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SINGLE SPIN ASYMMETRIES IN THE DRELL-YAN PROCESS*

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ABSTRACT

The angular distribution of lepton pairs produced via the $q\bar{q}$ annihilation process is studied through $O(\alpha_s^2)$ in perturbative QCD. Asymmetries for single spin experiments are found to be very small in this channel. This provides an interesting new QCD test in forthcoming $\pi^- p^\uparrow$ experiments. The smallness of the result is intimately related to the gauge structure of the theory and to the color coefficient associated with the $q\bar{q}$ subprocess.

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The treatment of lepton pair hadroproduction in quantum chromodynamics (QCD) has been the scene of continuous progress over the last years.[1] New experimental data have recently led to fairly comprehensive information on various differential cross sections.[2] The theoretical situation while becoming vastly more intricate and sophisticated is not yet settled. For example, testing the Drell-Yan mechanism for $d\sigma/d^4Q$ has become quite involved because of the phenomenological necessity of including higher twist and intrinsic transverse momentum effects. There are single-spin dependent quantities in $d\sigma/d^4Q d\Omega$, however, which vanish identically in the Drell-Yan picture since no imaginary (absorptive) phase is associated with the parton probability distributions. At the Born-term level, i.e., the usual applicable limit of perturbative QCD, these therefore provide a useful null test immune to the values of the parton distributions. The lowest order in which a parity conserving single-spin dependence can occur in the usual QCD framework can be understood as follows: one power of α_s is needed to provide $Q_T \neq 0$ and another power of α_s comes from loop integrals which can generate imaginary parts. Furthermore the complete calculation, as presented in Eq. (5), yields a coefficient of α_s^2 proportional to $(C_F - N_c/2)$ in the $q\bar{q}$ channel. These terms conspire in QCD to practically cancel so the null result of the parton model is maintained for, e.g., $\pi^- p$ lepton pair production.

Since single spin-proposals are under consideration at CERN and FNAL, it is useful and urgent to clarify the QCD expectations in some detail. In this paper we study the spin dependence of the differential cross section

$$\frac{d\sigma}{d^4Q d\Omega} \left(A^{\uparrow}\ B \rightarrow \mu^+\mu^-\ X \right)$$

where Q^{μ} is the lepton pair 4-momentum and Ω the angles of the leptons in a given frame. Correlations between Ω and a single hadron spin probe directly[3] the imaginary part of interfering amplitudes. We consider the $q\bar{q}$ fusion subprocess, known to dominate in π^-p (and $\bar{p}p$) collisions. The reason for this is that the spin dependent parton distribution functions are only known for quarks.[4] Needless to say the calculation can be extended to include the contribution of the qg subprocess with some assumptions for the spin-dependent gluon distribution function.

Let us first remind the reader about the basic formalism we are going to use. Calling $L^{\mu\nu}$ and $W^{\mu\nu}$ the usual leptonic and hadronic tensors, such that

$$e^2 W^{\mu\nu} = \int d^4x\ e^{iQ\cdot x} \left\langle P_A S_A P_B \right| J^{\mu}(0) J^{\nu}(x) \left| P_A S_A P_B \right\rangle$$

$$L^{\mu\nu} = -\ 2Q^2 \left(g^{\mu\nu} - \frac{Q^{\mu}Q^{\nu}}{Q^2} - \frac{k^{\mu}k^{\nu}}{k^2} \right) \qquad (1)$$

where k is the difference between the leptons' momenta k_1 and k_2, one gets

$$\frac{d\sigma}{d^4Q d\Omega} = \frac{\alpha^2}{2(2\pi)^4} \frac{1}{Q^2 s} \left(\delta^{ij} - \frac{k_1^i k_1^j}{\vec{k}_1^2} \right) W_{ij} \qquad (2)$$

in a lepton pair rest frame. Constructing in this frame the following basis[5]

$$Z = P_A\, Q\cdot P_B - P_B\, Q\cdot P_A$$

$$X = P_A Q^2\, Z\cdot P_B - P_B Q^2\, Z\cdot P_A + Q\left(Q\cdot P_B\, Z\cdot P_A - Q\cdot P_A\ \ Z\cdot P_B \right)$$

$$Y^{\mu} = \varepsilon^{\mu\alpha\beta\gamma}\, P_{A\alpha}\, P_{B\beta}\, Q_{\gamma}$$

which defines the angles θ and φ, the differential cross section for $Q_T \neq 0$ may be written under the form[5]

$$\frac{d\sigma}{d^4Q d\Omega} = \frac{\alpha^2}{2(2\pi)^4} \frac{1}{Q^2 s} \Big\{ 2\left(W_{00} + Y\cdot S_A T_{00}\right) + \frac{1}{3}\left[\left(1 - 3\cos^2\theta\right)\left(W_{20} + Y\cdot S_A T_{20}\right)\right]$$

$$+ \sin 2\theta \cos\varphi\left(W_{21} + Y\cdot S_A T_{21}\right) + \frac{1}{2}\left[\sin^2\theta \cos 2\varphi \left(W_{22} + Y\cdot S_A T_{22}\right)\right]$$

$$+ \sin 2\theta \sin\varphi\left(X\cdot S_A T^T_{2-1} + Z\cdot S_A T^L_{2-1}\right)$$

$$+ \sin^2 2\theta \sin 2\varphi\left(X\cdot S_A T^T_{2-2} + Z\cdot S_A T^L_{2-2}\right)\Big\} \tag{3}$$

where we have averaged over polarizations of hadron B but kept the spin vector S_A of hadron A. It is obvious that to define the angle φ one must have $Q_T \neq 0$. Straightforward power counting in QCD reveals that

$$\begin{aligned} W_{22}\ ,\ T_{22}\ ,\ T^L_{2-2}\ ,\ T^T_{2-2} &\propto \frac{Q_T^2}{\hat{s}} \\ W_{21}\ ,\ T_{21}\ ,\ T^L_{2-1},\ T^T_{2-1} &\propto \frac{Q_T}{\sqrt{\hat{s}}} \end{aligned} \tag{4}$$

when compared to W_{00}, $\hat{s}$ being the usual subprocess Mandlestam variable. Moreover, usual chiral properties suppress transverse spin effects by $1/\sqrt{\hat{s}}$ so that T_{21} and T^T_{2-1} should be small. Helicity effects can on the other hand be large. Indeed deep inelastic polarized experiments show that quarks remember fairly well the proton's helicity.[4] We thus have chosen to first calculate the largest helicity term T^L_{2-1}, at lowest nontrivial order, i.e., at $O(\alpha_s^2)$ for the $q\bar{q}$ channel.

Let us now present the main steps of the calculation and outline the basic features of the result. We assume some factorization of the long-distance dynamics from the short-distance perturbatively calculated QCD diagrams. We use the dimensional regularization procedure and work in the Feynman gauge. The goal is to calculate the imaginary part of the interference between Born graphs (Fig. 1.a) and higher order graphs (some of which are drawn in Fig. 1.b-d) summing over the antiquark helicity states while taking the difference between the quark helicity states. It is straightforward to see that only graphs with a loop may have an imaginary part. Moreover one can show that only those in Fig. 1.b-d actually contribute. The computation thus requires, apart from some rather large traces which have been performed with the help of the symbolic program REDUCE,[6] the calculation of integrals of the form

$$\mathcal{I}m \ \frac{1}{i} \int d^n m \ \frac{\left(1,\ m^\mu,\ m^\mu m^\nu,\ m^\mu m^\nu m^\rho\right)}{m^2 (p-m)^2 \ (Q-m-r)^2 \ (m+r)^2}$$

where $n = 4 - \varepsilon$. The resulting expression at the subprocess level may be cast into the form

$$\hat{T}^L_{2-1} = 4\pi e_q^2 \alpha_s^2 \frac{C_F}{N_c}\left(C_F - \frac{N_c}{2}\right)\left[A\left(\frac{2}{\varepsilon} - \ell n\,\frac{Q^2}{\mu^2}\right) + B\,\ell n\,\frac{\hat{s}}{Q^2} + C\,\ell n\left(1+\frac{Q_T^2}{Q^2}\right) + D\,\ell n\left(\frac{Q\cdot p}{Q\cdot r}\right) + E\right].$$

The infrared finiteness of the result at this order requires that A be zero, which we indeed find. This is in itself a nontrivial result ensuring the applicability of perturbative QCD. An imaginary divergence would not have been factorizable into the usual quark distribution. The B - E coefficients have fairly compact forms

$$B = \frac{\Delta\left(-v^4 + 3\Sigma v^3 - \left(2\Delta^2 + 3\Sigma^2\right)v^2 + \Delta^2\Sigma v + 2\Delta^4\right)\ \hat{s}^2}{v^2\left(v^2 - \Delta^2\right)}$$

$$C = \frac{\Delta\hat{s}^4(v-\Sigma)^2}{\left(v^2 - \Delta^2\right)}$$

$$D = \frac{-4(v-\Sigma)\left(\Sigma v - \Delta^2\right)\ \hat{s}^2}{\left(v^2 - \Delta^2\right)} \tag{6}$$

$$E = \frac{2\Delta\left[-4\Sigma v^4 - 2v^3\left(\Sigma^2+\Delta^2\right) + v^2\Sigma\left(\Delta^2+3\Sigma^2\right) + v\left(4\Sigma^4 - 3\Sigma^2\Delta^2 + 3\Sigma^4\right) + 2\Delta^2\Sigma\left(\Delta^2 - \Sigma^3\right)\right]}{v\left(\Sigma^2 - \Delta^2\right)}$$

with $\Delta = Q\cdot p - Q\cdot r$, $\Sigma = Q\cdot p + Q\cdot r$, $v = \hat{s} - \Sigma$, $\ln(Q\cdot p/Q\cdot r)$ being twice the rapidity of the pair in the subprocess c.m.s.

For this discussion, we define an integrated asymmetry $\mathscr{A}$ for the physical process by

$$\mathscr{A} = \frac{\int_0^1 d\cos\theta \left[\int_0^\pi d\varphi - \int_\pi^{2\pi} d\varphi\right] \frac{d\sigma^\uparrow}{d^4Q d\Omega}}{\int_0^1 d\cos\theta \left[\int_0^\pi d\varphi + \int_\pi^{2\pi} d\varphi\right] \frac{d\sigma}{d^4Q d\Omega}} \tag{7}$$

where $d\sigma^\uparrow$ indicates positive helicity λ for proton A [since $\mathscr{A}$ is linear in λ, one can replace $d\sigma^\uparrow$ by $(d\sigma^\uparrow - d\sigma^\downarrow)/2$]. As a preliminary step one can define $\hat{\mathscr{A}}$, the asymmetry at the subprocess level, by an obvious modification of Eq. (7). It is easy to show that

$$\mathscr{A} = \frac{Z\cdot S_A\ T^L_{2-1}}{3\pi\ W_{00}} \tag{8}$$

and a similar expression for $\hat{\mathscr{A}}$.

In Fig. 2, we plot the subprocess asymmetry as a function of the lepton pair transverse momentum Q_T for $Q^2 = 25\ \mathrm{GeV}^2$, $\hat{s} = 200$ and $400\ \mathrm{GeV}^2$ and the photon rapidity $y > 0$. [Note that, at this level, the differential cross section $d\sigma/d^4Qd\Omega$ is proportional to a $\delta(|y| - y_0)$ term and that $\hat{\mathscr{A}}$ is odd in y.] This asymmetry is quite small; indeed this might have been anticipated for the $q\bar{q}$ channel, as we will now show.

Let us consider the theoretical expression for $\hat{T}^L_{2-1}$, Eq. (5). The coefficients B, C and D are imaginary parts of doubly logarithmic terms while E is the imaginary part of a single logarithm, coming from relations like

$$\mathscr{I}m\ \ell n^2(-s-i\eta) = -i\pi\ \ell n\ s$$

$$\mathscr{I}m\ \ell n(-s-i\eta) = -i\pi$$

It is well known that the leading (i.e., $\ell n^2\ s$) real corrections can be obtained order by order by "soft" gluon approximations, i.e., by integrating gluon internal momenta from $O(\sqrt{s})$ down to fixed values as $Q^2, s \to \infty$. Similarly infrared divergent real terms of single log order come from the various collinear regions determined by the external legs. For the purposes of locating imaginary terms, one can replace the original Feynman integrals by their approximations in these regions, neglecting the loop momentum dependence of propagators that become far off-shell.

The analytic properties of this replacement can then be easily pinpointed. For instance, the soft and collinear regions of Fig. 1.d have no thresholds, except for a gauge term that annihilates with Figs. 1.b–c using the Ward identity. The B - E coefficients should therefore multiply the color factor of Fig. 1.b, i.e., $(C_F - N_c/2)$. The overall color coefficient of the complete calculation, Eq. (5) thus has a simple explanation.

The fact that $C_F - N_c/2 = -1/6$ in QCD has important consequences. The results of the complete $O(\alpha_s^2)$ calculation can be summarized in the limit Q^2/s, Q_T^2/Q^2, y fixed and $s \to \infty$ by the estimate[7]

$$|\mathscr{A}| \simeq \left|C_F - \frac{N_c}{2}\right| \frac{\alpha_s^2 \left\langle \left(\frac{Q_T}{\sqrt{\hat{s}}}\right) \lambda_q(x)\; q(x)\; \bar{q}(x) \right\rangle}{\alpha_s \langle q(x)\; \bar{q}(x) \rangle} \sim 1\% \tag{9}$$

where the brackets indicate the parton convolutions. The near cancellation between the color factors therefore suppresses the QCD contribution to $\mathscr{A}$ substantially. Numerical integration of our result using the NA_3 parton distributions[8] and $\lambda_q(x) = 0.94\sqrt{x}$ from the SLAC-Yale experiment[4] give $\mathscr{A} = 2.2\ \alpha_s\%$ $(1.8\ \alpha_s\%)$ at $Q^2 = 25\ \text{GeV}^2$, $Q_T = 4$ GeV (3 GeV) and $y = 1$ for $\pi^- p$ experiments at $\sqrt{s} = 27$ GeV.

We conclude that the null value for $\mathscr{A}$ of the naive Drell-Yan model including intrinsic effects is not upset at $O(\alpha_s^2)$ in QCD because of a fortunate cancellation in the color algebra for the $q\bar{q}$ subprocess. Since $\pi^- p \to \mu\bar{\mu} + X$ is dominated[9] by the $q\bar{q}$ channel, observation of a nonzero asymmetry in this experiment would be difficult to explain within the present framework.

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Figure Captions

Fig. 1. a) The lowest order graph for $q\bar{q} \to \gamma^* g$

b) The graphs contributing in Feynman gauge to the imaginary part of $W^{\mu\nu}$ at order α_s^2 (crossed graphs have been omitted).

Fig. 2. The asymmetry at the subprocess level, as defined in Eq. (7), in units of $(N_c - 2C_F)\ \alpha_s(Q^2)$ for $\hat{s} = 200\ \text{GeV}^2$ (full curve) and $\hat{s} = 400\ \text{GeV}^2$ (dashed curve).

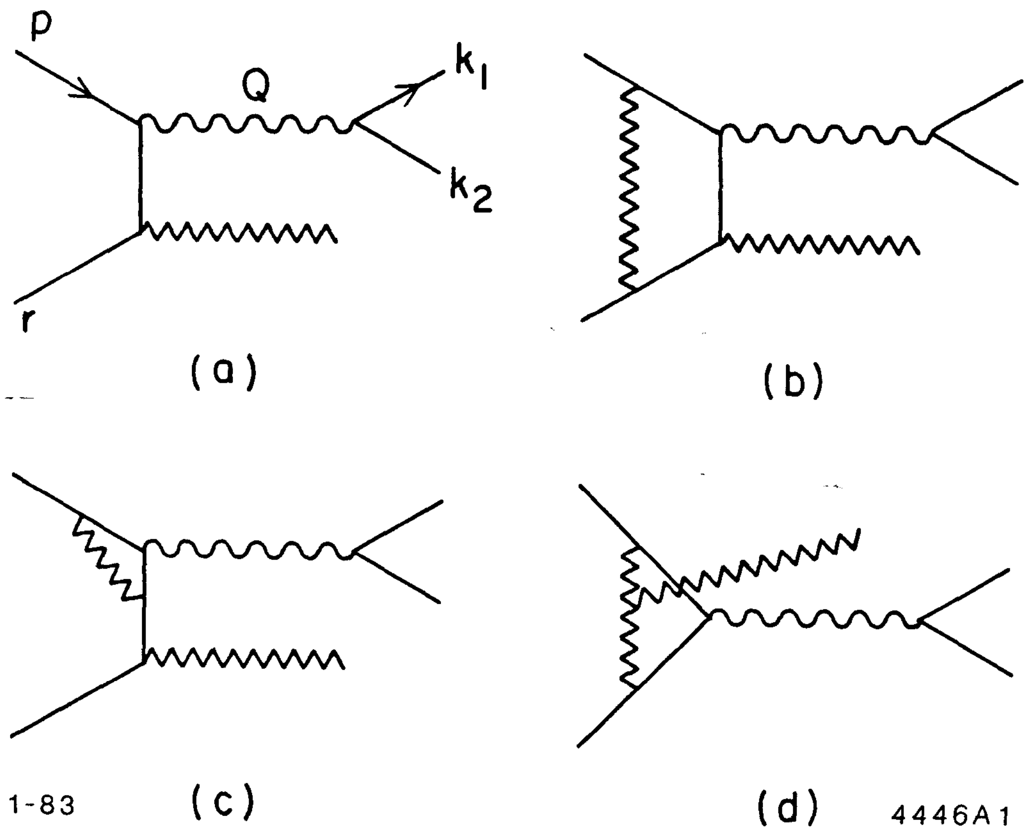

Fig. 1

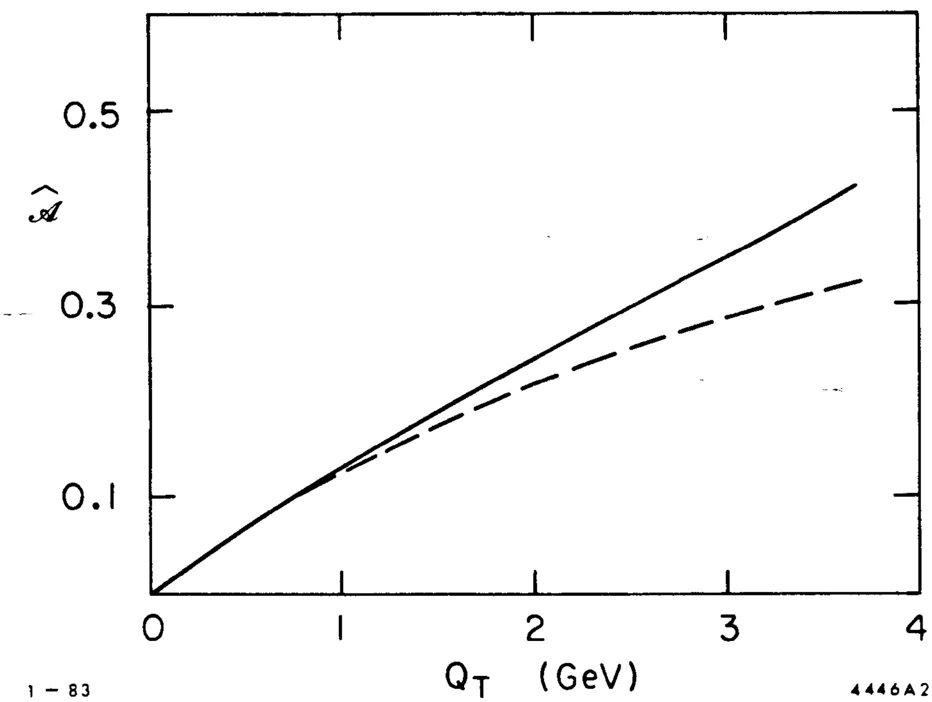

Fig. 2