SLAC-PUB-3061
February 1983
T/E

AN EFFECTIVE LAGRANGIAN FOR SUPERSYMMETRIC QCD*

Michael E. Peskin
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

I present a Lagrangian which describes the spontaneous breaking of chiral symmetries in strongly interacting supersymmetric Yang-Mills theory with matter fields. This Lagrangian predicts that supersymmetry is spontaneously broken if the matter fields have precisely zero mass.

Lecture presented at the Workshop on
"Problems in Unification and Supergravity"
La Jolla Institute, La Jolla, California
January 12-16, 1983

*Work supported by the Department of Energy under contract DE-AC03-76SF-00515.

Over the past few years, as our understanding of weakly coupled supersymmetric theories has steadily increased, the dynamics of strongly coupled supersymmetric Yang-Mills theory has come to appear more and more mysterious. Initially, it was tempting to regard these theories as having qualitatively the same behavior as ordinary gauge theories of fermions. Using this hypothesis, Dine, Fischler, and Srednicki[1] and Dimopoulos and Raby[2] argued that these theories should show spontaneous supersymmetry breaking. This conclusion, however, was apparently contradicted when Witten derived a striking constraint on dynamical supersymmetry breaking.[3] This contradiction has left workers in this field more than a little puzzled and has led to a consensus that the pattern of chiral symmetry breaking in supersymmetric Yang-Mills theory must be an unusual one. However, it need not be so. In this lecture, I will demonstrate this by exhibiting an effective Lagrangian describing the spontaneous breaking of chiral symmetry in supersymmetric Yang-Mills theory which is consistent both with the physical picture of Dine, Fischler, Srednicki, Dimopoulos and Raby and with the constraints proved by Witten. This conclusion differs from that of a recent paper by Taylor, Veneziano, and Yankielowicz;[4] I will clarify the difference between my analysis and theirs as I proceed.

I will restrict my attention in this lecture to theories in which a gauge supermultiplet (A_μ, λ, D) couples to matter fields which belong to a real representation of the gauge group. For most of the analysis of this paper, I will take this representation to comprise n copies each of a complex representation r and its complex conjugate $\bar{r}$. The matter supermultiplets are, then, of the form:

$$(A_{ri}, \psi_{ri}, F_{ri}) + (A_{\bar{r}i}, \psi_{\bar{r}i}, F_{\bar{r}i})$$

where $i = 1, \ldots, n$. ψ denotes a left-handed fermion; the other fields are complex bosons. These models are essentially supersymmetric versions of QCD with n flavors; I will refer to them as SSQCD. They are the models to which Witten's theorem applies most directly.

At the classical level, for zero mass matter fields, SSQCD has the global symmetry $U(n) \times U(n) \times U(1)$, where the last $U(1)$ corresponds to R-invariance. In the quantum theory, one $U(1)$ symmetry is destroyed by anomalies; the full global symmetry is, then, $U(n) \times U(n)$. One can give mass to the matter fields by adding to the Lagrangian a superpotential of the form:

$$W(A) = \sum_{i=1}^{n} m \; A_{ri} A_{\bar{r}i} \tag{2}$$

This potential breaks $U(n) \times U(n)$ explicitly to (vectorial) $U(n)$. In ordinary QCD, the formation of fermion pair condensates causes a spontaneous breaking of the chiral symmetry of the zero-mass theory; I see no good reason why this same physics should not appear also in the supersymmetric theory. Such fermion-pair condensates would give rise in SSQCD to the pattern of spontaneous symmetry-breaking:

$$U(n) \times U(n) \to U(n) \, . \tag{3}$$

I will argue that the symmetry-breaking pattern (3) is consistent with the constraints of supersymmetry by exhibiting a supersymmetric effective Lagrangian which gives a low-energy phenomenological description of this symmetry breaking. This Lagrangian should be the appropriate generalization to SSQCD of the description of the low-energy dynamics of QCD by a nonlinear sigma model.[5] More specifically, this Lagrangian has the following properties: First, it obeys a number of requirements which follow from exact properties of SSQCD:

1. The Lagrangian has the form

$$\mathcal{L} = \mathcal{L}_0 + tr\ m\Delta \tag{4}$$

where $\mathcal{L}_0$ is invariant to $U(n) \times U(n)$ and Δ , which represents the matter-field mass term, transforms as an $(\bar{n}, n)$ under $U(n) \times U(n)$.

2. The Lagrangian is manifestly supersymmetric.

3. Supersymmetry is not spontaneously broken for any value of $m \neq 0$.

Requirement (3) follows from Witten's theorem.[3] Secondly, the Lagrangian is consistent with a number of intuitive requirements of the physical picture of chiral symmetry breaking by fermion pair condensates:

4. The pattern of spontaneous symmetry breaking is

$$U(n) \times U(n) \rightarrow U(n)\ ;$$

the associated Goldstone bosons appear as elementary fields of the phenomenological Lagrangian.

5. The gluino is heavy and irrelevant to low-energy physics; the gluino field does not appear in the Lagrangian.

6. The Lagrangian implies that $< \psi_r \psi_{\bar{r}} > \neq 0$ by insuring that, in the presence of the symmetry-breaking perturbation $tr\ m\Delta$, the Goldstone bosons receive $(mass)^2$ propertional to m.

7. The bosonic variables of the model live on a compact space.

8. The Lagrangian satisfies decoupling: sending one eigenvalues of m to infinity reduces the $U(n) \times U(n)$ version of the Lagrangian to the $U(n-1) \times U(n-1)$ version.

Since the requirements (6) and (7) are not completely obvious, and since they will play a crucial role in my analysis, I comment on them briefly. The authors of Refs. 1 and 2 argue that my assumption (6) already implies that supersymmetry is spontaneously broken. Their argument makes use of the Ward identity, valid if supersymmetry is manifest:

$$\left\langle \frac{1}{2}\psi_r \cdot \psi_{\bar{r}} + A_r \cdot F_{\bar{r}} \right\rangle = 0$$

If one eliminates F using the equations of motion, one finds

$$-\frac{1}{2} < \psi_r \cdot \psi_{\bar{r}} >= m < |A_r|^2 > \tag{5}$$

If $< |A|^2 >$ is regular as $m \to 0$, supersymmetry implies $< \psi_r \psi_{\bar{r}} >= 0$. But such regularity is not necessary, or even to be expected. In ordinary QCD one can cast $< \bar{\psi}\,\psi >$ into the form

$$< \bar{\psi}\,\psi >= m\left\langle tr\left(\frac{1}{-D^2 + m^2 + \frac{g}{4}\sigma_{\mu\nu}F^{\mu\nu}}\right)\right\rangle \tag{6}$$

where the expectation value is to be taken over configurations of the gauge field.[6] The object inside the trace is formally quite similar to the A_r propagator. If the right-hand side of (6) can remain nonzero as $m \to 0$, why should the right-hand side of (5) not also show this behavior? * I feel that the assumption (6) does not unduly prejudice the theory I will construct toward spontaneous supersymmetry breaking.

My assumption (7) would not be a strong assumption in ordinary field theories with global symmetries. However, in supersymmetric theories it is a very strong assumption, because supersymmetric nonliner sigma models with variables on

* I thank Giorgio Parisi for this observation.

compact spaces are not obtainable as limits of linear sigma models[7,8] I will simply assume that the nonperturbative dynamics of SSQCD gives rise to a compact manifold of possible vacuum states. It is here that my analysis differs from that of Taylor, Veneziano, and Yankielowicz[4]; those authors chose a set of dynamical variables which could be obtained from a supersymmetric linear sigma model.

The formulation of a supersymmetric nonlinear sigma model with the symmetry-breaking pattern (3) appears at first sight problematical, for the following reason: Nonlinear sigma models describing the spontaneous breakdown of a symmetry group G to H normally have variables which live on the coset space G/H. Such a model can be made sypersymmetric only if this space is a Kähler manifold.[9] However, the space suggested by (3) is

$$\frac{G}{H} = \frac{U(n) \times U(n)}{U(n)} \approx U(n) \tag{7}$$

which is not even a complex manifold, and therefore is not Kähler. I choose to interpret this difficulty as a requirement from supersymmetry that there be additional light bosons in the theory beyond the required Goldstone bosons. The spectrum of these particles should be determined by embedding (7) in a larger space which is a Kähler manifold.[8] The smallest such homogeneous space with (7) as a subspace is

$$\frac{U(2n)}{U(n) \times U(n)} \ . \tag{8}$$

There are many embeddings of (7) into (8); for the purpose of this lecture, I will choose one and work out its implications. Let me, then, label the $U(n) \times U(n)$ subgroup of $U(2n)$ appearing in (8) as $[U(n) \times U(n)]_D$ and the isomorphic group appearing in (7) as $[U(n) \times U(n)]_N$. If I take $U(2n)$ in (8) to be generated by arbitrary Hermitian $2n \times 2n$ matrices, I can represent an embedding of (7) into (8) by identifying as the generators of the various subgroups of this $U(2n)$ matrices of the following forms:[10]

$$
\begin{aligned}
&[U(x) \times U(n)]_D : && \left(\begin{array}{c|c} t_1 & 0 \\ \hline 0 & t_2 \end{array}\right) \\
&[U(n) \times U(n)]_N : && V \left(\begin{array}{c|c} t_1 & 0 \\ \hline 0 & t_2 \end{array}\right) V^{-1} \\
&U(n) : && \left(\begin{array}{c|c} t_1 & 0 \\ \hline 0 & t_1 \end{array}\right)
\end{aligned}
\tag{9}
$$

where t_1, t_2 are $n \times n$ Hermitian matrices and

$$
V = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} 1 & 1 \\ \hline -1 & 1 \end{array}\right) \; \epsilon \; SU(2)
\tag{10}
$$

$[U(n) \times U(n)]_D$ and $[U(n) \times U(n)]_N$ coincide precisely on the $U(n)$ subgroup generated by the last line of (9); this group will play the role of the conserved vector $U(n)$ of SSQCD.

The manifold (8) has $2n^2$ dimensions, so the number of light bosons in the model is doubled from the number of Goldstone bosons associated with the symmetry-breaking (3). The $2n^2$ coordinates form two adjoint representations of the vectorial $U(n)$. The particle spectrum of this model may be given a plausible physical interpretation as follows: Since the theory with fermionic matter fields alone must contain Goldstone bosons composed of two fermions and the theory with bosonic matter fields only should contain Goldstone bosons composed of two bosons, the full SSQCD should contain two light pseudoscalar mesons, both of which supersymmetry could well require to be massless. These mesons, with the quantum numbers of

$$(\psi_{ri} \cdot \psi_{\bar{r}j}) \quad \text{and} \quad (A_{ri} \cdot A_{\bar{r}j}) , \tag{11}$$

do form two adjoint representations of $U(n)$.

The most general $U(2n)$-invariant Lagrangian with coordinates in (8) has been constructed some time ago by Zumino[8] and Aoyama[11] ; it may be written:

$$\mathcal{L} = \int d^4\theta \; f_\pi^2 \; tr \; log\,(1 + A\bar{A}) \tag{12}$$

where A is an $n \times n$ complex matrix. Under an infinitesimal $U(2n)$ transformation

$$U = 1 + iT \, , \qquad T = \left(\begin{array}{c|c} t_1 & t \\ \hline \bar{t} & t_2 \end{array} \right) \; . \tag{13}$$

A transforms according to

$$\delta A = i(At_2 - t_1 A) + t + A\bar{t}A \, . \tag{14}$$

In principle, one could add terms to (12) to break its symmetry explicitly to $[U(n) \times U(n)]_N$; however, I will study only the simplest kinetic energy term (12) here.

Equation (12) describes a theory with manifest supersymmetry and $2n^2$ massless boson-fermion pairs. However, this theory is not yet an acceptable one, because it does not satisfy the requirement (6) above. One might try to give masses to the particles of this theory by adding to (12) a symmetry-breaking F term of the form

$$\int d^2\theta \; tr(m\Delta(a)) \, . \tag{15}$$

However, this term produces $(mass)^2$ for the Goldstone bosons proportional to m^2, a signal that $< \psi_r \cdot \psi_{\bar{r}} >= 0$. This problem can only be remedied by adding to (12) an F term of zeroth order in m. There is no such term invariant under all of $U(2n)$, but one can find an F term invariant to $[U(n) \times U(n)]_N$. The generators of this subgroup can be rewritten from (9) in the form

$$T = \left(\begin{array}{c|c} t_a & t_b \\ \hline t_b & t_a \end{array} \right) \ , \tag{16}$$

where t_a and t_b are Hermitian. For these generators, (14) specializes to

$$\delta A = i[A, t_a] + t_b + A t_b A \ . \tag{17}$$

There is a unique F term constructed from A which is invariant to (17):

$$\int d^2\theta \ h f_\pi \cdot tr \ (tan^{-1} A) \tag{18}$$

where h is a constant. In addition, there is a unique structure which transforms linearly as an $(\bar{n}\,, n)$ under $[U(n) \times U(n)]_N$:

$$\int d^2\theta \ f_\pi \ tr \ m \Big(\frac{i}{1 - iA} \Big) \ . \tag{19}$$

The bosonic part of (18) plus (19) has the following form:

$$f_\pi F^y \ tr \ X_y \Big(\frac{h}{1 + A^2} + \frac{1}{1 - iA} (-m) \frac{1}{1 - iA} \Big) \tag{20}$$

where F is the auxiliary field associated with A and the X_y span a complete set of $n \times n$ matrices.

Let us first study the Lagrangian (12) plus (18), with $m = 0$. Eliminating F yields the potential energy:

$$V(A) = h^2 \ tr \left(\frac{1}{(1 + A^2)} \ (1 + A\bar{A}) \ \frac{1}{(1 + \bar{A}^2)} \ (1 + \bar{A} A) \right) . \tag{21}$$

If A is Hermitian, this $V(A) = h^2$; this choice gives the minimum of (21). That fact poses a severe problem for the theory: Supersymmetry is spontaneously broken. The minimum, though, does have some redeeming features. First, the space of minima of $V(A)$ is isomorphic to $U(n)$, so the vacuum degeneracy is that expected from (3). Secondly, if one attempts to give the Goldstone bosons mass by adding the mass term (19) and treating it only as a first-order perturbation of this theory, one finds a correction to the potential (for Hermitian A)

$$\Delta V = tr\ (mA^2) \tag{22}$$

which, properly, gives the Goldstone bosons $(mass)^2$ proportional to m.

The problem I have noted is, however, neatly resolved by a more careful examination of the full theory. If one takes $m \neq 0$, in (20), one can find a point at which the coefficient of F in this term vanishes by setting

$$\begin{aligned} 0 &= \frac{h}{1+A^2} - \frac{1}{1-iA}\ m\ \frac{1}{1-iA} \\ &= \frac{h}{1-iA}\left(\frac{1-iA}{1+iA} - \frac{m}{h}\right)\frac{1}{1-iA} \end{aligned} \tag{23}$$

Thus, by setting

$$A = -i\left(\frac{1-m/h}{1+m/h}\right) \tag{24}$$

one finds a supersymmetric vacuum state. Note that if m is a matrix with one large eigenvalue, the corresponding eigenvalue of A is drawn to the value (24); this verifies the decoupling requirement (8) above.

This supersymmetric vacuum states exists for any $m \neq 0$, but as $m \to 0$ it is separated from the set of states with A Hermitian by a potential energy barrier whose height grows as m^{-1}. If we speak in terms of the index of Witten[3], the model I have constructed has index equal to 1 for any nonzero value of m but a zero index at $m = 0$. This discontinuous change in the index at $m = 0$, or, equivalently, the inaccessibility of the supersymmetric state as $m \rightarrow 0$, violates an explicit assumption made by Witten in extending his proof the absence of spontaneous sypersymmetry breaking in SSQCD to the massless case.[3] This method of evading Witten's conclusion was suggested earlier by Srednicki;[12] I thought at the time that it could never be realized in an explicit model of SSQCD.

I have, then, presented an effective Lagrangian which describes the low-energy of dynamics of supersymmetric Yang-Mills theory with matter fields in complex-conjugate-pair representations, assuming that the pattern of chiral symmetry breaking is that observed in the familiar strong interactions. This Lagrangian has a supersymmetric vacuum state for any nonzero value of the matter field mass, but it has spontaneously broken supersymmetry for matter fields of precisely zero mass.

One can straightforwardly extend the analysis of this paper to more general forms of the non-linear sigma model action and to the case of matter fields in real representations. In all cases, the physics of the generalized models is qualitatively the same as that described here.[13]

I am grateful to Chong-Leong Ong, Giorgio Parisi, Gabriele Veneziano, Edward Witten, and Shimon Yankielowicz, for discussions of the properties of SSQCD, to I. M. Singer for valuable advice on geometry, and, especially, to Joe Polchinski, for asking all the right questions. I thank Glennys Farrar and Frank Henyey for organizing this stimulating conference.

REFERENCES

1. M. Dine, W. Fischler, and M. Srednicki, Nucl. Phys. B189, 575 (1981).
2. S. Dimopoulos and S. Raby, Nucl. Phys. B192, 353 (1981).
3. E. Witten, Nucl. Phys. B202, 513 (1981).
4. T. R. Taylor, G. Veneziano, and S. Yankielowicz, CERN preprint TH 3460 (1980).
5. S. Weinberg, Phys. Rev. Lett. 29, 1698 (1967), Phys. Rev. 166, 1568 (1968).
6. T. Banks and A. Casher, Nucl. Phys. B169, 102 (1980).
7. B. A. Ovrut and J. Wess, Phys. Rev. D25, 409 (1982).
8. C.-L. Ong, SLAC preprint SLAC-PUB-2956 (1982).
9. B. Zumino, Phys. Lett. 87B, 203 (1979).
10. I. M. Singer, personal communication.
11. S. Aoyama, Nuov. Cim. 57A, 176 (1980).
12. M. Srednicki, personal communication.
13. M. E. Peskin, in preparation.