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A LOWER BOUND ON $|\epsilon'/\epsilon|$ *

FREDERICK J. GILMAN AND JOHN S. HAGELIN
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

In the Kobayashi-Maskawa (K-M) model, a lower bound on the CP violating product $s_2c_2s_3s_\delta$ follows from the imaginary part of the short-distance $K^0-\bar{K}^0$ mixing amplitude together with a conservative upper bound on the short-distance contribution to $K_L \to \mu\mu$. This leads to a lower bound on $|\epsilon'/\epsilon|$ in terms of a matrix element of a single $(V-A)\times(V+A)$ type operator. Familiar current algebra and bag model estimates for this operator give $|\epsilon'/\epsilon| > 2\times10^{-3}$. We also observe that the experimental upper bound on the branching ratio for the b-quark into u-quarks fixes the sign of $s_2c_2s_3s_\delta$ and ϵ'/ϵ both to be positive. Allowances for QCD corrections and long distance effects are included throughout our analysis.

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With three generations of quarks, mixing between weak eigenstates and quark mass eigenstates is parametrized by a 3×3 unitary (K-M) matrix[1] with three Cabibbo-like angles θ_i and a phase δ, which if non-zero generally results in CP violation. In the neutral Kaon system, the only place where CP violation has been observed[2] so far, there can be CP violating effects both in the off-diagonal element M_{12} of the $K^0 - \bar{K}^0$ mass matrix and in the $K^0 \rightarrow \pi\pi$ decay amplitude. Both effects are due to virtual transitions to charm (c) and top (t) quarks and involve amplitudes proportional to $\sin\theta_2 \cos\theta_2 \sin\theta_3 \sin\delta \equiv s_2 c_2 s_3 s_\delta$.

It has previously been suggested[3] that the presence of CP violation in the decay amplitude, which results in a non-zero value of the CP violation parameter ϵ', might well be detectable in experiments of improved accuracy which are underway or planned. A value of $\epsilon' \neq 0$ would distinguish the explanation of CP violation based on the K-M model from models which are "superweak" wherein CP violation occurs solely in the $K^0 - \bar{K}^0$ mass matrix and only the parameter ϵ is non-zero. Naturally the question arises: In the K-M model how small could ϵ' be? In this paper we attempt to answer this by putting a lower bound on $s_2 c_2 s_3 s_\delta$ using the short-distance contribution to the imaginary part of the $K^0 - \bar{K}^0$ mass matrix together with a conservative upper bound on the short-distance contribution to $K_L^0 \rightarrow \mu\mu$.

As just noted, there are two possible contributions to the observed CP violation in the neutral Kaon system. "Direct" CP violation in $K^0 \rightarrow \pi\pi$ can give rise to different phases for the weak amplitudes for $K \rightarrow \pi\pi$ $(I = 0)$ and $K \rightarrow \pi\pi$ $(I = 2)$, A_0 and A_2 respectively. This is parametrized[4] by the quantity ϵ' where

$$\epsilon' = \frac{1}{\sqrt{2}} \, e^{i(\pi/2 + \delta_2 - \delta_0)} \, \frac{Im A_2}{A_0} \qquad (1)$$

in the standard phase convention where A_0 is chosen real and positive. "Superweak" CP violation contributes only to the CP impurity parameter ϵ which measures the departure of the mass eigenstates K_L^0 and K_S^0 from being CP eigenstates:

$$K_{L,S} = \left[2(1+|\epsilon|^2)\right]^{-\frac{1}{2}}\left[(1+\epsilon)K^0 \pm (1-\epsilon)\bar{K}^0\right] , \tag{2}$$

where

$$\epsilon = i\,\frac{\frac{1}{2}\,Im\Gamma_{12} + iImM_{12}}{\frac{1}{2}\Delta\Gamma + i\Delta M} \tag{3}$$

and M_{12} and Γ_{12} are the off-diagonal elements of the $K^0 - \bar{K}^0$ mass matrix. In the standard convention where A_0 is real, $Im\Gamma_{12}$ is negligible, which together with the experimental[5] relationship $-\Delta M = -(M_S - M_L) \approx \frac{1}{2}(\Gamma_S - \Gamma_L) = \frac{1}{2}\Delta\Gamma$, allows one to write

$$\epsilon \approx \frac{e^{i\pi/4}}{\sqrt{2}}\,\frac{ImM_{12}}{\Delta M} . \tag{4}$$

The experimental values[5] for the strong interaction $\pi\pi$ phase shifts, δ_0 and δ_2, in Eq. (1) make ϵ and ϵ' nearly parallel or antiparallel in the complex plane.

However, if we start with the conventional choice of quark field phases and K-M matrix (where the weak couplings among light quarks are real), A_0 is not real since the effective Hamiltonian for $\Delta S = 1$ weak decays contains CP violating terms. These arise from "penguin" diagrams involving virtual c and t quarks generated when strong interaction corrections to the weak interaction Hamiltonian are taken into account. Insofar as the CP violation enters through such induced penguin-type operators in the resulting effective Hamiltonian, it is characterized by $\Delta I = 1/2$ and only contributes to the $I = 0$ final state in

$K^0 \to \pi\pi$. As a result, the weak amplitude A_0, which would have been real in this basis were there no CP violation, picks up a small imaginary part, ImA_0. Defining $\xi \equiv ImA_0/A_0$, and using the fact that it is a small quantity,

$$(A_0)_{quark} \approx A_0(1+i\xi) \approx A_0 e^{i\xi} \tag{5}$$

where the added subscript on A_0 is used to emphasize the quark phase convention.

The standard phase convention where A_0 is real is restored simply by redefining the phases of the K^0 and $\bar{K}^0$ states:

$$|K^0> \to e^{-i\xi}\,|K^0>$$

$$|\bar{K}^0> \to e^{i\xi}\,|\bar{K}^0>$$

so that $(A_0)_{quark} \to e^{-i\xi}(A_0)_{quark} = A_0$. At the same time, the previously (in the quark basis) real amplitude A_2 picks up a phase $e^{-i\xi}$ and is complex in the basis where A_0 is real. Thus from Eq. (1),

$$\left|\frac{\epsilon'}{\epsilon}\right| = \frac{1}{\sqrt{2}}\,\frac{|\xi|}{|\epsilon|}\,\left|\frac{A_2}{A_0}\right| = 15.6\,|\xi|\,, \tag{6}$$

where we have used the experimental values[5] of $|A_2/A_0| = 1/20$ and of $|\epsilon| = 2.27 \times 10^{-3}$.

The effective $\Delta S = 1$ Hamiltonian which is responsible for K^0 decay has been extensively studied elsewhere.[6] The CP-violating contribution to $K^0 \to \pi\pi$ $(I=0)$, decay is dominated by the contribution from a single $(V-A)\times(V+A)$ operator, Q_6, in the effective Hamiltonian, $\mathcal{H} = \sum_{i=1}^{6} C_i Q_i$. ImC_6 is proportional to the combination of K-M parameters $s_2 c_2 s_3 s_\delta$, in addition to the usual factor of $\frac{G_F}{\sqrt{2}} s_1$ characteristic of $\Delta S = 1$ weak amplitudes. Thus we write

$$\begin{aligned}
\xi &= \frac{Im < \pi\pi(I=0)|\ \mathcal{H}\ |K^0>}{A_0} \\
&\approx \frac{Im C_6 < \pi\pi(I=0)|\ Q_6\ |K^0>}{A_0} \\
&= (s_2 c_2 s_3 s_\delta)(Im\,\tilde{C}_6)\frac{G_F}{\sqrt{2}}\ s_1 \frac{< \pi\pi(I=0)|\ Q_6\ |K^0>}{A_0}\ ,
\end{aligned} \tag{7}$$

where $G_F s_1/\sqrt{2}$ and A_0, the $K_0 \rightarrow \pi\pi(I=0)$ amplitude, have values directly determined by experiment, which we will use. $Im\,\tilde{C}_6$ and $< \pi\pi(I=0)|Q_6|K^0>$ have been studied and discussed elsewhere, and to these we shall return. Our objective is to establish a lower bound on the CP violating product $s_2 c_2 s_3 s_\delta$ to which we now proceed.

To constrain the product $s_2 c_2 s_3 s_\delta$ we return to the expression for ϵ in Eq. (4) in the basis where A_0 is real. Here much previous work used a short distance analysis[7] for both ReM_{12} and ImM_{12}. However it is difficult to justify neglecting the long distance contributions to ReM_{12}, and we shall simply use the experimental value for $\Delta M = 2ReM_{12}$ in the denominator of Eq. (4). On the other hand, one can argue that ImM_{12} is given almost entirely by the short distance contribution in the phase convention where A_0 is real.[8] Traditionally, this short-distance (sd) contribution (from the box diagram involving heavy quarks and W's) is computed in the quark basis. In the basis where A_0 is real, $ImM_{12}^{sd} = (ImM_{12}^{sd})_{quark} + 2\xi\, ReM_{12}^{sd}$, and Eq. (4) becomes

$$\epsilon \approx \frac{e^{i\pi/4}}{\sqrt{2}}\left[\frac{(ImM_{12}^{sd})_{quark}}{\Delta M} + 2\xi\frac{ReM_{12}^{sd}}{\Delta M}\right]. \tag{8}$$

Inserting the familiar expression[7] for the short-distance contribution to $Im M_{12}$, we have:

$$\epsilon = \left[\frac{BG_F^2 f_K^2 m_K}{12\sqrt{2}\,\pi^2 \Delta M_K}\ Im\left(\eta_1 \lambda_c^2 m_c^2 + \eta_2 \lambda_t^2 m_t^2 + 2\eta_3 \lambda_c \lambda_t m_c^2 \ell n \frac{m_t^2}{m_c^2}\right) + \sqrt{2}\ \xi \frac{Re M_{12}^{sd}}{\Delta M}\right] e^{i\pi/4} . \tag{9}$$

In Eq. (9), $\lambda_q \equiv U_{qs}^* U_{qd}$ is a product of K-M matrix elements, B parametrizes the matrix element of the $\Delta S = 2$ operator ($B = +1$ for vacuum insertion),[7] and η_1, η_2, η_3 take account of the strong interaction corrections to the effective $\Delta S = 2$ Hamiltonian relevant to $K^0 - \bar{K}^0$ mixing. These latter parameters have the values 0.7, 0.6 and 0.4, respectively, for $M_W = 80\ GeV$ and $\Lambda_{QCD} = 0.1\ GeV$.[9–10] The value of B has recently been extracted by relating the matrix element of the $\Delta S = 2$ operator to the measured contribution of the $\Delta I = 3/2$ operator to $K \to \pi\pi$ decay using $SU(3)$ and current algebra, with the result[11] $B = 0.33$. Although not written expressly in Eq. (9), we also include the effect[12] of higher powers of m_t^2/M_W^2. Inserting experimental masses and dropping terms which are second order in sine θ_i compared to those of zero order, Eq. (9) becomes

$$\epsilon = \left[\left(\frac{1.94}{GeV^2}\right) s_1^2 (s_2 c_2 s_3 s_\delta)\left(-0.7 m_c^2 + 0.6 m_t^2 \left(\frac{Re\lambda_t}{s_1}\right) + 0.4 m_c^2 \ell n \left(\frac{m_t^2}{m_c^2}\right)\right) + \sqrt{2}\ \xi \frac{Re M_{12}^{sd}}{\Delta M}\right] e^{i\pi/4} \tag{10}$$

where $(Re\lambda_t/s_1) = s_2(c_1 s_2 c_3 + c_2 s_3 c_\delta)$. Eq. (7) for ξ shows that it is proportional to $s_2 c_2 s_3 s_\delta$ and considerably smaller than the first term inside the brackets of Eq. (10). It is negligible in the domain we are investigating of a lower bound on $|\epsilon'/\epsilon|$ and hence on $|\xi|$. Here we also point out that the experimental bound[13] on the b-quark branching ratios into up versus charm quark final states $B(b \to$

$u)/B(b \to c) < 0.09$ eliminates the sign ambiguity in $Re\lambda_t$ and hence in $s_2c_2s_3s_\delta$. Specifically, it was previously possible[14] that if $c_\delta < 0$ and $s_3 > s_2$, $Re\lambda_t$ could become sufficiently negative that the entire coefficient of $s_2c_2s_3s_\delta$ in Eq. (10) would become negative. Then $s_2c_2s_3s_\delta$ must also become negative to preserve the observed phase of ϵ. However, the constraint $B(b \to u)/B(b \to c) < 0.09$ requires[15] $7.7\ |U_{ub}|^2 < 0.09 \times 2.75|U_{cb}|^2$ or that $s_3^2 < s_2^2 + 2s_2s_3c_\delta + s_3^2$, which in turn requires $s_2^2 + 2s_2s_3c_\delta > 0$. Thus, if $c_\delta < 0$ we observe that $Re\lambda_t/s_1 \simeq s_2^2 + s_2s_3c_\delta > s_2^2 + 2s_2s_3c_\delta > 0$ and therefore $Re\lambda_t$ is positive. ($Re\lambda_t$ is trivially positive if $c_\delta > 0$.) This fixes the sign of $s_2c_2s_3s_\delta$ *also to be positive.*[16] It now follows that the parameter ξ is negative. This comes from the known phase of C_6 ($ImC_6/ReC_6 < 0$) and the requirement that C_6Q_6 contribute *constructively* to A_0 which is positive by definition – i.e. that "penguins" contribute to a $\Delta I = 1/2$ enhancement rather than a suppression. This in turn fixes ϵ'/ϵ through Eq. (6) to be *positive.*[16] Therefore we drop the modulus on $|\epsilon'/\epsilon|$ and treat ϵ'/ϵ as a positive quantity. Also, since ξ is negative, its presence in Eq. (10) only strengthens the bound on $s_2c_2s_2s_\delta$ which one obtains by dropping it. Therefore we drop the term involving ξ, and use the experimental values for ϵ and s_1^2 to obtain

$$s_2c_2s_3s_\delta \geq \frac{2.4 \times 10^{-2}\ GeV^2}{\left[-0.7m_c^2 + 0.6m_t^2\left(\frac{Re\lambda_t}{s_1}\right) + 0.4m_c^2\ \ln\frac{m_t^2}{m_c^2}\right]} . \tag{11}$$

A lower bound on $s_2c_2s_3s_\delta$ obviously requires an upper bound on $m_t^2 Re\lambda_t$, for which we turn to $K_L \to \mu\mu$ decay. The relevant constraint arises from requiring that the short-distance contribution to the $K_L \to \mu\mu$ branching ratio should not exceed the total minus the calculable absorptive contribution from $K_L \to \gamma\gamma \to \mu\mu$ with on-shell intermediate photons. The only uncertainty here is the long-distance contribution from the dispersive amplitude involving off-shell intermediate photons, which might interfere destructively with the short-distance contribution permitting the latter to be larger.[17] However, the present experimental situation[5] regarding the analogous decay $\eta \to \mu\mu$ (but without the short-distance contribution) indicates that the long-distance dispersive contribution is no bigger than the absorptive one (and consistent with zero). This strongly suggests that there is at most a factor of two uncertainty in the maximum size of the short-distance contribution, even if we allow for complete destructive interference with the long-distance dispersive contribution. *Without* this factor of two, the constraint from $K_L \to \mu\mu$ reads[18]

$$(Re\lambda_t/s_1)C(x_t)\ \eta \leq 0.9 \times 10^{-2} \tag{12}$$

where $x_t = m_t^2/m_W^2$, and

$$C(x_t) = x_t + \frac{3}{4}\frac{x_t^2}{1-x_t} + \frac{3}{4}\frac{x_t^2\ \ell n\, x_t}{(1-x_t)^2}$$

and η is a correction factor due to strong interactions which we incorporate from QCD ($\eta \approx 0.9$).[18] The upper bound on $m_t^2 Re\lambda_t$ which follows from Eq. (12), with and without allowance for long-distance uncertainties is plotted in Fig. 1 as a function of m_t ($m_t^2 Re\lambda_t$ would be independent of m_t if non-leading terms in m_t^2/m_W^2 and the m_t dependence of QCD corrections were neglected).

The lower bound on $s_2c_2s_3s_\delta$ which now follows from inserting the upper bound on $m_t^2 Re\lambda_t$ in Fig. 1 into Eq. (11) is shown in Fig. 2. With maximum allowance for the uncertainties due to long-distance contributions to $K_L \to \mu\mu$, we observe that $s_2c_2s_3s_\delta \geq 2 \times 10^{-4}$. In constructing this bound we have used $m_c = 1.5\ GeV$ in the denominator of Eq. (11). However the value of m_c

matters very little: using the upper bound on $m_t^2 Re\lambda_t$, the t quark contribution completely dominates CP violation in the $K^0 - \bar{K}^0$ mass matrix and the other terms in the denominator of Eq. (11).

Having obtained a lower bound on $s_2 c_2 s_3 s_\delta$, we direct our attention to $Im\,\tilde{C}_6$. The Wilson coefficients of the operators appearing in the effective $\Delta S = 1$ weak Hamiltonian have been derived in a number of analyses[6] of QCD corrections to the weak interaction, usually computed in the leading logarithm approximation to all orders in the strong interaction. These analyses[6] give $Im\,\tilde{C}_6 \approx -0.1$.[19] Since ImC_6 in particular is generated at momentum scales between m_t and m_c, it is truly a short-distance effect susceptible to such a leading logarithm calculation in QCD and is quite stable with respect to changes in parameters (e.g., Λ_{QCD}).

Finally we consider $< \pi\pi(I=0)|\, Q_6\, |K^0 >$ where Q_6 is the $(V-A)\times(V+A)$ "penguin" operator,

$$[\bar{s}_\alpha\, \gamma^\mu(1-\gamma_5)d_\beta]\; [\bar{u}_\beta\, \gamma_\mu(1+\gamma_5)u_\alpha + \bar{d}_\beta\, \gamma_\mu(1+\gamma_5)d_\alpha + \bar{s}_\beta\, \gamma_\mu(1+\gamma_5)s_\alpha]\ ,$$

through whose coefficient CP non-invariance primarily enters the $K \to \pi\pi$ amplitude. Here also we favor a conservative approach by using the bag model value for this matrix element. In particular, we make ***no assumption about the origin of the*** $\Delta I = 1/2$ ***rule*** being due to "penguin" contributions to A_0, which would require much larger values of $< \pi\pi|\, Q_6\, |K^0 >$ if the usual calculations[6] of the real part of the Wilson coefficient C_6 are also employed.

To use the bag model matrix element in the literature, we observe that Q_6 is related to the operator O_5 used by Donoghue *et al.*[20–21] by a factor of 9/16 when matrix elements between color singlet states are taken. Therefore,

$$\left|\left\langle \pi\pi(I=0)\right|\, Q_6\, \left|K^0\right\rangle\right| = \frac{9\sqrt{3}}{16}\left|\left\langle \pi^0\pi^0(I=0)\right| O_5 \left|K^0\right\rangle\right| = 1.4\,GeV^3 \qquad (13)$$

where we have used directly the value for $< \pi^0\pi^0|O_5|K^0 >$ from the compendium[21] of bag-model matrix elements. With the same normalization, $A_0 = 4.70 \times 10^{-4}\ MeV$, so that

$$\left|\frac{G_F}{\sqrt{2}}\, s_1 \frac{< \pi\pi(I=0)|\,Q_6\,|K^0 >}{A_0}\right| = 5.4\,. \tag{14}$$

It should be noted that the value of the matrix element for $< \pi^0\pi^0|\,O_5\,|K^0 >$ is actually derived from that of $< \pi^0|\,O_5\,|K^0 >$ by use of current algebra, assuming the former matrix element is a quadratic form in the external four-momenta, matching to the soft pion conditions, and demanding the amplitude vanish in the $SU(3)$ limit. As noted in Ref. 15 there is some ambiguity in what four-momentum squared to assign the remaining external K and π states in the soft-pion limit if they are to remain bag model states at rest.

Combining $Im\,\tilde{C}_6$, the bag model matrix element in Eq. (13), and the lower bound on $s_2c_2s_3s_\delta$, results in the lower bound shown in Fig. 3, from which we observe

$$\epsilon'/\epsilon \geq 2 \times 10^{-3}(0.33/B)|Im\,\tilde{C}_6\,/0.1|\;|< \pi\pi(I=0)|Q_6|K^0 > /1.4\,GeV^3| \tag{15}$$

where the dependence of ϵ'/ϵ upon B, $|Im\,\tilde{C}_6\,|$, and the matrix element of Q_6 are explicitly shown. We conclude with a discussion of these uncertainties.

We have eliminated a major uncertainty by establishing a reliable lower bound on $s_2c_2s_3s_\delta$, the product characteristic of CP violation throughout the K-M model. This bound depends inversely on the parameter B, but if typical of other results which use $SU(3)$ and current algebra in their derivation, the value of 0.33 ought to be reliable to within $O(20\%)$. Furthermore, our allowance for long-distance dispersive contributions to $K_L \rightarrow \mu\mu$ is probably an overestimate of the actual uncertainty, and so we regard our bound on $s_2c_2s_3s_\delta$ as conservative.

Concerning the Wilson coefficient $Im\,\tilde{C}_6$, there is good agreement among the extant renormalization group analyses.[19] As this CP violating coefficient is generated at momentum scales above m_c, there is no dependence on an infrared cutoff and little sensitivity to Λ_{QCD}. The uncertainties associated with this coefficient are probably $O(30\%)$.

The largest remaining uncertainty concerns the $K \to \pi\pi$ matrix element of Q_6 in Eq. (13). We note that matrix elements of $(V-A)\times(V+A)$ type operators such as Q_6 are much more certain in the bag model than those of $(V-A)\times(V-A)$ operators: the integrals contributing enter with the same sign rather than opposite signs and there are no delicate cancellations which can lead to major uncertainties.[20] Moreover, we do not assume that the $\Delta I = 1/2$ rule is due to penguin contributions to the amplitude for $K \to \pi\pi$. This would require effectively "boosting" the matrix element of Q_6 (and hence ϵ'/ϵ) by at least a factor of two given most calculations[6] of ReC_6. We have taken a more conservative approach in seeking a lower bound on ϵ'/ϵ, by choosing an independent evaluation of the matrix element. While it is still possible that the actual matrix element is smaller than what we have used, smaller matrix elements make our understanding of the observed $\Delta I = 1/2$ dominance increasingly problematic.

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16. Since the θ_i are chosen to lie in the first quadrant by convention, this fixes s_δ to be positive. We emphasize that the positivity of ϵ'/ϵ distinguishes the K-M model from models where CP violation is predominantly in the $\Delta S = 1$ interaction [such as in the model of S. Weinberg, Phys. Rev. Letters 37, 657 (1976)] where ϵ'/ϵ is negative, see J. S. Hagelin, Phys. Lett. 117B, 441 (1982).

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FIGURE CAPTIONS

1. Upper bound on $Re\lambda_t m_t^2$ as a function of m_t from $K_L \to \mu^+\mu^-$ including subleading terms in m_t^2/m_W^2 and perturbative QCD: (a) with no allowance for long-distance dispersive contributions (dashed); (b) with maximal allowance for long-distance dispersive contributions (solid).
2. Lower bound on the product $s_2 c_2 s_3 s_\delta$ from $|\epsilon|$ and $K_L \to \mu^+\mu^-$ decay shown for $B = 0.33$: (a) with no allowance for long-distance dispersive contributions in $K_L \to \mu^+\mu^-$ (dashed); (b) with maximal allowance for long-distance dispersive contributions (solid). The bound scales inversely with B.
3. Lower bound on ϵ'/ϵ in the standard model as a function of m_t, shown for $B = 0.33, |Im\,\tilde{C}_6| = 0.1$ and $| < \pi\pi(I = 0)|Q_6|K^0 > | = 1.4\ GeV^3$: (a) with no allowance for long-distance dispersive contributions in $K_L \to \mu^+\mu^-$ (dashed); (b) with maximal allowance for long-distance dispersive contributions (solid). The bound scales proportionally with $Im\,\tilde{C}_6$, $< 2\pi(I = 0)|Q_6|K^0 >$, and inversely with B.

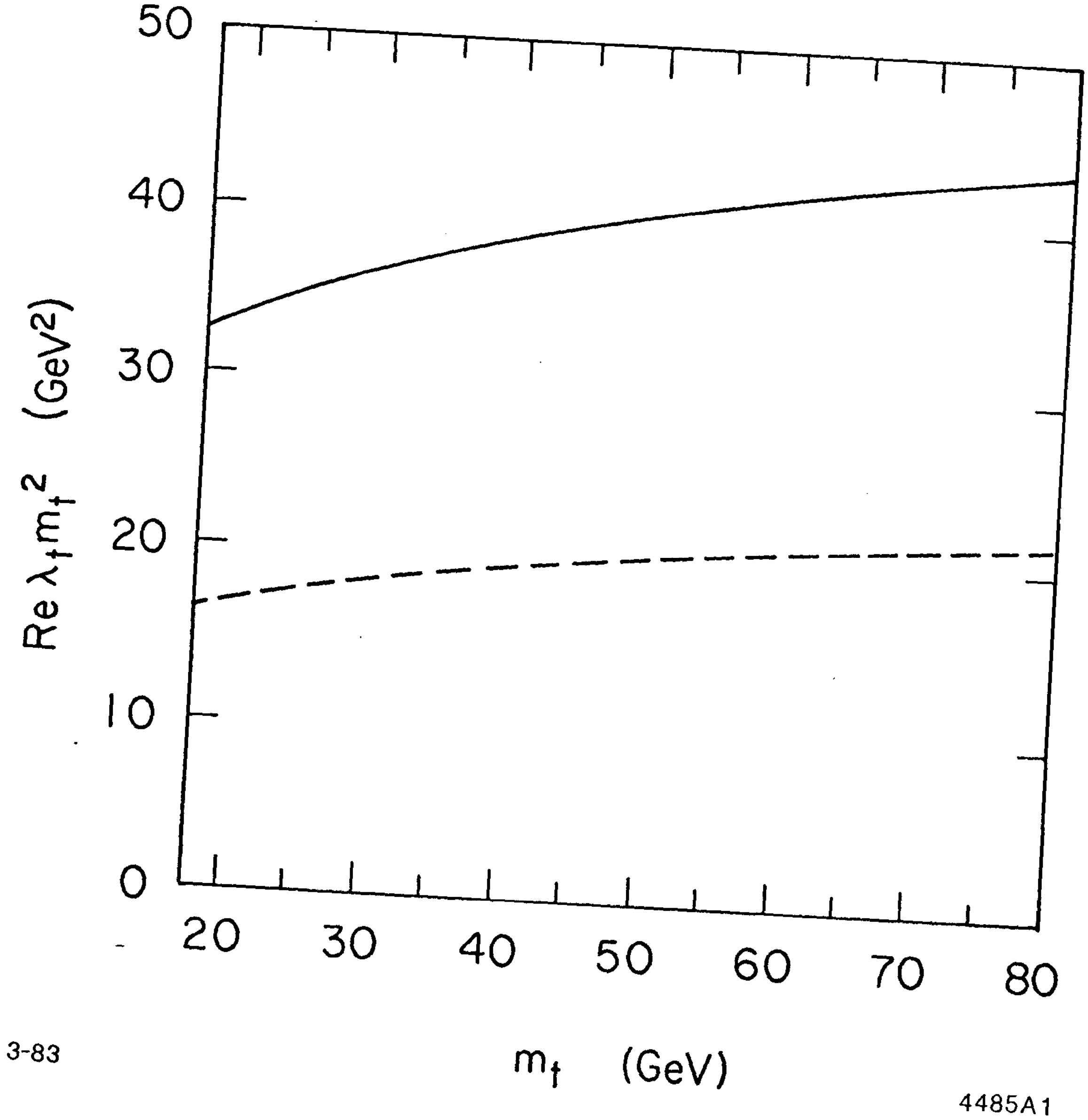
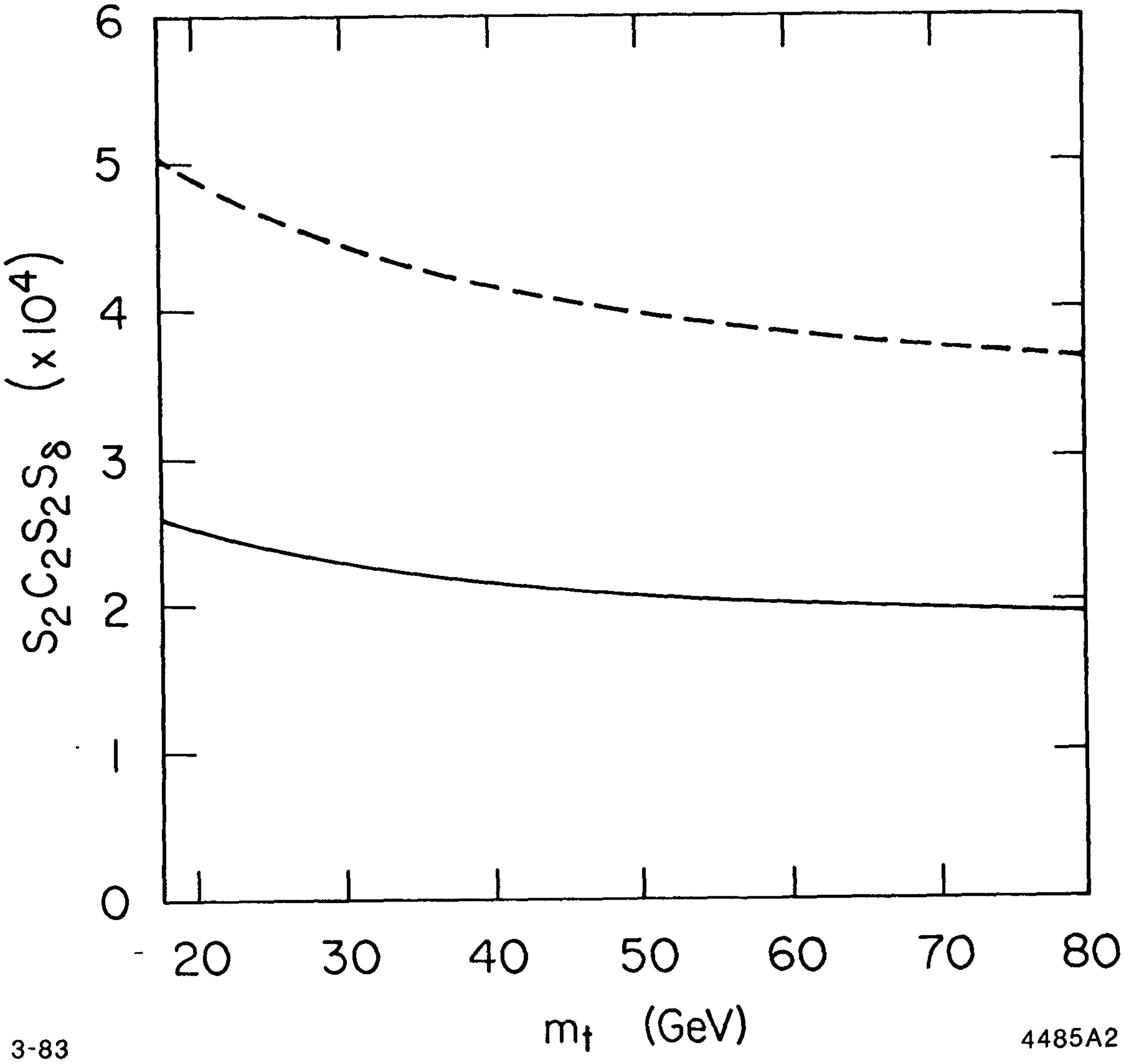

50
40
30
20
10
0
Re $\lambda_t m_t^2$ (GeV^2)
20
30
40
50
60
70
80
3-83
m_t (GeV)
4485A1

Fig. 1



$S_2C_2S_2S_\delta$ $(\times 10^4)$
0
1
2
3
4
5
6
20
30
40
50
60
70
80
m_t (GeV)
3-83
4485A2

Fig. 2

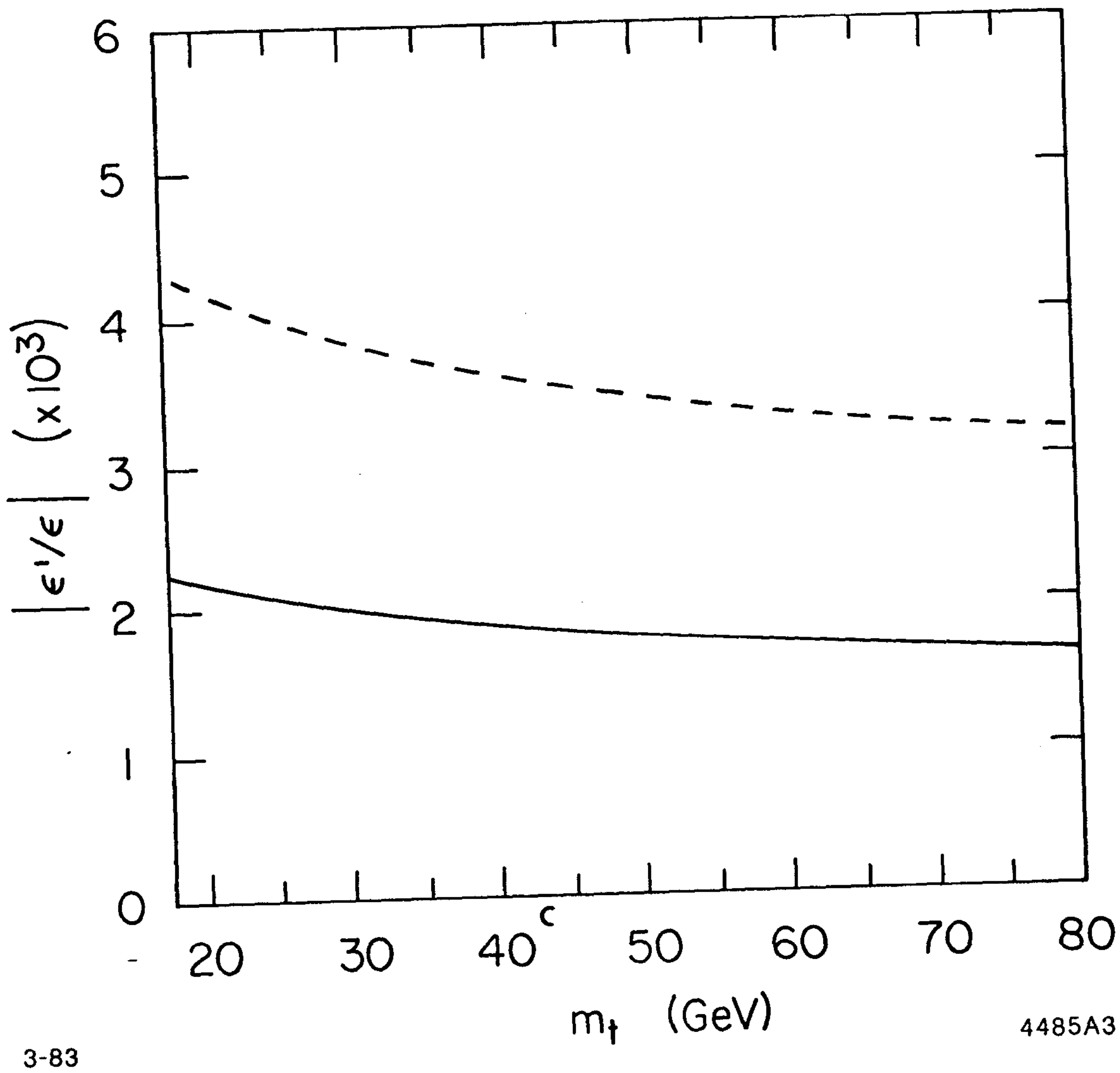


Fig. 3