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SEARCH FOR SUPERSYMMETRY IN TOPONIUM DECAYS*

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ABSTRACT

Although cosmology and unification suggest that the mass of the gluino is larger than about 14 GeV, in many theories of broken supersymmetry the photino $\tilde{\gamma}$, gluino $\tilde{g}$ and one of the spin-zero partners $\tilde{t}$ of the t quark can be lighter than m_t. We calculate the decay rates for toponium $\rightarrow \tilde{g}\tilde{g}$, $\tilde{\gamma}\tilde{\gamma}$ and $\bar{\tilde{t}}\tilde{t}$ and show that in many models they can be as large as, or larger than, the rate for toponium $\rightarrow e^+e^-$ decay. If it is kinematically accessible, the decay $t \rightarrow \tilde{t} + \tilde{\gamma}$ dominates the decays of toponium as well as those of naked top states.

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One obstacle to the experimental search [1] for supersymmetry (susy) is our theoretical ignorance about the likely masses of supersymmetric particles. This means that we cannot be sure where susy will first appear, and must plan a broad-band search. The production and detection of supersymmetric particles in hadron-hadron collisions [2], lepton-hadron collisions [3,4] and e^+e^- annihilation [3,5,6] have been extensively discussed. In the case of e^+e^- annihilation, there have been discussions of the production of squarks and gluinos in the continuum and of gluinos in heavy quarkonium decays [3,5,6]. The first calculated rates [3] were for gluino pair production in association with $\bar{q}q$ pairs or gluons and did not depend on unknown squark masses which introduce additional uncertainties. Calculations have also been made [5] for $e^+e^- \to \tilde{g}\,\tilde{g}$ through loop diagrams. Recently calculations have also been made [6] of quarkonium $\to g\,\tilde{g}\,\tilde{g}$ and $g\,\tilde{g}\,\tilde{\gamma}$ due to squark exchange. We know [1] that the light quarks can only have much heavier squark partners, but this need not be the case for the t quark. In some models [7] for the spontaneous breaking of susy and of the weak gauge symmetry the scale of susy breaking can be much less than m_W while the t quark mass may be $O(m_W)$ [8]. In this class of models $^3S_1$ orthotoponium Θ can have exclusive decays into pairs of gluinos or $\tilde{t}$ squarks whose branching ratios rival or even dominate the familiar $\Theta \to e^+e^-$ decays.

In this paper we first discuss the possible masses of gluinos and squarks. We point out that in models where $SU(3) \times SU(2) \times U(1)$ is eventually embedded in a unifying group the cosmological limit [9] $m_{\tilde{\gamma}} \geq m_\tau = 1.8\ GeV$ suggests that the gluino mass

$$m_{\tilde{g}} \;\geq\; 14\ GeV \tag{1}$$

Gluinos may still be light enough to be pair-produced in Θ decays, since experiment tells us that $m_\Theta \geq 38\ GeV$. Then we discuss the $\tilde{t}$ squark mass matrix and show that in some models $m_{\tilde{t}} = O(m_t)$ and it could be that one of the two stop mass eigenstates could even be lighter:

$$m_{\tilde{t}} < m_t \tag{2}$$

Next we present general formulae for the ratios $\Gamma(\Theta \rightarrow \tilde{g}\,\tilde{g})/\Gamma(\Theta \rightarrow e^+e^-)$ and $\Gamma(\Theta \rightarrow \bar{\tilde{t}}\,\tilde{t})/\Gamma(\Theta \rightarrow e^+e^-)$. The gluino pair branching ratio can be larger than the leptonic branching ratio in models where $m_{\tilde{t}} = O(m_t)$. If it is kinematically accessible, the $\Theta \rightarrow \bar{\tilde{t}}\,\tilde{t}$ decay rate due to gluino exchange can even be $O(100) \times \Gamma(\Theta \rightarrow e^+e^-)$ and is enhanced by a supersymmetric analogue of a Coulomb binding singularity if $m_{\tilde{g}} \ll m_t \approx m_{\tilde{t}}$. For this reason, the $\Theta \rightarrow \bar{\tilde{t}}\,\tilde{t}$ decay rate need not exhibit the β^3 threshold P-wave phase space factor characteristic of $\bar{\tilde{t}}\,\tilde{t}$ production in the e^+e^- continuum. If it is kinematically accessible, the decay $t \rightarrow \tilde{t} + \tilde{\gamma}$ will be even faster, and will dominate the decays of toponium as well as those of t-flavored hadrons.

Let us first discuss the gluino mass. The negative results of searches for their decay products in beam dump experiments suggest [2] that the gluino mass is larger than about 2 GeV, though the limit depends on the lifetime and decay modes of the gluino. A much more stringent limit (1) comes from cosmological considerations. It has been shown [9] that either

$$m_{\tilde{\gamma}} \leq O(1)\, keV \quad \text{or} \quad m_{\tilde{\gamma}} \geq 1.8\, GeV\ . \tag{3}$$

In leading order, the renormalization group equations for gaugino masses are identical with those for the gauge couplings $\alpha_i(Q) : i = 3,2,1$ for the low energy $SU(3) \times SU(2) \times U(1)$ gauge group. If there is an underlying (grand) unifying group the $\alpha_i(Q)$ all become equal at some scale m_X, as do the effective gaugino masses at that scale. Therefore at lower energies we have

$$\frac{m_{\tilde{i}}}{\alpha_i} = \text{independent of } i \tag{4}$$

since the photino is a mixture of $SU(2)$ and $U(1)$ gauginos, its mass [7] is related to that of gluino by a modicum of algebra:

$$\frac{m_{\tilde{g}}}{m_{\tilde{\gamma}}} = \frac{3}{8} \frac{\alpha_3}{\alpha_2 \sin^2 \theta_W} \tag{5}$$

with α_3 presumably being evaluated at a momentum scale $Q = O(m_{\tilde{g}})$. Numerically, eq. (5) becomes

$$\frac{m_{\tilde{g}}}{m_{\tilde{\gamma}}} = 50\ \alpha_3\ (m_{\tilde{g}})\ . \tag{6}$$

We now combine this with the cosmological limit (3), taking the representative values $\alpha_3 = 1$ for the lower mass range and $\alpha_3 = 0.15$ for the upper mass range, and obtain

$$m_{\tilde{g}}\ \leq\ 50\ keV \quad \text{or} \quad \geq 14\ GeV\ . \tag{7}$$

The lower branch of the range (7) is excluded by experiment, and we are left with the conclusion (1).

Now we discuss the masses of the spin-zero $\tilde{t}$ squarks. There are two states $\tilde{t}_{L,R}$ associated in chiral supermultiplets $Q, \bar{T}$ with the left-handed (t, b) quark doublet and the left-handed t^c conjugate quark singlet respectively. Their interactions can be described by superpotential terms

$$W \ni \frac{m_t}{v} Q\,\bar{T}\,H + \epsilon\,\bar{H}\,H \tag{8}$$

where H and $\bar{H}$ are the two light Higgs supermultiplets, $v, \bar{v} \equiv \langle 0 | H, \bar{H} | 0 \rangle$ while ϵ is an unknown mass parameter expected to be $O(m_W)$, and by susy breaking $(\text{mass})^2$ terms given by $\tilde{m}$:

$$\mathcal{L}_{susy\,X} \ni -L^2\,\tilde{m}^2\ |\tilde{t}_L|^2 - R^2\,\tilde{m}^2\ |\tilde{t}_R|^2 - \hat{A}\,m_t\,\tilde{m}\,\tilde{t}_L\,\bar{\tilde{t}}_R \tag{9}$$

where $L^2 \neq R^2$ in general [7] thereby violating parity, and A is an unknown parameter which is $O(1)$. The interactions (8,9) give us a mass matrix

$$\left(\bar{\tilde{t}}_L, \bar{\tilde{t}}_R\right)\begin{pmatrix} L^2\,\tilde{m}^2 + m_t^2 & A\,\tilde{m}\,m_t \\ A\,\tilde{m}\,m_t & R^2\,\tilde{m}^2 + m_t^2 \end{pmatrix}\begin{pmatrix}\tilde{t}_L \\ \tilde{t}_R\end{pmatrix} \tag{10}$$

where we have written $A \equiv \hat{A} + (\epsilon\,\bar{v}\,/\,\tilde{m}\,v)$. The $\tilde{t}$ mass eigenstates $\tilde{t}_{1,2}$ are rotated relative to $\tilde{t}_L$ and $\tilde{t}_R$ by an angle θ: $\cos\theta \equiv c$, $\sin\theta \equiv s$ and

$$\tan 2\theta = \frac{-2Am_t}{(L^2 - R^2)\,\tilde{m}} \tag{11}$$

and have squared masses

$$m^2_{\tilde{t}_{1,2}} = m_t^2 + \left[\frac{(L^2+R^2)\,\tilde{m}^2 \pm \sqrt{(L^2-R^2)^2\,\tilde{m}^4 + 4A^2m_t^2\,\tilde{m}^2}}{2}\right]. \tag{12}$$

In the cases of lighter quarks q, $m^2_{\tilde{q}_{1,2}} \approx (L^2, R^2)\,\tilde{m}^2 \gg m_q^2$. However, in some models [7] $\tilde{m}^2 \ll m_W^2$ while [7,8] $m_t \geq O(m_W)$, so that in these cases $\tilde{m}_{\tilde{t}_{1,2}} \approx m_t$ and θ can be large. In particular, it is clear from eq. (12) that the lighter $\tilde{t}$ may actually weigh less than the t quark. For given values of L, R, and A the minimum mass occurs when

$$\frac{\tilde{m}^2}{m_t^2} = \frac{A^2}{LR(L+R)^2} \tag{13}$$

in which case

$$m^2_{\tilde{t}_1}/m_t^2 = 1 - \frac{A^2}{(L+R)^2}\,. \tag{14}$$

The first line of the table displays the minimum values of $m_{\tilde{t}}/m_t$ for the five models of ref. 7.

We now turn to the Θ decay modes involving gluinos and $\tilde{t}$ squarks. In the non-relativistic limit the relevant $\bar{t}\,t \rightarrow \tilde{g}\,\tilde{g}$ interaction generated by the $\tilde{t}$ exchanges of fig. 1 is (where λ is the gluino field)

$$\frac{g_3^2}{24}\,\frac{C}{m_t^2}\,(\bar{t}\,\gamma_\mu t)\,(\bar{\lambda}\,\gamma_\mu\gamma_5\lambda) \tag{15}$$

where C is a coefficient related to the parameters of the model (8,9,10):

$$\begin{aligned} C &= (c^2 - s^2)\left[\frac{1}{(m_t^2 - m_{\tilde{g}}^2) + m_{\tilde{t}_1}^2} - \frac{1}{(m_t^2 - m_{\tilde{g}}^2) + m_{\tilde{t}_2}^2}\right] m_t^2 \\ &= \frac{(L^2 - R^2)\,\tilde{m}^2\, m_t^2}{L^2R^2\,\tilde{m}^4 + (L^2 + R^2)\,\tilde{m}^2\,(2m_t^2 - m_{\tilde{g}}^2) - A^2\,\tilde{m}^2\, m_t^2 + (2m_t^2 - m_{\tilde{g}}^2)^2} \end{aligned} \tag{16}$$

This coefficient is maximized if

$$\tilde{m}^2 = \frac{(2m_t^2 - m_{\tilde{g}}^2)}{LR} \tag{17}$$

in which case

$$C = \frac{(L^2 - R^2)}{\left(2 - \frac{m_{\tilde{g}}^2}{m_t^2}\right)(L + R)^2 - A^2}\ . \tag{18}$$

We see in the interaction (15) that the final state gluinos are in a P-wave, as required by their Majorana nature. This means that $^3S_1$ orthotoponium can only decay into them if parity is violated, as is the case if $L \neq R$ in eq. (9). This requirement is mirrored in the numerator of the expression (18) for the coefficient C. Using the operator (13) we find

$$\frac{\Gamma(\Theta \to \tilde{g}\,\tilde{g})}{\Gamma(\Theta \to \gamma^* \to e^+e^-)} = \left(\frac{\alpha_3}{\alpha}\right)^2 C^2 \left(1 - \frac{m_{\tilde{g}}^2}{m_t^2}\right)^{3/2} \tag{19a}$$

$$\approx\ 400\ C^2 \tag{19b}$$

if we assume $m_{\tilde{g}} \ll m_t$ and take $\alpha_s \approx 0.15$. The ratio (19) is therefore larger than 1 if $C \geq 0.05$. The second line of the table displays the maximal values of C for the five models of ref. [7]. These give values of the ratio (19) of decay rates which can easily be $\gg 1$. If $m_\Theta \approx O(m_{Z^o})$ the rate for $\Theta \to e^+e^-$ receives an important contribution from Z^o exchange and there are other effects [10] on Θ decays, but $\Theta \to \tilde{g}\,\tilde{g}$ may still be a dominant decay mode. Perhaps orthotoponium will decay predominantly into gluino pairs?

Figure 1 can also be used to calculate the $\Theta \to \tilde{\gamma}\tilde{\gamma}$ decay rate. We find

$$\frac{\Gamma(\Theta \to \tilde{\gamma}\tilde{\gamma})}{\Gamma(\Theta \to \gamma^* \to e^+e^-)} = \frac{8}{9}\ C^2 \tag{20}$$

if $m_{\tilde{\gamma}} \ll m_t$. This could be substantial for some of the models [7] listed in the Table. We would expect the photinos to escape as invisible neutral energy. One way to trigger on such events is to look for $e^+e^- \to \Theta'$, $\Theta' \to \pi\pi + (\Theta \to$ missing neutrals) as has previously been discussed [11] as a way of counting neutrinos. Formula (20) and the table indicate that $\Theta \to \bar{\nu}\nu$ decays may be an insignificant background to $\Theta \to \tilde{\gamma}\tilde{\gamma}$ decays.

The corresponding effective interaction for non-relativistic $^3S_1\ \bar{t}t$ annihilation into $\bar{\tilde{t}}_1\,\tilde{t}_1$ squark pairs due to the gluino exchange of fig. 2 is:

$$\frac{2}{9}\ g_3^2 \left(\frac{1}{m_{\tilde{g}}^2 + m_t^2 - m_{\tilde{t}_1}^2}\right) (\bar{t}_i\,\gamma_\mu t^j)\ (\bar{\tilde{t}}_1^i \overset{\leftrightarrow}{\partial_\mu} \tilde{t}_{1j})\ . \tag{21}$$

From this interaction we calculate the ratio of decay rates

$$\frac{\Gamma(\Theta \to \bar{\tilde{t}}_1\,\tilde{t}_1)}{\Gamma(\Theta \to \gamma^* \to e^+e^-)} = \frac{4}{3}\left(\frac{\alpha_3}{\alpha}\right)^2 \Phi\Big(m_{\tilde{g}/m_t}, m_{\tilde{t}_1/m_t}\Big) \tag{22a}$$

$$\approx\ 500\ \Phi \tag{22b}$$

where Φ is a factor containing finite mass corrections:

$$\Phi = \left(1 + \frac{m_{\tilde{g}}^2}{m_t^2} - \frac{m_{\tilde{t}_1}^2}{m_t^2}\right)^{-2} \left(1 - \frac{m_{\tilde{t}_1}^2}{m_t^2}\right)^{3/2}\ . \tag{23}$$

This formula exhibits a supersymmetric analogue of the Coulomb binding singularity when $m_{\tilde{g}}/m_t \to 0$:

$$\phi \approx \left(1 - \frac{m_{\tilde{t}_1}^2}{m_t^2}\right)^{-1/2}\ ! \tag{24}$$

It is readily apparent from eq. (22) that if $m_{\tilde{t}_1} < m_t$, as is seen from eq. (14) and from the Table to be a definite possibility, then $\Theta \to \bar{\tilde{t}}\tilde{t}$ may be the dominant decay mode unless $m_{\tilde{g}} \gg m_t$.

There is another interesting possibility for Θ decays which exists if the decay $t \to \tilde{t} + \tilde{\gamma}$ is kinematically accessible. If so, we find

$$\Gamma(t \to \tilde{t} + \tilde{\gamma}) = \frac{\alpha}{9m_t^3}\left[(m_t + m_{\tilde{\gamma}})^2 - 2m_{\tilde{\gamma}}m_t(1 + sin2\theta) - m_{\tilde{t}_1}^2\right]\ \lambda^{1/2}\left(m_t^2, m_{\tilde{\gamma}}^2, m_{\tilde{t}_1}^2\right) \tag{25}$$

where λ is the usual kinematical function. This is sufficient to dominate toponium decays:

$$\frac{\Gamma(\Theta \to t\bar{\tilde{t}}_1\tilde{\gamma} + \bar{t}\tilde{t}_1\tilde{\gamma})}{\Gamma(\Theta \to \gamma^* \to e^+e^-)} \approx 300\ m_t\ \text{(GeV)}\left(1 - \frac{m_{\tilde{t}_1}^2}{m_t^2}\right)^2 \tag{26}$$

if we take $\Gamma(\Theta \to \gamma^* \to e^+e^-) \sim 5$ keV and neglect $m_{\tilde{\gamma}}$. The electromagnetic $t \to \tilde{t} + \tilde{\gamma}$ decay also dominates free t decay:

$$\frac{\Gamma(t \to \tilde{t}_1 + \tilde{\gamma})}{\Gamma(t \to bf\bar{f})} \approx 100\ \left(\frac{m_W}{m_t}\right)^4\ \left(1 - \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}}^2}\right)^2 \tag{27}$$

for $m_{\tilde{\gamma}} \approx 0,\ m_t \ll m_W$, and the decay $t \to \tilde{t} + \tilde{\gamma}$ is also competitive with $t \to b + W$ if $m_t > m_W$. This would modify the conventionally expected signatures for naked t decay.

Our analysis has shown that ${}^3S_1$ orthotoponium Θ may have very unexpected dominant decay modes in some supersymmetric theories [7,8] Similar results would apply to other toponium states, such as the ${}^3P_1$ state which could be produced in Z^0 annihilation in e^+e^- annihilation and also decay into e^+e^- via Z^0 exchange. These possible bizarre final states in Θ decay should perhaps be taken into account when interpreting experimental searches in e^+e^- annihilation and elsewhere. We would expect the decay modes

$$\tilde{g} \to (\bar{q}q) + \tilde{\gamma} \tag{28}$$

to dominate $\tilde{g}$ decays. The dominant decay modes of the $\tilde{t}$ may be

$$\begin{aligned}\tilde{t} \to b + \tilde{W}^* & \\ & \to \bar{\ell} + (\tilde{\ell} \to \ell + \tilde{\gamma}) \\ & \qquad \text{or} \\ & \to (\bar{q}q, \bar{\ell}\ell) + \tilde{\gamma}\end{aligned} \tag{29}$$

In both of these cases the final state would have missing energy and an unusual structure containing 4 hadronic jets (25a) or 6 assorted leptons and jets (25b). If toponium were found to decay in such a manner, it could be the first experimental evidence for susy.

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TABLE I

The minimal $\tilde{t}$ squark masses and maximal $\tilde{g}\,,\tilde{\gamma}$ decay coefficients C in the 5 models shown in the 5 columns of table I of ref. 7.

Models		1	2	3	4	5
$m_{\tilde{t}_1}/m_t \geq$		0.50	0.62	0.76	0	0.44
$\lvert C\rvert \leq$	$m_{\tilde{g}} = 0$	0.06	0.03	0.19	0.14	0.05
	$m_{\tilde{g}} = m_t$	0.32	0.10	0.51	0.19	0.31

FIGURE CAPTIONS

1. Diagram for $\tilde{t}$ exchange which contributes to $\Theta \rightarrow \tilde{g}\,\tilde{g}\,,\tilde{\gamma}\,\tilde{\gamma}$ decays.

2. Diagram for $\tilde{g}$ exchange which contributes to $\Theta \rightarrow \bar{\tilde{t}}_1\,\tilde{t}_1$ decays.

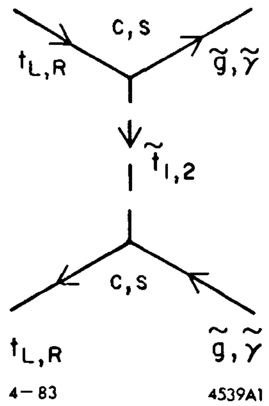
c, s
$t_{L,R}$
$\tilde{g}, \tilde{\gamma}$
$\tilde{t}_{1,2}$
c, s
$t_{L,R}$
$\tilde{g}, \tilde{\gamma}$
4–83
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Fig. 1

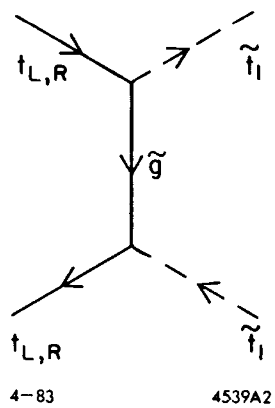

Fig. 2