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SPONTANEOUS CP VIOLATION IN EXTENDED TECHNICOLOR MODELS WITH HORIZONTAL $U(2)_L \otimes U(2)_R$ FLAVOR SYMMETRIES (II)*

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ABSTRACT

The results presented in Part I of this paper are extended to include previously neglected electro-weak degrees of freedom, and are illustrated in a toy model. A problem associated with colored technifermions is identified and discussed. Some hope is offered for the disappointing quark mass matrix obtained in Part I.

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1. INTRODUCTION

In a previous paper [1], hereafter referred to as I, we investigated spontaneous CP violation in a class of extended technicolor (ETC) models [2,3,4,5]. We constructed the effective Hamiltonian generated by broken ETC interactions and minimized its contribution to the vacuum energy, as called for by Dashen's Theorem [6]. As originally pointed out by Dashen, and more recently by Eichten, Lane and Preskill in the context of the ETC program, this aligning of the chiral vacuum and Hamiltonian can lead to CP violation from an initially CP symmetric theory [7,8].

The ETC models analyzed in I were distinguished by the global flavor invariance of the color-technicolor forces:

$$G_F = \Pi\, U(2)_L \otimes U(2)_R \tag{1}$$

For these models, we found that, in general, the occurrence of spontaneous CP violation is tied to CP nonconservation in the strong interactions and, thus, to an unacceptably large neutron electric dipole moment. However, strong CP violation is avoided by imposing a discrete invariance, denoted by GP, on the effective Hamiltonian.

We also showed that if CP violating phases do arise when the vacuum is aligned, their size scales with a ratio of ETC gauge boson masses rather than being on the order of one as expected by Eichten, Lane and Preskill.

Here, we address several problems deferred in I. First, the models considered in I did not include electro-weak degrees of freedom. The chiral symmetry group, Eq. (1), was "horizontal," i.e., operated in the space of fermion generations. Of course, when the electro-weak interactions are included, G_F must be enlarged to contain $G_w \equiv SU(2)_L \otimes U(1)_Y$. In sect. 2 we argue that the analysis carried out in I remains applicable when G_w is embedded in G_F.

In sect. 3, we exhibit the results of I in the context of a toy model based on an $SO(10)_{ETC}$ group.

In sect. 4, we raise and discuss a possible flaw in this mechanism for CP violation related to the contribution of techniquarks to strong CP violation. Finally, we note that conditions exist under which the GP symmetry constraint can be relaxed, allowing the development of a realistic quark mass matrix without sacrificing its hermiticity. *

2. INCLUSION OF ELECTRO-WEAK DEGREES OF FREEDOM

In considering models with G_F given by (1), the requirement that the low-energy effective ETC Hamiltonian be a G_w singlet forced us to identify flavor doublets as generational rather than electro-weak. The alternative would have led to a trivial problem with only CP conserving solutions. But, in reality, G_F contains G_w as a subgroup. Therefore, in the analysis of I, we simply ignored electro-weak degrees of freedom. Here we argue that this simplification is as reasonable as it is expedient.

If we assume all left-handed fermions occur in weak $SU(2)_L$ doublets, the multiplicity of each irreducible representation of $G_{TC} \otimes G_C$ must be doubled. This yields

$$G_F = \Pi\ U(4)_L \otimes U(4)_R \ .$$

Now, the analysis carried out in I depended crucially on G_F being a product of $U(2)_L \otimes U(2)_R$ factors. However, it is easy to see that if the chiral vacuum is to conserve electric charge, its alignment must be specified by an element of the subgroup.

$$\Pi\ [U(2)_L \otimes U(2)_R]_{up} \otimes [U(2)_L \otimes U(2)_R]_{down} \ ; \tag{2}$$

that is, up-like and down-like fermions are to be rotated independently. Otherwise, a non-zero vacuum expectation value with the G_w properties of $\bar{u}_L d_R$ will appear, spontaneously breaking electric charge.

* See I, sect. 3.1.

More precisely, the vacuum is specified by the expectation values

$$\langle \bar{\psi}_L^{(\rho)r} \psi_R^{(\sigma)r'} \rangle_0 = \Delta^\rho \delta^{\rho\sigma} W_{rr'}^{(\rho)}$$

where ρ and σ denote irreducible representations of $G_{TC} \otimes G_C$, r,r' is a flavor index, and $W^{(\rho)}$ is a 4×4 unitary matrix, $W^{(\rho)} = W^{R(\rho)} W^{L(\rho)\dagger}$ ($G_{TC} \otimes G_C$ indices have been suppressed). Clearly a conserved electric charge exists only if

$$W^{(\rho)} = \begin{pmatrix} W^{(\rho,u)} & 0 \\ 0 & W^{(\rho,d)} \end{pmatrix} \tag{3}$$

in some basis, where $W^{(\rho,u)}$ and $W^{(\rho,d)}$ are 2×2 unitary matrices. Thus, if we restrict our interest to charge conserving vacua, the effective flavor invariance is given by (2) and the *CP* character of charge conserving critical points of the vacuum energy is determined by the analysis promulgated in I. In particular, a *GP* operation can be defined and its conservation by the effective Hamiltonian will imply that the *CP* symmetry of the strong interactions is unmolested. Also, phases will be suppressed through the mechanism identified in I.

Of course, in a given model, the true, global minimum might spontaneously break charge conservation. In this case, the model would be disqualified on grounds unrelated to the fate of *CP*.

3. A TOY MODEL

In this section, we describe a simple toy model which displays the spontaneous *CP* violation discussed in I. The model includes the essential properties of *CP* and *GP* symmetry at the level of the effective Hamiltonian, and spontaneously generates *CP* violating phases suppressed by a ratio of *ETC* mass scales. Our purpose here is to go some way towards demonstrating the feasibility and internal consistency of this mechanism, rather than to present a realistic model into which it is incorporated. As we will see, the model is either pathological or incomplete in several important respects, which, however, do not obviously bear on the matter at hand.*

* An evident exception to this is the unrealistic quark mass spectrum implied by *GP* symmetry (cf. sect. I.3.1).

Initially, the unbroken local gauge symmetry is $G_{ETC} \otimes G_c \otimes G_w$, where $G_{ETC} = SO(10)_{ETC}$ and $G_c \otimes G_w$ is the standard color-electro-weak sector. Fermions occur in three representations:.

$$\psi_L \sim (16,3,2,\frac{1}{6}) \; ; \; \psi_R^{(u)} \sim (16,3,1,\frac{2}{3}) \; ; \; \psi_R^{(d)} \sim (16,3,1,-\frac{1}{3}) \; . \tag{4}$$

The last label for each field is the weak hypercharge.

Immediately, three comments must be made:

1. In general, $[G_{ETC}, G_c] = 0$ and/or $[G_{ETC}, U(1)_Y] = 0$ implies that some chiral symmetries are not gauged and therefore leads to massless Goldstone bosons. This was originally pointed out by Eichten and Lane who concluded that quarks and leptons must occur together in irreducible representations of G_{ETC}.[4] Here this constraint is "solved" by omitting leptons altogether, thus eliminating unwanted chiral symmetries. Were we to introduce color singlets, $SO(10)_{ETC} \otimes SU(3)_c \otimes U(1)_Y$ would, presumably, have to be embedded in some larger, simple group.

2. The fermion content, (4), leads to vectorial *ETC* interactions possessing a global $SU(2)_L \otimes SU(2)_R$ symmetry. After chiral symmetry breaking, assuming the vacuum alignment conserved electric charge and ignoring the weak gauging of $SU(2)_L$ the model retains a residual vector isospin symmetry and, therefore, can only generate equal masses for up- and down-like quarks. *

3. By embedding $G_c \otimes G_w$ in an $SO(10)$, and adding leptons to fill out the standard multiplet, a variation of the "vertical-horizontal symmetric" $SO(10)_V \otimes SO(10)_H$ grand unifying model proposed by Davidson, Wali and Mannheim is obtained.[9] Here, the major departures from that model is the choice of technicolor group ($SU(3)$ rather than $SU(4)$) and of fermion representation (16,16) rather than (16,10) + (10,16)). (In fact, these variations are related since, for

*A non-vectorial realization of the model is obtained by assigning right-handed up-like fermions to the $\underline{16}$, and right-handed down-like fermions to the inequivalent $\underline{16}^*$ of $SO(10)_{ETC}$. However, this complication tends to obscure our program in exchange for extremely meager compensation.

$G_{TC} = SU(4)$, the (16,16) contains no technicolor singlets.) Since we are primarily interested in the breaking of G_{ETC}, which occurs at a mass scale well below grand unification, and in light of the constraint mentioned in comment 1, we do not further emphasize this aspect of the model.

At a mass scale M'_{ETC}, G_{ETC} is broken to $SO(6) \otimes SO(4) \equiv SU(4)_{TC} \otimes SU(2)_I \otimes SU(2)_{II}$. A second stage of symmetry breaking occurs at $M_{ETC} < M'_{ETC}$ leaving intact only the technicolor group, $SU(3)_{TC}$. Under $SU(3)_{TC} \otimes SU(2)_I \otimes SU(2)_{II}$, the fermionic 16-plets transform as

$$\underline{16} \equiv (3,2,1) + (3^*,1,2) + (1,2,1) + (1,1,2) \; .$$

Thus, there are four generations of both techniquarks and quarks. We denote these fields by

$$\begin{aligned} &Q^{Ar}_{aL(R)} \sim (3,2,1) \; ; \; Q'^{Ar}_{aL(R)} \sim (3^*,1,2) \\ &q^{r}_{aL(R)} \sim (1,2,1) \; ; \; q'^{r}_{aL(R)} \sim (1,1,2) \end{aligned} \tag{5}$$

where $A = 1,2,3$ labels technicolor, $r = 1,2$ is the $SU(2)_I$ or $SU(2)_{II}$ index and $a = 1,2$ distinguishes up- and down-like fermions. The suppressed color degree of freedom plays a trivial role in what follows.

Now, ideally, G_{TC} would mimic $SU(3)_c$ with $\Lambda_{TC} \simeq 10^3 \Lambda_c$. Unfortunately, in this model the technicolor β -function is positive, indicating that G_{TC} is not asymptotically free. Nevertheless, we will assume that, in analogy to the presumed low energy behavior of the color forces, $SU(3)_{TC}$ spontaneously breaks chiral symmetry at an energy scale of 1 TeV.

Since the fields, (5), are all triplets under G_c, the $G_{TC} \otimes G_c$ couplings are invariant under the action of the global symmetry group

$$G_F = [U(4)_L \otimes U(4)_R]_Q \otimes [U(4)_L \otimes U(4)_R]_{Q'} \otimes [U(8)_L \otimes U(8)_R]_{q,q'} \; .$$

By imposing $G_{TC} \otimes G_c \otimes G_w \otimes SU(2)_R$ invariance, we can write down the most general four-fermion operator which breaks G_F and contributes non-trivally to the vacuum energy:

$$
\begin{aligned}
\mathcal{H}' = & t^{(m)}_{ABCD} \Gamma^{Q(m)}_{rr'ss'} \bar{Q}^{Ar}_{aL} Q^{Br'}_{bR} \bar{Q}^{Cs}_{bR} Q^{Ds'}_{aL} \\
& + t^{(m)}_{ABCD} \Gamma^{Q'(m)}_{rr'ss'} \bar{Q}'^{Ar}_{aL} Q'^{Br'}_{bR} \bar{Q}'^{Cs}_{bR} Q'^{Ds'}_{aL} \\
& + t^{(m)}_{ABCD} \Gamma^{QQ'(m)}_{rr'ss'} (\bar{Q}^{Ar}_{aL} Q^{Br'}_{bR} \bar{Q}'^{Cs}_{bR} Q'^{Ds'}_{aL} + h.c.) \\
& + \Gamma^{Qq}_{rr'xy} (\bar{Q}^{Ar}_{aL} Q^{Ar'}_{bR} \bar{q}^{x}_{bR} q^{y}_{aL} + h.c.) \\
& + \Gamma^{Q'q}_{rr'xy} (\bar{Q}'^{Ar}_{aL} Q'^{Ar'}_{bR} \bar{q}^{x}_{bR} q^{y}_{aL} + h.c.) \\
& + \Gamma^{q}_{xx'yy'} \bar{q}^{x}_{aL} q^{x'}_{bR} \bar{q}^{y}_{bR} q^{y'}_{aL} \,.
\end{aligned}
\tag{6}
$$

In this expression, all Γ-tensors are real; $m = 1,2$ with $t^{(1)}_{ABCD} = \delta_{AB}\delta_{CD}$, $t^{(2)}_{ABCD} = \delta_{AD}\delta_{BC}$. The indices x, x', y, y' take on the values $1,2,3,4$, with, e.g., $q^x = q^r$, for $x = 1,2$, and $q^x = q'^r$, for $x = 3,4$. q and q' will eventually decouple in $\mathcal{H}'$. *

Our program is to compute the Γ-tensors to lowest order in g^2_{ETC} from single boson exchange interactions. *ETC* gauge bosons make up a <u>45</u> of $SO(10)_{ETC}$;

* The observant reader will have noticed that the technifermions possess a $U(24)_L \otimes U(24)_R$ global invariance when their color interactions are neglected. This symmetry is weakly and explicitly broken by color to the factors appearing in G_F. The larger invariance group implies relations amongst the several technifermion condensates (see eqs. (12) and (13)) but, since color is strictly conserved, does not admit additional terms in eq. (6) for $\mathcal{H}'$.

together with their transformation properties under $SU(3)_{TC}\otimes SU(2)_I\otimes SU(2)_{II}$ and the fermionic currents to which they couple, they are:

$$
\begin{aligned}
&C_\mu \sim (1,1,1) : J^0_\mu = \frac{1}{\sqrt{6}}(\bar{Q}\,\gamma_\mu Q - 3\,\bar{q}\,\gamma_\mu q - \bar{Q}'\,\gamma_\mu Q' + 3\,\bar{q}'\,\gamma_\mu q')\\
&\vec{X}_\mu \sim (1,3,1) : \vec{J}^I_\mu = \bar{Q}^r\,\gamma_\mu\,\vec{\tau}_{rs}\,Q^s + \bar{q}^r\,\gamma_\mu\,\vec{\tau}_{rs}\,q^s\\
&\vec{Y}_\mu \sim (1,1,3) : \vec{J}^{II}_\mu = \bar{Q}'^r\,\gamma_\mu\,\vec{\tau}_{rs}\,Q'^s + \bar{q}'^r\,\gamma_\mu\,\vec{\tau}_{rs}\,q'^s\\
&D^A_\mu \sim (3,1,1) : K^A_\mu = \bar{Q}'^A\,\gamma_\mu q' + \bar{q}\,\gamma_\mu Q^A\\
&\bar{D}^A_\mu \sim (3^*,1,1) : K^{A\dagger}_\mu\\
&A^{A\kappa}_\mu \sim (3,2,2) : J^{A\kappa}_\mu = P^\kappa_{sr} J^{Ars}_\mu\\
&\qquad\qquad = P^\kappa_{sr}\Big[\epsilon_{ABC}\,\bar{Q}^{Br'}\,i\tau^2_{r'r}\gamma_\mu Q'^{Cs}\\
&\qquad\qquad\quad + \bar{Q}'^{Ar'}\,i\tau^2_{r's}\gamma_\mu q^r - \bar{q}'^{r'}\,i\tau^2_{r's}\gamma_\mu Q^{Ar}\,\Big]\\
&\bar{A}^{A\kappa}_\mu \sim (3^*,2,2) : I^{A\kappa}_\mu = P^\kappa_{rs}(J^{A\dagger}_\mu)^{rs}\\
&H_\mu \sim (8,1,1)\ .
\end{aligned}
\tag{7}
$$

In eq. (7), suppressed indices are traced and $\kappa = 1,2,3,4$ with

$$
P^\kappa = \frac{1}{2}\begin{pmatrix} 1+\tau^3 \\ 1-\tau^3 \\ \tau^1 - i\tau^2 \\ \tau^1 + i\tau^2 \end{pmatrix}\ .
$$

All gauge bosons, except the H_μ, acquire mass at one, or both, stages of G_{ETC} breaking. Using the residual $SU(3)_{TC}$ symmetry we have

$$
\begin{aligned}
\mathcal{L}_{mass} = &\mu_0^2 C_\mu C^\mu + (\mu_I^2)_{ij} X^i_\mu X^{\mu j} + (\mu_{II}^2)_{ij} Y^i_\mu Y^{\mu j}\\
&+ [(\mu^2_{I,II})_{ij} X^i_\mu Y^{\mu j} + (\mu^2_{I,0})_i X^i_\mu C^\mu + (\mu^2_{II,0})_i Y^i_\mu C^\mu + h.c.]\\
&+ \mu_4^2 D^A_\mu\,\bar{D}^{\mu A} + (\mu'^2)_{\kappa\lambda} A^{A\kappa}_\mu\,\bar{A}^{\mu A\lambda} + [(\mu''^2)_\kappa A^{A\kappa}_\mu\,\bar{D}^{\mu A} + h.c.]\ .
\end{aligned}
$$

Here μ_0^2, μ_4^2, μ_I^2 and μ_{II}^2 are real and symmetric and of order M_{ETC}^2. μ'^2 is hermitian and of order $M_{ETC}'^2$. In this model, we take $\mu_{I,II}^2 = \mu_{I,0}^2 = \mu_{II,0}^2 = \mu''^2 = 0$, which is a consistent choice provided $SO(10)_{ETC}$ is broken only by objects transforming as $(1,1)$ or $(1,3) \oplus (3,1)$ under $SU(2)_I \otimes SU(2)_{II}$.

The effective current-current Hamiltonian is

$$\begin{aligned}
\mathcal{H}' =& g_{ETC}^2 \, (\mu_I^{-2})_{ij} \, J_\mu^{Ii} J^{\mu Ij} + g_{ETC}^2 \, (\mu_{II}^{-2})_{ij} \, J_\mu^{IIi} J^{\mu IIj} \\
&+ g_{ETC}^2 \mu_0^{-2} J_\mu^0 J^{\mu 0} + g_{ETC}^2 \mu_4^{-2} K_\mu^A K^{\mu A\dagger} \\
&+ g_{ETC}^2 \, (\mu'^{-2})_{\kappa\lambda} \, J_\mu^{A\kappa} I^{\mu A\lambda} \ .
\end{aligned}$$

Comparing this expression to eq. (6) with the aid of a Fierz transformation, we find

$$\begin{aligned}
\mathcal{H}' =& \Gamma^Q_{\alpha\beta} \tau^\alpha_{rs'} \tau^\beta_{sr'} \, \bar{Q}^{Ar}_{aL} \, Q^{Br'}_{bR} \, \bar{Q}^{Bs}_{bR} \, Q^{As'}_{aL} \\
&+ \Gamma^{Q'}_{\alpha\beta} \tau^\alpha_{rs'} \tau^\beta_{sr'} \, \bar{Q}'^{Ar}_{aL} \, Q'^{Br'}_{bR} \, \bar{Q}'^{Bs}_{bR} \, Q'^{As'}_{aL} \\
&- \Gamma^{10}_{\alpha\beta} \tau^\alpha_{rs'} \tau^\beta_{sr'} \left(\bar{Q}^{Ar}_{aL} \, Q^{Br'}_{bR} \, \bar{Q}'^{Cs}_{bR} \, Q'^{Ds'}_{aL} \, \epsilon_{ADF} \epsilon_{BCF} + h.c. \right) \\
&+ \Gamma^4_{\alpha\beta} \tau^\alpha_{rs'} \tau^\beta_{sr'} \left(\bar{Q}^{Ar}_{aL} \, Q^{Ar'}_{bR} \, \bar{q}^s_{bR} \, q^{s'}_{aL} + \bar{Q}'^{Ar}_{aL} \, Q'^{Ar'}_{bR} \, \bar{q}'^s_{bR} \, q'^{s'}_{aL} + h.c. \right) \qquad (8) \\
&- \Gamma^{10}_{\alpha\beta} \tau^\alpha_{rs'} \tau^\beta_{sr'} \left(\bar{Q}^{Ar}_{aL} \, Q^{Ar'}_{bR} \, \bar{q}'^s_{bR} \, q'^{s'}_{aL} + \bar{Q}'^{As}_{aL} \, Q'^{As'}_{bR} \, \bar{q}^r_{bR} \, q^{r'}_{aL} + h.c. \right) \\
&+ \Gamma^q_{\alpha\beta} \tau^\alpha_{rs'} \tau^\beta_{sr'} \, \bar{q}^r_{aL} \, q^{r'}_{bR} \, \bar{q}^s_{bR} \, q^{s'}_{aL} \\
&+ \Gamma^{q'}_{\alpha\beta} \tau^\alpha_{rs'} \tau^\beta_{sr'} \, \bar{q}'^r_{aL} \, q'^{r'}_{bR} q'^s_{bR} q'^{s'}_{aL}
\end{aligned}$$

with

$$\Gamma^Q = 2 \begin{pmatrix} \frac{1}{3}\mu_0^{-2} & 0 \\ 0 & 2\mu_I^{-2} \end{pmatrix}$$

$$\Gamma^{Q'} = 2 \begin{pmatrix} \frac{1}{3}\mu_0^{-2} & 0 \\ 0 & 2\mu_{II}^{-2} \end{pmatrix}$$

$$\Gamma^4 = 2\mu_4^{-2} \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}$$

$$\Gamma^{10} = 2 \begin{pmatrix} \mu'^{-2}_{34} - \mu'^{-2}_{33} & -i(\mu'^{-2}_{13} - \mu'^{-2}_{14}) & 0 & 0 \\ & -\mu'^{-1}_{12} - \mu'^{-2}_{11} & 0 & 0 \\ & & \mu'^{-2}_{12} - \mu'^{-2}_{11} & \mu'^{-2}_{13} + \mu'^{-2}_{14} \\ & & & -\mu'^{-2}_{34} - \mu'^{-2}_{33} \end{pmatrix} = \Gamma^{10\dagger}$$

$$\Gamma^q = 2 \begin{pmatrix} 3\mu_0^{-2} & 0 \\ 0 & 2\mu_I^{-2} \end{pmatrix}$$

$$\Gamma^{q'} = 2 \begin{pmatrix} 3\mu_0^{-2} & 0 \\ 0 & 2\mu_{II}^{-2} \end{pmatrix} \tag{9}$$

In eq. (9), Γ^{10} has been been given in the basis $(\tau^0, \tau^2, \tau^1, \tau^3)$, and we have assumed that $\mathcal{H}'$ is invariant under the action of a GP operation defined with respect to the subgroup of G_F

$$\begin{aligned} H_F = \prod_{a=u,d} & [U(2)_L \otimes U(2)_R]_{Q,a} \otimes [U(2)_L \otimes U(2)_R]_{Q',a} \\ & \otimes [U(2)_L \otimes U(2)_R]_{q,a} \otimes [U(2)_L \otimes U(2)_R]_{q',a} \end{aligned} \tag{10}$$

In other words, $\mathcal{H}'$ is invariant under

$$\begin{aligned} Q^{(\prime)Ar}_{L(R)a} &\to i\tau^2_{rr'}\, CPQ^{(\prime)Ar'}_{L(R)a}\, (CP)^{-1} \\ q^{(\prime)r}_{L(R)a} &\to i\tau^2_{rr'}\, CPq^{(\prime)r'}_{L(R)a}\, (CP)^{-1} \end{aligned} \tag{11}$$

This assumption has enabled us to restrict Γ^{10} to the form given in (9).

Now, when their couplings become strong, $G_{TC} \otimes G_C$ interactions break G_F to $S_F = [U(4)_V]_Q \otimes [U(4)_V]_{Q'} \otimes [U(8)_V]_{q,q'}$ and generate non-zero vacuum expectation values for fermion bilinears:

$$\begin{aligned}
&\langle \bar{Q}^{Ar}_{aL} Q^{Br'}_{bR} \rangle_0 = \langle \bar{Q}'^{Ar}_{aL} Q'^{Br'}_{bR} \rangle_0 = \Delta^Q \delta^{AB} \delta_{ab} \delta_{rr'} \; ; \; \Delta^Q = \Delta^{Q^*} \\
&\langle \bar{q}^x_{aL} q^y_{bR} \rangle_0 = \Delta^q \delta_{ab} \delta_{xy} \; ; \; \Delta^q = \Delta^{q^*}
\end{aligned} \tag{12}$$

and for four-fermion operators:

$$\begin{aligned}
\langle \bar{Q}^{Ar}_{aL} Q^{Br'}_{bR} \bar{Q}^{Bs}_{bR} Q^{As'}_{aL} \rangle_0 =& \langle \bar{Q}'^{Ar}_{aL} Q'^{Br'}_{bR} \bar{Q}'^{Bs}_{bR} Q'^{As'}_{aL} \rangle_0 \\
=& \Delta^{QQ} \delta_{rr'} \delta_{ss'} + \Delta'^{QQ} \delta_{rs'} \delta_{sr'} \; , \\
\Delta^{QQ} =& \Delta^{QQ^*} \; , \; \Delta'^{QQ} = \Delta'^{QQ^*} \; ;
\end{aligned}$$

$$\langle \bar{Q}^{Ar}_{aL} Q^{Br'}_{bR} \bar{Q}'^{Cs}_{bR} Q'^{Ds'}_{aL} \rangle_0 \epsilon_{ADF} \epsilon_{BCF} = \Delta^{QQ'} \delta_{rr'} \delta_{ss'} \; , \; \Delta^{QQ'} = \Delta^{QQ'^*} \; ;$$

$$\begin{aligned}
\langle \bar{Q}^{Ar}_{aL} Q^{Ar'}_{bR} \bar{q}^x_{bR} q^y_{aL} \rangle_0 =& \langle \bar{Q}'^{Ar}_{aL} Q'^{Ar'}_{bR} \bar{q}^x_{bR} q^y_{aL} \rangle_0 \\
=& \Delta^{Qq} \delta_{rr'} \delta_{xy} \; , \; \Delta^{Qq} = \Delta^{Qq^*} \; ;
\end{aligned}$$

$$\begin{aligned}
\langle \bar{q}^x_{aL} q^{x'}_{bR} \bar{q}^y_{bR} q^{y'}_{aL} \rangle_0 =& \Delta^{qq} \delta_{xx'} \delta_{yy'} + \Delta'^{qq} \delta_{xy'} \delta_{yx'} \; , \\
\Delta^{qq} =& \Delta^{qq^*} \; , \; \Delta'^{qq} = \Delta'^{qq^*} \; .
\end{aligned} \tag{13}$$

The vacuum of eqs. (12,13) is invariant under the action of the vector subgroup of G_F. * The true chiral vacuum, determined by minimizing the ground state energy, is parameterized by $U(4)$ matrices W and W' corresponding to chiral transformations of Q^r_a and Q'^r_a, and a $U(8)$ matrix associated with q^x_a. However, as pointed out in sect. 2, by assuming that the true vacuum is electrically neutral, the effective flavor symmetry is reduced to H_F, eq. (10). Furthermore, the

* The equalities $\Delta^{Q'} = \Delta^Q$ in (12) and $\Delta^{QQ} = \Delta^{Q'Q'}$, $\Delta^{Qq} = \Delta^{Q'q}$ in (13) follow from the $SU(24)_L \otimes SU(24)_R$ global invariance of the technicolor sector when color is neglected and are accurate up to small QCD corrections.

residual vector isospin symmetry of $\mathcal{H}'$ implies $W^{(\rho,u)} = W^{(\rho,d)}$ at the minimum of the vacuum energy (cf. eq. (2)). Thus

$$
\begin{aligned}
W &= W^R W^{L\dagger} = \delta_{ab} W_{rr'} = \delta_{ab} e^{i\phi}(w_0 + i\,\vec{w}\cdot\vec{\tau})_{rr'} \\
W' &= W'^R W'^{L\dagger} = \delta_{ab} W'_{rr'} = \delta_{ab} e^{i\phi'}(w'_0 + i\,\vec{w}'\cdot\vec{\tau})_{rr'}
\end{aligned}
\tag{14}
$$

In addition, with $\mathcal{H}'$ diagonal with respect to (q, q') and invariant under the GP transformation, eq. (11), charge conserving critical points correspond to independent rotations of q and q':

$$
\begin{aligned}
U &= U^R U^{L\dagger} = \delta_{ab} e^{i\chi}(u_0 + i\,\vec{u}\cdot\vec{\tau})_{rr'} \\
U' &= U'^R U'^{L\dagger} = \delta_{ab} e^{i\chi'}(u'_0 + i\,\vec{u}'\cdot\vec{\tau})_{rr'}
\end{aligned}
\tag{15}
$$

Combining eqs. (8), (13), (14) and (15) and using, from I, eqs. (I.17), (I.A.1) and (I.A.7), we obtain the vacuum energy

$$
\begin{aligned}
E(W, W', U, U') =& const. + w_\alpha \lambda^{Q}_{\alpha\beta} w_\beta + w'_\alpha \lambda^{Q'}_{\alpha\beta} w'_\beta \\
&+ 2h cos\,(\phi - \phi')\, w_\alpha \lambda^{QQ'}_{\alpha\beta}\, w'_\beta + 2g\Big[cos(\phi - \chi) w_\alpha \lambda^{Qq}_{\alpha\beta} u_\beta \\
&- h cos(\phi' - \chi) w'_\alpha\, \tilde{\lambda}^{Q'q}_{\alpha\beta}\, u_\beta + cos(\phi' - \chi') w'_\alpha \lambda^{Qq}_{\alpha\beta} u'_\beta \\
&- h cos(\phi - \chi) w_\alpha\, \lambda^{Q'q}_{\alpha\beta} u'_\beta\Big] + g^2\Big[u_\alpha \lambda^{q}_{\alpha\beta} u_\beta + u'_\alpha \lambda^{q'}_{\alpha\beta} u'_\beta\Big]
\end{aligned}
\tag{16}
$$

In this expression, $h = M^2_{ETC}/M'^2_{ETC} < 1$ and $g = \Delta^{Qq}/\Delta^{QQ} \sim 10^{-9}$. The λ -matrices are all of comparable magnitude and are given by

$$
\begin{aligned}
&\lambda^{Q} = 4\Delta^{QQ}\lambda^{I}\ ,\ \lambda^{Q'} = 4\Delta^{QQ}\lambda^{II}\ , \\
&g^2\lambda^{q} = 4\Delta^{qq}\lambda^{I}\ ,\ g^2\lambda^{q'} = 4\Delta^{qq}\lambda^{II}\ , \\
&h\lambda^{QQ'} = 4\Delta^{QQ'}\lambda^{10}\ ,\ hg\lambda^{Q'q} 4\Delta^{Qq}\lambda^{10}\ ,\ g\lambda^{Qq} = 4\Delta^{Qq}\mu_4^{-2}\, 1\ ,
\end{aligned}
\tag{17}
$$

where

$$
\lambda^{I} = \begin{pmatrix} Tr\mu_I^{-2} & 0 \\ 0 & (\mu_I^{-2})_{ij} \end{pmatrix} \quad ; \tag{18a}
$$

$$\lambda^{II} = \begin{pmatrix} Tr\mu_{II}^{-2} & 0 \\ 0 & (\mu_{II}^{-2})_{ij} \end{pmatrix} \tag{18b}$$

$$\lambda^{10} = \begin{pmatrix} \mu'^{-2}_{11} - \mu'^{-2}_{33} & 0 & 0 & 0 \\ 0 & \mu'^{-2}_{12} + \mu'^{-2}_{34} & 0 & 0 \\ 0 & 0 & -\mu'^{-2}_{12} + \mu'^{-2}_{34} & 2\mu'^{-2}_{14} \\ 0 & 0 & 2\mu'^{-2}_{13} & \mu'^{-2}_{11} - \mu'^{-2}_{22} \end{pmatrix} \tag{19}$$

Constant terms have been extracted from both λ^I and λ^{II} in (18a,b) (cf. eq. (I.36)). The variables w_α, w'_α, u_α and u'_α are real and satisfy $w_\alpha w_\alpha = w'_\alpha w'_\alpha = u_\alpha u_\alpha = u'_\alpha u'_\alpha = 1$. To avoid anomalous elements of G_F the phases must satisfy

$$exp\ 24i(\phi + \phi') = exp\ 24i(\chi + \chi') = 1$$

Equation (13) is, of course, the desired result: its minimization follows the program described in I. Strong CP conservation is assured since vacuum alignment takes place within a subgroup, H_F, with respect to which, $\mathcal{H}'$ is GP symmetric. Whether or not spontaneous CP violation occurs depends only on the matrices λ^I, λ^{II} and λ^{10} and on the sign of Δ^{QQ}, as discussed in sections I.3.3 and I.3.4.

Thus far, the following assumptions have been made about the breakdown of $SO(10)_{ETC}$:

(i) The vacuum which minimizes the effective potential conserves electric charge. This assumption is plausible, though its complete vindication would require explicitly minimizing E with respect to the full flavor symmetry, G_F. Still, we expect that for some range of values of λ^I, λ^{II} and λ^{10}, charge conservation is obtained. What is not clear is that this range is consistent with the assumptions which follow.

(ii) $\mu^2_{I,II} = \mu^2_{I,0} = \mu^2_{II,0} = \mu''^2 = 0$. As was mentioned, this is realized if $SO(10)_{ETC}$ is broken only by objects transforming as $(1,1)$ or $(1,3)+(3,1)$ under the $SU(2)_I \otimes SU(2)_{II}$ subgroup.

(iii) $\mathcal{H}'$ is CP and GP symmetric.

If spontaneous CP violation is to occur, two additional criteria must be met.

(iv) The largest eigenvalue of μ_I^2 is $(\mu_I^2)_{22}$ and the largest eigenvalue of μ_{II}^2 is not $(\mu_{II}^2)_{22}$, or vice-versa.

(v) $\Delta^{QQ} > 0$. Otherwise E is minimized by $w = w' = u = u' = 1$. This is not a constraint on ETC breakdown and, for purposes of this dicsussion, will simply be imposed. *

To demonstrate the feasibility and consistency of (i), (iii) and (iv), we resort to the Higgs mechanism to break $SO(10)_{ETC}$. The Higgs sector we'll introduce is not very aesthetic, even as these things go, and the pattern of vacuum expectation values is, unfortunately, not general. However, this scheme has the advantage of quickly and easily justifying (ii), (iii) and (iv).

Higgs scalars comprise four representations of $SO(10)_{ETC}$:

$$\xi \sim \underline{54}\ ;\ \phi \sim \underline{45}\ ;\phi' \sim \underline{45}\ ;\chi \sim \underline{126}\,,$$

and, under $SU(4)_{TC} \otimes SU(2)_I \otimes SU(2)_{II}$,

$$\langle\xi\rangle_0 \sim (1,1,1)\ ;\ \langle\phi\rangle_0 \sim \langle\phi'\rangle_0 \sim (1,3,1)+(1,1,3)\ ;\ \langle\chi\rangle_0 \sim (10,3,1)+(10^*,1,3)\ .$$

$\langle\xi\rangle_0$ is of the order of M'_{ETC} and breaks $SO(10)_{ETC}$ to $SU(4)_{TC} \otimes SU(2)_I \otimes SU(2)_{II}$. It produces

$$(\mu'^2)_{\kappa\lambda} \propto M'^2_{ETC}\delta_{\kappa\lambda}$$

$\langle\phi\rangle_0$, $\langle\phi'\rangle_0$ and $\langle\chi\rangle_0$ are order M^2_{ETC}. $\langle\chi\rangle_0$ must transform as the $SU(3)$ singlet found in the $\underline{10}$ and $\underline{10^*}$ of $SU(4)$, and gives $\mu_0^2 \sim \mu_4^2 \sim M^2_{ETC}$. Now, if the

---

* Calculation of Δ^{QQ} is a strong interaction problem, however it looks suspiciously like the positive definite square of a mass operator and in simple approximations this impression is affirmed. On the other hand, in ref. 8, the author suggests that $\Delta^{\rho\sigma} < 0$ but states that a proof has not been found. If this is true, a vector model will not generate CP violation, though a non-vectorial model of G_{ETC} can.[10] Of course, G_{ETC} must be non-vectorial for unrelated reasons, i.e., to produce up-down mass splittings.

triplets under $SU(2)_I \otimes SU(2)_{II}$ are denoted by

$$\langle\phi\rangle_0 : \vec{v} \sim (1,3,1)\,,\ \vec{w} \sim (1,1,3)\ ;$$

$$\langle\phi'\rangle_0 : \vec{v}' \sim (1,3,1)\,,\ \vec{w}' \sim (1,1,3)\ ;$$

$$\langle\chi\rangle_0 : \vec{v}'' \sim (10,3,1)\,,\ \vec{w}'' \sim (10^*,1,3)\ ;$$

then each $(3,1)+(1,3)$ contributes, for example,

$$(\mu_I^2)_{ij} \propto \vec{v}^2\,\delta_{ij} - v_i v_j\ ,\ (\mu_{II}^2) \propto \vec{w}^2\,\delta_{ij} - w_i w_j\ ,$$

$$\mu'^2 \propto -(\vec{v}^2+\vec{w}^2)1 - 2\begin{pmatrix} v_3w_3 & v_-w_- & v_3w_- & w_3v_- \\ v_+w_+ & v_3w_3 & -w_3v_+ & -v_3w_+ \\ v_3w_+ & w_3v_- & -v_3w_3 & v_-v_+ \\ w_3v_+ & -v_3w_- & v_+w_- & -v_3w_3 \end{pmatrix} = (\mu'^2)^\dagger\ ,$$

where $v_\pm = v_1 \pm iv_2$, $w_\pm = w_1 \pm iw_2$. By choosing

$$\vec{v} = (v_1, 0, v_3)\ ,\ \vec{w} = (w_1, 0, w_3)\ ,\ \vec{v}' = (0, v_2', 0)\ ,$$

$$\vec{w}' = (0, w_2', 0)\ ,\ \vec{v}'' = (0,0,v_3'')\ ,\ \vec{w}'' = (0,0,w_3'') \qquad (20)$$

we readily obtain *CP* and *GP* conservation. For μ_I^2 and μ_{II}^2 we find

$$\mu_I^2 \propto \begin{pmatrix} v_1^2+v_3^2+v_3''^2 & 0 & 0 \\ 0 & v_3^2+v_2'^2+v_3''^2 & -v_1v_3 \\ 0 & -v_1v_3 & v_1^2+v_2'^2 \end{pmatrix}\ ,$$

$$\mu_{II}^2 \propto \begin{pmatrix} w_1^2+w_3^2+w_3''^2 & 0 & 0 \\ 0 & w_3^2+w_2'^2+w_3''^2 & -w_1w_3 \\ 0 & -w_1w_3 & w_1^2+w_2'^2 \end{pmatrix}$$

in the (τ^2, τ^1, τ^3) basis. A straightforward computation of eigenvalues shows that v_2' and w_2' are easily adjusted to meet criterion (iv), above. Specifically, *CP* is spontaneously violated when

$$v_2'^2 \mathop{}^{>}_{<} \frac{1}{2}(v_1^2+v_3^2+v_3''^2) - \frac{1}{2}\sqrt{(v_1^2+v_3^2+v_3''^2)^2 - 4v_1^2v_3''^2}$$

and

$$w_2'^2 \lessgtr \frac{1}{2}(w_1^2 + w_3^2 + w_3''^2) - \frac{1}{2}\sqrt{(w_1^2 + w_3^2 + w_3''^2)^2 - 4w_1^2 w_3''^2} \tag{21}$$

provided $\Delta^{QQ} > 0$.

This result suggests that spontaneous CP violation occurs for a sizable range of vacuum expectation values. However, it must be pointed out that, although the $VEV's$ $\vec{v}$, $\vec{w}$, $\vec{v}''$ and $\vec{w}''$ may always be brought into the forms of eq. (20), our choice of $v_1' = v_3' = w_1' = w_3' = 0$ is very special. We conclude that our ansatz is not to be taken too seriously; in particular, the constraint (ii), introduced here to simplify exposition, is far too strong.

4. A PROBLEM WITH TECHNIQUARKS

When CP is spontaneously broken, strong CP violation is typically signalled by the appearance of an anti-hermitian component in the quark mass matrix. Putting the matrix in real diagonal form then requires an anomalous axial $U(1)$ transformation which induces a change in the Lagrangian

$$\delta \mathcal{L} = -i\theta_{eff}\frac{g_c^2}{16\pi^2} TrF_c \cdot \tilde{F}_c \ , \ \theta_{eff} = arg\ det\ M_{quark} \ ,$$

where F_c is the color field strength tensor and $\tilde{F}_c$ its dual. If the model contains colored technifermions, i.e., techniquarks, evidently θ_{eff} has an additional component when their mass matrix is not hermitian.

Now, the light quark mass matrix is readily identified in the effective low energy theory obtained after integration over heavy technicolor and broken ETC degrees of freedom. It is hermitian to a part in 10^9 provided the matrix

$$\sum_{\sigma} \sum_{rr'ss'} \sum_{m} \Delta^{q\sigma,m} \Gamma^{q\sigma,m}_{rr'ss'} W^{(q)}_{r't} W^{(\sigma)\dagger}_{s's} \equiv \Delta^q \mathcal{M}^q_{rt}$$

is hermitian.[7] This is the statement of eqs. (I.6) and (I.7).

However, though the corresponding techniquark operator, $\mathcal{M}^Q$, will generally be hermitian whenever $\mathcal{M}^q$ is [7,10,11], its connection to θ_{eff} is not as straightforward. In fact, there is no reason to believe that techniquarks renormalize

θ_{eff} primarily through this operator. Thus, in a model with techniquarks, the Eichten, Lane, Preskill criterion ($\mathcal{M}^{\rho} = \mathcal{M}^{\rho\dagger}$) appears to be insufficient for predicting the fate of strong CP conservation.

In this section, we investigate the techniquark contribution to θ_{eff} in the context of models described by eq. (1) (and by straightforward extension, those of sect. 2). Our results suggest that suppression of this contribution requires an additional constraint on the effective ETC Hamiltonian beyond those invoked in I.

For simplicity, we consider a model with no technileptons, i.e., color singlet technifermions. This restriction will in no way affect our conclusions. We will also ignore effects at the level of a part in 10^9. Explicitly, then, the CP violating technifermion operator which renormalizes θ_{eff} is just the four-techniquark terms of $\mathcal{H}'$:

$$\mathcal{H}'_Q = \sum_{\rho\sigma} \sum_{\substack{rr'ss' \\ tt'=1}}^{2} \Gamma^{\rho\sigma}_{rtt's'} W^{(\rho)}_{tr'} W^{(\sigma)\dagger}_{st'} \bar{Q}^{(\rho)r}_L Q^{(\rho)r'}_R \bar{Q}^{(\sigma)s}_R Q^{(\sigma)s'}_L$$

where color and technicolor indices have been suppressed, and the sum is over inequivalent, non-trivial representations, ρ, σ, of $G_{TC} \otimes G_C$. CP violating phases reside in the matrices $W^{(\rho)}$. On dimensional grounds, we expect the leading contribution to θ_{eff} from this operator to be of order (assuming phases are $O(1)$)

$$g^2_{ETC} \frac{\langle \bar{Q} Q \rangle_0}{M^2_{ETC}} \frac{1}{\Lambda_{TC}} \simeq \frac{m_q}{\Lambda_{TC}} \simeq 10^{-3}$$

which is, clearly, unacceptably large.

A model calculation of θ_{eff} can be done, based on the single ETC boson exchange contribution to $\mathcal{H}'_Q$ and the vacuum graphs in fig. 1. If the double solid lines in fig. 1 represent exact fermion propagators in the presence of color instantons, * these vacuum amplitudes include a term proportional to $TrF_c \cdot \tilde{F}_c$. The coefficient vanishes in perturbation theory since $TrF_c \cdot \tilde{F}_c$ is a total divergence,

* Or any other effect which gives rise to a non-vanishing $F_c \cdot \tilde{F}_c$.

but can be extracted in the dilute gas approximation by expanding the fermion propagators in powers of the field strength.[12,13]

The massive *ETC* boson exchanged in fig. (1) couples to broken currents

$$J^{a}_{L(R)\mu} = \sum_{\rho\sigma}\sum_{rr'}\sum_{ij} \bar{Q}^{(\rho)r}_{iL(R)} T^{L(R)a}_{\rho ri,\sigma r'j} \gamma_\mu Q^{(\sigma)r'}_{jL(R)} + \cdots \tag{22}$$

where i, j are color indices and technicolor indices have been suppressed. Omitted in this expression are possible additional terms which, however, do not contribute to $\mathcal{H}'_Q$. For the generators we write

$$T^{L(R)a}_{\rho ri,\sigma r'j} = \sum_{\alpha=0}^{3} \left(T^{L(R)a}_{\rho i,\sigma j}\right)_\alpha \tau^{\alpha}_{rr'} \ .$$

Now, the alignment of the chiral, technicolor vacuum with respect to the currents, (22), is specified by a set of matrices. $W^{(\rho)} = W^{R(\rho)} W^{L(\rho)\dagger}$ which, presumably, harbor *CP* violating phases. We introduce this *CP* violation into the graphs of fig. 1 by choosing $W^{L(\rho)} = 1$ and rotating the currents:

$$\begin{aligned} &J^{a}_{L\mu} \to J^{a}_{L\mu} \\ &J^{a}_{R\mu} \to \sum_{\rho\sigma}\sum_{rr'}\sum_{ij}\sum_{\alpha,\beta} \bar{Q}^{(\rho)r}_{iR} \left(T^{Ra}_{\rho i,\sigma j}\right)_\alpha \Phi^{\rho\sigma}_{\alpha\beta} \tau^{\beta}_{rr'} \gamma_\mu Q^{(\sigma)r}_{jR} \ , \end{aligned} \tag{23}$$

where

$$\Phi^{\rho\sigma}_{\alpha\beta} = \frac{1}{2} Tr(W^{(\rho)\dagger}\tau^\alpha W^{(\sigma)}\tau^\beta) \ ; \ \Phi^{\rho\sigma *} = \Phi^{\sigma\rho} \ ; \ \Phi^{\rho\sigma\dagger}\Phi^{\rho\sigma} = 1 \ . \tag{24}$$

Having introduced the chiral rotations, $W^{(\rho)}$, into the broken *ETC* currents (i.e., into $\mathcal{H}'_Q$), the appropriate fermion propagator is that obtained in the vacuum

$$\langle \bar{Q}^{(\rho)r}_{iL} Q^{(\sigma)r'}_{jR} \rangle_0 = \delta_{ij}\delta^{\rho\sigma}\delta^{rr'}\Delta^\rho \ . \tag{25}$$

When the techniquarks carry momenta $\leq \Lambda_{TC}$ this condensate induces a mass term in the propagator. Above Λ_{TC}, though, the propagator is γ_5 -even (there is no bare techniquark mass) and the matrices $W^{(\rho)}$ can be rotated away. Thus, the loop integrals for fig. 1 are naturally cut off.

To perform this calculation, however, we introduce an explicit "hard" mass into the techniquark propagators,

$$(m_\rho)_{rr'} = m_\rho \delta_{rr'}$$

where $m_\rho = O(\Lambda_{TC})$; the flavor structure follows from eq. (25) and the fact that we work to lowest order in g^2_{ETC}. Loop integrals will be cut off with a Pauli-Villars regulator. The result contains a logarithmic divergence associated with the techniquark self-energy, which we ignore in view of the natural cut-off.

Using the vertices defined by eqs. (22), (23) and (24), the CP violating amplitude, $\mathcal{M}_{\rho\sigma}$, is

$$\begin{aligned} \mathcal{M}_{\rho\sigma} =& g^2_{ETC} \tau^{\alpha}_{rs'} \tau^{\beta}_{sr'} \Big[\left(V^a_{\rho i,\sigma \ell}\right)_\alpha \left(A^b_{\sigma k,\rho j}\right)_\beta \\ & Tr\ S_F^{\rho ri,\rho r' j}(x-y)\gamma_\mu S_F^{\sigma sk,\sigma s' \ell}(y-x)\gamma_\nu\gamma_5 D^{\mu\nu}_{ab}(x-y) \\ & + \left(A^a_{\rho i,\sigma \ell}\right)_\alpha \left(V^b_{\sigma k,\rho j}\right)_\beta \ Tr\ S_F^{\rho ri,\rho r' j}(x-y) \\ & \gamma_\mu\gamma_5 S_F^{\sigma sk,\sigma s' \ell}(y-x)\gamma_\nu D^{\mu\nu}_{ab}(x-y) \Big] \end{aligned} \tag{26}$$

Here and below, all repeated indices, except ρ and σ, are summed. The trace is over Dirac indices and space-time parameters and

$$V^a_\alpha \ , \ A^a_\alpha = \frac{T^{La}_\alpha \pm (T^{Ra}\Phi)_\alpha}{2}$$

$D^{\mu\nu}_{ab}$ is the massive ETC boson propagator in 't Hooft-Feynman gauge:

$$D^{\mu\nu}_{ab} = -g^{\mu\nu}\left(\frac{1}{k^2 - M^2}\right)_{ab} .$$

Lastly,

$$iS_F^{\rho ri,\rho' r' j}(x-y) = \delta^{\rho\rho'}\delta^{rr'}\langle x \left| \left(\frac{i}{i\not{D} - m_\rho}\right)_{ij} \right| y\rangle \ ;$$

$$\not{D}_{ij} = \gamma_\mu(\partial^\mu - ig_c A_c^\mu)_{ij} \tag{27}$$

is the exact fermion propagator. From (26), (27) and well known properties of the Dirac matrices, we obtain

$$\begin{aligned}
\mathcal{M}_{\rho\sigma} = &- g_{ETC}^2 m_\rho m_\sigma \\
&\cdot \left[\left(T^{La}_{\rho i,\sigma\ell} \right)_\alpha \left(T^{Rb}_{\sigma k,\rho j} \Phi^{\rho\sigma *} \right)_\alpha - \left(T^{Ra}_{\rho i,\sigma\ell} \Phi^{\rho\sigma} \right)_\alpha \left(T^{Lb}_{\sigma k,\rho j} \right)_\alpha \right] \\
&\cdot Tr\gamma_5 \langle x \left| (\not{D}^2 + m_\rho^2)^{-1}_{ij} \right| y \rangle \gamma_\mu \langle y \left| (\not{D}^2 + m_\sigma^2)^{-1}_{k\ell} \right| x \rangle \gamma_\nu D^{\mu\nu}_{ab}(x-y)
\end{aligned}$$

where we have dropped a term which is independent of $\Phi^{\rho\sigma}$.

To proceed, we expand $(\not{D}^2 + m_\rho^2)^{-1}$ to lowest non-vanishing order in the ratio of instanton size to m_ρ. After a bit more algebra (cf. fig. 2)

$$\begin{aligned}
\mathcal{M}_{\rho\sigma} = &g_{ETC}^2 g_c^2 m_\rho m_\sigma \left[T^{La}_{\rho i,\sigma\ell} \cdot T^{Rb}_{\sigma k,\rho j} \Phi^{\rho\sigma *} - T^{Ra}_{\rho i,\sigma\ell} \Phi^{\rho\sigma} \cdot T^{Lb}_{\sigma k,\rho j} \right] \\
&\cdot 8ig_{\mu\nu}\epsilon_{\gamma\delta\gamma'\delta'} Tr \Big[\langle x \left| (\partial^2 + m_\rho^2)^{-1} F^{\gamma\delta}_{cii'} (\partial^2 + m_\rho^2)^{-1} F^{\gamma'\delta'}_{ci'j} (\partial^2 + m_\rho^2)^{-1} \right| y \rangle \\
&\cdot \langle y \left| (\partial^2 + m_\sigma^2)^{-1} \delta_{k\ell} \right| x \rangle D^{\mu\nu}_{ab}(x-y) - \langle x \left| (\partial^2 + m_\rho^2)^{-1} \delta_{ij} \right| y \rangle \\
&\cdot \langle y \left| (\partial^2 + m_\sigma^2)^{-1} F^{\gamma\delta}_{ckk'} (\partial^2 + m_\sigma^2)^{-1} F^{\gamma'\delta'}_{ck'\ell} (\partial^2 + m_\sigma^2)^{-1} \right| x \rangle D^{\mu\nu}_{ab}(x-y) \Big]
\end{aligned} \tag{28}$$

The space-time integrals are carried out in ref. [13] with the result

$$\begin{aligned}
\mathcal{M}_{\rho\sigma} = &\frac{g_c^2}{16\pi^2} \frac{4g_{ETC}^2}{\pi^2} m_Q^2 \left(\frac{1}{\bar{M}^2} \right)_{ab} \\
&\cdot \left(T^{La}_{\rho i,\sigma\ell} \cdot T^{Rb}_{\sigma k,\rho j} \Phi^{\rho\sigma *} - T^{Ra}_{\rho i,\sigma\ell} \Phi^{\rho\sigma} \cdot T^{Lb}_{\sigma k,\rho j} \right) \\
&\cdot \left[(F_c \cdot \tilde{F}_c)_{ij} \delta_{k\ell} - (F_c \cdot \tilde{F}_c)_{k\ell} \delta_{ij} \right]
\end{aligned}$$

where

$$\left(\frac{1}{\bar{M}^2} \right)_{ab} \equiv \left(\frac{-1}{m_Q^2} \ell n \frac{M^2}{\Lambda^2} + \frac{2}{M^2} \ell n \frac{m_Q^2}{M^2} \right)_{ab}$$

and Λ is a Pauli-Villars mass. For convenience, we have set $m_\rho = m_\sigma = m_Q$ which, though by no means general, will not alter our conclusions.

Now recall, to lowest order in g^2_{ETC}, we had (cf. eqs. (I.2), (I.3), (I.17))

$$g^2_{ETC}\left(\frac{1}{M^2}\right)_{ab}\left(T^{La}_{\rho i,\sigma\ell}\right)_\alpha\left(T^{Ra}_{\sigma k,\rho j}\right)_\beta = \frac{1}{2}\sum_m \Gamma^{\rho\sigma,m}_{\alpha\beta}\, t^{\rho\sigma,m}_{ijk\ell} \tag{29}$$

where the $t^{\rho\sigma,m}_{ijk\ell}$ are $SU(3)_c$ singlets (again, technicolor is suppressed). In the same way, we can write

$$g^2_{ETC}\left(\frac{1}{\bar{M}^2}\right)_{ab}\left(T^{La}_{\rho i,\sigma\ell}\right)_\alpha\left(T^{Rb}_{\sigma k,\rho j}\right)_\beta = \frac{1}{2}\sum_m \bar{\Gamma}^{\rho\sigma,m}_{\alpha\beta}\, t^{\rho\sigma,m}_{ijk\ell}$$

It is easy to verify, using eq. (I.A.4) that $\bar{\Gamma}^{\rho\sigma,m} = \bar{\Gamma}^{\sigma\rho m,*}$. Thus, we find *

$$\begin{aligned} \mathcal{M}_{\rho\sigma} &= \frac{g_c^2}{16\pi^2}\left[\frac{4m_Q^2}{\pi^2}\left(r^{(\rho)} - r^{(\sigma)}\right)\sum_m i\ Im\ Tr\left(\bar{\Gamma}^{\rho\sigma,m}\,\Phi^{\rho\sigma *}\right)\right] Tr\ F_c\cdot\tilde{F}_c \\ &\equiv \frac{ig_c^2}{16\pi^2}\theta^{\rho\sigma} Tr F_c\cdot\tilde{F}_c \end{aligned} \tag{30}$$

where $r^{(\rho)}$ is the color dimensionality of $G_{TC}\otimes G_C$ representation ρ and $\theta^{\rho\sigma}$ is the contribution to θ_{eff}.

This result requires elaboration. First, and most obvious, $\theta^{\rho\sigma} = 0$ for $\rho = \sigma$. Equation (30) also implies $\theta^{\rho\sigma} = 0$ when ρ and σ are complex conjugate representations of color; in fact this is only true when $m_\rho = m_\sigma$ as we have assumed. Otherwise $r^{(\rho)}$ and $r^{(\sigma)}$ do not combine as simply as they do here. (In the model of sect. 3, eq. (12) implies $m_Q = m_{Q'}$, thus $\theta_{eff} = 0$ in this calculation.) Of course, for $\rho \neq \sigma, \sigma^*$, $\theta^{\rho\sigma}$ is generally non-vanishing and

$$\theta_{eff} = \sum_{\rho\neq\sigma}\theta^{\rho\sigma} = O\left(\frac{m_Q^2}{M^2}\,\ell n\,\frac{m_Q^2}{M^2}\right) .$$

There is one case in which the techniquark component of θ_{eff} vanishes trivially. This is when $\Gamma^{FF',m}_{\alpha\beta}$ is both GP symmetric and hermitian for all technifermion F and F'. Then it's not difficult to demonstrate that, not only $\bar{\Gamma}^{FF',m}$, but $\Phi^{FF'}$, is hermitian as well. Thus $Im\ Tr\ \bar{\Gamma}^{FF',m}\,\Phi^{FF'} \equiv 0$ and $\theta^{FF'}$ vanishes.

* Up to technicolor multiplicity factors.

We conclude that, in general, techniquarks are a dangerous source of strong CP violation, even when the Eichten, Lane, Preskill criterion (arg det $m_{quark} \simeq 10^{-9}$) is satisfied. Neutralization of this effect requires an additional restriction on the tensors $\Gamma^{\rho\sigma,m}$ appearing in the effective Hamiltonian. At the level of our investigation it's unclear how reasonable this added constraint is (though it's realized rather effortlessly in the model of sect. 3).

Though we will not go into details here,[10] it is interesting to note that when the matrices $\Gamma^{FF'}_{\alpha\beta}$ are both hermitian and GP invariant, it becomes possible to obtain arg det $m_q = O(10^{-9})$ without requiring that $\Gamma^{Fq}_{\alpha\beta}$ be GP symmetric. That is, GP symmetry can be broken by the quark mass matrix without its developing a large anti-hermitian part. This reopens the possibility, previously foreclosed by GP invariance (cf. eq. (I.16)), of quark mass splittings and Cabbibo mixing.

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FIGURE CAPTIONS

1. Diagrams contributing in lowest order in g^2_{ETC}/M^2_{ETC} to the strong *CP* violating phase θ_{eff}. Double solid lines represent exact techniquark propagators in the presence of color instantons. The exchanged gauge particle is a massive *ETC* vector boson.

2. Dilute gas approximation to fig. 1, corresponding to eq. (28). The fermion line is a free particle propagator and "x" represents the vertex $g_c\sigma_{\mu\nu}F_c^{\mu\nu}$.

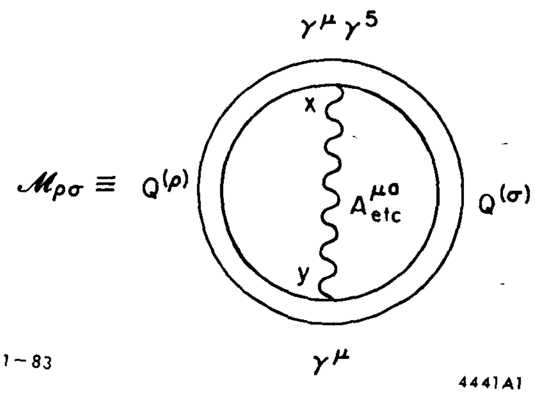
$\gamma^\mu \gamma^5$
$\mathcal{M}_{\rho\sigma} \equiv$
$Q^{(\rho)}$
x
$A^{\mu a}_{etc}$
y
$Q^{(\sigma)}$
1-83
γ^μ
4441A1

Fig. 1

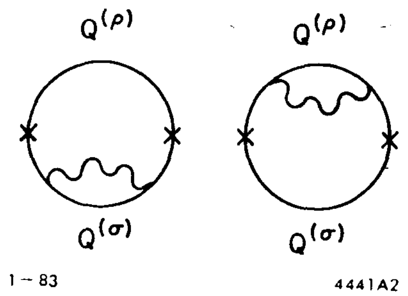

Fig. 2