

Theoretical Perspectives on Strange Physics

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1. General Overview

Kaons are heavy enough to have an interesting range of decay modes available to them, and light enough to be produced in sufficient numbers to explore rare modes with satisfying statistics. Kaons and their decays have provided at least two major breakthroughs in our knowledge of fundamental physics. They have revealed to us CP violation,¹ and their lack of flavor-changing neutral interactions warned us to expect charm.² In addition, $K^0 - \bar{K}^0$ mixing has provided us with one of our most elegant and sensitive laboratories for testing quantum mechanics.³ There is every reason to expect that future generations of kaon experiments with intense sources would add further to our knowledge of fundamental physics. This talk attempts to set future kaon experiments in a general theoretical context, and indicate how they may bear upon fundamental theoretical issues.

Figure 1 encapsulates some important trends in elementary particle physics. Two major philosophical approaches can be distinguished. The "onion-skin" philosophy emphasizes the search for more elementary constituents of matter as previous levels are shown to be composite. Many authors believe⁴ that the present "elementary" particles such as quarks, leptons, possibly gauge bosons and maybe Higgs fields are in fact composites of underlying preons on a distance scale $\leq O(10^{-16})$ cm. The unification merchants, on the other hand, emphasize⁵ the common origin and form of the different fundamental forces. It now seems established that the weak and electromagnetic interactions are at least partially unified in the Glashow-Salam-Weinberg (GSW) model,⁶ and that the nuclear forces are complicated manifestations of an underlying gauge theory QCD⁷ which is conceptually close to the GSW model. Together they constitute the "Standard Model" of elementary particles. Their family resemblance leads naturally to the hypothesis⁸ of a grand unified theory of all gauge interactions. If this exists, its grand symmetry must be broken at a very high energy scale⁹ within a few orders of magnitude of the Planck mass of $O(10^{19})$ GeV. At that scale there may be a final "superunification" with gravity. The provocative prefix "super" reflects my belief that such a final unification probably employs supersymmetry (SUSY) in some form.¹⁰ It may well be that SUSY makes an earlier appearance on the stage of unification. As we will see later, technical difficulties with GUTs would be alleviated if SUSY was effectively restored at an energy scale as low as 1 TeV.

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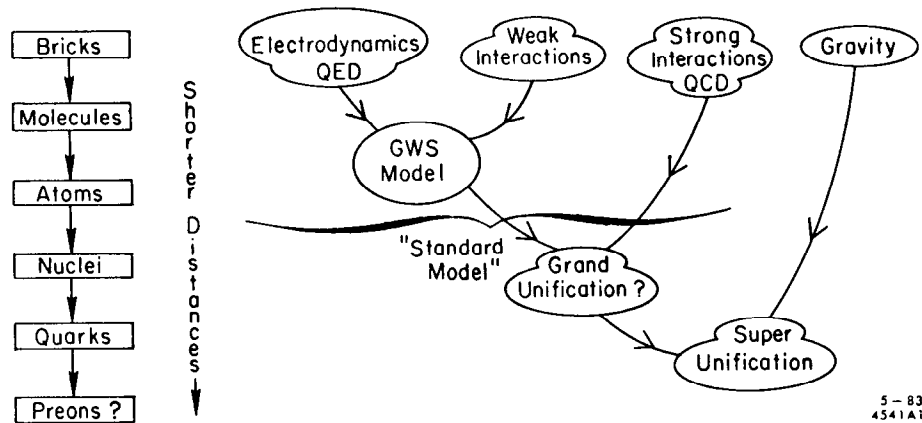


Fig. 1. A general overview of the “onion skin” and unification trends in elementary particle physics.

The different theoretical ideas introduced above are developed in greater detail in Sec. 2 of this talk. Then follows a survey of different experiments which would be done with an Intense Medium Energy Source of Strangeness, including rare K decays, probes of the nature of CP-violation, μ decays, hyperon decays and neutrino physics. Each experiment will be assessed for its interest as a test of the different theoretical ideas reviewed in Sec. 2. Section 3 concludes with a personal list of priorities for IMESS experiments. That terminates the physics content of this talk, and leaves us with two short sections of craziness. Section 4 discusses why and how quantum mechanics might be violated, and how one might test this in the $K^0 - \bar{K}^0$ system. Finally, Sec. 5 asks the unaskable question: how best should one proceed to explore strange physics in the future?

2. Survey of Different Theories

2.1 THE STANDARD MODEL

By this name we denote the gauge theory based on the group $SU(3)$ (for the strong interactions) $\times SU(2) \times U(1)$ (for the weak and electromagnetic interactions), with just 3 (or perhaps $N > 3$) identical generations of quarks and leptons, and with spontaneous gauge symmetry breaking furnished by a single doublet of Higgs fields.¹¹ As mandated by the suppression of flavor-changing neutral interactions of kaons, the Standard Model incorporates the GIM mechanism² so that neutral currents conserve flavor at the tree level, and flavor-changing neutral interactions at the one-loop level are suppressed by $O(\alpha m_q^2/m_W^2)$. The GIM mechanism is an automatic consequence of any theory¹² in which all quarks of the same charge and helicity have the same weak isospin and get their masses from the same Higgs doublet. The Standard Model provides not only a qualitative but also a quantitative explanation of the magnitudes of flavor-changing neutral interactions. For example, the magnitude of the $\Delta S = 2 K^0 - \bar{K}^0$ mass mixing was used¹³ to give an experimentally verified upper limit on the mass of

the charmed quark, and the c quark contribution to the $\Delta S = 1$ $K^0 \rightarrow \mu^+ \mu^-$ decay matrix element is small enough¹³ to be compatible with experiment.¹⁴ We will return later to these calculations in the 6-quark Kobayashi-Maskawa¹⁵ extension of the original 4-quark GIM model.² They will provide us with useful constraints on the angles θ_i ($i = 1, 2, 3$) characterizing the charged weak interactions of quarks,¹⁶ as well as on the mass of the t quark¹⁷ and thereby connect with the phase δ which is the sole source of CP violation in the Standard Model. There is no room for weak mixing angles between leptons in the Standard Model as the neutrinos are supposed to be massless. This means that the numbers $L_{e,\mu,\tau}$ of the different Lepton families are absolutely conserved, so that $\mu \not\rightarrow e\gamma$, $K^0 \not\rightarrow \mu e$, etc. Even if the neutrino masses are non-zero, upper limits¹⁸ on their values suppress $\Delta L \neq 0$ reactions to unobservably low rates unless there is some physics beyond the Standard Model.

2.2 MORE W 'S OR HIGGS?

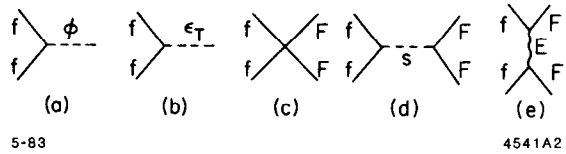
It is natural to entertain the possibility of extending the weak gauge group of the Standard Model, perhaps by making it more symmetric: $SU(2) \times U(1)$ becomes $SU(2)_L \times SU(2)_R \times U(1)$ with parity broken spontaneously, or perhaps by making it more unified: $SU(2)_L \times U(1)$ becomes $SU(3)_L$ or? In the absence of any attractive unified weak interaction models the unifiers⁵ have gone on to grand unification⁸ at a scale $\geq O(10^{15})$ GeV.⁹ Left-right symmetric models are still very much with us, as evidenced by discussions in the CP-violation section of this meeting.¹⁹ They predict two more W_R^\pm beyond the two W_L^\pm already discovered, and two neutral Z^0 bosons. Left-right symmetric models also expect a right-handed neutrino field and non-zero neutrino masses. The right-handed neutrino can acquire an $SU(2)_L \times U(1)$ invariant Majorana mass and may well be rather heavy. There are constraints²⁰ on the masses of W_R^\pm bosons and their mixing with W_L^\pm bosons which are considerably more restrictive if the ν_R are lighter than a kaon.²²

Why not have more Higgs bosons? They are needed in theories with larger gauge groups, and could easily be incorporated in the minimal $SU(2)_L \times U(1)$ theory. With two Higgs doublets one can implement²³ a global $U(1)$ symmetry which would solve the problem of strong CP-violation in QCD through the θ vacuum parameter.²⁴ SUSY theories actually require an even number of Higgs doublets in order to cancel out anomalies and give masses to all quarks and leptons.²⁵ Another motivation for multiple Higgses was to get an additional source of CP-violation²⁶ as discussed in the CP-violation session at this meeting.¹⁹ However, in models with a $U(1)$ symmetry and in SUSY theories with two Higgs doublets, the quarks of charge $+2/3$ get their masses from one Higgs doublet and the quarks of charge $-1/3$ from another. Thus these models incorporate the GIM mechanism,² the neutral Higgs couplings conserve flavor, and there is no extra Higgs source²⁶ of CP-violation. In general, theories with two Higgs doublets contain two charged bosons H^\pm and three neutrals including two scalars H^0 and H'^0 and one pseudoscalar a which becomes the light axion if a global $U(1)$ symmetry²³ is implemented.

2.3 DYNAMICAL SYMMETRY BREAKING

Many people regard Higgs fields as an unattractive wart on the face of gauge theory which they would prefer to burn out. One suggestion²⁷ is that Higgses are in fact composites of fermions bound together by some new “technicolor” gauge interaction which confines them within a range of $[\Lambda_{TC} = O(1 \text{ TeV})]^{-1}$. The role of the spontaneous symmetry breaking previously associated with the vacuum expectation of elementary Higgs fields is now usurped by dynamical symmetry breaking associated with condensates $\langle 0 | \bar{F} F | 0 \rangle$ of these “technifermions.” This mechanism gives masses to the vector bosons in a very economical way, but requires epicycles in order to give masses to quarks and leptons. One proposed solution²⁸ was to introduce new “extended technicolor” (ETC) interactions as shown in Fig. 2.

Fig. 2. (a) A Higgs- $f\bar{f}$ vertex metamorphoses into (b) a composite scalar $\epsilon_T - f\bar{f}$ vertex which requires (c) a four-fermion vertex that can be generated either by (d) scalar exchange or (e) vector ETC (E) exchange.

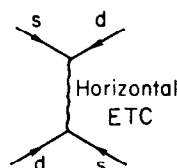
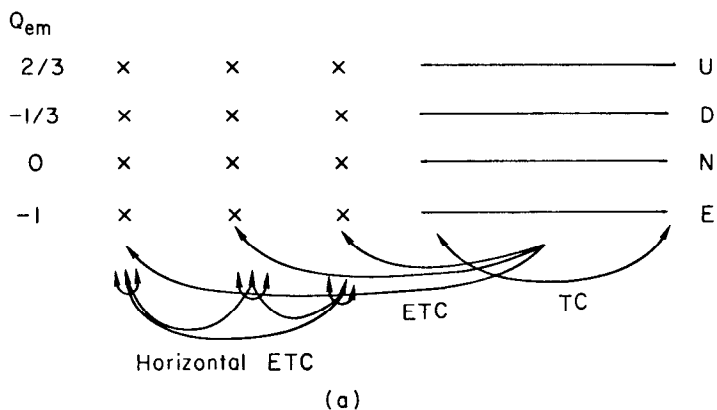


The conventional elementary Higgs- $\bar{q}q$ vertex is replaced by the composite Higgs- $\bar{q}q$ vertex which embodies a four-fermion $\bar{Q}Q\bar{q}q$ interaction generated by the exchange of massive ETC gauge bosons. One then has quark and lepton masses

$$m_{q,L} \approx \frac{\Lambda_{TC}^3}{m_{ETC}^2} \quad (1)$$

enabling the ETC boson masses to be estimated in terms of the known fermion mass spectrum. In a favored class²⁹ of ETC models there is one technigeneration (U, D, E, N) of techniquarks and leptons in parallel to the conventional generations (u, d, e, ν) etc. They are coupled to the conventional fermions by several different classes of ETC bosons, namely at least one per generation as indicated in Fig. 3a. In addition to these gauge bosons, a non-Abelian gauge theory of ETC must contain bosons coupling the different conventional generations to one another as seen in Fig. 3b. These “horizontal” ETC bosons have the same properties as the horizontal gauge bosons often postulated in the absence of an ETC motivation. However, in ETC theories the masses of these horizontal bosons are similar to those of the ETC bosons and can be estimated³⁰ using formula (1). There are other denizens of the technicolor zoo, some of whose masses are less tightly constrained. The theory contains²⁹ many “technipions” which are partners of the composite Higgs that escape being eaten by the W^\pm and Z^0 . They include color triplet bound states of techniquarks and technileptons called pseudoscalar leptoquarks P_{LQ} whose masses are expected²⁹ to be $O(150)$ GeV. There are also color singlet charged pseudoscalars P^\pm which should³¹ have masses $O(5 \text{ to } 14)$ GeV and have not been seen at PEP or PETRA,³² to the embarrassment of technicolor theorists. There should also be even lighter neutral

Fig. 3. (a) A sketch of the group structure of the simplest extended technicolor theories, and (b) a typical flavor-changing neutral interaction mediated by horizontal ETC boson exchange.



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(b)

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bosons $P^{0,3}$ whose masses can only³³ come from other interactions that we have not yet mentioned. One candidate³⁴ is vector leptoquark interactions of the Pati-Salam³⁵ type. These are expected^{34,36} to yield

$$m_{P^{0,3}} = O\left(1\frac{1}{2}\right) \text{ GeV} \times \left(\frac{300 \text{ TeV}}{m_{PS}}\right) \times O(2^{0\pm 1}) \quad (2)$$

and the non-observation of $K^\pm \rightarrow \pi^\pm P^{0,3}$ decay tells³⁶ us that

$$M_{P^{0,3}} \geq O(350) \text{ MeV} \quad (3)$$

which suggests via Eq. (2) an upper bound on m_{PS} of order 3000 TeV in such extended ETC theories. These vector leptoquarks will be met again in the discussion of rare K and Σ decays, along with the pseudoscalar leptoquarks.

Before leaving ETC theories, we should emphasize that models of the type described have severe problems with flavor-changing neutral interactions,³⁰ especially the magnitude of the $\Delta S = 2$ interaction responsible for $K^0 - \bar{K}^0$ mixing, and the absence so far of a $\Delta C = 2$ interaction leading to $D^0 - \bar{D}^0$ mixing. These problems can be traced^{30,36,37} to the failure of ETC theories to satisfy the usual conditions¹² for natural flavor conservation. People have not abandoned hope of solving these problems.³⁸ If and when a full solution is found, it may well affect some of the order of magnitude estimates of rare K decays that we make later. However, these estimates do apply to the models^{36,37} with partial solutions that do exist.

2.4 SUPERSYMMETRY

This is another response to the puzzles posed by Higgs fields. In order for the W^\pm and Z^0 bosons to have required masses of $O(100)$ GeV, there must be at least some Higgs bosons with comparable masses. However, elementary scalar fields are believed to receive contributions to their masses $\delta m_H = O(m_P \simeq 10^{19}$ GeV) when propagating (Fig. 4a) through the space-time foam that is believed to constitute the quantum gravitational vacuum.³⁹ More prosaically, they acquire $\delta m_H = O(m_X \simeq 10^{15}$ GeV) from interactions (Fig. 4b) with other Higgses in the grand unified theory vacuum.⁴⁰

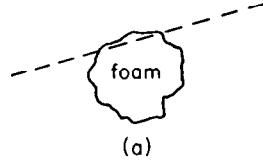
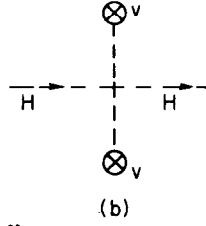


Fig. 4. A scalar field acquires large mass by propagating either through (a) space-time foam or (b) through the GUT vacuum.



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Even if these contributions to the light Higgs mass were to vanish or cancel miraculously, there would be radiative corrections as in Fig. 5a which would give $\delta m_H^2 = O(\alpha^n)(m_P^2 \text{ or } m_X^2)$. These must be cancelled out through $O(\alpha^{12})$, which requires some quite powerful magic. This can be provided either by “dissolving” Higgses so that they become composite on distance scales $\leq O(1 \text{ TeV}^{-1})$, as in technicolor theories,²⁷ or else by imposing supersymmetry (SUSY). In SUSY theories¹⁰ there are bosons and fermions with similar couplings. Since their quantum loops have opposite signs as indicated in Fig. 5b, fermions and bosons tend to cancel so that

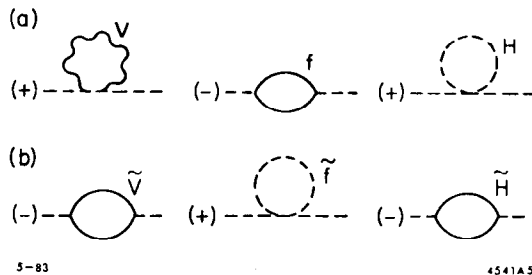
$$\delta m_H^2 \approx \frac{g^2}{16\pi^2} \int^{\Lambda \sim m_X \text{ or } m_P} d^4k \frac{1}{k^2} \approx \frac{g^2}{16\pi^2} \Lambda^2 \quad (4)$$

$$\rightarrow \frac{g^2}{16\pi^2} (m_B^2 - m_F^2) .$$

In order for δm_H to be less than the required Higgs masses $O(100 \text{ GeV})$ we see from Eq. (4) that one needs

$$(m_B^2 - m_F^2) \leq O(1 \text{ TeV}^2) . \quad (5)$$

Fig. 5. (a) Diagrams renormalizing the scalar mass, and (b) the diagrams which almost cancel them in a SUSY theory, with signs indicated in parentheses.



Thus the unseen SUSY partners of known particles cannot be very heavy. The basic building blocks of the simplest SUSY theories¹⁰ are supermultiplets containing pairs of particles differing in helicity by $\pm 1/2$:

$$\left. \begin{array}{l} \left(\begin{array}{l} 1 \\ \frac{1}{2} \end{array} \right) \text{ gauge boson} \\ \left(\begin{array}{l} \frac{1}{2} \\ 0 \end{array} \right) \text{ gaugino} \end{array} \right\} ; \left. \begin{array}{l} \left(\begin{array}{l} \frac{1}{2} \\ 0 \end{array} \right) \text{ quark } q, \text{ lepton } \ell, \text{ shiggs } \tilde{H} \\ \left(\begin{array}{l} 0 \\ 0 \end{array} \right) \text{ squark } \tilde{q}, \text{ slepton } \tilde{\ell}, \text{ Higgs } H \end{array} \right\} \quad (6)$$

Of this zoo of new particles, the lightest gaugino is likely to be the photino $\tilde{\gamma}$, while there may also be light neutral shiggs particles \tilde{H}^0 .⁴¹

The same flavor-changing neutral interactions that were the Nemesis³⁰ of technicolor theories also impose strong constraints on SUSY theories.⁴² The tree level couplings of all the new neutral particles must conserve flavor, and loop contributions to $\Delta F \neq 0$ interactions must be very small. The first requirement implies that the squark and slepton mass eigenstates must be spartners of pure quark and lepton mass eigenstates, and hence that the SUSY analogues of the Cabibbo-Kobayashi-Maskawa charged weak current mixing angles must be nearly identical with the familiar quark mixing angles. The suppression of loop diagrams requires a super-GIM mechanism, for example in the super-box diagram of Fig. 6 which is of order

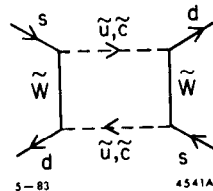
$$G_F \frac{\alpha}{4\pi} \sin^2 \theta_c \frac{m_c^2}{m_W^2} \quad (7)$$

only if

$$\frac{m_c^2 - m_u^2}{m_c^2 \text{ or } m_u^2} \simeq O\left(\frac{m_c^2}{m_W^2}\right) \leq O(10^{-3}) . \quad (8)$$

At first blush, these $\Delta F \neq 0$ neutral interaction conditions may seem difficult to satisfy, but in fact they emerge naturally in models with SUSY broken spontaneously. There all the squarks acquire universal m_q^2 of order $(30 \text{ GeV})^2$ or more, while all differences Δm_q^2 in squared masses are $O(\Delta m_q^2) \approx O(1) \text{ GeV}^2$ in the case of the first two generations. In these spontaneously broken SUSY theories super-loop contributions to $\Delta F \neq 0$ processes can be comparable⁴³ to the usual Standard Model contributions, and there may also be observable decays^{44,49,46} into light SUSY particles such as the $\tilde{\gamma}$ and \tilde{H}^0 , as we shall see later.

Fig. 6. A typical super-box diagram contributing to $\Delta F \neq 0$ neutral interactions in a SUSY theory which requires a super-GIM cancellation.

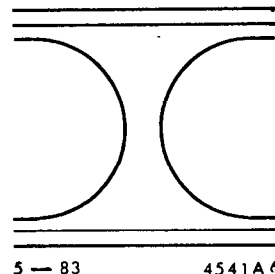


There are also SUSY models which have additional sources of symmetry violation beyond those in the Standard Model. For example, there exist possible additional sources of CP violation.⁴⁷ There is also the possibility of spontaneous lepton number violation due to vacuum expectation values for the spin-zero partners of neutrinos, whose effects are currently being investigated.⁴⁸ Indeed, we cannot even be sure that the photino and gluino couplings conserve flavor at the tree level. This is strongly suggested by the phenomenological constraints,^{44,45,46} but it can be argued⁴⁹ that one should take a more agnostic phenomenological viewpoint.

2.5 PREONS

It seems natural to suppose that the particles we currently regard as fundamental and elementary are in fact composite.⁴ We have already removed so many layers of the onion — why not one more? Moreover, we now know such an untidy profusion of quark and lepton flavors that it is very appealing to seek a simpler description of nature with fewer fundamental elements. We have already toyed with the idea of composite Higgs fields, so perhaps quarks, leptons and even gauge bosons are composite also on a distance scale $O(\Lambda^{-1})$ which may be as large as $O(1 \text{ TeV}^{-1})$. Suppose for example that the only preons are fermionic, with quarks and leptons containing at least three, while bosons contain two preons. Then one can visualize the observed interactions as being due to exchange forces as in Fig. 7, with the dominant forces of longest range occurring in channels corresponding to the lightest bosonic bound states with masses in $\ll \Lambda$. This is similar to the way π exchange is important in nuclear interactions because $m_\pi \ll 1 \text{ GeV}$. We would expect there to be additional forces in other channels corresponding to the exchanges of other bosons with masses $m = O(\Lambda)$ analogous to the ρ , ω and tensor meson exchanges of the conventional strong interactions.

Fig. 7. An exchange diagram which yields a new effective interaction in a preon theory.



This analogy with the conventional strong interactions is too glib and glosses over many technical puzzles.⁴ Why are the lightest bound states fermions with $m \ll O(\Lambda)$, whereas the fermionic bound states of QCD, the baryons, are heavy with masses $O(1 \text{ GeV})$? The underlying dynamics must obey consistency conditions⁵⁰ which are very difficult to satisfy. Why are there light bosons of spin 1, whereas the only light bosons in QCD have spin 0? It is very difficult to see how light gauge bosons would emerge unless the underlying dynamics already possessed the corresponding gauge invariance.

If we suspend our disbelief for a moment, we might expect that the exchanges of heavy bosons, or other dynamics on a scale of $O(1 \text{ TeV}^{-1})$ might generate all manner of novel interactions:

$$\frac{O(1)}{\Lambda^2} (\bar{q} q \bar{q} q , \bar{q} q \bar{\ell} \ell , qq\ell\ell , \dots) \quad (9a)$$

and more generally

$$\frac{O(1)}{\Lambda^{4-\frac{3n}{2}-m}} f^n B^m \quad (9b)$$

In general, we would not expect these new interactions to conserve conventional quantum numbers such as lepton number L , baryon number B , etc., since presumably different quark and lepton flavors share some preonic constituents in common, though some of the interactions (9) might be suppressed by some approximate chiral symmetry. The low-energy phenomenology of preon theories may in many ways resemble that of ETC theories, since observable new interactions of the form (9) probably involve the transformation of horizontal generation quantum numbers or the exchange of leptoquark quantum numbers.

3. Survey of Experimental Probes

3.1 RARE K DECAYS

There has been a lot of discussion⁴⁹ at this meeting of the whole gamut of rare K decays. Here I will only select a few possible experiments and concentrate on a subset of theories, treating composite models only cursorily and models with multiple gauge or Higgs bosons not at all. Each experiment will be discussed in turn for its interest within different frameworks, and the results are summarized in a table expressing my personal assessments.

3.1.1 $K_L^0 \rightarrow \mu e$ and $K^\pm \rightarrow \pi^\pm \mu e$ These two decays do not occur at all in the Standard Model, because it conserves separately both electron and muon lepton numbers L_e and L_μ . The same is also true in most SUSY models, though there is a possible escape route.⁴⁸ If any of the spin-zero sneutrino fields acquire a vacuum expectation value, the corresponding violation of lepton number would conceivably have repercussions in K decay, though the details are still being worked out.⁴⁸

In contrast, $K_L^0 \rightarrow \mu e$ and $K^\pm \rightarrow \pi^\pm \mu e$ are mainstream possibilities in technicolor theories,¹⁷ where there are many possible contributions to these decays. In the direct channel one can exchange a horizontal ETC gauge boson as in Fig. 8a. From the experimental upper limit⁵¹

$$\frac{\Gamma(K_L^0 \rightarrow \mu e)}{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)} < 0.63 \times 10^{-9} \quad (10)$$

one deduces³⁴ a lower limit on the squared mass of the corresponding gauge bosons:

$$m_{E_{d,s}}^2 \geq (3500, 170) \text{ TeV}^2 \quad (11)$$

which is exceedingly close to the estimates³⁰ obtained by using the formula (1). Therefore this mechanism should yield $K_L^0 \rightarrow \mu e$ decay at a rate very close to the experimental limit (10). Another contribution to $K_L^0 \rightarrow \mu e$ decay could come from the exchange of a Pati-Salam³⁵ vector leptoquark boson in the crossed channel as in Fig. 8b. In this case the limit (10) tells us³⁴ that

$$m_{PS} > O(300) \text{ TeV} \quad (12)$$

whereas we deduced earlier from the nonobservation of $K^\pm \rightarrow \pi^\pm P^{0,3}$ decay that $m_{P^{0,3}} > O(350) \text{ MeV}$ (3) and hence $m_{PS} < O(3000) \text{ TeV}$. Hence this mechanism should give rise to $K_L^0 \rightarrow \mu e$ decay at a rate within at most $O(10^{-4})$ of the present experimental upper limit (10). The exchanges of pseudoscalar bosons can also contribute to $K_L^0 \rightarrow \mu e$ decay. For example, the light pseudoscalars $P^{0,3}$ can be exchanged in the direct channel as in Fig. 8c, and the upper limit (10) tells us³⁰ that

$$m_{P^{0,3}}^2 > 2 \times 10^5 \theta^2 \text{ GeV}^2 \quad (13)$$

where the θ are some $\Delta F \neq 0$ mixing angles. Since the $P^{0,3}$ are expected on the basis of Eqs. (2) and (12) to weigh less than 3 GeV, the constraint (13) means that the mixing angles θ must be very small indeed. This possibility is not excluded within the ETC framework.³⁶ A contribution to $K_L^0 \rightarrow \mu e$ which is less easy to suppress is crossed channel pseudoscalar leptoquark (P_{LQ}) exchange as in Fig. 8d. The upper limit (10) tells us³⁴ that

$$m_{P_{LQ}} \geq O(150) \text{ GeV} \quad (14)$$

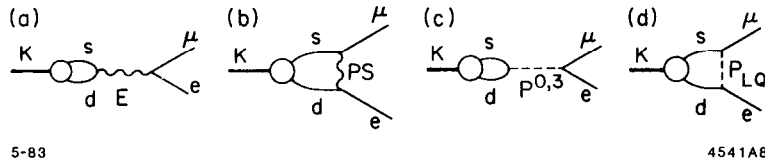


Fig. 8. Contributions to $K_L^0 \rightarrow \mu e$ decay from (a) direct channel HETC exchange, (b) cross channel Pati-Salam gauge boson exchange, (c) direct channel $P^{0,3}$ exchange and (d) crossed channel pseudoscalar leptoquark exchange.

which coincides with the best estimates^{29,33} of the mass of the P_{LQ} . Therefore ETC theories would also expect $K_L^0 \rightarrow \mu e$ via this mechanism to occur very close to the present experimental upper limit (10). This contribution is difficult to avoid because there is no way of preventing technileptons and techniquarks from binding to form the P_{LQ} , its mass is simply related by scaling to the $\pi^+ - \pi^0$ mass difference, and the $s \rightarrow \mu$ and $d \rightarrow e$ transitions are not expected to be suppressed by any small angle factors.

The above contributions all have analogues in the decays $K^\pm \rightarrow \pi^\pm \mu e$. The only difference is that the present experimental limit⁵²

$$\frac{\Gamma(K^\pm \rightarrow \pi^\pm \mu e)}{\Gamma(K^+ \rightarrow \pi^0 \mu^+ \nu_\mu)} < 1.5 \times 10^{-7} \quad (15)$$

is a weaker constraint than the $K_L^0 \rightarrow \mu e$ limit (10). For example, it means³⁰ that

$$m_{E_d}^2 > O(200) \text{ TeV}^2 \quad (16)$$

to be compared with the more stringent bound (11). However, the bound (15) is logically independent, since $K_L^0 \rightarrow \mu e$ proceeds via a pseudoscalar coupling to the quarks, whereas $K^+ \rightarrow \mu e$ proceeds via a vector or scalar coupling. Therefore in principle one decay could occur and not the other, though in practice models³⁷ predict comparable values for the ratios (10) and (15).

Searches for the decay $K_L^0 \rightarrow \mu e$ and also for $K^+ \rightarrow \pi^+ \mu e$ would be very interesting and topical from the point of view of technicolor theories of dynamical symmetry breaking.²⁹ These decays could also occur in preon models,⁴ but the rates are much more difficult to pin down. However, as we see later they may be the best ways to test preon models in rare K decays.

3.1.2 $K^+ \rightarrow \pi^+ + \text{Higgs}$ A parenthetic revival of this decay is appropriate here since an experiment to look for $K^+ \rightarrow \pi^+ \mu e$ decay could also be a useful Higgs search experiment. In the Standard Model there is just one physical neutral Higgs which should weigh more than 10 GeV.¹¹ However, there could be a lighter neutral Higgs in models with multiple Higgs doublets, such as supersymmetric models for example. We expect¹¹ the following to be the dominant decay modes of light neutral Higgs bosons:

$$\begin{aligned} 2m_e < m_H < 2m_\mu & : H \rightarrow e^+ e^- \text{ or } \gamma\gamma \\ 2m_\mu < m_H < 2m_\pi & : H \rightarrow \mu^+ \mu^- \\ 2m_\pi < m_H < 2m_K & : H \rightarrow \mu^+ \mu^- \text{ or } \pi^+ \pi^-, \pi^0 \pi^0 \\ 2m_K < m_H < 2m_\tau & : H \rightarrow K \bar{K} (n\pi) . \end{aligned} \quad (17)$$

Nuclear physics excludes¹¹ $m_H < O(15)$ MeV. There is a claim⁵³ that the absence of a Higgs peak in $\eta' \rightarrow \eta + \mu^+\mu^-$ excludes $m_H < O(400)$ MeV, but the experimental upper limit is not far below the calculation⁵⁴ of the $\eta' \rightarrow \eta + H$ decay rate which is not very reliable. The range $m_\pi < m_H < 2m_\mu$ is almost excluded by a search in the $K^+ \rightarrow \pi^+ + e^+e^-$ decay.⁵⁵ There is no information about lighter Higgses because of background problems, and heavier Higgses would have decayed into $\mu^+\mu^-$ and could not have been seen in this experiment. It would be interesting to perform sensitive searches for Higgses as spikes in $K^+ \rightarrow \pi^+ + (e^+e^- \text{ and } \mu^+\mu^-)$, which could be byproducts of $K^+ \rightarrow \pi^+\mu e$ experiments.

3.1.3 $K^+ \rightarrow \pi^+ + \text{Nothing}$ “Nothing” comes in many varieties of unobserved neutrals, including the following:

$K^+ \rightarrow \pi^+ + \text{axion}$ This is a two-body decay for which the experimental upper limit⁵⁷ is

$$\frac{\Gamma(K^+ \rightarrow \pi^+ + a)}{\Gamma(K^+ \rightarrow \text{all})} < 3.8 \times 10^{-8} \quad (18)$$

to be compared with a theoretical rate⁵⁸ which is generally $\geq O(10^{-6})$. It may be possible to arrange for a partial cancellation in the decay amplitude by a judicious choice of charged Higgs boson mass, but it seems very difficult to suppress the decay rate below the upper limit (18). This is just one of the many reasons why the conventional axion should be dead. However, its Frankenstein refuses to accept⁵⁹ the mortality of his creature: perhaps it is an undead zombie which still awaits the silver stake of another experiment to be driven through its heart?

$K^+ \rightarrow \pi^+ + \text{familon}$ Wilczek⁶⁰ has invented another light pseudoscalar for you, this time the exactly massless familon f , a Goldstone boson of a conjectured family or generation symmetry. He estimates⁶⁰ the branching ratio

$$B(K^+ \rightarrow \pi^+ + f) = O(10^{14}) \left(\frac{1 \text{ GeV}}{F} \right)^2 \quad (19)$$

where F is an analogue of the pion decay constant f_π . Cosmology⁶¹ tells us that $F \leq O(10^{12})$ GeV, whereas present experiments (18) tell us that $F \geq 5 \times 10^{10}$ GeV. This small gap could be closed by an experiment sensitive to a branching ratio of $O(10^{-10})$. Here is a theory which could be excluded by a forthcoming rare K decay experiment.

$K^+ \rightarrow \pi^+ + \sum_i \nu_i \bar{\nu}_i$ The experimental upper limit⁵⁷ on this three-body decay is

$$\frac{\Gamma(K^+ \rightarrow \pi^+ + \sum_i \nu_i \bar{\nu}_i)}{\Gamma(K^+ \rightarrow \text{all})} < 1.4 \times 10^{-7} \quad (20)$$

The GIM loop diagrams⁶² of the Standard Model give for each neutrino flavor i

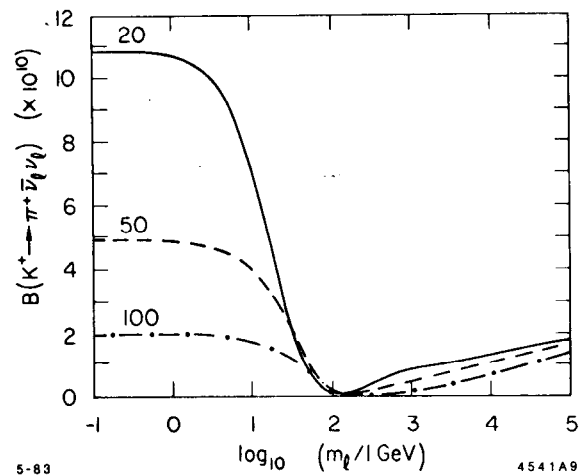
$$B(K^+ \rightarrow \pi^+ \nu_i \bar{\nu}_i) = 0.6 \times 10^{-6} \frac{|\tilde{D}_i|^2}{U_{us}^2} \times \text{QCD factor} \quad (21)$$

with

$$\tilde{D}_i = \sum_{j>1} U_{js}^* U_{jd} \bar{D} \left(\frac{m_{q_j}^2}{m_W^2}, \frac{m_{L_i}^2}{m_W^2} \right). \quad (22)$$

In these formulae, the QCD correction factor is reasonably well known, the U_{jk} denote Cabibbo-Kobayashi-Maskawa mixing matrix elements, and the \bar{D}_i are known⁶² kinematic functions of the quark and lepton masses. As can be seen in Fig. 9, \bar{D} vanishes when the mass of the lepton associated with ν_i is $O(1 \text{ to } 3) \times m_W$. The Cabibbo-Kobayashi-Maskawa mixing angle factors and the unknown t quark mass are constrained by other $\Delta F \neq 0$ amplitudes⁶³ such as $K_L^0 \rightarrow \mu^+ \mu^-$ which gives an upper bound¹⁷ on the $K^+ \rightarrow \pi^+ + \nu_e \bar{\nu}_e$ decay rate⁴⁴ as seen in Fig. 10. The known amount of $K^0 - \bar{K}^0$ mixing provides⁴⁴ a lower bound on the $K^+ \rightarrow \pi^+ + \nu_i \bar{\nu}_i$ (for light associated charged leptons, ℓ_i) which can also be seen in Fig. 10. Indicated explicitly in Fig. 10 is the uncertainty in the $K^0 - \bar{K}^0$ mixing constraint arising from our ignorance of the $\Delta S = 2$ operator matrix element $\langle \bar{K}^0 | (\bar{s} d)^2 | K^0 \rangle$, usually expressed as a coefficient R times the value obtained by inserting the vacuum intermediate state.

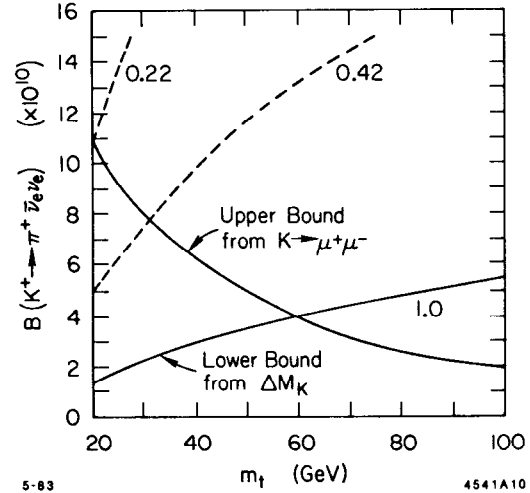
Fig. 9. Variation of the $K^+ \rightarrow \pi^+ \bar{\nu}_\ell \nu_\ell$ branching ratio with the mass of the lepton ℓ associated with ν_ℓ , for different values of the t quark mass in GeV.



There was a recent advance⁶⁴ on this point when it was realized that this matrix element would be related by $SU(3)$ and PCAC to the known $\Delta I = 3/2$ $K^+ \rightarrow \pi^+ \pi^0$ decay amplitude, yielding $R = 0.33$ with perhaps a (30 to 60)% uncertainty indicated by the dashed lines in Fig. 10. Another potential uncertainty arises in the $K_L^0 \rightarrow \mu^+ \mu^-$ constraint. In contrast to the $\Delta S = 2$ operator, it can get significant contributions from SUSY super-loop diagrams,^{43,65} which could lift the bound for a given value of m_t . If we take conservatively the solid lines in Fig. 10 corresponding to $R = 1$ and no SUSY contribution to $K_L^0 \rightarrow \mu^+ \mu^-$, we find they are only compatible for $m_t < 60$ GeV, the Buras¹⁷ bound. We get⁴⁴ the absolute upper bound

$$\sum_i B(K^+ \rightarrow \pi^+ + \nu_i \bar{\nu}_i) < N_\nu \times 1.1 \times 10^{-9} \quad (23)$$

Fig. 10. Upper and lower bounds on the $K^+ \rightarrow \pi^+ \bar{\nu}_e \nu_e$ decay rate for different values of m_t . The numbers refer to the matrix element factor R .



on the decay rate and a lower bound⁴⁴

$$\sum_i B(K^+ \rightarrow \pi^+ + \nu_i \bar{\nu}_i) > N_\nu^{light} \times 1.4 \times 10^{-10} \quad (24)$$

where N_ν^{light} denotes the number of neutrinos whose associated leptons are light. The numbers (23) and (24) neglect the contributions of the heavier quarks to the GIM loop diagrams. Because of the $K_L^0 \rightarrow \mu^+ \mu^-$ and $K^0 - \bar{K}^0$ mixing constraints, they can only slightly increase the upper bound on the $K^+ \rightarrow \pi^+ + \nu_i \bar{\nu}_i$ decay rate.⁴⁴ However, the lower bound can be destroyed by cancellations between the t quark and the 8th quark, though this cannot⁴⁴ happen if the 8th quark mixing angles are in the hierarchial ratio

$$\left| \frac{U_{j8}}{U_{jt}} \right|^2 \approx \frac{m_t}{m_8} \quad (25)$$

If we ignore all this uncertainty, the bound (24) can be used to establish an upper bound on the number of neutrinos whose associated charged leptons do not have masses too close to the W mass as seen in Fig. 11. The curve was plotted using the conservative value $R = 1$ for the $\Delta S = 2$ matrix element.

Our conclusion is that the decay $K^+ \rightarrow \pi^+ +$ unobserved neutrals is very interesting in the context of the Standard Model. If the branching ratio is significantly larger than the upper limit of 3.3×10^{-9} (23) this would be a signal for new physics: perhaps more than three neutrinos or? Paradoxically, a branching ratio significantly lower than 4.2×10^{-10} (24) could also be a signal for new physics, perhaps a cancellation between third and fourth generation quarks, or?

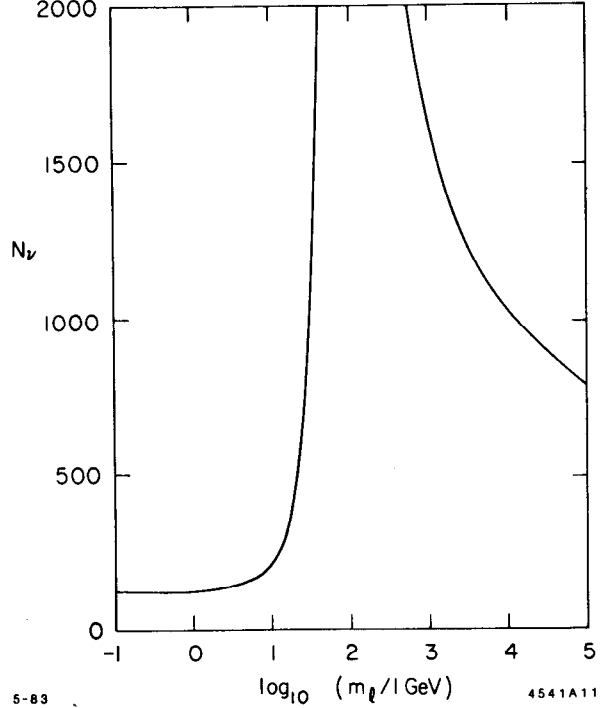


Fig. 11. The present upper bound on the number of neutrinos from $K^+ \rightarrow \pi^+ + \bar{\nu}_\ell \nu_\ell$ decay²² as a function of the mass m_ℓ of the charged lepton associated with ν_ℓ .

$K^+ \rightarrow \pi^+ + \nu_i \bar{\nu}_{j \neq i}$ So far we have concentrated on processes where the outgoing neutrino and antineutrino have the same flavor. ETC theories and preon models could also give neutrinos and antineutrinos of different flavors, by mechanisms similar to those by which they give $K^+ \rightarrow \pi^+ \mu e$. Unfortunately, since these decays are experimentally indistinguishable from $K^+ \rightarrow \pi^+ + \nu_i \bar{\nu}_i$, they will not be identifiable unless their branching ratios are above a few times 10^{-9} , which seems unlikely in view of the $K_L^0 \rightarrow \mu e$ bound (10).

$K^+ \rightarrow \pi^+ +$ photinos $\tilde{\gamma}$ or shiggses \tilde{H}^0 The light neutral particles expected in the SUSY theories of Sec. 2.4 could also be pair-produced in K^+ decays, either at the tree level,^{45,49} or more plausibly (?) via super-loop diagrams analogous to those for decays into $\nu_i \bar{\nu}_i$. We expect^{44,46} decays into gravitino pairs to be negligible, and we expect

$$B(K^+ \rightarrow \pi^+ + \tilde{\gamma} + \text{gravitino}) \leq 2B(K^+ \rightarrow \pi^+ + \tilde{\gamma} + \tilde{\gamma}) \quad (26)$$

and it is probably much smaller unless the scale of SUSY breaking is surprisingly small. The photino pair-production rate depends on the unknown spectrum of SUSY particles, but in general^{44,46}

$$B(K^+ \rightarrow \pi^+ + \tilde{\gamma} + \tilde{\gamma}) \leq O\left(\frac{1}{10}\right) B(K^+ \rightarrow \pi^+ + \nu_i \bar{\nu}_i) \quad (27)$$

so that it is unlikely to push the rate for $K^+ \rightarrow \pi^+ +$ nothing up substantially. However, in some theories the lightest SUSY particles may be neutral shiggses \tilde{H}^0 , and they could⁴⁴ be pair-produced with branching ratios comparable with those for a single neutrino flavor:

$$\frac{B(K^+ \rightarrow \pi^+ + \tilde{H}^0 \tilde{H}^0)}{B(K^+ \rightarrow \pi^+ + \nu_e \bar{\nu}_e)} = \frac{\left(-7 - 5 \ln \frac{m_t^2}{m_W^2}\right) v_1^2 + \left(-4 + 2 \ln \frac{m_t^2}{m_W^2}\right) v_2^2}{\left(-1 - 3 \ln \frac{m_t^2}{m_W^2}\right) (v_1^2 + v_2^2)} \quad (28)$$

where v_1 and v_2 are the vacuum expectation values of the two Higgs doublets in SUSY theories. We expect $v_1 \geq v_2$, and if $v_1 \ll v_2$ the ratio (28) becomes 0.88 for $m_t = 20$ GeV. We conclude that the decay $K^+ \rightarrow \pi^+$ + nothing can also be interesting for SUSY theories.

3.2 COMPARISONS

My personal assessments of the interest of different rare K decay experiments from the points of view of different theories are shown in Table 1. The crosses indicate frameworks where a given process does not occur. This does not necessarily mean that the experiment is not interesting from the point of view of that theory. As Sherlock Holmes has observed, the best clue is often the dog that does not bark!

Table 1. Testing Theories in Rare K Decays

Theories		Standard	Technicolor	Super-	Preon
		Model		Symmetry	Models
Experiments					
$K_L^0 \rightarrow \mu e$		×	√√√√	√?	√
$K^+ \rightarrow \pi^+ \mu e$		×	√√	√?	√
	light pseudo scalar	×	did not occur	×	×
$K^+ \rightarrow \pi^+$ + nothing	$\nu \bar{\nu}$	√√√√√	√	×	√
	SUSY particles	×	×	√√√	×

The number of checks in each box correspond to the amount of interest I personally have in confirming the non-zero prediction of the corresponding theory for that experiment. Do they correspond to the number of years of an experimentalist's life that it might be worth devoting to that test of the theory? The question marks denote cases where the theory is not yet completely clear.

Since Table 1 attaches considerable significance to $K^+ \rightarrow \pi^+ + \text{nothing}$ experiments, it is worthwhile to make comparisons with other nothing production experiments, in which the existence of nothing is inferred by tagging the event with a pion, photon, $\pi\pi$ or whatever.

$\pi^0 \rightarrow \text{Nothing}$ Clearly one way of tagging this is in $K^+ \rightarrow \pi^+ + (\pi^0 \rightarrow \text{nothing})$, and it has been discussed as a way of producing pairs of massive photinos⁶⁶ or neutrinos. There was some discussion at the Workshop of making and tagging the π^0 in another way, perhaps by $\pi^+ + d \rightarrow p + p + (\pi^0 \rightarrow \text{nothing})$, but it was concluded that background problems made this uncompetitive with K^+ decay.

$J/\psi \rightarrow \text{Nothing}$ This is in principle a reliable way to count neutrinos, using the $\psi' \rightarrow J/\psi + \pi^+ \pi^-$ decay to tag $J/\psi \rightarrow \text{nothing}$, but it is not very sensitive. From the expected⁶⁷ decay rate

$$\frac{B(J/\psi \rightarrow \bar{\nu} \nu)}{B(J/\psi \rightarrow e^+ e^-)} \simeq 2 \times 10^{-8} N_\nu \left[m_{J/\psi} \text{ (GeV)} \right]^2 \quad (29)$$

and the experimental upper limit⁶⁸ of 1/10 on this ratio we can infer that

$$N_\nu \leq 5 \times 10^5 . \quad (30)$$

The cosmologists⁶⁹ with their upper limit of three or four neutrinos should be laughing at us particle physicists! The constraints on N_ν from $K^+ \rightarrow \pi^+ + \text{nothing}$ ⁴⁴ and from the late stages of stellar evolution⁷⁰ are much more stringent than (30), even though somewhat uncertain. As for SUSY particles, the rate for $J/\psi \rightarrow \tilde{\gamma} \tilde{\gamma}$ depends sensitively on the unknown \tilde{c} squark masses,⁷¹ and, in fact, requires a difference between the masses of partners of the left- and right-handed \tilde{c} squarks.

$\Upsilon \rightarrow \text{Nothing}$ Many of the same remarks apply, except that the greater mass of the Υ means that one has better sensitivity than (29), but experimentalists have not yet got around to quoting an upper limit on $\Upsilon \rightarrow \text{nothing}$. If they could establish that this was $\leq O(1/10)B(\Upsilon \rightarrow e^+ e^-)$ then one would get a limit: $N_\nu \leq O(5000)$, which begins to be comparable with present limits from $K^+ \rightarrow \pi^+ + \text{nothing}$ decay.

$e^+ e^- \rightarrow \text{Nothing}$ This can be tagged⁷² by a bremsstrahlung γ , and there is currently a proposal⁷³ to look for these at PEP with a sensitivity corresponding to $N_\nu \leq O(10)$ at a center-of-mass energy of $\sqrt{s} = 29$ GeV. This search is also sensitive to photinos⁷⁴ if they weigh less than about 10 GeV and $M_{\tilde{e}} \leq O(50)$ GeV.

$Z^0 \rightarrow \text{Nothing}$ This is of course the neutrino counting experiment par excellence, and it is easy⁷⁵ to gain a sensitivity to $\Delta N_\nu \ll 1$ in the reaction $e^+ e^- \rightarrow Z^0 + \gamma$. In contrast, $Z^0 \not\rightarrow \tilde{\gamma} \tilde{\gamma}$ at the tree level.

The reactions listed above were arranged by order of increasing energy, and hence sensitivity to heavier “nothing” particles. Since different experiments see different mass ranges as well as having differing sensitivities to different species of “nothing” particle, a complete understanding of the spectrum of quasi-stable neutral particles will require performing all the experiments. For example, if one

found an apparent “ $N_\nu > 3$ ” in both $Z^0 \rightarrow \text{nothing}$ and $e^+e^- \rightarrow \text{nothing}$ at $\sqrt{s} = 29$ GeV, would $K^+ \rightarrow \pi^+ + \text{nothing}$ tell us $N_\nu = 3$, in which case we would suspect the existence of novel neutrals, probably with masses larger than 150 MeV? Or if we were satisfied from Z^0 and $e^+e^- \rightarrow \text{nothing}$ that $N_\nu = 3$, we could use the rate for $K^+ \rightarrow \pi^+ + \text{nothing}$ as a probe of Standard Model dynamics. Either way, the decay $K^+ \rightarrow \pi^+ + \text{nothing}$ has an important role to play.

The same probably cannot be said for the decay $\mu \rightarrow e + \text{nothing}$ which can be probed at a π or K factory.⁷⁶ One difficulty is that the dominant decay of the μ is $\mu \rightarrow e + (\text{nothing} = \nu\bar{\nu})$, so that there is a big background. One does not usually expect L_e and L_μ to be violated in spontaneously broken SUSY theories, so $\mu \not\rightarrow e + \tilde{\gamma} + \tilde{\gamma}$, whereas the analogous $K^+ \rightarrow \pi^+ + \tilde{\gamma} + \tilde{\gamma}$ decay can occur at $O(1/10)$ of the $K^+ \rightarrow \pi^+ + \nu + \bar{\nu}$ rate. For the same reason one does not expect $\mu \rightarrow e + \text{axion}$ decay, whereas the absence of $K^+ \rightarrow \pi^+ + \text{axion}$ has already been used to dispute the very existence of an axion. However, the two-body decay $\mu \rightarrow e + \text{familiar}$ could be interesting:

$$B(\mu \rightarrow e + f) \approx O(10^{12}) \left(\frac{1 \text{ GeV}}{F} \right)^2 \quad (31)$$

to be compared with the analogous $K^+ \rightarrow \pi^+ + f$ decay rate (19). For this reason a continued search for the two-body $\mu \rightarrow e + \text{nothing}$ decay down to a limit of order 10^{-12} may still be valuable.

3.3 MUON AND HYPERON DECAYS

Now that we have broached the subject of $\Delta L_\mu \neq 0$ processes, it is worthwhile to make a systematic comparison of one with another, and also with the rare K decays that were the subject of Sec. 3.1.1. Table 2 is a compilation of limits from various rare processes⁷⁷ on the possible masses of certain species of exotic beasts. It is adapted from a table developed by Shanker.⁷⁸ It is always difficult to achieve a consensus on the couplings to be assumed for imaginary particles, and I have made somewhat different assumptions than Shanker.⁷⁸ These do not affect the ratios of different limits on the same particle from different experiments so much as they affect the absolute values of the limits, especially for theories with many Higgs doublets. Shanker⁷⁸ assumed couplings to leptons of order $g_2 m_\tau/m_W$ and to quarks of order $g_2 m_b/m_W$, whereas I have chosen values three orders of magnitude smaller. These values correspond more closely to the masses of the light quarks and leptons involved in the processes we are considering. If they appear too small, recall that there are probably mixing angle factors which enter when we consider reactions that violate L_e and/or L_μ , and probably also when one changes quark generation: $s \rightarrow d$. For this reason it is difficult to interpret directly the order-of-magnitude limit on the Higgs mass coming from processes violating L_e and L_μ . However, it probably is reasonable to interpret Table 2 as indicating that among such processes, multiple Higgs theories are most sorely tested by anomalous muon conversion $\mu Z \rightarrow eZ$. This need not be the case for other theories of L_e and L_μ violation, which may well be more severely tested by $\mu \rightarrow e\gamma$ or $\mu \rightarrow e\bar{e}e$ searches.

Table 2. Comparison of Mass Limits From Rare Processes

Process	Multiple Higgs	Pseudoscalar Leptoquark	Vector Leptoquark	Experimental Limit
$B(\mu \rightarrow e\gamma)$	0.2	—	—	1.9×10^{-10}
$B(\mu \rightarrow e\bar{e}e)$	0.4	—	—	1.9×10^{-9}
$B(\mu Z \rightarrow eZ)$	11.0	$0.15 \times \theta$	$60 \times \theta$	7×10^{-11}
$B(K_L^0 \rightarrow \mu e)$	7.0	0.18	93	2×10^{-9}
$B(K_L^0 \rightarrow \mu\mu)$	4.7	$0.12 \times \theta$	$62 \times \theta$	9×10^{-9}
$B(K_L^0 \rightarrow ee)$	7.0	$0.18 \times \theta$	$95 \times \theta$	2×10^{-9}
$B(K^+ \rightarrow \pi^+\mu e)$	0.7	0.01	3.5	7×10^{-9}
Δm_K	150	—	—	$\approx 3.5 \times 10^{-15}$ (GeV)
	mass limits in GeV	mass limits in TeV	mass limits in TeV	

Table 2 features my personal guesses as to possible mixing angle factors in leptoquark interactions which would arise if one makes the normal identifications of lepton and quark generations: $s \leftrightarrow \mu$, $d \leftrightarrow e$. We see again that the best limits on leptoquark masses seem to come from the $K_L^0 \rightarrow \mu e$ decay limit discussed in Sec. 3.1.1, with $K^+ \rightarrow \pi^+\mu e$ providing less stringent limits as we expected. We also see that the decays $K_L^0 \rightarrow \mu\mu$ and ee give less interesting limits because of mixing angle factors. Since the decay $K_L^0 \rightarrow \mu\mu$ has already been observed at a rate close to the unitarity limit, after $K_L^0 \rightarrow \mu e$ the next most interesting of these leptonic K decays to look for may be $K_L^0 \rightarrow ee$.

Another decay which gives access to similar physics is $\Sigma \rightarrow p\mu e$. Unfortunately, it has been estimated⁷⁷ that the upper limit on $K \rightarrow \mu e$ decay already suggests that

$$B(\Sigma \rightarrow p\mu e) \leq O(10^{-12}) . \quad (32)$$

By comparison, the most stringent upper limits over rare Σ decays are around 10^{-6} , which prompted one experimentalist at this meeting to describe the range (32) as “an awful long ways to go.” K decays seem more sensitive probes of L_e and L_μ violating physics.

One point to be recalled in looking at Table 2 is that as the sensitivity to a rare decay branching ratio B is increased, the sensitivity to heavy boson masses only increases as

$$m_{heavy} \propto B^{-\frac{1}{4}} . \quad (33)$$

Thus an order of magnitude increase in sensitivity corresponds to a change in the limit on m_{heavy} by less than a factor of two. Moreover, this change is often

small compared with other uncertainties in the calculation, such as those in the values of coupling constants and of matrix elements. Progress will not be rapid, nor will its interpretation be unambiguous.

3.4 CP-VIOLATION

The discovery¹ of CP-violation in the neutral K^0 system has so far been one of the greatest contributions by kaons to our physical knowledge. Unfortunately, despite its cosmic significance⁷⁹ this original manifestation of CP-violation in the $K^0 - \bar{K}^0$ mass matrix is still the only CP-violating phenomenon observed experimentally. Table 3 presents a list of interesting CP-violating observables together with the corresponding predictions of the theoretical frameworks introduced in Sec. 2.

We see that four of the observables appear in K decays, while two involve hyperon decays. First we have the fundamental $K_1 - K_2$ mass mixing parameter ϵ . Its value can be fitted but not predicted in the Standard Model where

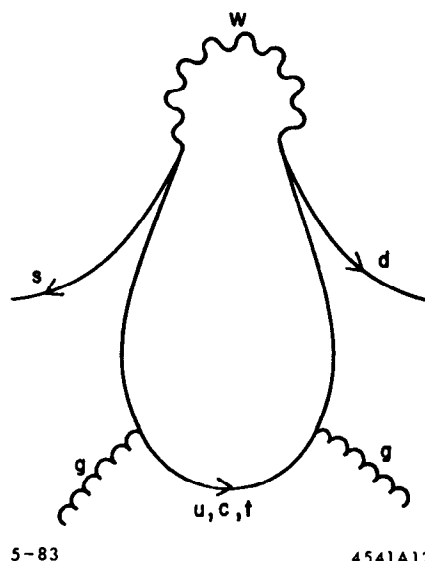
$$\epsilon \propto s_2 s_3 \sin\delta \text{ for small } \theta_2, \theta_3 . \quad (34)$$

The observed value of $\epsilon = O(10^{-3})$ can be understood very naturally if s_2 and s_3 are both considerably smaller than $\sin\theta_c$, as would be the case if the b quark lifetime turns out to be $O(10^{-12})$ seconds. In models with additional gauge bosons and left-right symmetric models in particular the value of ϵ is related to the masses of the heavier gauge bosons. In multiple Higgs models the value of ϵ is related to the spectra and couplings of the Higgs bosons. Technicolor models tend to predict magnitudes of the CP-conserving real part of the $\Delta S = 2$ $K_1 - K_2$ mass mixing which are much too large, and give no understanding why ϵ should be so small.³⁰ The last column in Table 3 introduces the non-perturbative QCD CP-violating θ vacuum angle,²⁴ which cannot contribute significantly to ϵ because of the severe upper limit of $O(10^{-9})$ on θ coming from the non-observation⁸⁰ of a neutron electric dipole moment. Next we turn to model predictions for intrinsic CP violation ϵ' in the $K \rightarrow 2\pi$ decay amplitude and the ratio ϵ'/ϵ . Figure 12 depicts a Penguin diagram, which plays a crucial role in calculations of this quantity. As reviewed by Wolfenstein,¹⁹ this meeting has witnessed considerable progress in the elucidation of the Standard Model⁸¹ and multi-Higgs model⁸² predictions. The Standard Model predicts⁸¹ a definite sign (positive) as well as a lower bound on the magnitude. Multiple Higgs models also predict⁸² a definite sign (negative) and a magnitude much larger than the Standard Model bound. These models are on the borderline of experimental exclusion, but may not yet have crossed it. Left-right symmetric gauge models are unfortunately rather less specific in their predictions for ϵ'/ϵ . The naive technicolor models seem⁸³ to predict too large a value for ϵ'/ϵ , but this defect might also be rectified if one could cure³⁸ the other flavor-changing neutral interaction problems³⁰ of such theories.

Table 3. CP Violating Observables and Model Predictions

	Standard Model	Multiple W	Multiple H	Technicolor	Super-symmetry	θ_{QCD}
ϵ	fit	$\leftrightarrow m_{WR}$	$\leftrightarrow m_H$	too big?	as for Standard Model	0
ϵ'/ϵ	$\geq 2 \times 10^{-3}$	$\sim 10^{-3}$	$\leq -2 \times 10^{-2}$	large ?		0
$\eta_{+-0} - \eta_{+--}$	$O(\epsilon')$	$O(10^{-5})$?	large ?		0
P_n^μ in $K \rightarrow \pi\mu\nu$	0	0	$\leq 5 \times 10^{-3}$	0		0
$\frac{B(\Lambda \rightarrow p) - B(\bar{\Lambda} \rightarrow \bar{p})}{B(\Lambda \rightarrow p) + B(\bar{\Lambda} \rightarrow \bar{p})}$	$\sim \frac{\epsilon'}{5}$?	?	?		0
$\frac{\alpha(\Lambda \rightarrow p) + \alpha(\bar{\Lambda} \rightarrow \bar{p})}{\alpha(\Lambda \rightarrow p) - \alpha(\bar{\Lambda} \rightarrow \bar{p})}$	$\sim 3\epsilon'$?	?	?		0
$d_n(e - cm)$	$< 10^{-30}$	$\sim 3 \times 10^{-27}$	$\sim 5 \times 10^{-26}$	$\sim 10^{-25}$	$\leq 10^{-25}$ (also for electron)	$3 \times 10^{-16}\theta$
$D^0 - \bar{D}^0$	very small	?	large ?	large ?	as for Standard Model	0
$B^0 - \bar{B}^0$	unobservable	?	large ?	large ?		0
Universe	too small	fit - unless CPX spontaneously	fit - unless CPX spontaneously	0	possible	\Rightarrow large θ ??

Fig. 12. A penguin diagram.



The next row¹⁹ features predictions of the different models for intrinsic CP-violation in the $K \rightarrow 3\pi$ decay amplitude. It is expected to be the same order of magnitude as ϵ' in the Standard Model, and also very small in a left-right symmetric gauge model, whereas the multiple-Higgs model prediction has not been developed. The muon transverse polarization P_n^μ is expected to be non-zero only in multiple-Higgs models, but is not expected to be very large. For completeness, two CP-violating observables in hyperon decay are listed, together with the Standard Model predictions for them which are both $O(\epsilon')$. The left-right gauge and multiple-Higgs model predictions have not yet been developed.

We now turn to CP-violating observables beyond the ambit of the strange physicist. The neutron electric dipole moment d_n is expected⁸⁴ to be unobservably small in the Standard Model, but can be close to the present experimental limit⁸⁰ of $6 \times 10^{-25} e\text{-cm}$ in left-right gauge models,⁸⁵ multiple-Higgs models,⁸⁶ and in technicolor theories.⁸⁷ This is the only observable for which the QCD θ vacuum parameter is likely to play a role, and the estimate⁸⁸ $d_n \simeq 3 \times 10^{-16} \theta e\text{-cm}$ tells us that

$$\theta \leq 2 \times 10^{-9} \quad (35)$$

which is why its effects are not observable elsewhere. Observation of a neutron electric dipole moment in the near future need not exclude the Standard Model of CP-violation. Its prediction could have been augmented by a contribution from θ close to the limit (35) which would not show up in any other phenomenological situation. The electron could also have an observable⁸⁹ electric dipole moment, which can be comparable with that of the neutron in some SUSY models.⁹⁰

It is natural to ask whether CP-violation could appear in the $D^0 - \bar{D}^0$ or $B^0 - \bar{B}^0$ systems in ways analogous to its manifestation in the $K^0 - \bar{K}^0$ system. According to the Standard Model, ϵ should be small in the D^0 system but could

be large in the B^0 system. This depends on the values of unknown Kobayashi-Maskawa angles, but unfortunately $B^0 - \bar{B}^0$ mixing is expected to be suppressed in domains of the angles where ϵ is large,⁹¹ so it is unlikely that CP-violation could actually be observed in the B^0 system if the Standard Model is correct.

Finally we come to the Universe. It is included here because of the idea⁷⁹ that the observed baryon asymmetry in the Universe may have originated from CP- and B -violation in GUT reactions⁹² when the Universe was $O(10^{-35})$ seconds old. This qualitative mechanism is not strong⁹³ enough in the Standard Model to produce the observed baryon-to-photon ratio of a few times 10^{-10} , but could be fit in more complicated models containing more Higgs multiplets and/or more gauge bosons. However, the connection with low energy physics is not clear, since the extra baryosynthetic structure need only appear at the GUT scale, and may not be light enough to show up in present-day accelerator experiments. Also, it is important to note that in many of these models with additional low energy structure CP is violated spontaneously, in which case no significant baryon asymmetry can be generated. For this reason technicolor theories⁸⁷ are not baryosynthetic. The QCD θ parameter also does not contribute directly to the baryon asymmetry,⁹⁴ although one can argue that most theories which generate enough baryons and do not possess some additional symmetry such as a Peccei-Quinn²³ $U(1)$ or SUSY will also have a value of θ close to the limit (35) and suggest a neutron electric dipole moment close to the present experimental limit.⁹⁵ Since cosmological CP-violation may be the reason we exist at this meeting, it provides a motivation for constructing a suitable extension of the Standard Model.

3.5 NEUTRINO PHYSICS

The death of a kaon is often the birth of a neutrino which can be used for high intensity and precision neutrino physics, as was discussed by a working group⁹⁶ at this meeting. One can imagine detailed measurements of $\nu_\mu e$, $\bar{\nu}_\mu e$ and $\nu_e e$ scattering which enable one to measure $d\sigma/dy$ as well as σ . In this way one might be able to measure $\sin^2\theta_W$ with a sensitivity comparable to that obtainable from experiments near the Z^0 peak. The comparison between low energy and high energy measurements is a crucial check to the radiative corrections⁹⁷ whose calculability was the prime motivation for spontaneously broken gauge theories. To get some idea of the precision required, let us recall that

$$\delta m_{Z^0} \approx 140 \text{ GeV} \times \delta(\sin^2\theta_W) \quad (36)$$

so that a determination of m_{Z^0} with a precision of about 300 MeV would fix $\sin^2\theta_W$ with an error of ± 0.002 . It is not clear that one would gain from a more accurate value of m_{Z^0} , because of uncertainties in the one loop correction due to strongly interacting particles and because of higher order radiative corrections to the Z^0 mass. Compare this precision with the shift⁹⁸ in the apparent value of $\sin^2\theta_W$ due to radiative corrections, which is $O(0.012)$. In this context, a "precision" low energy determination of $\sin^2\theta_W$ should mean an error $\ll 0.01$ and preferably $O(0.002)$.

As discussed by Shrock⁴⁹ at this meeting, another interesting class of low energy neutrino experiments involves searches⁹⁹ for massive neutrinos ν_H . One may look for their production in $\pi \rightarrow \mu\nu_H, e\nu_H$ and $K \rightarrow \mu\nu_H, e\nu_H$ decays via anomalous bumps in the lepton energy spectrum, in which cases one is sensitive to $|U_{\mu H}|^2$ or $|U_{eH}|^2$ respectively, where the $U_{\ell H}$ are mixing matrix elements, as seen in Fig. 13. Another possibility is to search for ν_H decays, as was mentioned at this meeting by Ferro-Luzzi.¹⁰⁰ One can look either for $\nu_H \rightarrow \nu_\ell e^+ e^-$ or for $\nu_H \rightarrow \nu_\ell \gamma \gamma$, and is generally sensitive to the product $|U_{eH}U_{eH}|$ or $|U_{eH}U_{\mu H}|$. Figure 13 is taken from a proposal¹⁰⁰ for such a decay experiment at CERN, and we see that very large ranges of mixing matrix elements are accessible to such an experiment. This range could be further improved using a more intense source, just as the μ or e spectrum bump-hunting experiments could also improve in sensitivity.

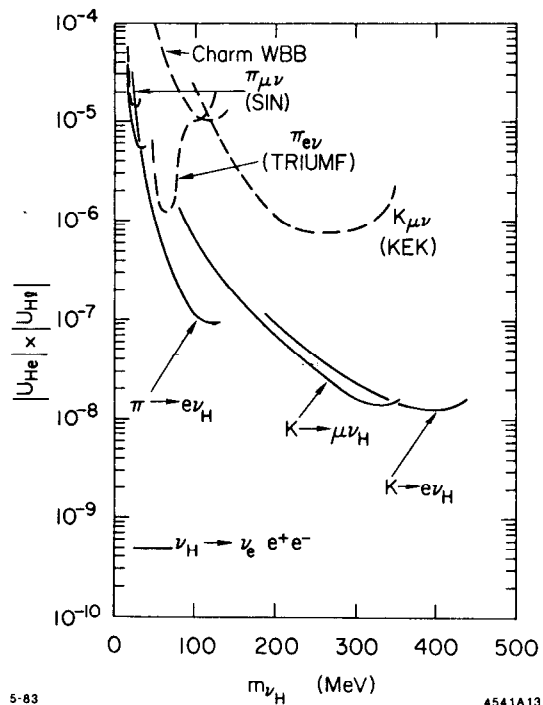


Fig. 13. Actual and potential bounds on heavy neutrino couplings as functions of their masses.

3.6 A PERSONAL PRIORITY LIST

It is always dangerous to put something like this into print, as it is likely to be over-simplified and over-interpreted. However, several participants have urged me to produce such a distillation, which I do not mind doing as long as everyone realizes it is just a personal opinion, and there are at least as many different opinions as there are theorists at this meeting.

Top of my priorities would be a measurement of the CP-violating parameter ϵ'/ϵ , because of the light it would cast on the mechanism(s) of CP-violation. Second would be $K^+ \rightarrow \pi^+$ + nothing experiments, because this is the last great frontier of $\Delta S \neq 0$ neutral interactions which will tell us a lot about the Standard Model as well as expose us to new physics. Next I come to the more

speculative searches for new interactions, at the top of which list I would place $K_L^0 \rightarrow \mu e$. This seems to be more sensitive and topical than the other searches such as $K^\pm \rightarrow \pi^\pm \mu e$, $\Sigma \rightarrow p \mu e$, and $\mu A \rightarrow e A$, $\mu \rightarrow e \gamma$ and $\mu \rightarrow e \bar{e} e$. At the bottom of my list come precision neutrino experiments, in part because of my doubts whether the errors could be reduced sufficiently to be really exciting. Then there are many other experiments which get an "incomplete."

4. Beyond Physics

The discussion of serious physics has terminated with the previous section. Now I would like to mention to you briefly a little philosophical speculation you may care to entertain: can one observe a violation of quantum mechanics?

It has been noticed¹⁰¹ that black holes correspond to mixed thermal states with finite entropy associated to the existence of an event horizon across which information can be lost. Hawking¹⁰² has further observed that quantum effects cause black holes to radiate particles with a mixed thermal spectrum. These discoveries may not be of purely philosophical or astrophysical significance in view of the old idea¹⁰³ that spacetime may have a foamy structure at short distances, with $O(1)$ mini black hole or other topological structure (instanton?) in every Planck volume. Thus we may imagine that event horizons constantly appear and disappear, and wonder whether this would have any implication for the purity of quantum mechanical wave functions. Hawking and collaborators³⁹ have performed calculations suggesting that indeed gravitational instantons may cause initially pure states to evolve with time into mixed states, and Hawking¹⁰⁴ has argued that the conventional laws of quantum mechanics should be modified as a consequence.

He entertains¹⁰⁴ the possibility that an initial density matrix ρ_{-D}^C may yield a final density matrix ρ_{+B}^A :

$$\rho_{+B}^A = \mathcal{S}_{BC}^A{}^D \rho_{-D}^C \quad (37)$$

where the superscattering operator $\mathcal{S}_{BC}^A{}^D$ does not factorize into the product of an S matrix and its adjoint:

$$\mathcal{S}_{BC}^A{}^D \neq S_C^A S_B^\dagger{}^D \quad (38)$$

as in conventional quantum mechanics. If this is indeed the case, one might expect¹⁰⁵ a non-standard equation for the time evolution of the density matrix:

$$\frac{d\rho}{dt} = \mathcal{H}(\rho) \quad (39)$$

where \mathcal{H} is a general hermitian linear operator which does not take the standard form:

$$\mathcal{H} = i[\rho, H] + \delta \mathcal{H} \quad : \quad \delta \mathcal{H} \neq 0 \quad (40)$$

it is easy to check that pure states may evolve into mixed states if the \mathcal{S} operator does not factor as in Eq. (38), or if \mathcal{H} takes the non-canonical form (40). We would expect¹⁰⁶ the conventional rules of quantum mechanics to apply on time scales $\delta t \ll |\delta \mathcal{H}|^{-1}$, but expect that if

$$\delta t \geq |\delta \mathcal{H}|^{-1} \quad (41)$$

then violations of quantum mechanics may be observable. This general expectation¹⁰⁶ is indeed borne out by calculations in simple examples¹⁰⁵ of modified quantum mechanical systems.

For example, one might expect that an initially pure K^0 state produced in a hadron-hadron collision could evolve into a mixed state which is approximately a K_L but with a small K_S admixture. This K_S component can come from a term $\delta \mathcal{H}$ in (40) which produces a slow ‘decay’ of the K_L into the K_S , appearing as a continuous ‘regeneration’ of K_S in vacuo (precisely what one would expect if the definite phase relationship between the K^0 and \bar{K}^0 components of a K_L beam were to become mixed in some way). This contributes to the downstream 2π yield in a way which is distinguishable from the usual CP impurity of the K_L , but this distinction requires a precise comparison of the CP parameters $|\eta_{+-}| \simeq |\epsilon|$, ϕ_{+-} , and $\delta \simeq 2Re\epsilon$. Based on existing data¹⁸ this comparison gives

$$|\delta \mathcal{H}| \leq 2 \times 10^{-21} \text{ GeV} . \quad (42)$$

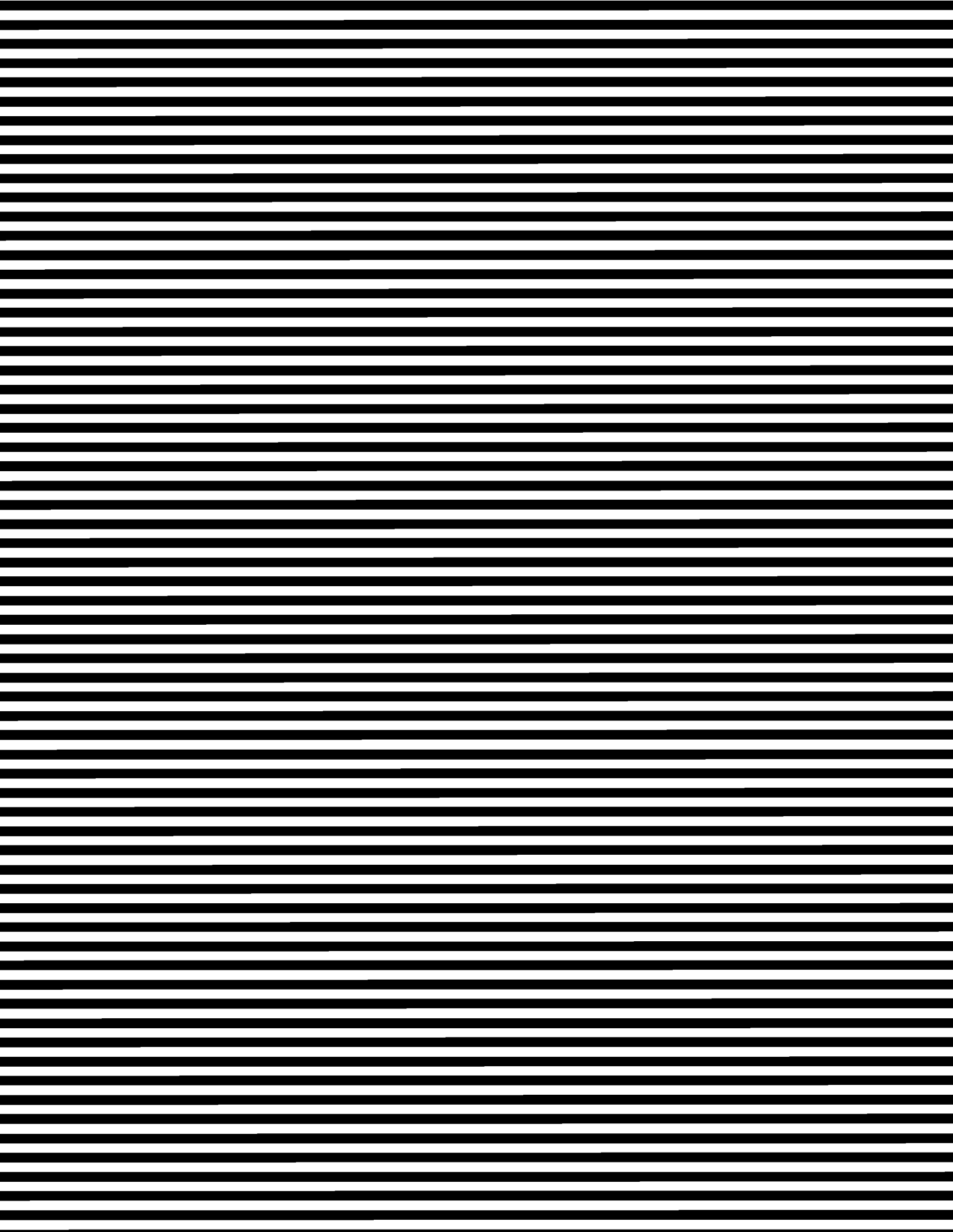
A constraint on $|\delta \mathcal{H}|$ for neutrons of similar magnitude can be deduced^{105,106} from the success of long baseline neutron interferometry experiments. Since neutral kaons are at the cutting edge of this issue, it might be worthwhile to consider how one might devise somewhat more sensitive tests of quantum mechanics in the $K^0 - \bar{K}^0$ system.

5. A Question

Figure 14 is an artist’s impression of the recent past history of elementary particle physics and its cousins which also features some possible extrapolations into the future. The horizontal lines depict fields of study which do not get more fundamental as time elapses. The diagonal line depicts the direction of elementary particle physics, including a few landmarks passed in the past as well as some possible landmarks of the future. As the diagonal and horizontal lines diverge, new fields of study open up between them. For example, the recent achievement of a consensus Standard Model of elementary particle physics bids fair to act as a node where a new line branches off horizontally. This may meet a new diagonal line branching off from traditional nuclear physics.

The old pion factories were mainly motivated by nuclear physics but have turned out to give some help in elucidating the Standard Model. Kaon factories would surely be useful for Standard Model physics. We can ask ourselves whether they will have significant impact on the particle physics of the 1990’s. Assuming that they do, we should also ask our political masters whether we are playing in a zero-sum game. Would any significant fraction of the cost of a kaon factory be charged against the elementary particle physics account? If so, we particle physics chauvinists have to wonder whether kaon factories would be a cost-effective means of pursuing our discipline.

The “weak” lobby gathered at this meeting is agreed that strange physics is great physics. Furthermore, there seems to be little transatlantic competition since we learn that there is to be no beam for charged kaon physics at CERN in the foreseeable future. One has the impression that a kind of Yalta philosophy



may be at work, according to which CERN concentrates on p physics and leaves strange physics to accelerators in the United States. Can the variety of strange physics we feel necessary be done "on the cheap"?

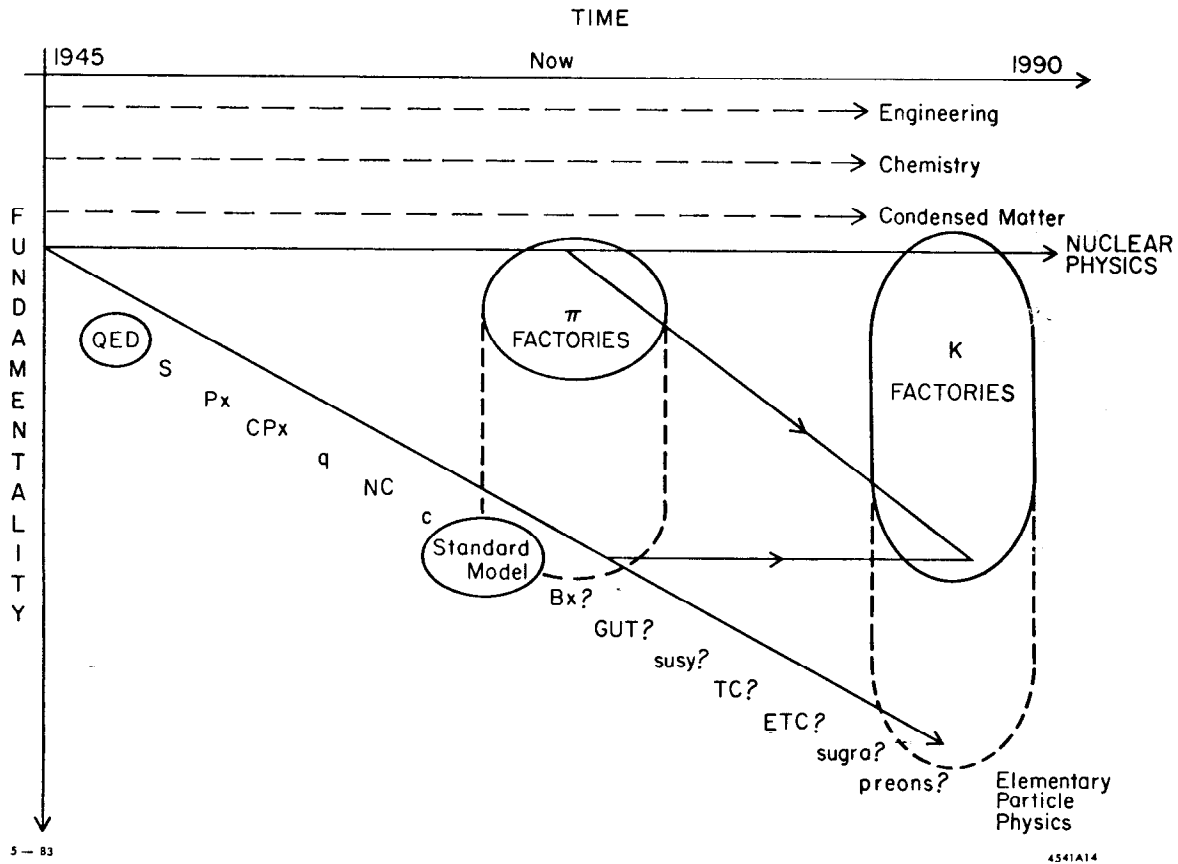


Fig. 14. A sketch of the past and possible future evolution of elementary particle physics and related disciplines.

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