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AN EVOLUTION CONDITION FOR ELECTROWEAK INTERACTIONS IN COMPOSITE MODELS*

Dedicated to the Memory of J. J. Sakurai.

PISIN CHEN† AND F. M. RENARD
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

We impose the requirement of good high energy behavior on $f\bar{f} \rightarrow W^+W^-$ scattering amplitudes for effective theory of electroweak interactions in composite models. We find that there exists a set of constraints on the form factors which we call the "evolution condition" any such theory should obey. According to this condition the electromagnetic and weak interactions are "unified" in a less stringent way than the so-called "unification condition." In addition, the effective theory should have a well-defined evolution governed by the constraints on the form factors. We have checked this condition on several $\gamma - W$ mixing schemes. Possible experimental tests are discussed.

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† Present address: Department of Physics, UCLA, Los Angeles, CA 90024.

1. INTRODUCTION

So far all the low energy phenomenology of electroweak interactions is consistent with the standard model predictions. Even the recent data from CERN[1] on the W-boson mass seems to be close to the value expected by the standard model. But there are also difficulties in the standard model: the "generation problem" associated with lepton-quark masses; the large number of parameters; the "naturalness" problem associated with the Higgs particles; the "desert" between 10^2 GeV and 10^{15} GeV; ... One wonders whether these difficulties could be avoided.

There are, on the other hand, radical alternatives (as opposed to extended gauge theories) to the electroweak interactions advocated by Bjorken[2] and by Hung and Sakurai.[3] In these non-gauge-theory alternative models, the low energy phenomenology can also be successfully reproduced. However, the price to pay is, aside from the non-renormalizability of their models, that the aesthetic beauty of unification between the electromagnetic and the weak interactions is not any more guaranteed as in the standard model. This is because in these non-gauge models extra parameters are present. For example, in the Hung-Sakurai model an extra parameter appears through the $\gamma - W^3$ mixing strength.

To find constraints on this extra freedom, Hung and Sakurai imposed requirements based on general principles. In particular, by imposing "asymptotic $SU(2) \times U(1)$ symmetry" on the neutral current Hamiltonian, they arrived at a constraint between EM and weak couplings which they called the "unification condition." Namely, the ratio of the two couplings $\frac{e}{g}$ is identical to the mixing strength λ between the photon and the neutral weak-boson intermediate states. It is under this unification condition that all the low energy predictions, including the Weinberg mass relation, are identical to that of the standard model. This is actually not very surprising since one also has the property of asymptotic $SU(2) \times U(1)$ gauge symmetry in the standard model when the energy is much larger than the scale of spontaneous gauge symmetry breaking.

What surprises us is that they get the same unification condition by imposing another independent requirement. If one uses the so-called "minimal substitution scheme" for $W^{\pm}$ couplings (see Sec. 3) and requires for good high energy behaviors of $\nu\bar{\nu} \to W^+W^-$ or $e^+e^- \to W^+W^-$ scattering amplitudes, one will again arrive at the same condition $\frac{\lambda g}{e} = 1$. Is this merely a coincidence? Or maybe the Hung-Sakurai model has mimicked the gauge coupling structure too much via the minimal substitution scheme?

In recent years, there has been a large industry of composite models of leptons and quarks[4], which aim at solving the difficulties associated with the standard model. Among widely different approaches, there is a class of composite models that treats the weak-bosons also as composite particles. As a result the weak interactions are never more mediated by the gauge fields. It is in this context that the Hung-Sakurai model has been largely used[5] at the composite effective level so as to reproduce the low energy phenomenology of electroweak interactions.

Again, the question of how EM interaction and weak interaction are unified arises. With rather strong assumption of complete W-dominance for the photon couplings, Kögerler and Schildknecht[6] and Kuroda and Schildknecht[7] were forced to impose the unification condition due to universality. Without this strong assumption, one wonders whether the unification condition can still be obtained in composite models. Notice that if we ask for asymptotic properties of a composite system, potential complexities will arise. In particular, when $s \geq \Lambda^2$, where Λ is the mass scale of the underlying dynamics of the subconstituents, many new effects are expected to turn on.

Nevertheless, if we assume that $\Lambda^2 \gg M_{W,Z}^2$ (which is the case from various constraints[8]) then there is still a very large energy range (i.e., $4M_W^2 < s < \Lambda^2$) where we don't see much "oasis" in the "desert." It is within this energy range that the study of asymptotic $SU(2) \times U(1)$ symmetry and good high energy behaviors of the $f\bar{f} \to W^+W^-$ scattering amplitudes can be carried out without involving unwanted complexities.

In this paper we shall show that, under the requirement of good high energy ($4M_W^2 \ll s \leq \Lambda^2$) behaviors of $f\bar{f} \to W^+W^-$, the W form factors due to compositeness should obey a set of constraints which we call the "evolution condition." This evolution condition tells us not only how EM and weak interactions should unify, but also how an effective electroweak interaction should evolve. Namely, we found that for any composite model with effective electroweak interaction the relation between e and g may deviate from the unification condition. In addition, this effective description of the electroweak interaction should evolve in the following way: At low energy it is allowed to be non-standard although all other predictions like G_F, $sin^2\theta_W$, and ρ should agree with experiments. As energy goes higher, the form factors of the W couplings should evolve into a Yang-Mills-like structure, so that this effective theory evolves into a gauge-like theory.

One may observe that our approach to the constraints on W couplings is in the same spirit as that of Llewellyn Smith and Cornwall et al.[9] The main difference is that we replace the assumption of point-like couplings by the form factor effects due to compositeness.

The contents of this paper is the following: in Sec. 2 we review briefly the $\gamma - W$ mixing formalism in general cases. The "unification condition" is introduced through asymptotic $SU(2) \times U(1)$ symmetry. In Sec. 3 we derive the evolution condition in a primitive model which assumes only one isotriplet of composite W bosons. Some consequences of the evolution condition are then illustrated. Next we re-examine the evolution condition in multi-boson cases in Sec. 4. We conclude that the essential features of the condition is unaltered. Section 5 is the summary and discussion.

2. THE $\gamma - W$ MIXING FORMALISM

In this section we briefly review the $\gamma - W$ mixing formalism in its simplest[3] and generalized[10] forms with attentions to their connections with composite models. The concept of the so-called "unification condition," which is the center of our concern in this paper, will also be established. The basic assumptions of the $\gamma - W$ mixing formalism are the following:

1. There exists a triplet of weak bosons $\vec{W}$ (can be either point-like or composite) whose interactions satisfy a global $SU(2)$ invariance. The kinetic and mass terms of their Lagrangian are

$$\mathcal{L}_0 = -\frac{1}{4}\vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu} + \frac{1}{2}M_W^2 \vec{W}_\mu \cdot \vec{W}^\mu \tag{2.1}$$

   where $\vec{W}_{\mu\nu} \equiv \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu$, and M_W are the (degenerate) masses of the $W's$.

2. There exists a mixing Lagrangian which transfers the photon and the neutral component of the $\vec{W}$ triplet into each other. Let the strength of this mixing be

λ, then the relevent Lagrangian is assumed to be

$$\mathcal{L}_{em} + \mathcal{L}_{mix} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \lambda \left(F^{\mu\nu} W^3_{\mu\nu} + W^3_{\mu\nu} F^{\mu\nu} \right) . \qquad (2.2)$$

Notice that the mixing strength λ can either be interpreted as a point-like coupling,[3] or it can be viewed as originated from W^3 substructures such as direct γ-subconstituent charge coupling and W^3 bound-state wave function at the origin.[11]

Using the propagator formalism (see Appendix A) one finds the physical photon and neutral Z-boson poles after wave function renormalization and mass matrix diagonalization. Their resultant masses and currents after mixing are

$$m_\gamma = 0 \ , \quad J^{em}_\mu \ \textit{unchanged} \ ;$$

$$M_Z^2 = \frac{M_W^2}{1-\lambda^2} \ , \quad J^Z_\mu = \frac{1}{\sqrt{1-\lambda^2}} \left[J^3_\mu - \lambda J^{em}_\mu \right] . \qquad (2.3)$$

Note that this is a zero width treatment of the W-bosons. Finite width effects (which are treated in Ref. 12) do not change the essential feature of our discussions, so we ignore them throughout this paper.

The outcome of this $\gamma - W$ mixing model is perfectly consistent with low energy charged current and neutral current experiments and phenomenology. Namely, the charged current Hamiltonian has the form

$$\mathcal{H}^{CC} = \frac{1}{2} \cdot \frac{g^2}{M_W^2 - s} J^{(+)} \cdot J^{\mu(-)} \qquad (2.4)$$

whereas the neutral current Hamiltonian has the form

$$\mathcal{H}^{NC} = \frac{1}{2} \left[-\frac{J^{em}_\mu J^{\mu em}}{s} + \frac{J^Z_\mu J^{\mu Z}}{M_Z^2 - s} \right] . \qquad (2.5)$$

Moreover, the EM current J^{em}_μ is related to the third component of isospin current I^3_μ through the following identity

$$J^{em}_\mu \equiv e(I^3_\mu + Y_\mu) \equiv e Q_\mu \ , \qquad (2.6)$$

where Y_μ is the $U(1)$ hypercharge current. Thus, with $J^3_\mu = gI^3_\mu$,

$$\begin{aligned} J^Z_\mu &= \frac{g}{\sqrt{1-\lambda^2}}\left[I^3_\mu - \frac{e\lambda}{g}(I^3_\mu + Y_\mu)\right] \\ &= \frac{g}{\sqrt{1-\lambda^2}}\left[\left(1-\frac{e\lambda}{g}\right)I^3_\mu - \frac{e\lambda}{g}Y_\mu\right] \ . \end{aligned} \tag{2.7}$$

When related to experimental measurable quantities, one has

$$\frac{8G_F}{\sqrt{2}} = \frac{g^2}{M^2_W} \quad , sin^2\theta_W = \frac{e\lambda}{g} \quad , \rho = 1 \ . \tag{2.8}$$

There is, however, one free parameter $\frac{g\lambda}{e}(\equiv F)$ which actually controls the values of M_W and M_Z and has not yet been fixed by experiments. (In the recent discovery of W at CERN[1], its mass measurement still needs better accuracy.) To be more specific, the mass relation of Eq. (2.3) can also be written as

$$M^2_Z = \frac{M^2_Z}{1 - F\ sin^2\theta_W} \tag{2.9}$$

and there's no *a priori* reason to tell what value that F should be. However, it is quite obvious to see that this model can accommodate the standard model if one imposes the "unification condition":[3]

$$F = \frac{g\lambda}{e} = 1 \ . \tag{2.10}$$

Under this condition Eq. (2.9) then turns into the well-known Weinberg mass relation.

Hung and Sakurai have shown that this "unification condition" can be obtained in two ways:

1. One may impose "asymptotic $SU(2) \times U(1)$ symmetry." This means that one requires no $I^3_\mu Y^\mu$ term in $\mathcal{H}^{NC}_{s\to\infty}$. This can be achieved in Eq. (2.5) and Eq. (2.7) by letting

$$\lambda g = e \ , \ or \ \ F = 1 \ .$$

2. One may instead ask for good high energy behavior in the process $\nu\bar{\nu} \to W^+W^-$ or $e^+e^- \to W^+W^-$ (neglecting m_e and scalar effects). This however requires additional assumptions about $W^\pm$ self-couplings (see the "minimal substitution scheme" in Sec. 3).

The $\gamma - W$ mixing formalism can be extended to a multi-boson case, see Refs. 10 and 18. Let us consider the case of N $\vec{W}_i$ triplets ($i = 1, \ldots, N$) and N' W_j^0 singlets ($j = 1, \ldots, N'$) under global $SU(2)$ invariance. One introduces N λ_i's for $\gamma - W_i^3$ junctions and N' λ_j''s for $\gamma - W_j^0$ junctions and proceeds with propagator renormalization and mass matrix diagonalization as before. Only that now we should work on $(N+N'+1) \times (N+N'+1)$ matrices (see Appendix A). The results are of similar nature. Namely, $\mathcal{H}^{CC}$ and $\mathcal{H}^{NC}$ are now given by a series of γ, Z_i and Y_j poles. However there is obviously a definite departure from the standard model because of the additional weak-boson poles. One can still ask for asymptotic $SU(2) \times U(1)$ symmetry in $\mathcal{H}^{NC}$ as s goes to infinity. The requirement of no $I_\mu^3 Y^\mu$ term now leads to

$$\left(e - \sum_{i=1}^{N} \lambda_i g_i\right)\left(e - \sum_{j=1}^{N'} \lambda_j' g_j'\right) = 0\,. \tag{2.11}$$

The original unification condition $e = \lambda g$ is now replaced by either $e = \sum_{i=1}^{N} \lambda_i g_i$ or $e = \sum_{j=1}^{N'} \lambda_j' g_j'$, both of which we shall call "generalized unification condition." It is surprising that actually only one generalized unification condition is sufficient to ensure the good asymptotic symmetry of $\mathcal{H}^{NC}$.

In some specific models these two constraints are in fact related. For example, in the composite models where W^3 and W^0 are made of the same subconstituents, then the effective couplings $\lambda_i g_i$ and $\lambda_i' g_i'$ may be related. Or, as we mentioned previously, in the case of W-dominance model [6,7], both conditions follow from universality, i.e., $\sum_{i=1}^{N} \lambda_i g_i = \sum_{j=1}^{N'} \lambda_i' g_i' = e$.

3. THE EVOLUTION CONDITION IN A PRIMITIVE MODEL

Having introduced the basic ideas of the $\gamma - W$ mixing formalism and the unification condition, we now want to derive the "evolution condition" in a primitive model. We first look at the high energy behavior of $f\bar{f} \to W^+W^-$ scattering amplitude in the case of one triplet of $\vec{W}$. The evolution condition is obtained as a set of constraints on the W form factors at high ($s \leq \Lambda^2$) energy. We then discuss its consequences.

3.1 The High and Low Energy Constraints

For a non-gauge effective theory of weak interaction based on some composite substructures, the $\gamma W^+ W^-$ and $W^3 W^+ W^-$ vertex structures are quite unrestricted other than some very general invariance principles. Consider the most general C, P, T, invariant forms that the $\gamma W^+ W^-$ vertex can have. They are[13]

$$
\begin{aligned}
I_1^\mu &= (\epsilon^+ \cdot \epsilon^-)(k^- - k^+)^\mu - (\epsilon^+ \cdot k^-)\epsilon^{-\mu} + (\epsilon^- \cdot k^+)\epsilon^{+\mu} , \\
I_2^\mu &= (\epsilon^- \cdot k^+)\epsilon^{+\mu} - (\epsilon^+ \cdot k^-)\epsilon^{-\mu} , \\
I_3^\mu &= (\epsilon^- \cdot k^+)(\epsilon^+ \cdot k^-)(k^- - k^+)^\mu ,
\end{aligned}
\tag{3.1}
$$

where $k^\pm$ are the four-momentum, and $\epsilon^\pm$ are the polarizations, of $W^\pm$, respectively. Notice that I_3^μ is associated with $W^\pm$ quadrupole moment and is known[9,14] to give an incurable bad high energy behavior unless it is attached to a vanishing form factor. Thus we ignore this term in the following discussion.

For the remaining two expressions, I_1 is associated with charge and $(I_1 + I_2)$ is associated with magnetic moment of $W^\pm$. We shall keep their explicit forms in both $\gamma W^+ W^-$ and $W^3 W^+ W^-$ vertices with *a priori* four independent form factors:

$$
\begin{aligned}
J_\mu^{em} &= e\,[I_{1\mu} F_1^{em}(s) + I_{2\mu} F_2^{em}(s)] \ , \\
I_\mu^3 &= g\,[I_{1\mu} F_1^3(s) + I_{2\mu} F_2^3(s)] \ .
\end{aligned}
\tag{3.2}
$$

The normalization conditions (at $s = 0$) for the form factors can be fixed by the "minimal substitution scheme," meaning that we make the following substitutions in the Lagrangians in Eq. (2.1) and Eq. (2.2):

$$
\partial_\mu \rightarrow \partial_\mu - ie\,Q\,A_\mu \qquad \text{for EM couplings,} \tag{3.3}
$$

and

$$
\partial_\mu \rightarrow \partial_\mu - ig\,\vec{T} \cdot \vec{W}_\mu \qquad \text{for } W^3 \text{ couplings .} \tag{3.4}
$$

By imposing (3.3) on the kinetic part of the $\vec{W}$ Lagrangian $\mathcal{L}_0$ (Eq. (2.1)), one identifies

$$F_1^{em}(0) = Q_W = 1 \quad , \tag{3.5}$$

but has nothing to say on $F_2^{em}(0)$. On the other hand, imposing (3.4) on $\mathcal{L}_{em} + \mathcal{L}_{mix}$ (Eq. (2.2)) one finds

$$F_2^{em}(0) = \frac{\lambda g}{e} \equiv F \quad , \quad F_1^3(0) = 1 \quad , \quad F_2^3(0) = 1 \quad . \tag{3.6}$$

In a non-gauge theory there is *a priori* no restriction on how the couplings should be written. Schemes other than the "minimal substitution" can surely be constructed. However, the minimal substitution scheme turns out to be the simplest scheme that accommodates universality, and serves well enough our purpose of discussing the origin of unification. Thus in the following we shall allow for four arbitrary s-dependent form factors with their corresponding normalizations given by Eqs. (3.5) and (3.6). Notice, however, that $\frac{\lambda g}{e} = F$ is still arbitrary.

After mixing with the photon we get the Z current,

$$\begin{aligned} J_\mu^Z &= \frac{1}{\sqrt{1-\lambda^2}} [J_\mu^3 - \lambda J_\mu^{em}] \\ &= \frac{g}{\sqrt{1-\lambda^2}} \left\{ I_{1\mu} \left[F_1^3(s) - \frac{e\lambda}{g} F_1^{em}(s) \right] + I_{2\mu} \left[F_2^3(s) - \frac{e\lambda}{g} F_2^{em}(s) \right] \right\} \quad . \end{aligned} \tag{3.7}$$

It is worth mentioning that in the Hung-Sakurai model all form factors are identical to 1 (without s-dependence). With $\lambda = \frac{e}{g} = sin\theta_W$, Eq. (3.7) reduces to

$$J_\mu^Z = \frac{e}{tan\theta_W} [I_{1\mu} + I_{2\mu}] \quad ,$$

whereas

$$J_\mu^{em} = e\,[I_{1\mu} + I_{2\mu}] \quad .$$

These currents have exactly the same expressions as in the standard model.

We now look to the high energy behavior of $\nu\bar{\nu} \rightarrow W^+W^-$ and $e^+e^- \rightarrow W^+W^-$ scattering amplitudes. We compute them with $e(\nu)$ exchange in the t-channel and γ, Z

formation in the s-channel (Fig. 1). With the help of J_μ^{em} (cf. Eq. (3.2)) and J_μ^Z (cf. Eq. (3.7)) the amplitudes are (with $m_e \simeq 0$),

for $\nu\,\bar{\nu} \to W^+W^-$:

$$
\begin{aligned}
R_{fi} = &-\frac{g^2\,[F_e(t)]^2}{4t}\bar{v}\,(p')\,\not\epsilon^-\,(\not k^- - \not p')\,\not\epsilon^+\,(1-\gamma_5)\,u(p) \\
&+\frac{g^2F_\nu^3(s)}{4(1-\lambda^2)(s-M_Z^2)}\bar{v}\,(p')\left\{\left[F_1^3(s)-\frac{e\lambda}{g}F_1^{em}(s)\right]\not I_1\right. \\
&+\left.\left[F_2^3(s)-\frac{e\lambda}{g}F_2^{em}(s)\right]\not I_2\right\}(1-\gamma_5)u(p)
\end{aligned}
\tag{3.8}
$$

for $e^+e^- \to W^+W^-$:

$$
\begin{aligned}
R_{fi} = &-\frac{g^2\,[F_\nu(t)]^2}{4t}\bar{v}\,(p')\,\not\epsilon^+\,(\not k^+ - \not p')\,\not\epsilon^-\,(1-\gamma_5)u(p) \\
&+\frac{e^2F_e^{em}(s)}{s}\bar{v}\,(p')\,\{F_1^{em}(s)\,\not I_1 + F_2^{em}(s)\,\not I_2\}\,u(p) \\
&+\frac{g^2}{(1-\lambda^2)(s-M_Z^2)}\bar{v}\,(p')\left\{-\frac{1}{4}(1+\gamma_5)F_e^3(s)-\frac{e\lambda}{g}F_e^{em}(s)\right\} \\
&\cdot\left\{\left[F_1^3(s)-\frac{e\lambda}{g}F_1^{em}(s)\right]\not I_1+\left[F_2^3(s)-\frac{e\lambda}{g}F_2^{em}\right]\not I_2\right\}u(p)
\end{aligned}
\tag{3.9}
$$

where $F_e(t)$ and $F_\nu(t)$ are the t-channel e and ν exchange form factors, respectively. And $F_f^3(s)$ and $F_f^{em}(s)$ are defined through the following couplings (see Appendix C):

$$
gF_f^3(s)\,\bar{f}\,\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)W_\mu^3 f \qquad \text{for } W^3 f\,\bar{f} \text{ coupling },
$$

and

$$
eF_f^{em}(s)\,\bar{f}\,\gamma^\mu A_\mu f \qquad \text{for } \gamma f\,\bar{f} \text{ coupling }.
$$

These amplitudes presumably will have bad high energy behaviors when one (to order $\sqrt{s}$) or both (to order s) polarizations $\epsilon^\pm$ are longitudinal. We shall list the asymptotic ($s \gg M^2_{W,Z}$) behaviors of these amplitudes by looking at the coefficients of their $1-\gamma_5$ part and vector part separately.

(i) For the $1-\gamma_5$ Part.

To order s:

$$[F_e(t)]^2 - \frac{F_\nu^3(s)}{1-\lambda^2}\left\{F_2^3(s) - \frac{e\lambda}{g}F_2^{em}(s)\right\} , \quad (\text{from } \nu\bar{\nu} \to W^+W^-) , \tag{3.10}$$

$$[F_\nu(t)]^2 - \frac{F_e^3(s)}{1-\lambda^2}\left\{F_2^3(s) - \frac{e\lambda}{g}F_2^{em}(s)\right\}, \quad (\text{from } e^+e^- \to W^+W^-) . \tag{3.11}$$

to order $\sqrt{s}$:

$$[F_e(t)]^2 - \frac{F_\nu^3(s)}{2(1-\lambda^2)}\left\{F_1^3(s) + F_2^3(s) - \frac{e\lambda}{g}[F_1^{em}(s) + F_2^{em}(s)]\right\} \quad (\text{from } \nu\bar{\nu} \to W^+W^-) , \tag{3.12}$$

$$[F_\nu(t)]^2 - \frac{F_e^3(s)}{2(1-\lambda^2)}\left\{F_1^3(s) + F_2^3(s) - \frac{e\lambda}{g}[F_1^{em}(s) + F_2^{em}(s)]\right\} \quad (\text{from } e^+e^- \to W^+W^-) . \tag{3.13}$$

(ii) For the vector part. (from $e^+e^- \to W^+W^-$ only)

To order s:

$$\frac{eF_e^{em}(s)}{1-\lambda^2}\{eF_2^{em}(s) - \lambda g F_2^3(s)\} , \tag{3.14}$$

to order $\sqrt{s}$:

$$\frac{eF_e^{em}(s)}{1-\lambda^2}\{e\,[F_1^{em}(s) + F_2^{em}(s)] - \lambda g\,[F_1^3(s) + F_2^3(s)]\} . \tag{3.15}$$

It is easy to check that with all form factors equal to one, all the above coefficients will vanish when $\frac{\lambda g}{e} = 1$ (the Hung-Sakurai result).

The existence of t-dependent form factors deserves a special discussion. First we notice that by subtracting Eq. (3.10) from Eq. (3.12), and Eq. (3.11) from Eq. (3.13), one gets t —independent constraints:

$$F_1^3(s) - F_2^3(s) - \frac{e\lambda}{g}[F_1^{em}(s) - F_2^{em}(s)] = 0 \quad (\text{from } \nu\bar{\nu} \to W^+W^-) \tag{3.16}$$

and

$$eF_2^{em}(s) - \lambda g F_2^3(s) = eF_1^{em}(s) - \lambda g F_1^3(s) = 0 \quad (\text{from } e^+e^- \to W^+W^-) \tag{3.17}$$

If one takes the non-trivial condition where $F_e^{em}(s)$ and $F_\nu^3(s)$ are not necessarily vanishing, then one has the solutions

$$F_1^{em}(s) = F_2^{em}(s) \ , \ F_1^3(s) = F_2^3(s) \ ,$$

and

$$\frac{\lambda g}{e} = \frac{F_1^{em}(s)}{F_1^3(s)} = \frac{F_2^{em}(s)}{F_2^3(s)} \equiv F \ . \tag{3.18}$$

Let us turn back to the coefficients of the $1-\gamma_5$ parts (i.e., Eq. (3.10) to Eq. (3.13)). Notice that these t —dependent constraints cannot be satisfied without additional contributions if $F(t) \neq 1$. So either we restrict our discussions to the "$t=0$ window," or we add effective "contact terms" to assure the good high energy behavior.

In the first case, at $t=0$ we let

$$F_e(t)_{t=0} = F_\nu(t)_{t=0} = 1 \ , \tag{3.19}$$

then the requirement for good high energy behavior translates into the following constraint:

$$F_1^3(s) \cdot F_\nu^3(s) = F_1^3(s) F_e^3(s) = 1 \ . \tag{3.20}$$

Note that even by choosing $F_\nu^3(s) \simeq F_e^3(s) \simeq F_1^3(s) \simeq 1$ we are still left with an undetermined $F_1^{em}(s) = F_2^{em}(s)$ asymptotic value. Thus our free parameter $\frac{\lambda g}{e} = F$ is still undetermined.

This "$t = 0$ window" can be compared to the "low p_T window" in the hadronic processes where the substructure effects do not directly show up. This is opposite to the "large p_T" case where the hard scatterings of subconstituents occur (e.g., the jets).

In the second case, we can just add, for example, to the amplitudes R_{fi} in Eqs. (3.8) and (3.9) an extra contact term

$$H(s,t)\,\bar{v}\,(p')\,[\not{\epsilon}_1 + \not{\epsilon}_2]\,(1-\gamma_5)u(p) \quad , \qquad 3.21)$$

with

$$H^{\nu\bar{\nu}}(s,t) = \frac{g^2}{4s}\left[F_e^2(t) - F_\nu^3(s)\left(\frac{F_2^3(s) - \frac{e\lambda}{g}F_2^{em}(s)}{1-\lambda^2}\right)\right] \,, \qquad (3.22)$$

and

$$H^{e^+e^-}(s,t) = -\frac{g^2}{4s}\left[F_\nu^2(t) - F_e^3(s)\left(\frac{F_2^3(s) - \frac{e\lambda}{g}F_2^{em}(s)}{1-\lambda^2}\right)\right] \,,$$

respectively, such that the t-dependent constraints are automatically satisfied. Notice that the t-independent constraints (Eq. (3.16), (3.17)) are not modified by this additional term.

This procedure would be similar to the one proposed a long time ago[15] for keeping gauge invariance and Ward identities while modifying QED with electron form factors. For example in the $e^+e^- \to \gamma\gamma$ scattering if one introduces form factors $F_e(t)$ and $F_e(u)$ for the electron exchange diagrams, one is lead to add a $e^+e^-\gamma\gamma$ contact (or "seagull") term which is very similar to the $H(s,t)$ in Eq. (3.21). This shows that if there is a substructure inside electrons, QED will not be the only possible solution; with $F_e(t) \neq 1$ and the additional contact term one has perfectly acceptable amplitudes.

Our problem is in fact very similar to this one. In our case, without form factors (i.e., all Fi's are equal to one) the theorem of Llewellyn Smith and Cornwall et al[9] would enforce the gauge model as being the only solution to the complete good high energy behavior of $WW, fW, f\bar{f}, \phi\phi, \phi W \ldots$ scatterings including scalar and vector bosons.

When we employ the possibility of some underlying substructures, we are allowed to find solutions to good high energy behaviors which depart from the gauge structure. As is demonstrated above, a general solution with arbitrary s and t can be easily achieved by adding the contact terms to the scattering amplitudes. However, the contact terms may actually simulate the complexities due to excited states like e^*, e^{**}, ... and spins $\frac{1}{2}$, $\frac{3}{2}$, ... etc., which we have assumed to neglect. Actually, since the t-independent constraints (plus the $t = 0$ constraint) give us another self-consistent solution, which is not modified by t-dependent considerations, we believe that it forms a close set of properties which are independent of the remaining unknowns. This closed sub-set of constraints then helps us to open up a "window" to look into the composite nature of the system.

We shall from now on call this closed set of constraints the "evolution condition." To be precise, the evolution condition consists of a set of constraints at high energy ($4M_W^2 \ll s \leq \Lambda$) which the W form factors should satisfy. Given initial (i.e., the normalization) conditions at $s = 0$ in a particular model, the form factors should evolve in such a way that the high energy constraints be satisfied. The detailed expressions of these constraints may have to be modified when we extend our consideration from this primitive version of the effective weak interaction to more sophisticated ones. But the essential feature of the evolution condition is unchanged, as we will show in the next section.

3.2 Consequences From the Constraints

Limiting ourselves on the "$t = 0$ window," we now discuss some consequences of the evolution condition. From Eq. (3.18) we have $\frac{\lambda g}{e} = F$ where F is *a priori* not equal to 1. This corresponds to non-unified, non-standard effective theories, however, prefectly consistent with the low energy phenomenology of charged current and neutral current. The experimental constraints

$$G_F = \frac{g^2\sqrt{2}}{8M_W^2}\ ,\ sin^2\theta_W = \frac{e\lambda}{g} = \frac{\lambda^2}{F}\ , \text{and}\ \ \rho = 1 \tag{3.23}$$

are satisfied for any value of F. Only that the W and Z masses

$$\begin{aligned} M_W^2 &= F\left(\frac{e^2\sqrt{2}}{8G_F sin^2\theta_W}\right)\ , \\ M_Z^2 &= \frac{M_W^2}{1 - F\ sin^2\theta_W} \end{aligned} \tag{3.24}$$

may differ from the standard model (which corresponds to $F = 1$). We now have $M_W^2 = F(M_W^2)_{S.M.}$ and the relation between Z and W masses is modified. Here the parameter F appears as the ratio of the W boson mass squared to the canonical value.

Furthermore, the form factor effects will produce departure from standard model predictions when energy increases. Consider for example the $W^\pm$ form factors. At low energy (i.e., $s \simeq 4M_W^2 \ll \Lambda$) $F_i(s) = F_i(0)$ which are normalized according to Eq. (3.5) and Eq. (3.6). In particular, the $W^\pm$ magnetic moment is

$$\mu_W = \frac{e}{2M_W}[1 + F_2^{em}(0)] \tag{3.25}$$

with $F_2^{em}(0) = \frac{\lambda g}{e} = F$ if we take the minimal substitution scheme. It differs from the standard value which is $\mu_W = \frac{e}{M_W}$.

Next we discuss the nature of the F parameter. In Eq. (3.18) F appears as the ratio of photon versus W^3 form factors at large value of s. It is possible that this ratio is independent of s at high energy. In any case, in a model where there is a mass scale Λ for the substructure and where the energy domain of interest is $4M_W^2 < s < \Lambda^2$, for definiteness we can take

$$F \equiv \frac{F_1^{em}(\Lambda^2)}{F_1^3(\Lambda^2)} .$$

Notice that if there is a hard core inside the W-boson (and perhaps also inside e and ν) of extension $\frac{1}{\Lambda}$ due to its substructure it is perfectly possible that the form factors tend to some non-zero constant values asymptotically. Then it is not unreasonable to assume that at high energy F is s-independent, i.e.,

$$F = \frac{F_1^{em}(s)}{F_1^3(s)} = const. \text{ for } s \gg 4M_W^2 . \tag{3.26}$$

Now we turn to the "evolution" of the form factors. As the energy increases the s-dependence of the form factors must evolve in such a way that the constraints in Eq.

(3.18) are satisfied. Consequently for large s ($\leq \Lambda^2$) we get

$$
\begin{aligned}
J_\mu^{em} &\rightarrow eF_1^{em}(s)\{I_{1\mu}+I_{2\mu}\}\ , \\
J_\mu^3 &\rightarrow \frac{g}{F}F_1^{em}(s)\{I_{1\mu}+I_{2\mu}\}\ , \\
J_\mu^Z &\rightarrow \frac{e}{\sqrt{F}\, sin\theta_W}(1-Fsin^2\theta_W)^{1/2}F_1^{em}(s)\{I_{1\mu}+I_{2\mu}\}\ .
\end{aligned}
\tag{3.27}
$$

Comparing with Eqs. (3.2) and (3.7) we see that the $W^\pm$ form factors should evolve in such a way that they look more closely like that of the standard model, apart from a different overall normalization and a different weak angle.

This specific feature can be best illustrated by the cross section $\sigma(e^+e^- \rightarrow W^+W^-)$. To simplify our computation of $\sigma(s)$, we take the simplest non-trivial form factors satisfying Eq. (3.18) and Eq. (3.20):

$$
F_\nu(t)=1,\ \ F_1^3(s)=F_2^3(s)\equiv f(s),\ \ F_2^{em}(s)=Ff(s),\ \ F_1^{em}(s)=\frac{A+Fs}{A+s}f(s)\ , \tag{3.28}
$$

and

$$
F_e^3(s)=\frac{1}{f(s)},\ \ \text{with}\ \ f(s)=\frac{A}{A+s}
$$

where A is a very large parameter associated with Λ. With a t-channel ν -exchange diagram and two s-channel γ and Z formation diagrams (whose helicity amplitudes are given in Appendix B), the cross section as a function of s is shown in Fig. 2. This cross section differs from that of the standard model at low energy. Large differences could be seen on the angular distribution in the backward direction, even when $F=1$. This is so for two reasons: Firstly (and obviously) because M_W and M_Z values are not the standard ones. Secondly because the $W^\pm$ form factors are not the standard ones. Nevertheless, it is well-behaved when s increases. This is because it evolves (along with the evolution of the form factors) in such a way that it behaves for large s like a standard model, only that the weak angle is "rotated" from $sin^2\theta_W$ to $\frac{1}{F}sin^2\theta_W$.

This large s behavior may very well be a temporary one for $4M_W^2 \ll s \leq \Lambda^2$. When $s \geq \Lambda^2$ new effects coming from direct subconstituent hard scatterings may appear. As a result the picture will be further modified.

These are the main consequences of the evolution condition under which an effective theory based on any composite model should behave.

4. MULTI-BOSON CASES

In this section we want to investigate the "evolution condition" in multi-boson cases. We will show that the basic feature of the condition is unaltered.

The reason for going to multi-boson cases are the following: Firstly, the contents of the composite system was overly simplified in the previous section. So long as $\vec{W}$'s are composite and act as an isotriplet of a global $SU(2)$ symmetry, there is *apriori* no reason why the isosinglet partner W^o should not be around. In particular, this neutral vector particle W^o should participate in the neutral current Hamiltonian, thus it may potentially modify a lot of the "evolution condition." Secondly, there are versions of extended gauge theories[10] in which several triplets of gauge W-bosons are allowed. In case these weak-bosons have masses much lower than the mass scale Λ, will the "evolution condition" still work? Incidentally, these multi-bosons should not be confused with composite excited states of the lowest lying $\vec{W}$ and W^o since we expect them to have masses at least of the order Λ.[16] Since we are working in the energy domain $4M_W^2 \ll s \leq \Lambda^2$, we shall ignore these excited state in the following discussions.

Let us consider the two cases separately.

4.1 THE $Y - Z$ SYSTEM

We now have an isotriplet $\vec{W} \equiv (W^+, W^-, W^3)$ and an isosinglet W^o weak-bosons before mixing. Following the same "minimal substitution scheme," we assume the coupling constant of $W^oW^+W^-$ vertex to be g' and the strength of $\gamma - W^o$ junction to be λ'. At this level we have to take the 3×3 matrix $\gamma - W^3 - W^o$ mixing formalism (see Appendices A and C for the general treatment). After mixing, W^o turns into a physical vector boson Y and W^3 turns into Z.

We then add the Y intermediate state to $\nu\bar{\nu} \to W^+W^-$ and $e^+e^- \to W^+W^-$ amplitudes and write again the constraints for a good high energy behavior like we did in Sec. 3. The essential feature of the W^o contribution is that it has a change of sign against the W^3 contribution when passing from $\nu\bar{\nu}$ to e^+e^- because of the isosinglet nature of W^o. So in order to satisfy both $\nu\bar{\nu}$ and e^+e^- constraints the W^o contribution to the amplitudes should by itself be well-behaved. This gives the constraint (see Appendix C for the detailed treatment)

$$g' H(s) = 0 \tag{4.1}$$

for $W^o W^+ W^-$ coupling times a possible form factor.

The solutions are either a zero $W^o W^+ W^-$ coupling at any s or a vanishing form factor $H(s)$ at large s. If we were dealing with a gauge theory, then the requirement for gauge invariance should tell us that g' must be associated with the hypercharge operator which vanishes when applied to $W^{\pm}$. Actually, if we insist on the "minimal substitution scheme," we also expect to have $H(0) = 0$ in our non-gauge theory, since both the hypercharge and isospoin charge of W^o are zero.

So we conclude that the constraints on the Z boson form factors in Sec. 3 are not affected by the presence of Y-boson. The only change in the form of the Z-boson couplings comes from the different mixing parameters. For example, the factor $\sqrt{1-\lambda^2}$ is now replaced by $\sqrt{1-\lambda^2-\lambda'^2}$. But otherwise everything is the same, and we see that the "evolution condition" is unchanged.

4.2 Multi- $Y - Z$ System

Next we consider the case of N isotriplets of $\vec{W}_i$ and N' isosinglets of W_j^o with a $(N + N' + 1) \times (N + N' + 1)$ matrix mixing scheme (see Sec. 2 and Appendix A). The generalization of the method of Sec. 3 is straightforward, although tedious to write explicitly. We give some steps of the calculation in Appendix C.

We consider the amplitudes of the processes $\nu \bar{\nu} (e^+ e^-) \rightarrow W_n^+ W_m^-$ where n and m run from 1 to N independently. So in these processes there will be t-channel $e(\nu)$ exchanges and $\gamma; Z_1, \cdots, Z_N; Y_1, \cdots, Y_{N'}$ formations in the s-channel. Asking for well-vehaved amplitudes at large s produces eight sets of t-independent sum rules and one set of t- dependent sum rules.

The simplest non-trivial solution (without having all form factors vanish) we found is the following (at $4M_W^2 \ll s \leq \Lambda^2$):

$$\begin{aligned} F_1^{nm,em}(s) &= F_2^{nm,em}(s) \quad \text{for} \quad \gamma W_n^+ W_m^- \ , \\ F_1^{inm,3}(s) &= F_2^{inm,3}(s) \quad \text{for} \quad W_i^3 W_n^+ W_m^- \ , \\ H^{jnm}(s) &= 0 \quad \text{for} \quad W_j^o W_n^+ W_m^- \ , \end{aligned} \tag{4.2}$$

and

$$\sum_{i=1}^{N} \lambda_i g_i F_2^{inm,3}(s) = e F_2^{nm,em}(s) \ . \tag{4.3}$$

Again, these generalized conditions deviate from the "generalized unification condition" (cf. Eq. (2.11)) found in Sec. 2 on the basis of asymptotic symmetry. They may be identical only if

$$F_2^{inm,3}(s) = F_2^{nm,em}(s) \ \ , \text{ at } 4M_W^2 \ll s \leq \Lambda^2 \ . \tag{4.4}$$

But there is no *apriori* reason for this condition to hold unless we are in a complete W dominance model[6,7] in which photon couplings are completely dominated by W^3.

We can also find other solutions (to the good high energy behavior) under less general conditions. For example, if we assume the vanishing of the form factors for non-diagonal $W_i^3 W_n^+ W_m^-$ couplings ($n \neq m \neq i$), then we have (at $4M_W^2 \ll s \leq \Lambda^2$):

$$\begin{aligned} F_1^{mn,em}(s) &= F_2^{mm,em}(s) \quad \text{for} \quad \gamma W_m^+ W_m^- \ , \\ F_1^{mmm,3}(s) &= F_2^{mmm,3}(s) \quad \text{for} \quad W_m^3 W_m^+ W_m^- \end{aligned} \tag{4.5}$$

and

$$\frac{\lambda_m g_m}{e} = \frac{F_2^{mm,em}(s)}{F_2^{mmm,3}(s)} \equiv F_m \quad \text{for any} \quad m = 1, \cdots, N \ . \tag{4.6}$$

In this case the unification condition $\sum_i \lambda_i g_i = e$ can be satisfied only if $\sum_m F_m = 1$ which effectively requires for delicate arrangements among the form factors.

From both solutions we see that the constraints on the relation between EM and weak couplings are again departing from the "generalized unification condition." Moreover, comparing Eq. (4.2) and Eq. (4.5) with their corresponding normalization conditions (see Appendix C), one again sees that the form factors should evolve in different fashion such that the conditions at the two extremes (i.e., at $s = 0$ and $s \leq \Lambda^2$) can be consistent.

Finally, Eq. (4.2) and Eq. (4.5) also tell us that at high energy (i.e., $4M_W^2 \ll s \leq \Lambda^2$) the system should evolve into a gauge-like model. We thus conclude that the essential features of our "evolution condition" is unchanged in the multi-boson cases.

5. CONCLUSION

So far we have demonstrated, in the case of one $\vec{W}$ triplet and in the cases of multi-bosons, that any composite model (with large mass scale Λ) at its effective level should obey the evolution condition.

In summary, we started with the following assumptions:

1. The leptons, quarks, and weak-bosons are composite particles whose underlying dynamics has a mass scale Λ, which is much larger than the W-boson mass M_W.
2. The observed electroweak interaction is an effective interaction at the composite level. The connection between EM and weak interaction is through the $\gamma - W$ mixing mechanism.
3. The scattering amplitudes of $f\bar{f} \rightarrow W^+W^-$ are well- behaved at high energy.

We then derived a set of constraints at high energy which we called the evolution condition. There are two important implications of this condition:

1. The EM and weak interactions need not be unified in the same way as in the standard model and Hung-Sakurai model, although the standard way of unification can be accommodated. For the single $(\vec{W}, W^o)$ case we have $\frac{\lambda g}{e} = F$ and for multi-boson case we have $\sum_i \gamma_i g_i F_2^{inm,3}(s) = eF_2^{nm,em}(s)$.
2. Given initial (i.e., the normalization) conditions at $s = 0$ in a particular model, the form factors should evolve in such a way that the high energy constraints be satisfied.

So the landscapes that we shall see as suggested by the evolution condition are the following:

1. At low energy ($s \ll \Lambda^2$) the model has a non-standard, non- unified effective description of electroweak interactions.

2. At high energy ($4M_W^2 \ll s \leq \Lambda^2$) before hard substructure effects show up, this effective description is well-behaved, and it progressively evolves into a "rotated" gauge-like model.

3. At super-high energy ($s \geq \Lambda^2$) many new substructure effects appear. There will be new excited states, new channels, subconstituent jets ... etc., many "oases" prospering in the "desert."

How can we test this picture at the present energy range or at the future colliders?

First of all, a direct test will be the mass relation between W and Z. If we take the data of the recent CERN experiment,[1] we can fix the parameter F (cf. Eq. (3.24)), and look for M_Z at the right place. Other tests could be the universality and form factors in $Zf\bar{f}$ and $Wf\bar{f}$ couplings (cf. Appendix C). Furthermore, we can test the $W^\pm$ self-couplings and EM form factors, e.g., through the process $e^+e^- \rightarrow W^+W^-$. There are also possibilities of additional couplings[17] such as quadrupole moment (associated with vanishing form factors at high energy) which should not exist according to the standard model. Also, if there is a non-vanishing contribution to the isoscalar current, then it would be a good indication of the composite idea.

Finally, we would like to make some remarks on the parameter F. First notice that $F = 1$ does not mean that every feature of our picture should be identical to the standard model. This just means that M_W and M_Z should fall on the right place that the standard model predicts. All the possible departures due to form factors and additional couplings may still occur. Secondly, it is likely that the experimental value[1] for M_W will be close to the standard prediction up to a few percent. Even if this deviation can be made up by the radiative corrections within the standard model, it can equally well be explained by the substructure effects. In this sense it is not unreasonable to relate the dimensionless quantity $(F - 1)$ to the ratio $\frac{M_W}{\Lambda}$.

MEMORIUM

A portion of the ideas in this paper were originally raised by Professor J. J. Sakurai who passed away on November 1, 1982. We deeply regret that he could not contribute further along this line of thought.

APPENDIX A

Propagator Formalism for $\gamma - W$ Mixing

In this section we closely follow the method used by Hung and Sakurai[3] in the case of a single W and by de Groot and Schildknecht[10] in the multiboson case. We just write the formulae and the sum rules in a slightly more general form so that they can be applied to the case of N $\vec{W}_i$ triplets and N' W_j^o singlets with arbitrary coupling constants.

Let us put $n = N + N'$ and write the propagator in an $(n+1) \times (n+1)$ matrix form with $\phi^T = (\gamma, W_1^3, \ldots W_N^3, W_1^o, \ldots W_{N'}^o)$ representing the boson fields. From the kinetic and mixing terms defined in Sec. 2 we get the inverse propagator matrix:

$$D = \bar{M}^2 - s\Lambda \tag{A.1}$$

with

$$\bar{M}^2 = \begin{pmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \bar{M}_1^2 & & & & \\ \cdot & & \cdot & & 0 & \\ \cdot & & & \cdot & & \\ \cdot & & 0 & & \cdot & \\ 0 & & & & & \bar{M}_n^2 \end{pmatrix} = \text{the squared masses before mixing ,} \tag{A.2}$$

and

$$\Lambda = \begin{pmatrix} 1 & \lambda_1 & \cdot & \cdot & \cdot & \lambda_n \\ \lambda_1 & 1 & & & & \\ \cdot & & \cdot & & 0 & \\ \cdot & & & \cdot & & \\ \cdot & & 0 & & \cdot & \\ \lambda_n & & & & & 1 \end{pmatrix} = \text{the mixing matrix .} \tag{A.3}$$

We introduce a "wave function renormalization" matrix D and a "diagonalization unitary matrix" R defined by

$$D\Lambda D^{+} = 1 \quad \text{i.e.,} D = \begin{pmatrix} \frac{1}{\sqrt{\kappa}} & -\frac{\lambda_1}{\sqrt{\kappa}} & \cdot & \cdot & \cdot & -\frac{\lambda_n}{\sqrt{\kappa}} \\ 0 & 1 & & & & \\ \cdot & & \cdot & & 0 & \\ \cdot & & & \cdot & & \\ \cdot & & 0 & & \cdot & \\ 0 & & & & & 1 \end{pmatrix} \tag{A.4}$$

with

$$\kappa = \Lambda - \sum_{1}^{n} \lambda_i^2 \tag{A.5}$$

and

$$R^{+}(D\bar{M}^2 D^{+})R = M^2 \equiv \begin{pmatrix} 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & M_1^2 & & & & \\ \cdot & & \cdot & & 0 & \\ \cdot & & & \cdot & & \\ \cdot & & 0 & & \cdot & \\ 0 & & & & & M_n^2 \end{pmatrix} \tag{A.6}$$

with $R^{+} = R^{-1}$. M_ℓ^2 ($\ell = 1, \ldots n$ are the masses of the physical weak bosons. The inverse propagtor now becomes:

$$\mathcal{D} = (R^{+}D)^{-1}\,[M^2 - s]\,(D^{+}R)^{-1} \tag{A.7}$$

so that any transition amplitude going through γ ($\ell = 0$) and weak boson ($\ell = 1, \ldots, n$) formation can be written:

$$R_{fi} = \langle f|J\mathcal{D}^{-1}J|i\rangle = \sum_{\ell=0}^{n} \frac{g_{f\ell}\, g_{i\ell}}{M_\ell^2 - s} \tag{A.8}$$

with $g_{f\ell} = \sum_{k=0}^{n} \bar{g}_{fk} C_{k\ell}$ (and similarly for $g_{i\ell}$). $\bar{g}_{fk} \equiv \langle f|J|k\rangle$ are the coupling constants before mixing. $C_{k\ell} \equiv (D^{+}R)_{k\ell}$ are real mixing coefficients. Notice, as shown in Ref. 10, that $C_{00} = 1$ and $C_{k0} = 0$. In the single W case ($n = N = 1$, $N' = 0$) we have explicitely:

$$C_{00} = 1 \quad C_{10} = 0 \quad C_{01} = -\frac{\lambda}{\sqrt{1-\lambda^2}} \quad C_{11} = \frac{1}{\sqrt{1-\lambda^2}} \; . \qquad (A.9)$$

In the multiboson case, from $D^+D = \Lambda^{-1}$ and $R^+R = 1$ we get the sum rules

$$\begin{aligned}
&\sum_{\ell=1}^{n} (C_{0\ell})^2 = \frac{1-\kappa}{\kappa} \\
&\sum_{\ell=1}^{n} C_{0\ell} C_{k\ell} = -\frac{\lambda_k}{\kappa} \\
&\sum_{\ell=1}^{n} g_{f\ell} C_{0\ell} = -\frac{1}{\kappa} \sum_{k=1}^{n} \bar{g}_{fk} \lambda_k \\
&\sum_{\ell=1}^{n} g_{j\ell} C_{j\ell} = \bar{g}_{fj} + \frac{\lambda_j}{\kappa} \sum_{k=1}^{n} \bar{g}_{fk} \lambda_k \; .
\end{aligned} \qquad (A.10)$$

These sum rules will be used in Sec. 4 and Appendix C.

APPENDIX B

$e^+e^- \to W^+W^-$ Helicity Amplitudes and Cross Section

We consider the case of a single $\vec{W}$ triplet with $\gamma - W^3$ mixing. The $e^+e^- \to W^+W^-$ amplitude due to ν -exchange and γ, Z formation has been given in Sec. 3, Eq. (3.9). In order to easily compute the cross section we give below the complete set of helicity amplitudes. We use the formalism and notations of Ref. 13. $\mu = \pm\frac{1}{2}$; $\mu' = \pm\frac{1}{2}$; $\tau = 0, \pm 1$; $\tau' = 0, \pm 1$ are, respectively, the e^-, e^+, W^-, W^+ helicity states. $\ell = \sqrt{s}/2$, $p = (\frac{s}{4} - M_W^2)^{1/2}$ are the $e^\pm$ and $W^\pm$ momenta in the center of mass. θ is the (e^-, W^-) scattering angle.

ν exchange in the t-channel (only $\mu = -\frac{1}{2}$, $\mu' = +\frac{1}{2}$ contribute):

$$\mathcal{F}^t_{\tau,\tau} = \frac{g^2[F_\nu(t)]^2}{2t}(p - \ell\; cos\theta) sin\theta$$

$$\mathcal{F}^t_{\tau,-\tau} = \frac{g^2[F_\nu(t)]^2}{4t}\sqrt{s}(cos\theta - \tau) sin\theta$$

$$\mathcal{F}^t_{0,0} = -\frac{g^2[F_\nu(t)]^2}{2M_W^2 t}\left[2\ell^3 cos\theta - p\left(M_W^2 + \frac{s}{2}\right)\right] sin^2\theta \tag{B.1}$$

$$\mathcal{F}^t_{0,\tau'} = \mathcal{F}^t_{-\tau',0} = -\frac{g^2[F_\nu(t)]^2}{2\sqrt{2}M_W t}$$

$$\cdot\left[\left(\frac{s}{2} - M_W^2\right)cos\theta - p\sqrt{s} + \tau'\left(\frac{s}{2}cos^2\theta - M_W^2 - p\sqrt{s}\,cos\theta\right)\right]$$

with $t = M_W^2 - \sqrt{s}\left(\frac{\sqrt{s}}{2} - p cos\theta\right)$.

γ, Z formation in the s-channel (only $\mu = -\mu' = \pm\frac{1}{2}$ contribute):

$$\mathcal{F}^s_{\tau,\tau} = 2\mu e\left[A_1^\gamma + A_1^Z(a + 2\mu b)\right]2p sin\theta$$

$$\mathcal{F}^s_{\tau,-\tau} = 0\;.$$

$$\mathcal{F}^s_{0,0} = 2\mu e\left[2M_W^2 A_1^\gamma + sA_2^\gamma + (a + 2\mu b)\left(2M_W^2 w A_1^Z + sA_2^Z\right)\right]\frac{p sin\theta}{M_W^2} \tag{B.2}$$

$$\mathcal{F}^s_{0,\tau'} = \mathcal{F}^s_{-\tau',0} = 2\mu e\left[A_1^\gamma + A_2^\gamma + (a + 2\mu b)\left(A_1^Z + A_2^Z\right)\right]$$

$$\cdot\frac{p\sqrt{s}(\tau' cos\theta - 2\mu)}{\sqrt{2}M_W}\;.$$

with:

$$A_1^\gamma = -\frac{eF_e^{em}(s)F_1^{em}(s)}{s} \quad A_2^\gamma = -\frac{eF_e^{em}(s)F_2^{em}(s)}{s}$$

$$A_1^Z = \frac{g^2\left[F_1^3(s) - \frac{e\lambda}{g}F_1^{em}(s)\right]}{(1-\lambda^2)(s - M_Z^2)} \quad A_2^Z = \frac{g^2\left[F_2^3(s) - \frac{e\lambda}{g}F_2^{em}(s)\right]}{(1-\lambda^2)(s - M_Z^2)}$$

and

$$a = -\frac{F_e^3(s)}{4} + \frac{e\lambda}{g} F_e^{em}(s) \quad b = \frac{F_e^3(s)}{4} \ .$$

The differential cross section is now given by:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha p}{32\pi\sqrt{s}} \sum_{\substack{\mu,\mu'=\pm\frac{1}{2} \\ \tau,\tau'=0,\pm 1}} \left| \mathcal{F}^t_{\tau,\tau'} + \mathcal{F}^s_{\tau,\tau'} \right|^2 \ . \qquad (B.3)$$

APPENDIX C

High Energy Contraints in the Multiboson Case

We suppose that there exists N isotriplets $\vec{W}_i$ and N' isosinglets W_j^0. The neutral states W_i^3 and W_j^0 mix with the photon (see Sec. 2 and Appendix A) through junctions λ_i and λ'_j (we put $\kappa \equiv 1 - \sum_i \lambda_i^2 - \sum_j \lambda_j'^2$). We introduce form factors for their couplings to fermions $f \equiv \binom{\nu}{e}$:

$$\bar{f}\gamma_\mu \frac{1-\gamma^5}{2} \left[g_i F_{if}(s)\, \vec{\tau} \cdot \vec{W}_i^\mu + g'_j H_{jf}(s) W_j^{0\mu} \right] f \qquad (C.1)$$

$$eF_f^{em}(s) \ \bar{f}\gamma_\mu A^\mu f \qquad (C.2)$$

and for their couplings to W^+W^-:

$$W_j^0 W_n^+ W_m^- \ : \qquad g'_j H^{jnm}(s) \left[(\epsilon_n^+ \cdot k^-)\epsilon_{m\mu}^- + (\epsilon_m^- \cdot k^+)\epsilon_{n\mu}^+ \right] \epsilon_j^{0\mu} \qquad (C.3)$$

$$W_i^3 W_n^+ W_m^- \ : \qquad g_i \left[F_1^{inm,3}(s) I_{i\mu} + F_2^{inm,3}(s) I_{2\mu} \right] \epsilon_i^{3\mu} \qquad (C.4)$$

$$\gamma W_n^+ W_m^- \ : \qquad e \left[F_1^{nm,em}(s) I_{1\mu} + F_2^{nm,em}(s) I_{2\mu} \right] A^\mu \qquad (C.5)$$

I_1 and I_2 have been defined in Sec. 3; i, n, m run from 1 to N; and j runs from 1 to N'. We then consider the amplitudes of the reactions $\nu\bar{\nu} \to W_n^+ W_m^-$ and $e^+e^- \to W_n^+ W_m^-$ with $e(\nu)$ exchange in the t-channel and γ, $Z_1, \ldots Z_N$, $Y_1, \ldots Y_{N'}$ in the s-channel. We ask for a good high energy behavior (like in Sec. 3) and we get the constraints for s large $\left(s \gg M_{W_N}^2, M_{W_{N'}^0}^2\right)$:

Vector part (t-independent constraints in e^+e^-):

$$\frac{eF_e^{em}(s)}{\kappa}\left\{eF_2^{nm,em}(s) - \sum_i \lambda_i g_i F_2^{inm,3}(s)\right\} = 0 \,. \qquad (C.6)$$

$$\frac{eF_e^{em}(s)}{\kappa}\left\{e\,[F_2^{nm,em}(s) - F_1^{nm,em}(s)] - \sum_i \lambda_i g_i \left[F_2^{inm,3}(s) - F_1^{inm,3}(s)\right]\right\} = 0 \qquad (C.7)$$

$$\frac{eF_e^{em}(s)}{\kappa}\sum_j \lambda_j' g_j' H^{jnm}(s) = 0 \,. \qquad (C.8)$$

Left-handed Part (in $\nu\bar{\nu}$ and e^+e^-):

t-independent constraints ($f \equiv \nu$ or e):

$$\frac{\sum_i \lambda_i g_i F_{if}(s)}{\kappa}\left\{e\,[F_2^{nm,em}(s) - F_1^{nm,em}(s)] - \sum_i \lambda_i g_i \left[F_2^{inm,3}(s) - F_1^{inm,3}(s)\right]\right\}$$
$$- \sum_i g_i^2 F_{if}(s)\left[F_2^{inm,3}(s) - F_1^{inm,3}(s)\right] = 0 \,. \qquad (C.9)$$

$$\frac{\sum_j \lambda_j' g_j' H_{jf}(s)}{\kappa}\left\{eF_2^{nm,em}(s) - \sum_i \lambda_i g_i F_2^{inm,3}(s)\right\} = 0 \qquad (C.10)$$

$$\frac{\sum_j \lambda_j' g_j' H_{jf}(s)}{\kappa}\left\{e\,[F_2^{nm,em}(s) - F_1^{nm,em}(s)] - \right.$$
$$\left.\sum_i \lambda_i g_i \left[F_2^{inm,3}(s) - F_1^{inm,3}(s)\right]\right\} = 0 \,. \qquad (C.11)$$

$$\frac{\sum_i \lambda_i g_i F_{if}(s)}{\kappa} \sum_j \lambda'_j g'_j H^{jnm}(s) = 0 \; . \qquad (C.12)$$

$$\frac{\sum_j \lambda'_j g'_j H_{jf}(s)}{\kappa} \sum_j \lambda'_j g'_j H^{jnm}(s) + \sum_j g'^2_j H_{jf}(s) H^{jnm}(s) = 0 \; . \qquad (C.13)$$

<u>t-dependent constraints</u> $\quad (f \equiv \nu \; or \; e : \; f' \equiv e \; or \; \nu)$:

$$g_n g_m \left[F_{nf'}(t) F_{mf'}(t)\right] + \frac{\sum_i \lambda_i g_i F_{if}(s)}{\kappa} \Big\{ eF_2^{nm,em}(s) - \sum_i \lambda_i g_i F_2^{inm,3}(s) \Big\} - \sum_i g_i^2 F_{if}(s) F_2^{inm,3}(s) = 0 \; . \qquad (C.14)$$

The non-trivial (i.e., with non-vanishing $F_e^{em}(s)$ and $F_{if}(s)$) general solution of the t-independent constraints is <u>for s large</u>:

$$\begin{aligned} F_1^{nm,em}(s) &= F_2^{nm,em}(s) \\ F_1^{inm,3}(s) &= F_2^{inm,3}(s) \\ H^{jnm}(s) &= 0 \\ eF_2^{nm,em}(s) &= \sum_i \lambda_i g_i F_2^{inm,3}(s) \end{aligned} \qquad (C.15)$$

If there was no additional contact term the t-dependent constraint would be:

$$g_n g_m F_{nf'}(t) F_{mf'}(t) = \sum_i g_i^2 F_{if}(s) F_2^{inm,3}(s) \qquad (C.16)$$

(see the discussion of Sec. 3).

<u>Extended minimal scheme:</u> Using the covariant derivative $\partial_\mu - ieQA_\mu - i\sum_i g_i \vec{T} \cdot \vec{W}_{i\mu} - i \sum_j g'_j W^0_{j\mu}$ in the kinetic and mixing parts of the fermion and boson Lagrangian

one gets preditions for the various couplings defined above. Supposing they are valid at low energy and transfer this gives:

$$\begin{aligned}
F_{if}(0) &= H_{jf}(0) = 1 \\
F_f^{em}(0) &= Q_f \\
F_1^{nm,em}(0) &= \delta_{nm} \\
F_2^{nm,em}(0) &= \frac{1}{2e}(\lambda_n g_m + \lambda_m g_n) \\
H^{jnm}(0) &= \delta_{nm} \\
F_1^{inm,3}(0) &= \delta_{nm}\left[\frac{1+\delta_{in}}{2}\right] + (1-\delta_{nm})\frac{\delta_{in}g_m - \delta_{im}g_n}{2g_i} \\
F_2^{inm,3}(0) &= \frac{\delta_{in}g_m + \delta_{im}g_n}{2g_i} \ .
\end{aligned} \tag{C.17}$$

One can easily check for $N > 1$ that it is impossible to keep these values for large s if one wants the constraints to be satisfied; for example $F_1^{nm,em}(0) = F_2^{nm,em}(0)$ gives $\frac{1}{2e}(\lambda_n g_m + \lambda_m g_n) = \delta_{nm}$ impossible to satisfy non-trivially if $N > 1$. s-dependent form factors (i.e., an evolutional model) are then required for this scheme.

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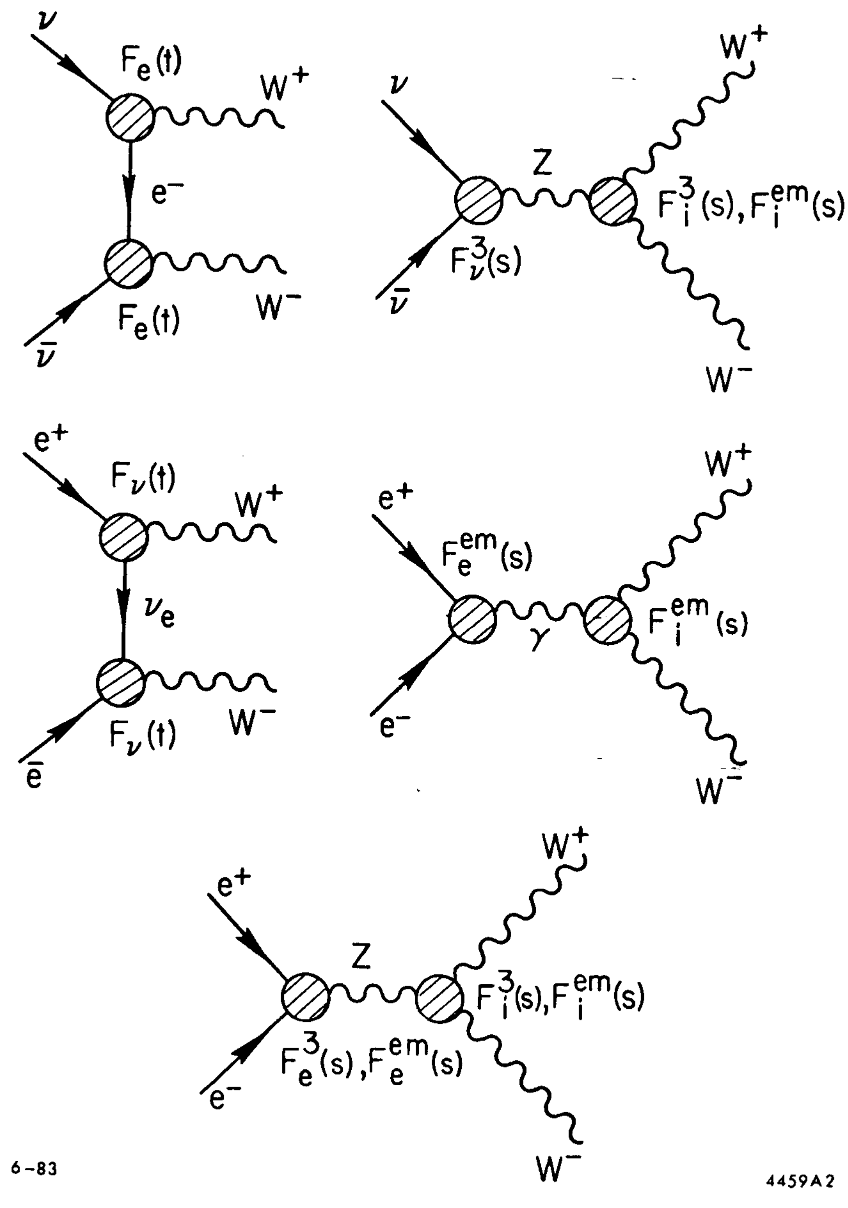
FIGURE CAPTIONS

1. Feynman diagrams for $\nu\bar{\nu} \to W^+W^-$ and $e^+e^- \to W^+W^-$ including form factor effects.

2. s-dependence of $\sigma(e^+e^- \to W^+W^-)$ for several choices of the parameter $F \equiv \frac{\lambda g}{e}$ with $A = 1$ TeV^2 and $sin^2\theta_W = 0.22$. Full curves (——) represent the mixing model and dashed curves (- - -) the "standard model" with $sin^2\theta_W/F$. For $F = 0.7$, 1, and 1.3, we have $(M_W, M_Z) = (67,81)$, $(80,91)$ and $(91,100)$ GeV, respectively.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W+
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Fe(t)
ν̄
W−
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Z
F3i(s), Femi(s)
F3ν(s)
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W+
W−
e+
Fν(t)
W+
νe
Fν(t)
ē
W−
e+
Feme(s)
γ
Femi(s)
e−
W+
W−
e+
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e−
W+
W−
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Fig. 1

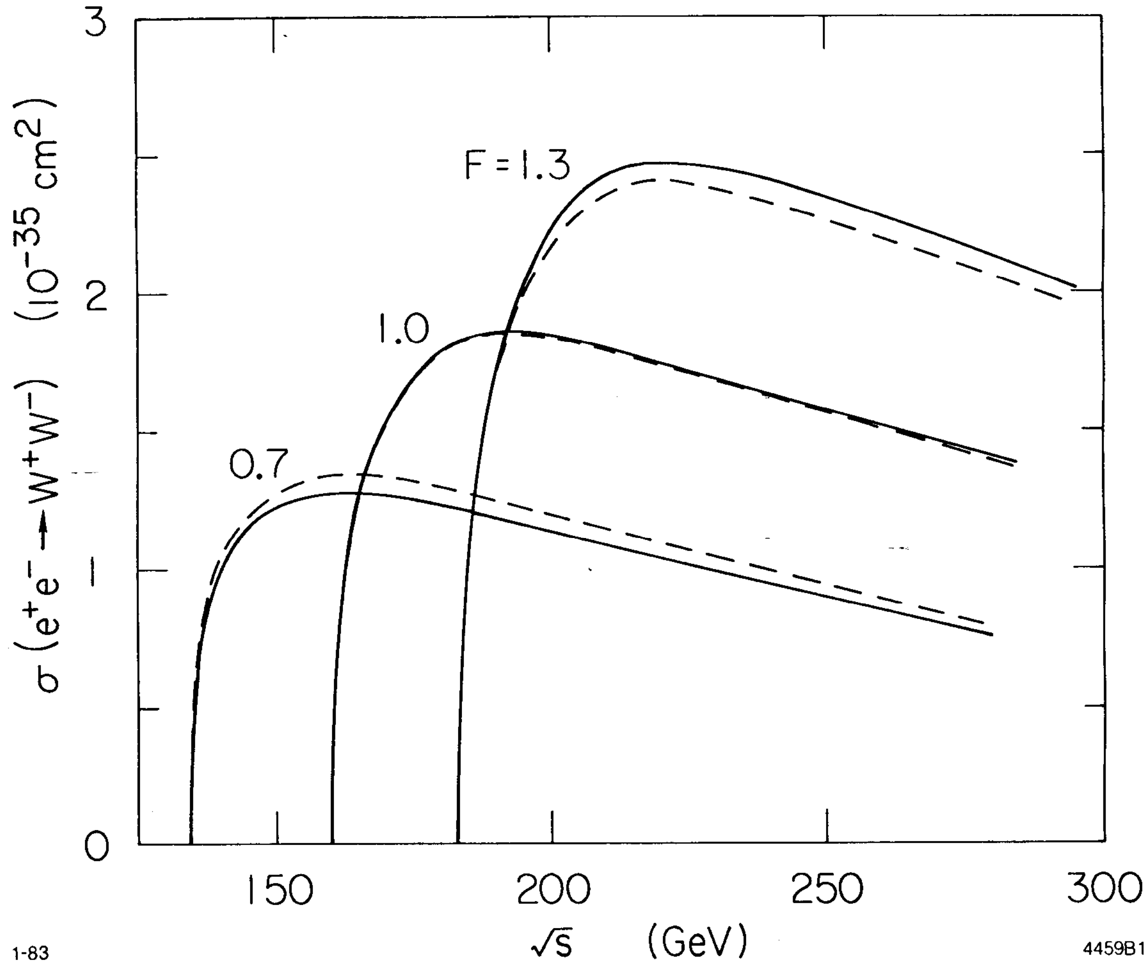

Fig. 2