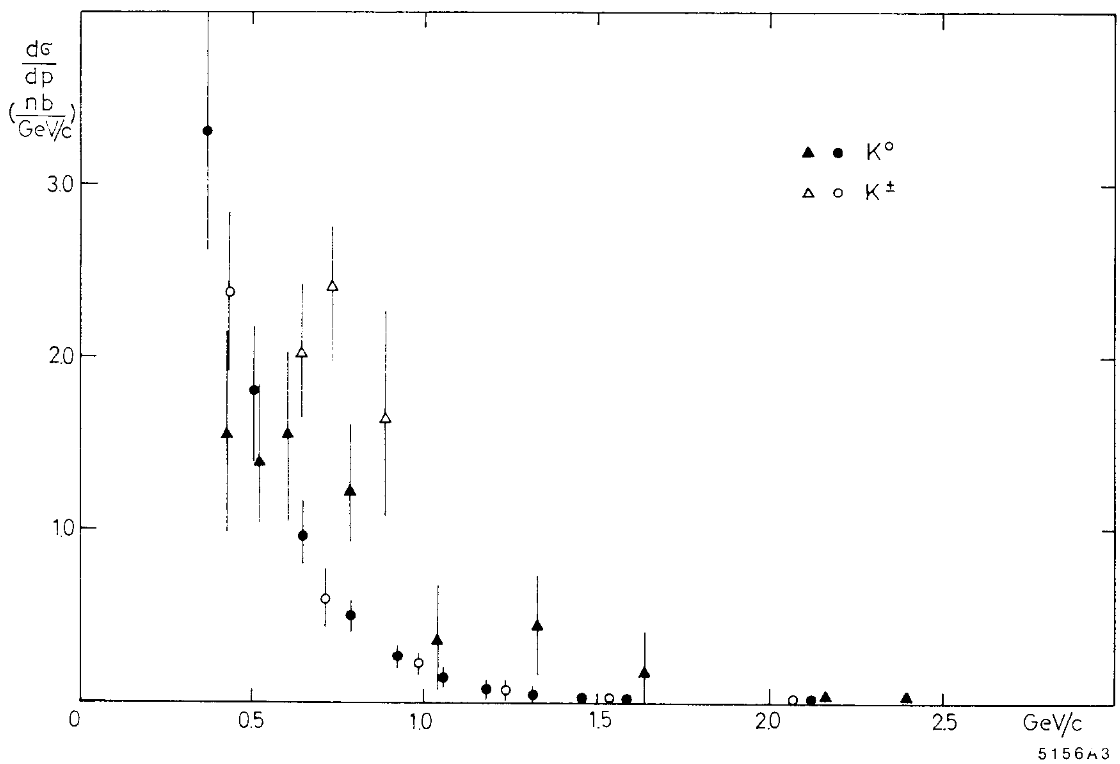

Fig. 3

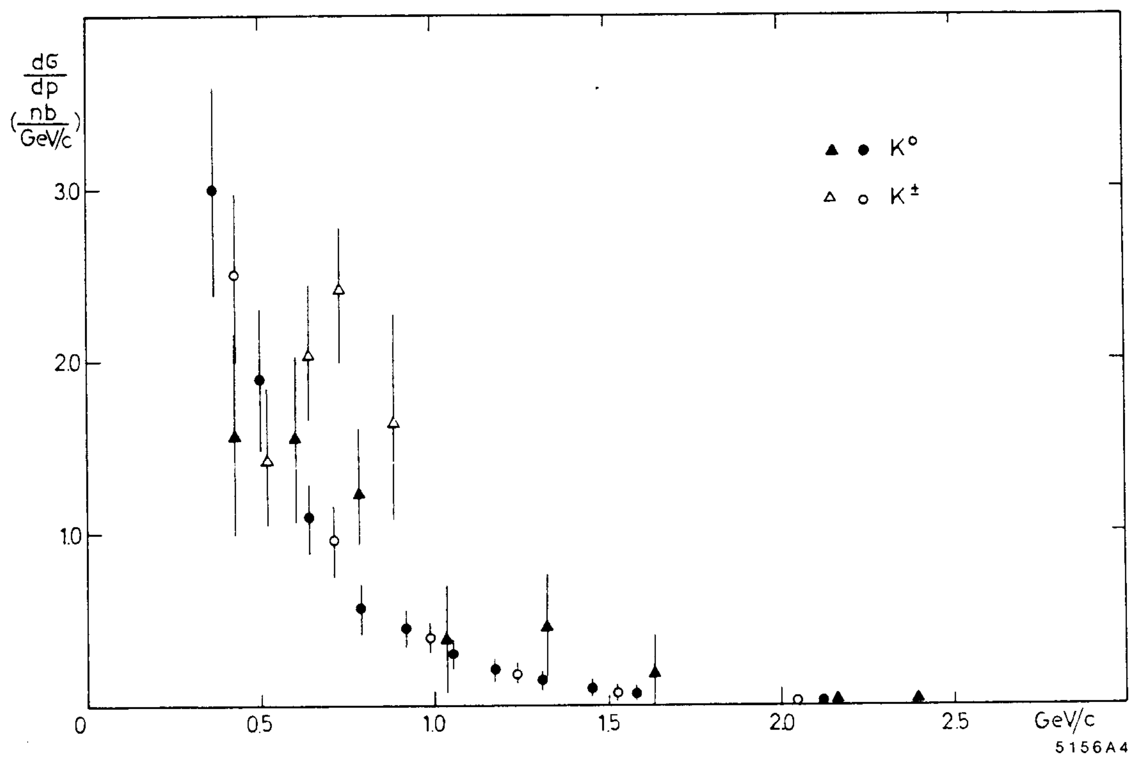

Fig. 4