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# STORAGE RING DESIGN OPTIMIZATION FOR FEL OPERATION*

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An electron beam in a storage ring seems to be a perfect driver for a free electron laser. The understanding of storage rings is very well advanced to project designs with high peak current, small beam sizes and short damping times. Since the beam is recycled after passage through the optical cavity we can expect a relatively high overall efficiency. On the other hand, however the fact that we do want the beam to circulate and interact with the laser field every turn puts a limit on the allowable perturbation of the beam due to the laser field. This limits the peak laser power we can get from a storage ring beam. The main advantage of a storage ring for FEL operation therefore is realized for continuous high power laser generation.

In this paper we will discuss some of the limitations in storage ring beam parameter as they pertain to FEL operation. The interaction of a FEL field with the beam, however, is not discussed here.

## I. BASIC PHYSICS OF STORAGE RINGS

Before we discuss limitations we will briefly review some of the basic processes of particle dynamics in a storage ring.

A string of bending magnets aligned on a closed loop establishes an ideal design orbit as the reference orbit against which lateral positions of all particles are measured. An additional set of quadrupole magnets interspersed between the bending magnets then produce stable orbits defined by

$$u_o(s) = \eta(s) \cdot \frac{\Delta E}{E_o} \tag{1}$$

This means for every energy $E = E_o + \Delta E$ there is a different orbit and the scaling function $\eta(s)$ is called the DISPERSION FUNCTION, since it determines at every point s the lateral dispersion due to the energy spread in the beam. The form of the dispersion function is determined by the actual position and strength of the quadrupoles and bending magnets. It is however always periodic

$$\eta(s) = \eta(s + C) \tag{2}$$

where C is the circumference of the storage ring. The orbit function $u_o(s)$ can be in the horizontal ($u_o = x_o$) or vertical ($u_o = y_o$) plane depending on whether we use the horizontal or vertical dispersion function. Since $\eta(s)$ depends on the bending fields we obviously have $\eta_y(s) = y_o(s) \equiv 0$ if the storage ring does not involve vertical bending magnets.

In addition to the lateral equilibrium positions defined by the stable orbit we also need a reference position for the longitudinal coordinate. A stable position on the longitudinal coordinate is established by the rf system used to compensate the energy loss of the particles due to synchrotron radiation and interaction with a laser field. The rf-frequency and the revolution frequency have to have an integer ratio

$$f_{rf} = k \cdot f_{rev} \qquad k\text{: integer} \tag{3}$$

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In this case the ideal particle circulating around the storage ring on the ideal design orbit will always arrive at the accelerating cavities at the same rf-phase.

Betatron and Synchrotron Oscillation

Individual particles will not in general have the ideal energy $E_o$, or ideal longitudinal position $s_o(t)$ relative to the rf-field, or travel exactly on and along the ideal orbits $x_o$ and $y_o$. This is acceptable as long as the particles stay close to or oscillate about the equilibrium position:

$$P_o(s,t) = [x_o(s),\ y_o(s),\ E_o, s_o(t)] \qquad (4)$$

Transverse oscillations about $[x_o(s), y_o(s)]$ are called BETATRON OSCILLATIONS and longitudinal oscillation about $[E_o,\ s_o(t)]$ are called SYNCHROTRON OSCILLATIONS.

The restoring force for the betatron oscillations are provided by quadrupole magnets which act as focusing elements for charged particle beams in a similar way as optical lenses do for light beams. The deviation $u_\beta(s)$ of a particular particle from the ideal orbit $u_o(s)$ ($u_\beta$ or $u_o$ stands for $x_\beta$ or $y_\beta$ and $x_o$ or $y_o$ respectively) is described by

$$u_\beta(s) = a\sqrt{\beta(s)}\ \cos[\psi(s) + \delta] \qquad (5)$$

Here $\beta(s)$ is the so called BETATRON FUNCTION which depends only on the quadrupole strengths and positions and is periodic $\beta(s) = \beta(s+C)$ like the dispersion function. The phase $\psi(s)$ is the BETATRON PHASE and is defined as $\psi(s) = \int_o^s d\tau/\beta(\tau)$. The quantity $\nu = \psi(C)/2\pi$ is called the TUNE or the OPERATING POINT of the storage ring and its value is equal to the number of lateral betatron oscillations per tune. The quantities a and $\delta$ are arbitrary integration constant and are different for every particle. From eq. (5) we find for the envelope of an ensemble of many particles

$$E(s) = a_{max}\ \sqrt{\beta(s)} \qquad (6)$$

The quantity $a^2_{max}$ of a beam is called the BEAM EMITTANCE.

In the longitudinal direction it is the accelerating rf-field together with certain lattice properties which provides the restoring force. The oscillations caused by this force are described by

$$\begin{aligned} \tau &= b\ \sin(\omega_s t + \rho) \\ \Delta E &= c\ \cos(\omega_s t + \sigma) \end{aligned} \qquad (7)$$

where b, c, $\rho$, $\sigma$ again are integration constants, $\omega_s$ is the synchrotron frequency, and $\tau$ is the longitudinal deviation of a particle from the center of the bunch.

The equations for the betatron (eq. 5) and synchrotron (eq. 7) oscillations are stable for all amplitudes which obviously is not true in a real environment. In a well designed storage ring we find the maximum betatron amplitude to be limited by the vacuum chamber and the maximum synchrotron amplitudes limited by the maximum voltage in the accelerating cavities.

Beam Sizes

The actual amplitudes of an electron beam in a storage ring are determined by the quantized emission of synchrotron radiation photons and a damping effect for all oscillation amplitudes. Both quantum excitation and damping lead to an equilibrium where all particles of a beam have gaussian distribution in all 6 phase space coordinates:

$$x,\ x',\ y,\ y',\ \Delta E,\ \Delta s$$

For such a beam we define:

$$\begin{aligned} \sigma_x^2/\beta_x &= \epsilon_x && \text{horizontal beam emittance} \\ \sigma_y^2/\beta_y &= \epsilon_y && \text{vertical beam emittance} \\ \sigma_E/E_o & && \text{energy spread} \\ \sigma_s & && \text{bunch length} \end{aligned} \tag{8}$$

What happens if we clip the gaussian tails due to limitations in the vacuum chamber of rf-voltage? Obviously we loose the particles in those tails but there is also a continuous loss since those tails are being refilled again. These losses will be one determining factor for the beam lifetime. This lifetime called quantum lifetime can be calculated and is given by /1/:

$$\tau_q = 1/2\ \tau_D\ e^{\xi}/\xi \tag{9}$$

where $\tau_D$ is the damping time and $\xi = 1/2\ n^2$ with n being the aperture limit in units of $\sigma_x$, $\sigma_y$, $\sigma_E$ or $\sigma_s$. Typically one needs an aperture which allows oscillation amplitudes of at least seven sigma's:

$$n \gtrsim 7 \tag{10}$$

in order to get a quantum lifetime of 50 hours or more.

Damping

In the previous sections we have made use of a damping effect in an electron storage ring. The physics behind the damping of synchrotron oscillations is simple, since the energy loss per turn $\Delta U$ due to synchrotron radiation depends on the particle energy $\Delta U \sim E^4$. Therefore a particle with too high an energy radiates more and a particle with too low an energy radiates less than the ideal particle. In either case the energy deviation is reduced or damped. Because energy and the longitudinal position of a particle are conjugate coordinates we also have a damping in the longitudinal coordinate.

The transverse oscillations are damped because the emission of a photon in general means a loss of momentum in the longitudinal as well as in the transverse direction. However, in the rf-accelerating cavity the lost momentum is compensated only in the longitudinal direction. We have therefore a net loss in transverse momentum or damping in the transverse plane.

The damping times in a separated function lattice (focusing and bending is done in separate magnets) are given by /1/:

$$\begin{aligned} \tau_E &= \tau_s = E\ T_{rev}/\Delta U \\ \tau_x &= \tau_y = 2E\ T_{rev}/\Delta U \end{aligned} \tag{11}$$

Where

$$\Delta U(\text{GeV}) = 8.85\cdot 10^{-5} E^4(\text{GeV}^4)/\rho(\text{m}) \tag{12}$$

is the energy loss per turn and $\rho$ the bending radius.

The damping times are changed if we have magnets where focusing and bending occurs in the same magnet. One example is the wiggler magnet needed for the gain expanded FEL proposed by MADEY /6/. Here we have to modify the damping times like:

$$\begin{aligned} \tau_E' &\rightarrow \tau_E\ 2/(2 + \mathcal{D}) \\ \tau_x &\rightarrow \tau_x/(1 - \mathcal{D}) \\ \tau_y &\rightarrow \tau_y \end{aligned} \tag{13}$$

where

$$\mathcal{D} = \oint \eta \left(\frac{1}{\rho^3} + 2k \frac{1}{\rho}\right) ds \Big/ \oint 1/\rho^2 ds \qquad (14)$$

and k is the quadrupole strength defined by $k \cdot \rho = (\partial B_y/\partial x)B_y$.

Obviously if $\mathcal{D} = 1$ we have lost all damping in the horizontal plane which is unacceptable for a beam to be stored for a long time. Conversely if $\mathcal{D} \leq -2$ we have lost the longitudinal damping. In praxis we can allow the quantity $\mathcal{D}$ to vary like

$$-1 \lesssim \mathcal{D} \lesssim 0.5 \qquad (15)$$

For presently assumed parameters for a gain expanded FEL one can easily stay within these limits.

It is, however, not possible to simplify the ring lattice by using combined function magnets only, like in synchrotrons. Horizontal antidamping would prevent a stable beam.

Beam Emittance

The beam emittance is determined by the balancing of damping and quantum excitation. In an earlier section we found that a proper storage ring lattice establishes a unique orbit for every energy a particle may have. We also found that each particle performs betatron oscillations about its orbit. Now the physics of quantum excitation becomes clear. The moment a particle emits a photon, it losses energy and therefore instantly starts performing betatron oscillations about a different orbit. The betatron oscillation amplitude therefore is changed instantly which leads to an increased beam emittance. We can determine the effect of the quantum emission on the beam emittance by the way the focusing lattice is chosen. Specifically we can minimize this effect which results in a minimum beam emittance for a socalled FODO lattice given by /2/:

$$\varepsilon_{xo}(\text{rad m}) = \sigma_x^2/\beta_x = 1.9\cdot10^{-6}E^2(\text{GeV}^2)\cdot\theta^3(\text{rad})R(\text{m})/\rho(\text{m}) \qquad (16)$$

Here $\theta$ is the bending angle between quadrupoles, R the average radius and $\rho$ the bending radius of the storage ring. It is obvious that a small bending angle reduces the beam emittance at the cost of more magnets and a larger beam circumference. In eq. (11) we have assumed a flat storage ring. The vertical beam emittance in such a ring is determine only by coupling of horizontal betatron oscillations into the vertical plane due to rotated or misaligned quadrupoles. The coupling is determined by a coupling constant defined by:

$$\begin{aligned} \varepsilon_x &= \varepsilon_{xo}/(1 + K^2) \\ \varepsilon_y &= \varepsilon_{yo}K^2/(1 + K^2) \qquad \text{or} \\ K^2 &= \varepsilon_y/\varepsilon_x \end{aligned} \qquad (17)$$

In a well aligned storage ring the coupling can be as small as

$$K_{min} \approx .05 \text{ to } .10 \qquad (18)$$

## II. BEAM CURRENT DENSITY LIMITATIONS

In this section we will discuss the major limitations on beam current density. The beam current limitations are discussed in another paper at this conference /3/. To relate the limitations to real storage ring designs we have compiled in Table I some parameters for two storage rings. One of these storage rings (Ring I in Table I) has been specifically designed for FEL operation. The other example is a design by the author for a damping ring to be used to produce an electron pulse with an extremely small emittance at a high repetition rate. Here the state of the art has been pushed to its limits to minimize both emittance and damping time. This damping ring is being constructed now at the Stanford Linear Accelerator Center and will be ready for testing during spring 1983.

Table I
Storage Ring Parameter

| Storage Ring | | I FEL-Storage Ring | II Damping Ring |
|---|---|---|---|
| Energy | E(GeV) | 1.0 | 1.0 |
| Circumference | C(m) | 94.56 | 35.27 |
| Damping Time | $\tau_E$(msec) | 6.77 | 2.7 |
| Beam Emittance | $\epsilon_{xo}$(mm-mrad) | .017 | .013 |
| FODO-Cell Length | $L_c$(m) | 1.60 | 1.29 |
| Bending Angle/Magnet | $\theta$ (deg) | 9.0 | 9.0 |
| Peak Beam Current | $\hat{I}$ (amp) | 270 | 160 |
| Touschek Lifetime (fully coupled beam) | $\tau_t$(h) | 1.0 | 0.14 |

Touschek Effect /4,5/

When two particles performing transverse betatron oscillations collide some of their transverse momentum is transformed into a longitudinal momentum change. One particle would gain and the other particle would lose an equivalent amount of longitudinal momentum. Both particles are lost if the momentum change is larger than the momentum acceptance of the storage ring. This effect is called the Touschek Effect. If this happens often enough we will notice a reduction of lifetime. Obviously the probability for collisions increases with the beam density and therefore limits the current density or beam emittance. For a fully coupled beam we have a limitation on the instantaneous bunch current ($\hat{I} = I_{av} \cdot C/\sqrt{2\pi}\,\sigma_\ell$, where C is the circumference of the storage ring, $\sigma_\ell$ the bunch length and $I_{av} = e\, f_{rev}\, N$ with $f_{rev}$ the revolution frequency and N the number of particles per bunch)

$$\hat{I}\text{ (amp)} \leq 3.8\cdot 10^{17}\ \frac{E^2\sqrt{\beta_x\beta_y}\,\epsilon_{xo}}{D(\epsilon)\tau_t}\left(\frac{\Delta E}{E}\right)^3_{max} \qquad (19)$$

Here E(GeV), is the beam energy, $\tau_t$(sec) the desired beam lifetime due to the Touschek effect, $D(\epsilon)$ a complicated function with $D(\epsilon) \approx 0.2 \pm .1$ for most storage rings,/5/, $(\Delta E/E)_{max}$ the maximum energy acceptance and $\beta_x$, $\beta_y$ the average values of the betatron functions. For order of magnitude calculations we may take $\bar{\beta}_x \approx \bar{\beta}_y \approx 2$ to 5m, (Fig. 1 and 2), $\epsilon_{xo}$(rad m) is the beam emittance.

It should be noted here that this limitation is a one bunch limitation. As far as the Touschek effect is concerned there is no limitation on the number of bunches that are filled to this maximum bunch current density. From eq. (19) we conclude that the peak current depends strongly on the energy acceptance. We assume that the rf-voltage is large enough to provide an energy acceptance at least as large as limited by other effect. In most cases the energy acceptance of a storage ring will be limited by chromatic and geometric aberrations in a storage ring.

Like in ordinary light optics the focusing of the beam in a storage ring is not independent of the energy (wavelength in light optics) of the particles. These are the chromatic aberrations some of which can be sufficiently compensated by sextupolar magnets. But there are also aperture or beam size dependent aberrations mostly due to the nonlinear fields of the sextupole magnet. A compromise therefore has to be made between both kinds of aberrations when a sextupole lattice is designed for a particular storage ring. Whatever the design will be there is a maximum energy deviation a particle may have to be still stable in a storage ring. In particular designs for FEL storage rings (ring I of Table I) it has been shown that this maximum energy deviation can be as large as

$$\left(\frac{\Delta E}{E}\right)_{max} \lesssim \pm (4 \text{ to } 6\%) \qquad (20)$$

Note that all particles of a gaussian beam have to be within this limit which means $(\sigma_E/E)_{max} \lesssim \pm (.6 \text{ to } .8\%)$.

At this point it should be discussed how we can maximize the peak beam current having FEL operation in mind. When the beam interacts with the laser field we expect an increased energyspread in the beam and as a consequence a longer bunch. On the other hand during injection we have a shorter bunchlength since the laser is off and therefore a higher particle density or a shorter Touschek lifetime. There is obviously a bottleneck for the maximum possible peak current at the end of the injection process. This bottleneck can be avoided. The ratio of the maximum peak current with laser on or off is given by:

$$\hat{I}_L/\hat{I}_I = (\varepsilon_{xoL}/\varepsilon_{xoI}) \cdot (\sigma_{\ell L}/\sigma_{\ell I}) \qquad (21)$$

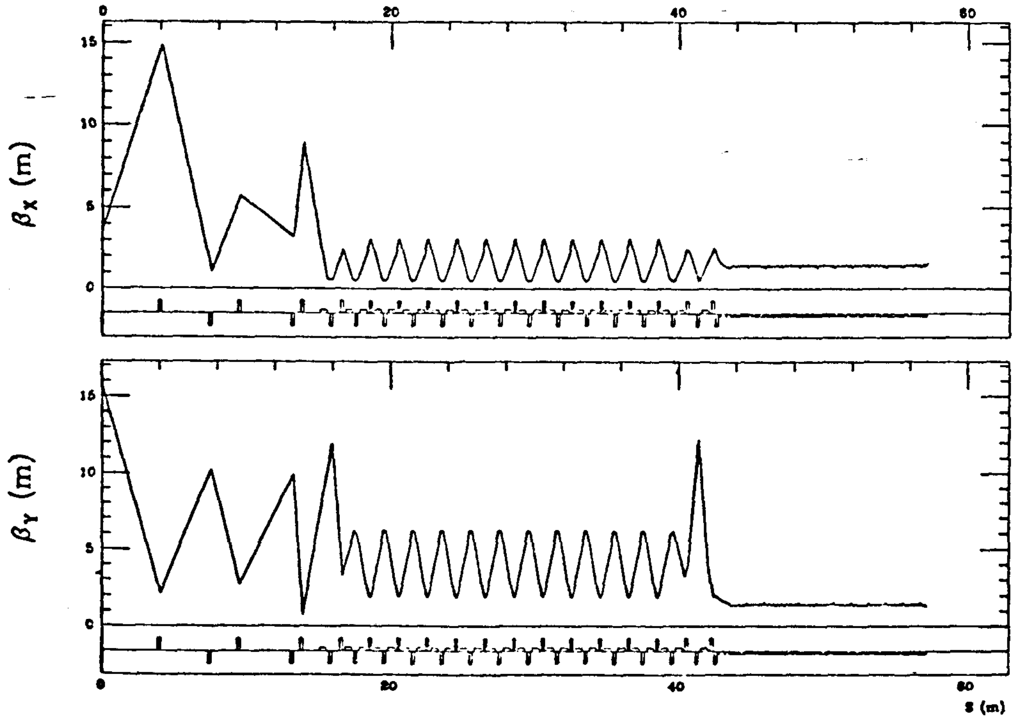


Fig. 1 : Betatron functions of one half of ring I (Table I). On the left side is the straight section for injection, rf etc., followed by the arc lattice and on the right is a long undulator for a gain expanded FEL.

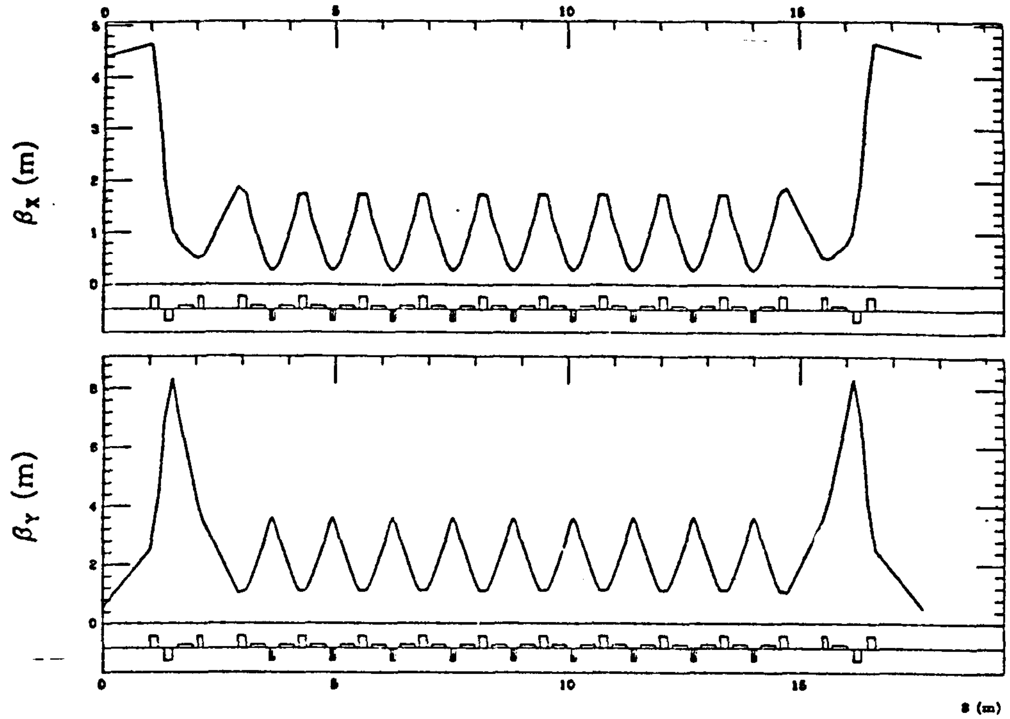


Fig. 2 : Betatron function of the damping ring (ring II) in Table I.

where the subscripts "L" refers to laser on and "I" to laser off. We expect $\sigma_{\ell L}/\sigma_{\ell I} > 1$. To make now $\hat{I}_L = \hat{I}_I$ would mean to manipulate the focusing lattice such that $\varepsilon_{xoL}/\varepsilon_{xoI} < 1$ or equal to the inverse of the ratio of the bunchlengths. This manipulation of the lattice has to be possible without crossing dangerous beam resonances since the beam now would be injected into one configuration and used with the laser turned on in another configuration. If the magnet lattice is designed properly this can be done (ring I of Table I). In this particular case it is possible to change the beam emittance continously by at least a factor of five.

With this provision the peak current is limited by the bunch volume as determined by the interaction with the laser. In the storage ring I of Table I the maximum achievable peak beam current due to the Touschek effect alone can be as high as about 900 amp at 1 GeV. Other effects may set a lower limit.

Maximum Gain/Pass

The maximum gain per pass of the FEL is related to the beam parameters like /6/

$$\frac{\text{GAIN}}{\text{PASS}} \sim \frac{\hat{I}}{\gamma^2 \varepsilon_x \varepsilon_y} \qquad (22)$$

Obviously this gain will be limited by the desired beam lifetime (Touschek Effect). Combining Eqs. (16), (19) and (22) we get:

$$\frac{\hat{I}}{\gamma^2 \varepsilon_x \varepsilon_y} \leq C_D \cdot \frac{\sqrt{\beta_x \beta_y}\,(\rho/R)}{\tau_t \theta^3 E^2\, D(\varepsilon)} \left(\frac{\Delta E}{E}\right)^3_{max} \qquad (23)$$

For the storage ring I in Table I this quantity is:

$$\frac{\hat{I}}{\gamma^2 \varepsilon_x \varepsilon_y} \approx 1 \cdot 10^8 \frac{\text{amp}}{\text{cm}^2} \qquad (24)$$

assuming $(\Delta E/E)_{max} = 2.5\%$. Higher values can be achieved with the "mismatch" scheme described in the previous section.

This is a factor of 100 larger than the expected value for the SLAC LINEAR COLLIDER (SLC) /7/ where the state of the art in linacs is employed to maximize this quantity. As a matter of fact in order to make the SLC a feasible colliding beam facility a storage ring is employed to reduce the beam emittance due to damping. With this damping storage ring (ring II of Table I) it is expected to reach a beam current density of $\hat{I}/(\gamma^2 \varepsilon_x \varepsilon_y) \approx 1 \cdot 10^8 \text{amp/cm}^2$. First measurement of this quantity are expected to be performed in summer 1983.

Beam Emittance and Damping

The RENIERI CRITERION /8/ relates the maximum laser power $P_L$ to the synchrotron radiation power $P_{syn}$. The latter however is just the particle energy E divided by the synchrotron damping time $\tau_E$. We have therefore

$$P_L \approx \frac{3}{2} \left(\frac{\sigma_E}{E}\right)_{max} \cdot \frac{E}{\tau E} \cdot N \qquad (25)$$

where N is the total number of electrons in the storage ring. This calls for a short damping time to maximize the laser power. From eqs. (11) and (12) we get

$$\tau_E(\text{sec}) = 2.4 \cdot 10^{-4} \frac{\rho(m)\ R(m)}{E^3(\text{GeV}^3)} \qquad (26)$$

Obviously a small bending radius $\rho$ or a small average radius R reduces the damping time, but doing so we have to watch the increase in beam emittance (eq. 16). We therefore calculate the product $\varepsilon_{xo} \cdot \tau_E$ and get

$$\varepsilon_{xo} \cdot \tau_E = C_\tau \cdot R^2 \theta^3 / E \qquad (27)$$

with $C_\tau = 4.5 \cdot 10^{-10}$ GeV sec/m.
Again we find that the bending angle per magnet should be small leading to a big ring. Better yet because of the $R^2$ factor one should design the storage ring with very short cells e.g. small distances between the quadrupoles. In this case it is also possible to use maximum bending fields or a minimum bending radius to minimize $\tau_E$ and still keeping the beam emittance small. With these criteria in mind the following design parameters have been achieved. Ring I of Table I.

$$\varepsilon_{xo} \cdot \tau_E = 11.5 \cdot 10^{-11} \text{rad m sec}$$

Ring II of Table I

$$\varepsilon_{xo} \cdot \tau_E = 3.5 \cdot 10^{-11} \text{rad m sec}$$

If we combine eqs. (25) and (26) and express the number N of particle in the storage ring by the total beam current we finally get for the total laser power

$$P_L(w) = 1330\ (\sigma_E/E)_{max}(\%) \cdot E^4(\text{GeV}^4) \cdot I_{av}(\text{amp})/\rho(m) \qquad (28)$$

Conclusion

The basic physics of electron storage rings and some limitations have been discussed. We found that the main limitation for high beam current density and short damping times comes from the Touschek effect. In spite of this limitation peak currents of the order of a few hundred amperes per bunch can be expected at a beam energy of 1 GeV. To maximize the total laser power more than one bunch should be filled which does not change the Touschek lifetime limit.

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