SLAC-PUB-3007
October, 1982
(T)

EFFECTS OF PAIR CREATION ON THE EXPANSION OF VACUUM BUBBLES * †

Hideaki Aoyama
California Institute of Technology, Pasadena, California 91125
and
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

The real-time behavior of true-vacuum bubbles nucleated in the false-vacuum background is studied in a $\lambda\varphi^4$ - theory. Classically, the bubble expansion rate has been known to approach the velocity of light asymptotically. A quantum effect, creation of Higgs and fermion pairs, is studied by a semiclassical method, and is shown to lead to a slower expansion rate. Within the thin-wall approximation, several possible asymptotic behaviors are examined. A solution that behaves like the classical one but with different coefficients is shown to be stable. The first order corrections to the coefficients are calculated approximately. The created Higgs pairs are found to remain inside the bubble and carry an energy of approximately $10^{-3}\lambda$ of the energy released from the false vacuum. The fermions that are massless in the false vacuum go out of the bubble and have energy $\sim 10^{-3}\, g^2$, where g is the Yukawa coupling constant. It is shown that the resulting asymptotic state is Lorentz-invariant.

Submitted to Nuclear Physics B

---

*Work supported in part by the Department of Energy, under contracts DE-AC03-81-ER40050 (Caltech) and DE-AC03-76SF00515 (SLAC).

†Revised version for CALT-68-925.

1. Introduction

In recent years there has been a growing interest in the very early universe in connection with the grand unified theories of the elementary particles. In most of the grand unified models, the symmetry of the theory is restored above a critical temperature T_c of the order of the grand unifying scale $\sim 10^{14\sim 15}$ GeV [1]: at such a high temperature, the state with zero expectation value of the Higgs field tends to have the lower free energy than states with nonzero Higgs expectation value. Therefore, the very early universe experiences a phase transition from the symmetric phase to a nonsymmetric phase as it expands and cools down to a temperature below T_c. This phase transition may occur via nucleation of bubbles of the nonsymmetric phase in the symmetric background, their eventual expansion and coalescence.

The nucleation mechanism for a phase transition was initially studied by Volosin and others [2] in a model of a real Higgs field φ. Their model has a potential $U(\varphi)$ with two minima, φ_+ and φ_-, the former being metastable and the latter stable with $U(\varphi_+)-U(\varphi_-) \equiv \varepsilon > 0$ (see figure 1). This model abstracts the real situation at a temperature less than T_c. Assuming that bubbles have thin walls, that is, $\langle\varphi\rangle = \varphi_-$ inside the bubbles and $\langle\varphi\rangle = \varphi_+$ outside, they gave the energy E^b of such a bubble relative to the homogeneous, i.e., complete stable vacuum:

$$E^b = \frac{4\pi S_1 R^2}{\sqrt{1-\dot{R}^2}} - \frac{4\pi\varepsilon}{3} R^3 \quad , \tag{1.1}$$

where $R(t)$ is the radius of the bubble in the c.m. frame and $\dot{R} \equiv dR/dt$. The first term includes the surface-tension and the Lorentz factor due to the motion of the wall. S_1 is the rest energy of the bubble wall per unit area. The second term is the energy gain due to the difference in the energy density between the stable and the meta-stable vacua. They assumed that the bubble energy E^b is conserved, $\dot{E}^b = 0$, and obtained a following classical solution,

$$R^{c\ell}(t) = \sqrt{R_0^2 + t^2} \tag{1.2}$$

where $R_0 = 3S_1/\varepsilon$ is the radius of the bubble at rest $(t=0)$. Therefore, once a bubble is created it blows up as $t \to \infty$. This is because all the energy released from the false vacuum is used to accelerate the wall. Coleman and Callan [3,4] have studied essentially the same model using the path integral formalism and gave the prescription for the calculation of the nucleation rate. Coleman [3] has also shown that the behavior (1.2) of the bubble wall is reproduced by the solution of the classical field equation, and noted that no excitations of the Higgs field, i.e., no Higgs particles are left inside the bubble ("no roiling sea of mesons"). The nucleation rate at finite temperature was first studied by Linde [5].

Guth and others [6] have discussed a possible scenario of the phase transition, via nucleation of the bubbles as mentioned above, in the early universe. They showed that the horizon and the flatness problems are avoided if the nucleation rate is small enough that the universe undergoes an exponential expansion, $R(t) \propto e^{\chi t}$, where the expansion rate χ is given by $[(8\pi/3)\, G\rho_0]^{1/2}$ in terms of the energy density ρ_0 of the meta-stable state. In this scenario, the primordial monopole problem is also solved because the universe is dominated by a few large bubbles. A difficulty of this scenario is that if Coleman's conclusion of no "roiling sea of mesons" is to be believed, then the energy of meta-stable vacua should be released only when the walls of the few large bubbles collide. This leads to a large scale inhomogeneity and anisotropy [7]. Linde [8] has also pointed out that for the horizon and the flatness problems to be actually solved in this scenario, the period of the exponential expansion has to be so long that the transition temperature has to be unrealistically small. To avoid these difficulties, several other scenarios have been proposed [8,9].

It should be noted that Volosin et al., and Coleman's argument on the real-time behavior of a bubble is a classical one. One of the possible quantum phenomena is pair production. There is no reason to believe that particles that couple to the Higgs field are not created by violent expanding bubbles. If particles are created, then the energy carried by the created pairs has to be included in the energy conservation law. In fact, some fluctuations of the Higgs field around the background of the bubble expanding as (1.2) have been shown to grow [10]. This indicates that particle creation may have a significant effect on the behavior of the bubble expansion. Since Coleman's conclusion is one of the bases of constructing the scenario for the very early universe,* it is important to take quantum effects into account and to see whether the energy obtained by the conversion is still concentrated on the wall and whether the bubble still expands as (1.2).

The purpose of this paper is to study the effect of pair creation on the behavior of the bubble expansion in a $\lambda\varphi^4$-theory with a fermion field.

---

*For example, matter creation, regardless of its cause, tends to suppress the supercooling. From this viewpoint, the pair creation due to the gravitational effect of the exponential expansion were studied by Horibe and Hosoya [11].

Specifically, by using a semiclassical method, we study the self-consistent behavior of the bubble expansion in the asymptotic region, $t \to \infty$. We obtain that the energy released from the false vacuum is distributed to the wall, the created Higgs particles and the created fermions according to a ratio that is constant in time. The ratio is given to the first nontrivial orders of the coupling constants of the model. We find that the created Higgs particles remain inside the bubble (see figure 3).

2. The model and the method

The classical bubble expansion (1.2) was obtained from the energy conservation law (1.1). In presence of pair creation, equation (1.1) becomes

$$0 = E^b(T) + E^p(T) \quad , \tag{2.1}$$

where $E^p(T)$ is the total energy carried by the pairs at time T. If we assume that the bubble is still parametrized by R(t), (2.1) would lead to a self-consistent equation for R(t), whose solution should yield a slower expansion rate than (1.2). However, the self-consistent equation so obtained is nonlocal and nonlinear, and thus seems difficult to be solved. Therefore, we limit ourselves to study the leading asymptotic behaviour of $E^p(T)$ for several possible asymptotic behaviours of R(t). Then, by looking at the energy balance (2.1), we find possible self-consistent asymptotic R(t). This limit is nevertheless interesting, because the asymptotic behaviour of the bubble expansion is important for cosmological problems.

In this paper, we study the model with a single real Higgs field φ and a fermion field ψ of spin $\frac{1}{2}$. The action of the Higgs field is the same as the one used by Coleman [3],

$$S_\varphi = \int d^4x \left\{ \frac{1}{2} \left(\partial_\mu \varphi\right)^2 - U(\varphi) \right\} . \qquad (2.2)$$

In order that the theory be renormalizable, the potential $U(\varphi)$ is a forth order polynomial in φ, which, up to a constant, is parametrized as follows,

$$U(\varphi) = \frac{\lambda}{8} \left(\varphi^2 - a^2\right)^2 + \frac{\varepsilon_0}{2a} (\varphi - a) , \qquad (2.3)$$

where λ, a and ε are positive. For small ε_0, the minima of the above potential are,

$$\varphi_\pm = \pm a - \frac{\varepsilon_0}{2\lambda a^3} + O\left(\varepsilon_0^2\right) . \qquad (2.4)$$

The meta-stable state φ_+ is called the false vacuum, and the stable state φ_- the true vacuum. The energy density difference between the true and the false vacuum is equal to ε_0 at the lowest order of the ε_0-expansion,

$$\varepsilon \equiv U\left(\varphi_+\right) - U\left(\varphi_-\right) = \varepsilon_0 + O\left(\varepsilon_0^3\right) .$$

Coleman has shown that for small ε_0, the action (2.2) is minimized by the following solution,

$$\varphi^{c\ell}(t,r) \approx \begin{cases} \varphi_- & \text{for } r < R^{c\ell}(t) \\ \operatorname{atanh} \dfrac{\mu\left(\sqrt{r^2 - t^2} - R_0\right)}{2} + O\left(\varepsilon_0\right) & \text{for } r \sim R^{c\ell}(t) \\ \varphi_+ & \text{for } r > R^{c\ell}(t) \end{cases} \qquad (2.5)$$

where $\mu \equiv a\sqrt{\lambda}$ and $R_0 \equiv \mu^3/\lambda\varepsilon_0$. Since

$$\mu\left(\sqrt{r^2 - t^2} - R_0\right) \sim \frac{\mu\sqrt{t^2 + R_0^2}}{R_0} \left[r - R^{c\ell}(t)\right] + 0 \left[r - R(t)\right]^2 .$$

the classical solution (2.5) gives the thickness of the wall $1/\mu_t$ at time t to be

$$\frac{1}{\mu} \frac{R_0}{\sqrt{t^2 + R_0^2}} \equiv \frac{1}{\mu_t} . \tag{2.6}$$

Therefore, the thin-wall approximation is valid when

$$1 << R_0 \mu = \frac{\mu^4}{\lambda \varepsilon_0} .$$

The time dependence of the wall thickness (2.6) can be understood as the result of a Lorentz contraction. In fact, at any time t,

$$\frac{1}{\mu_t} = \frac{1}{\mu} \sqrt{1 - R^2(t)} \tag{2.7}$$

is satisfied by $R(t) = R^{c\ell}(t)$.

We assume that pair creation only changes the expansion rate and not the shape of the bubble except for the factor of the Lorentz contraction in the wall thickness. It seems plausible to expect that the bubble would keep its shape, because a wider surface costs energy. Furthermore, if a shape is distorted, the deformed area would tend to create more pairs than a plain spherical surface. This effect would tend to reduce the distortion. According to the above assumption, the expectation value of the scalar field φ^b for any r is expressed in terms of R(t) as follows,

$$\varphi^b = a \tanh \frac{\mu_t\left(r - R(t)\right)}{2} + 0\left(\varepsilon_0\right) \tag{2.8a}$$

where μ_t' is given by (2.7). Since the bubble is to be created at $t \sim 0$, (2.8a) should apply for $t \gtrsim 0$. For $t \lesssim 0$,

$$\varphi^b = \varphi_+ \quad . \tag{2.8b}$$

The above bubble (2.8) is created without any particles associated with it. Right after its nucleation, regardless of whether it is spontaneous or stimulated, the space is clean. As the bubble expands and gains acceleration, it begins to create particles. The effect of pair creation can be significant in the asymptotic region $t \gg R_0$, where we concern ourselves. In this region, the bubble is a semiclassical object; that is, it acts merely as an external source for the particles. Thus, we are allowed to use the usual Feynman-graph method to calculate the pair creation amplitude. An analogous situation is β-decay of nuclei. Once an electron is emitted, as Coleman [3] put it, "it propagates classically" with a definite momentum. Imagine an electric field in some asymptotic region so that some of the electrons get accelerated and radiate photons. In order to find the asymptotic behavior of electrons, we merely have to solve the problem of a "semiclassical" electron coupled to the quantum photon field without worrying about the initial quantum origin of the electron.

The fluctuations φ^f of the Higgs field on the background φ^b satisfy the following equation,

$$0 = \left[\partial_\mu \partial^\mu + \frac{d^2U(\varphi^b)}{d\varphi^{b^2}}\right] \varphi^f + O\left(\varphi^{f^2}\right) \equiv \left[\partial_\mu \partial^\mu + \mu^2 + V^H(r,t)\right] \varphi^f + O\left(\varphi^{f^2}\right) \quad . \tag{2.9}$$

where $V^H(r,t)$ is, from (2.8a,b),

$$V^H(r,t) = -\frac{3\mu^2}{2}\ \theta(t)\,\mathrm{sech}^2\,\frac{\mu_t\left[r - R(t)\right]}{2} + O\left(\varepsilon_0\right)\ . \tag{2.10}$$

As seen in (2.9), the bubble background acts as an external space-time dependent mass term for the fluctuation. However, there is a complication here about separation of "particles" from the background. In order for the fluctuations to be understood as particles, they have to be far from the wall. Otherwise they are rather a part of a deformed wall and the calculation based on the assumption (2.3) would not be valid. We assume here that only fluctuations that are separable from the wall are excited. The consistency of the assumption and the result will be shown in the next section.

Note that in this particular model (1.2), the mass of the φ-particles in the false and true vacua differs only at order ε. Therefore, at the lowest order in ε, we assign the mass μ to the propagator in the loop graphs, and use the $O(\varepsilon_0^0)$ term of $V^H(r,t)$ as the vertices. The energy density difference ε appears only in the expression for the bubble energy $E^b(T)$ of (1.1).

The graph (a) illustrated in figure 2 gives the lowest nontrivial contribution to the energy $E^{Hp}(T)$ carried by the Higgs pairs as follows,

$$E^{Hp}(\infty) = \int d^4x \int d^4x' \int \frac{d^4k}{(2\pi)^4}\ \left|k^0\right| 2\,\mathrm{Im}K^H\left(k^2\right) e^{ik(x-y)}\, V^H(r,t)\, V^H(r',t')\ , \tag{2.11}$$

where the four-momentum k carried by the vertices gives the total momentum of the pair, and

$$\mathrm{Im}K^H\left(k^2\right) = \frac{1}{32\pi}\ \theta\left(k^2 - 4\mu^2\right) \sqrt{1 - \frac{4\mu^2}{k^2}}\ . \tag{2.12}$$

Since $\mathrm{Im}K'^{H}(k^2)$ is not a polynomial in k^2 as seen above, its Fourier transform is nonlocal. This causes difficulty in finding the $E^{Hp}(T)$ for finite T: if $E^{Hp}(\infty)$ is expressed by a one-fold integration over a time coordinate t, $E^{Hp}(T)$ is given by the finite integration of the same integrand with upper limit T. This prescription is not applicable to our case, because our $E^{Hp}(\infty)$ is essentially a two-fold integration over two time coordinates t and t'. In order to define $E^{Hp}(T)$ for a finite T, we introduce a cutoff function

$$e^{-\frac{t}{T}} \tag{2.13}$$

for each of the vertices in (2.11). Since this cutoff function gradually turns off the vertices after $t \sim T$, the resulting integration seems appropriate for the definition of $E^{Hp}(T)$, the energy carried away by the created pairs by time T.[*]

The integral over x and x' in (2.11) is straightforward,

---

[*] Another reason for choosing (2.13) is that the integral in (2.11) is actually divergent because of the upper limit $t \sim t' \sim \infty$. Consider the case when $E^{Hp}(T)$ is a single-fold integration over t and is divergent, for example,

$$E^{Hp}(\infty) = A \int_0^\infty dt\, t^n \qquad (n > 0) \quad ,$$

In this case, exact E^{Hp} for finite T is given by

$$E^{Hp}(T) = A \int_0^\infty dt\, t^n = \frac{AT^{n+1}}{n+1} \quad .$$

The cutoff function (2.13) gives the following approximation for the above $E^{Hp}(T)$,

$$E^{Hp}(T) \simeq A \int_0^\infty dt\, t^n \exp\left\{-\frac{t}{T}\right\} = A\, n!\, T^{n+1} \quad .$$

Therefore, the asymptotic behavior of $E^{Hp}(T)$ is correctly reproduced. However, the example also shows that the numerical factor in the result obtained under our assumption should not be rigorously believed in.

$$\tilde{V}_T^H(k_r,k_0) \equiv \int d^4x \, \exp\left\{-\frac{t}{T}\right\} e^{ikx} \, V^H(x,t)$$

$$= -\,24\pi^2\mu^2 \int_0^\infty dt \, \exp\left\{-\frac{t}{T}+ik_0t\right\} \frac{R_t}{\mu_t^2} \sin k_rR_t \, \operatorname{cosech}\frac{\pi k_r}{\mu_t} + \ldots\,, \qquad (2.14)$$

with $R_t \equiv R(t)$ and $k_t \equiv |k|$, where we have kept only the leading term in the thin-wall approximation. The following formula has been used to derive (2.14) [12],

$$\int_{-\infty}^{\infty} dr \, \cos k_r r \left(\operatorname{sech}\frac{\mu_t r}{2}\right)^{2n} = \frac{4^n \pi k_r}{(2n-1)!\,\mu_t^2} \operatorname{cosech}\frac{\pi k_r}{\mu_t} \prod_{t=1}^{n-1}\left(\frac{k_r^2}{\mu_t^2}+\ell^2\right). \quad (2.15)$$

For the fermion field ψ of spin ½ we assume that it couples to the Higgs field as

$$S_\psi = \int d^4x \left\{ \bar{\psi} i\partial\!\!\!/\psi + g\left(\varphi-\varphi_+\right)\psi\bar{\psi} \right\} \quad . \qquad (2.16)$$

where $g > 0$ for simplicity. This fermion is massless in the false vacuum and has a mass of $g(\varphi_+ - \varphi_-) \sim 2g\mu$ in the true vacuum, reflecting the actual situation: in actual grand unified models, the false vacuum has higher symmetries than the true vacua, therefore particles, especially fermions, tend to be massless in the false vacuum. The self-energy graph (a) with a fermion loop in figure 2 gives the energy E^{fp} carried by the created fermions. For reasons explained later in section 4, the created fermions are thought to exist mainly outside the bubble, i.e., in the false vacuum. Thus, we take the fermion propagators in graph (a) to be massless. The resulting E^{fp} is similar to (2.11), and have V^H replaced by

$$V^f(r,t) \equiv g\left[\varphi^b(r,t) - \varphi_+\right]$$

and $K^H(k^2)$ replaced by the following $K^f(k^2)$,

$$\mathrm{Im}K^f\left(k^2\right) = \frac{k^2}{48\pi}\;\theta\left(k^2\right) \quad . \tag{2.17}$$

Corresponding to (2.14), we have the following,

$$\tilde{V}^f_T\left(k_r,k_0\right) = 8\pi^2 a\frac{b}{\mu}\int_0^\infty dt\;\exp\left\{-\frac{t}{T}+ik_0t\right\}\left(-\frac{R_t}{k_t}\cos k_rR_t+\frac{\sin k_rR_t}{k_t^2}\right)$$

$$\times\;\mathrm{cosech}\,\frac{\pi k_t}{\mu_t} \quad . \tag{2.18}$$

In the following sections, we integrate over t for several possible asymptotic behaviors of R(t), and then carry out the momentum integrations in order to evaluate $E^{Hp}(T)$ and $E^{fp}(T)$.

3. Higgs pair creation

In this section, we calculate the asymptotic behaviour of $E^{Hp}(T)$ for three possible asymptotic behaviours of R(t). The classical expansion rate asymptotically approaches one, the velocity of light, as follows,

$$R^{c\ell}(t) \to t + \frac{R_0^2}{2t} \;\ldots \quad . \tag{3.1}$$

Since we expect that pair creation slows down the expansion, we study the following three cases. (i) The bubble expansion has an asymptotic velocity β less than one. (ii) The asymptotic rate of the bubble expansion is one, but it approaches it more slowly: $R(t) \to t - c + b^2/2T$ with $b > R_0$ and $c > 0$. (iii) Similar to the case (ii), but $R(t) \to t - \eta T^{\xi}$ with $\eta\xi > 0$ and $1 > \xi > -1$. Even though the case (iii) includes (ii) as a special case, (ii) is studied in detail to illustrate the method of approximations.

(i) First we assume that the asymptotic expansion rate approaches β of $1 > \beta > 0$:*

$$R(t) \to \beta t + \dots \qquad \text{as } t \to \infty \quad . \tag{3.2}$$

In this case, (2.14) is approximated as follows,

$$\tilde{V}^H_T\left(k_r, k_0\right) = -24\pi^2 \frac{\mu^2}{\mu_\infty^2} \operatorname{cosech} \frac{\pi k_r}{\mu_\infty} \left[\frac{-2 i k_o k_r \beta}{\left(k_0^2 - k_r^2 \beta^2\right)^2} + 0\left(\frac{1}{T}\right) \right] \quad . \tag{3.3}$$

The pair energy E^{H_p} is calculated from the following formula,

$$E^{Hp}(T) = 288\pi^4 \left(1-\beta^2\right)^2 \beta^4 \int_{2\mu}^{\infty} k_0 \, dk_0 \int_0^{\sqrt{k_0^2 - 4\mu^2}} k_t^2 \, dk_t \sqrt{1 - \frac{k^2}{4\mu^2}}$$

$$\times \left(\operatorname{cosech} \frac{\pi k_r}{\mu_\infty} \right)^2 \frac{k_0^2 k_r^2}{\left(k_0^2 - k_r^2 \beta^2\right)^4} + 0\left(\frac{1}{T}\right) \quad . \tag{3.5}$$

This yields a finite result. The integrand in (3.5) has no singularity because the denominator, $k_0^2 - k_r^2 \beta^2$, is always greater than $4\mu^2$ within the integration region. For $k_0 \to \infty$, the k_r-integration is finite because of the $(\text{cosech})^2$. Thus, the k_0-integration becomes

$$\sim \int^{\infty} k_0 \, dk_0 \frac{k_0^2}{k_0^8} \left(\mu_\infty\right)^5 \quad . \tag{3.6}$$

*There are no *ad hoc* reasons to exclude the case of $\beta = 0$. We may find a self-consistent solution among the R(t)'s that asymptotically behave as follows,

$$R(t) \to \infty \ , \qquad R(t) \to 0 \ ,$$

One example of such cases is, $R(t) \to a t^\gamma$ with $1 > \gamma > 0$. These need a separate treatment. Nevertheless, the analysis in this section shows that the solution in the case (ii) is the only stable one among R(t)'s in (iii) and therefore even if a solution exists among the above cases, it is never achieved.

which is finite for $\beta < 1$. The pair energy $E^{Hp}(\infty)$ is approximately

$$E^{Hp}(\infty) \sim \frac{9}{64} \frac{\beta^4}{\sqrt{1-\beta^2}} \mu \quad . \tag{3.7}$$

Since $E^{Hp}(T)$ is finite for $T \to \infty$, we conclude that creation of the Higgs pairs cannot make asymptotic rate of the bubble expansion less than one. Consider the r.h.s. of equation (2.1),

$$\frac{4\pi S_1 R_T^2}{\sqrt{1-R_T^2}} - \frac{4\pi\varepsilon}{3} R_T^3 + E^{Hp}(T) \quad . \tag{3.8}$$

Using the asymptotic form (3.2) for R_T, as $T \to \infty$, the leading orders of the first and second terms are of $O(T^2)$ and $O(T^3)$, respectively. Since we have found that $E^{Hp}(T)$ is of 091) in this case, it is impossible to balance the released energy (the second term) with the pair energy, i.e., we cannot have zero coefficient for the leading $O(T^3)$ term in (3.8). Therefore, the asymptotic behaviour (3.2) cannot be a self-consistent asymptotic solution of (2.1).

(ii) Next, we consider the case when the asymptotic expansion rate is one. Naively, (3.7) gives a divergent expression and therefore suggests that pair creation may affect the expansion rate significantly. We assume the following asymptotic behaviour for R(t) similar to the classical one,

$$R(t) \to t - c + \frac{b^2}{2t} + \ldots \quad . \tag{3.9}$$

Since, in this case

$$\dot{R}(t) \to 1 - \frac{b^2}{2t^2} \quad . \tag{3.10}$$

we expect that $b > R_0$ and $c > 0$, so that the expansion is slowed down by pair creation. The vertex $\tilde{V}^H$ of (2.14) is now,

$$\tilde{V}_T^H(k_r, k_0) \simeq -24\pi^2 b^2 \int_0^\infty dt \, \exp\left\{-\frac{t}{T} + ik_0 t\right\} \frac{\sin k_r t}{t} \operatorname{cosech} \frac{\pi b k_r}{\mu t} \quad , \tag{3.11}$$

where we have used

$$\mu_t \to \mu \frac{t}{b} \quad . \tag{3.12}$$

In order to estimate (3.19) and $E^{Hp}(T)$ of (3.4), we divide the momentum space integral in (3.4) into two parts.

(1) $k_r \lesssim \frac{\mu T}{\pi b}$

The integration is dominated by the region where the argument of cosech is small. Thus, by approximating cosech $x \sim 1/x$, we get

$$\tilde{V}_T^H(k_r, k_0) \simeq -24\pi b \mu \frac{1}{\left(k_0 + \frac{i}{T}\right)^2 - k_r^2} \quad .$$

Therefore, the total energy $E^{Hp,1}(T)$ of the pairs created in this region is given by:

$$E^{Hp,1}(T) \simeq \frac{18}{\pi^2} \mu^2 b^2 \int_0^{\frac{\mu T}{\pi b}} k_r^2 dk_r \int_{\sqrt{k_r^2 + 4\mu^2}}^\infty k_0 \, dk_0 \sqrt{1 - \frac{4\mu^2}{k^2}} \left| \frac{1}{\left(k_0 + \frac{i}{T}\right)^2 - k_r^2} \right|^2 \quad . \tag{3.13}$$

The k_0-integration is finite for $T \to \infty$. An approximate value $1/8\mu^2$ is obtained if we neglect the $\surd$-function that is $\simeq 1$ except near the boundary, $k^2 = 4\mu^2$. Note that the main contribution to the k_0-integration comes from the peak of the integrand at $k^2 = 5\mu^2$, and in the k_r-integration the integrand k_r^2 gives the largest contribution at the upper limit $k_r \sim (\mu T/\pi b)$. The approximate value of the leading order term of (3.13) is,

$$E^{Hp,1}(T) \simeq \frac{3}{4\pi^5} \frac{\mu^3}{b} T^3 \quad . \tag{3.14}$$

(2) $k_r \gtrsim \frac{\mu T}{\pi b}$

Using $\operatorname{cosech} x \sim 2e^{-x}$ for $x >> 1$, we approximate (3.11) as follows,

$$\tilde{V}_T^H\left(k_r, k_0\right) \simeq -48\pi^2 b^2 \int_0^\infty dt \, \exp\left\{-\frac{t}{T} + ik_0 t\right\} \frac{1}{t} \sin k_r t \, \exp\left\{-\frac{\pi b k_r}{\mu t}\right\} .$$

This integration is given in terms of a modified Bessel function of the third kind, $K_0(z)$. The argument z is such that

$$|z| > 2\sqrt{2\pi\mu b} > 2\sqrt{2\pi\mu R_0} >> 1 \quad .$$

By using the asymptotic expansion of $K_0(z) \sim (\pi/2z)^{\frac{1}{2}} e^{-z}$, we get

$$E^{Hp,2}(T) \simeq 72\pi b^4 \int_{\frac{\mu T}{\pi b}}^\infty k_r^2 \, dk_r \int_{\sqrt{k_r^2 + 4\mu^2}}^\infty k_0 \, dk_0 \sqrt{1 - \frac{4^{\;2}}{k^2}} \frac{e^{-y}}{\sqrt{2}\, y} \exp - \frac{8\pi^2 b^2 k_r^2}{\mu^2 T^2 y^3} \, .$$

where $y \equiv 2[k_r(k_0 - k_r)(2\pi b/\mu)]^{\frac{1}{2}}$. Finally, on changing the integration variable k_0 to y, we obtain the leading order term,

$$E^{Hp,2}(T) \simeq \frac{9}{4\pi^{3/2}} (\mu b)^{7/2} \exp\left\{-2\sqrt{\pi\mu b}\right\} \mu^4 T^3 \quad . \tag{3.15}$$

We can now discuss the consequences of the energy conservation. The asymptotic behavior (3.9) leads to the following leading terms for each piece of (3.8),

$$\text{the surface energy:} \quad \frac{4\pi S_1 R_T^2}{\sqrt{1-R_T^2}} \to \frac{4\pi S_1}{b} \; T^3 \; ,$$

$$\text{the volume energy:} \quad -\frac{4\pi\varepsilon}{3} \; R_T^3 \to -\frac{4\pi\varepsilon}{3} \; T^3 \; .$$

The pair energy $E^{Hp,2}(T)$ is considerably smaller than $E^{Hp,1}(T)$ because of the factor $\exp\{-2(\pi\mu b)^{\frac{1}{2}}\}$, therefore we neglect $E^{Hp,2}(T)$ and write

$$E^{Hp}(T) \to \frac{3}{4\pi^5} \, \frac{\mu^3}{b} \; T^3 \; .$$

If we include only the effect of the Higgs pair creation, the total energy conservation law (2.1) implies the following equation for b,

$$\frac{4\pi S_1}{b} - \frac{4\pi\varepsilon}{3} + \frac{3}{4\pi^5} \, \frac{\mu^3}{b} = 0 \quad . \qquad (3.16)$$

Thus, we obtain

$$b_{sol} = R_0 \left(1 + \frac{9\lambda}{16\pi^6}\right) \; , \qquad (3.17)$$

in agreement with the naive expectation $b_{sol} > R_0$. Since the bubble does not create many pairs initially, the value of c in (3.9) is to be found by connecting the asymptotic R(t) of (3.9) with $b = b_{sol}$ with the $R^{c\ell}(t)$ at $t \sim R_0$. The resulting c is of order of $(9\lambda/16\pi^6)R_0$.

The solution $(b,c) = (b_{sol}, c_{sol})$ is stable. If the bubble expansion at a certain time $T\ (\gg R_0)$ is described by (3.9) with $b = R_0$, and is thus slower than the one with b_{sol}, then more pairs are created than given by energy balance. This causes the rate of acceleration of the expansion to decrease and b to decrease. On the other hand, if the value of b is greater than b_{sol}, less pairs are created than necessary to balance the energies. As a result, b decreases.

From (3.17), we also learn that the ratio of the energy used to create pairs and the released energy is given by the following,

$$\frac{\frac{9\lambda}{16\pi^6}}{1+\frac{9\lambda}{16\pi^6}} \quad . \qquad (3.18)$$

The spectrum of the created particles can be obtained from the following argument: in deriving $E^{Hp,1}(T)$ of (3.14), we first learned that the integration in (3.13) is dominated by the peak at $k^2 = 5\mu^2$ of the integrand. This is mainly because of the fractional function in the integrand, which for $T \to \infty$ is $1/(k_0^2 - k_r^2)^2$. If we leave an angle θ between x and k unintegrated, (2.14) gives

$$\tilde{V}_T \propto \int d(\cos\theta)\ \frac{1}{\left(k_0 - k_r\ \cos\theta\right)^2} \quad .$$

Therefore, the integrand of (3.13) is proportional to the following,

$$\int d(\cos\theta) \int d(\cos\theta')\ \frac{1}{\left(k_0 - k_r\ \cos\theta\right)^2 \left(k_0 - k_r\ \cos\theta'\right)^2} \quad .$$

Thus, the integrand in (3.15) peaks around $\theta \sim \theta' \sim 0$. Therefore, we conclude that pair creation is a local phenomenon and that the direction of the total three-momentum of a pair is along the radius vector to the place where the pair is created, and outward. The momentum of the individual pair is obtained by noting that pair creation occurs isotopically in the rest frame of the center of mass system of the pairs. Note that it is <u>not</u> the rest frame of the portion of the bubble wall where a pair is created. Since $k^2 \sim 5\mu^2$ and $k_r \sim (\mu T/\pi b)$, where b is given by (3.17), the speed v of the rest frame of the center of mass system of the pairs at time T is given as follows,

$$v = \frac{k_r}{k_0} \sim 1 - \frac{5\pi^2 b^2}{2T^2} \quad . \tag{3.19}$$

Comparing this with (3.10), we learn that v is smaller than R. In this rest frame, the particle has an energy of $(\sqrt{5}/2)\mu$ and the velocity is $1/\sqrt{5}$. Therefore, the highest velocity of a particle going outward is given by

$$1 - \frac{1 - \frac{1}{\sqrt{5}}}{1 + \frac{1}{\sqrt{5}}} \frac{5\pi^2 b^2}{2T^2} \sim 1 - 9.4 \frac{b^2}{T^2} \quad . \tag{3.20}$$

which is still smaller than R. From this, we conclude that the created pairs are left inside the bubble, even though they are moving outward. They have high energies of order of $(\mu T/\pi b)$. This is consistent with the cutoff (2.13): since the high energy component is dominant, the short time scale is important, which is not affected by the cutoff (2.13). Also, from (3.12), we learn that $k_r \sim (\mu T/\pi)$. This agrees with the naive expectation that the thickness of the wall at T determines the scale of the energy spectrum of created paris at T. Figure 3 illustrates some examples of motion of the created pairs.

It is now straightforward to show that the "particles" can be clearly distinguished from the wall and thus the result is consistent with our assumption that only the fluctuation that corresponds to particles and not wall deformations are excited. One way to show this is to calculate the mean distance $D(T)$ between the c.m. of particles and the wall at time T and compare it with the thickness of the wall. The first step for this is to recognize that the particle creation rate per time period dt is proportional to tdt. This is obtained by dividing $[dE^{Hp}(t)/dt]$ by the typical energy per pair, $k_0 \propto t$. The c.m. of a pair created at time t is at radius $r(T,t) \sim R(t) + v(t)(T-t)$ at time T. Using (3.9) and (3.19), we obtain the following $D(T)$,

$$D(T) \quad \frac{\int^T \left[R(t) - r(T,t)\right] \, tdt}{\int^T tdt} = 5\pi^2 b^2 \, \frac{\ln T}{T} + O\left(\frac{1}{T}\right) \; .$$

On the contrast, the thickness of the wall at T is $b/\mu T$, which is considerably smaller than $D(T)$ due to T being large and the thin wall assumption. Therefore, even though the particles stay closer to the wall as T increases, they exist distinctly far from the wall.

(iii) The third, and more general case is,

$$R(t) \to t - \eta t^{\xi} + \ldots \quad . \tag{3.21}$$

Since $\dot{R}(t) \to 1 - \eta\xi t^{\xi-1}$, the parameters η and ξ have to satisfy $\eta\xi > 0$ and $1 > \xi$. Also since the bubble is expected to expand slower than the classical rate, we expect that $\xi > -1$. As we did in (ii), we divide the momentum space integral into two parts and take the region where $k_r < \mu_T/\pi$. In this region, by using that

$$\mu_t \to \mu \, \frac{t^{\frac{1-\xi}{2}}}{\sqrt{2\eta\xi}} \quad , \tag{3.22}$$

we obtain the following

$$\tilde{V}_T\left(k_r, k_0\right) = -12\pi\sqrt{2\eta\xi}\ \frac{\mu}{k_r}\ e^{i\frac{1+\xi}{4}}\ \Gamma\left(\frac{3+\xi}{2}\right)\left[\left(k_0 + k_r\right)^{-\frac{3+\xi}{2}} - \left(k_0 - k_r\right)^{-\frac{3+\xi}{2}}\right] \tag{3.23}$$

In the above, the $(k_0 - k_r)$-term has a peak at the boundary $k_0 \sim k_r \sim \mu_T/\pi$ of the momentum space, and therefore is dominant. By taking only this term into account, we obtain the following $E^{Hp}(T)$,

$$E^{Hp}(T) = \frac{9\eta\xi\mu^2}{\pi^2}\ \Gamma^2\left(\frac{3+\xi}{2}\right)\ \frac{1}{(2+\xi)(4+\xi)}\ \frac{1}{\left(2\mu^2\right)^{2+\xi}}\left(\frac{\mu_T}{\pi}\right)^{4+\xi} \propto T^{\frac{1-\xi}{2}(4+\xi)}\ . \tag{3.24}$$

Since the energy released from the false vacuum behaves like T^3, the power of T in the above has to be 3 in order to balance the energy. The only solution for ξ within the range $1 > \xi > -1$ is $\xi_{sol} = -1$. Since this reduces (3.21) to (3.9), we learn that $\eta_{sol} = -b^2_{sol}/2$.

The power of T in (3.24) decreases monotonically from $\xi \sim -1$ up to +1 as figure 3 shows. From this, we find that the solution (3.17) is not only stable among other R(t)'s in (3.9), but also stable in a wider class of R(t)'s in (3.21). The reasoning is similar to the one given below (3.9). That is, if ξ is bigger than -1 and the bubble is expanding slower than (3.9) with any b, too few pairs are created to balance the energy. Then, the bubble expansion gets more accelerated, so that ξ decreases. For the region $\xi < -1$, even though it is difficult to imagine how such behavior is achieved, similar discussion applies and the final ξ would increase.

4. Fermion pair creation

The analysis in the previous section showed that the created scalar particles move outward, but remain inside the bubble due to their mass and the high rate of expansion of the bubble. However, it may not be true for particles that are massless outside the bubble, like fermions in the grand unified models. In such a case, the created pairs would go out of the bubble due to their high velocity. In this section, we study the effect of creation of such massless fermions in the toy model described in section 2.

The lowest nontrivial contribution to the energy E^{fp} carried by the created fermion pairs comes from the graph (b) in figure 2. Since we assume that the created fermions go out of the bubble, we take the fermion propagators to be massless. We have to confirm that this assumption is consistent with the results.

(i) It is rather straightforward to show that, in case of (3.2), the energy E^{fp} carried by fermion pairs is finite for $T \to \infty$. Therefore, again it is impossible for the bubble to attain a terminal expansion rate less than one.

(ii) In case of (3.9), $R(t) \to t - c + b^2/2t$, the region of momentum space $k_r \lesssim \mu T/\pi b$ is again dominant and yields the following $\tilde{V}_T^f(k_r,k_0)$.

$$\tilde{V}_T^f\left(k_r,k_0\right) = 16\pi a \frac{1}{\left[\left(k_0+\frac{i}{T}\right)^2 - k_r^2\right]^2} \quad . \tag{4.1}$$

The above and (2.17) lead to the following,

$$E^{fp}(T) = \frac{8}{3\pi^2} \frac{g^2\mu^2}{\lambda} \int_0^{\frac{\mu T}{\pi b}} k_r^2\, dk_r \int_{k_r}^{\infty} k_0\, dk_0\, k^2 \left[\frac{1}{\left(k_0 + \frac{i}{T}\right)^2 - k_r^2} \right]^4 . \qquad (4.2)$$

The k_0-integral in the above yields an approximate value $T^2/16k_r^2$. This comes mainly from a peak of the integrand at $k^2 \sim 2k_r/T$. The energy $E^{fp}(T)$ gets another power of T from k_r-integration. The resulting pair energy is

$$E^{fp}(T) = \frac{1}{6\pi^3} \frac{g^3\mu^3}{\lambda b} T^3 . \qquad (4.3)$$

The total energy conservation law (2.1) with $E^p = E^{Hp} + E^{fp}$ gives the following solution for b,

$$b_{sol} = R_0 \left(1 + \frac{9\lambda}{16\pi^6} + \frac{g^2}{12\pi^4} \right) . \qquad (4.4)$$

From (4.4), we find that created massless fermion pairs carry

$$\frac{\dfrac{g^2}{12\ ^2}}{1 + \dfrac{9\lambda}{16\pi^6} + \dfrac{g^2}{12\pi^4}} \qquad (4.5)$$

of the energy released from the false vacuum. In contrast to the case of the Higgs pairs, the k_r-distribution of the fermion pairs is flat from 0 to $\sim \mu T/\pi b_{sol}$. However we can show that most of the created pairs go out of the bubble as assumed. At a fixed total four-momentum of a pair, the probability that both of the particles go out of the bubble is given by β_r, the radial velocity of the c.m. system relative to the wall. Since $k^2 \sim 2k_r/T$, we know that

$$\beta_r = \frac{k_r - \frac{T}{R_0^2}}{k_r + \frac{T}{R_0^2}} \quad . \tag{4.6}$$

for large k_r. Also since $\mu T/\pi b >> T/R_0^2$ due to the thin-wall approximation, $\beta_r \sim 1$ for most of the region of k_r, $0 \sim \mu T/\pi b$. Therefore, most of the particles go out of the bubble (see figure 5). In fact, the ratio of the particles actually going outside and all the created particles is given by

$$1 - \frac{2b_{sol}}{\mu R_0^2} \ \ell n \ \frac{\mu R_0^2}{2b_{sol}} \quad . \tag{4.7}$$

For example, the above yields 0.922 for $\mu R_0^2/b_{sol} = 100$, when, for $b_{sol} \sim R_0$, the thickness of the initial bubble wall is 1/100th of the radius. For the creation of the massive fermions that remain inside the bubble, a similar calculation shows that the resulting E^{fp} is

$$E^{fp} \sim 10^{-7} \ \frac{g^2 \lambda^2}{\left(R_0 \mu\right)^2} \left(\frac{R_0}{b}\right)^3 \quad .$$

Therefore, the effect of the massive fermion creation is small,* in agreement with the above discussion.

(iii) In this case, the Fourier-transform of the vertex V is ξ- independent and given by (4.1). The upper limit of the k_r-integration μ_t/π leads to the following,

---

*The main reason why these behave differently from the quantities in the previous section lies in the difference of the integration retion. The k_0-integral in (4.2) is divergent at the boundary $k^2 = 0$ for $T = \infty$, while the one in (3.13) is convergent within the integral region $k^2 \geq 4\mu^2$.

$$E^{fp}(T) \propto T^{\frac{5-\xi}{2}} \quad . \tag{4.8}$$

Since the power of T is monotonically decreasing as ξ increases,* as in (3.24), we conclude that the solution (4.4) is stable among a wide class of possible behaviors of R(t), (3.21).

5. Conclusion and discussion

We have studied pair creation by vacuum bubbles in a $\lambda\varphi^4$-theory. By using the thin-wall approximation, we have shown that the Higgs particles and spin ½ fermions, which have a Yukawa coupling with the Higgs field, are created locally by the expanding bubble wall. As a result, the bubble expansion rate approaches the velocity of light slower than that given by a classical analysis.

Classically, the radius of the bubble was shown to behave like

$$R^{c\ell}(t) \to t + \frac{R_0^2}{2t} \cdots \quad , \tag{3.1}$$

as $t \to \infty$. We have found that the only possible asymptotic solution among a wide class of the possibilities, (3.2) and (3.21), is

$$R(t) \to t - c_{sol} + \frac{b_{sol}^2}{2t} + \cdots \quad , \tag{3.9}$$

with

$$b_{sol} = R_0\left(1 + \frac{9\lambda}{16\pi^4} + \frac{g^2}{12\pi^4}\right), \qquad c_{sol} \sim R_0\left(\frac{9\lambda}{16\pi^6} + \frac{g^2}{12\pi^4}\right) \quad , \tag{4.4}$$

---

*See preceeding footnote.

and where g is the Yukawa coupling constant. We have also found that the solution (3.9) with (4.4) is stable.

The energy released by the false vacuum as the bubble expands is distributed among the wall, the Higgs particles and the fermions according to the following ratio,

$$1 : \frac{9\lambda}{16\pi^6} : \frac{g^2}{12\pi^4} \quad . \tag{5.1}$$

which is constant in time. The Higgs pairs created at time T have $k^2 \sim 5\mu^2$ and $k_r \sim \mu_T/\pi \sim \mu T/\pi b_{sol}$, where k is the total four-momentum of the pair and k_r is the total radial momentum of the pair. They remain inside the bubble moving outward. The fermion pairs created at time T have an almost flat distribution in k_r from $\sim T/R_0^2$ to $\sim \mu T/\pi b_{sol}$ and their k^2 is $\sim 2k_r/T$. They go out of the bubble, where they are taken to be massless. Typical behaviour of the Higgs particles is illustrated in figure 3. We should note that the numerical factors in (4.4) and (5.1) are not necessarily exact. In fact, if we had used cutoff functions other than (2.13), we would have obtained numerical factors different from those in (4.4) and (5.1). To be modest, we should only say that they are of order of 10^{-3}.

We have so far neglected the quantum corrective to the constants of the model and worked with the bare quantities. The corections due to the real parts of the graphs in figure 2 are of $O(\lambda)$ and $O(g^2)$ and thus should be treated properly. We can, however, show that at our order of the perturbation we merely need to replace the bare constants in (4.4) and (5.1) by the renormalized constants. First, let us take the Higgs particles. The real part of the graph (a) yields the renormalized constants μ^r, λ^r, ε_0^r. Neglecting the pair creation, the corrected classical bubble solution depends on the renormalized constants as the original classical solution does on the bare constants. Especially, $R_0^r = \mu^{r^3}/\lambda^r\varepsilon_0^r$. The constants in the expression of bubble energy (1.1) are also replaced by the renormalized constants. Therefore, the energy balance equation (3.16) becomes as follows,

$$\frac{4\pi S_1^r}{b} - \frac{4\pi\varepsilon^r}{3} + \frac{3}{4\pi^5}\,\frac{\mu^3}{b} = 0 \quad ,$$

which leads to

$$b = R_0^r\left[1 + \frac{9\lambda^r}{16\pi^6}\left(\frac{\mu}{\mu^r}\right)^3\right] \quad .$$

Since $\mu^r = \mu[1 + O(\lambda^r)]$, the quantum corrections are of higher order in λ^r, and thus can be neglected. A similar discussion applies to the fermion part, and the correction is of higher order of g. Therefore, (4.4) and (5.1) hold for the renormalized constants.

The asymptotic state we have found is Lorentz-invariant in the sense that Lorentz-transformations affected only the unknown nonleading terms. It is easy to see that r.h.s. of (3.9) is invariant, since (3.9) can be understood as the first three terms of the expansion of an hyperbola, $\left[(t - c_{sol})^2 + b_{sol}{}^2\right]^{\frac{1}{2}}$. In order to discuss the behaviour of the created particles, we first note that the Higgs pairs are created at time t with $k_r = (\mu t/\pi b)$, $k^2 = 5\mu^2$ at a rate of $\propto (1/t)dtdS$, where dS is an area element of the wall. Since on (3.9) $dt/t \simeq (1/b_{sol})d(\text{proper time})$, we find that the pair production rate is constant in the rest frame of the wall. In the same frame, the radial total momentum k_r' is given by a constant

$$k_r' = \frac{k_r - \dot{R}k_0}{\sqrt{1 - \dot{R}^2}} = -\frac{5\pi^2 - 1}{2\sqrt{2}\,\pi}\,\mu \quad .$$

Therefore, these Higgs particles are created Lorentz-invariantly. The fermions discussed in section 4 have a wide spectrum as shown in figure 5. Since the same argument applies to each slice of k_r, they too are created invariantly.

This asymptotic Lorentz-invariance strongly supports our expectation that our method used in this paper is applicable to the spontaneously nucleated bubbles. By using the path-integral method, Coleman [3] showed that the nucleation rate is obtained without integration over the Lorentz-group and thus the divergence appeared in an earlier calculation [2] is avoided. This observation was supported by the fact that his classical bubble solution possesses Lorentz-invariance: the final states related by a Lorentz-transformation are quantum-mechanically indistinguishable from each other, thus they should not be counted separately for the total nucleation rate. In our case, we have found that our asymptotic states have Lorentz-invariance too, and thus are indistinguishable from their Lorentz-transformed states. Therefore, our result is compatible with Coleman's formula for the nucleation rate.

In this paper, we have taken the lowest order pair creation probability into account. As for the higher order effects, the Lorentz-invariance of our solution seems to suggest that time dependence of the physical quantities remain unchanged. That is,

$$
\begin{aligned}
k_r &\propto t \\
\text{Pair creation rate} &\propto \frac{1}{t}\, dt\, dS \\
\text{Total energy of pairs} &\propto t^3 \\
\text{Bubble radius} &\propto (3.9) \quad .
\end{aligned}
\tag{5.2}
$$

Otherwise, the above discussion on compatibility of our results and the finiteness of the nucleation rate would fail.

In constructing a scenario for the very early universe, we need to investigate more realistic models such as a Coleman-Weinberg type model at a finite temperature. An analytical analysis parallel to that given in this paper is impossible in such a model since even a classical bubble solution that corresponds to (2.5) is not known. We could, however, make a qualitative guess about results: in order for the asymptotic Lorentz-invariance to hold, (5.2) seems to be a general property of the results. Qualitative features can be quite different. Especially, the ratios of the energy given to the particles are not necessarily as small as (5.1). If they are not, the pair creation phenomena investigated in this paper would play an important role in the very early universe.

Acknowledgements

I thank Dr. Makoto Kobayashi for pointing out the problem studied in this paper to the author several years ago. I appreciate the encouragement and useful discussions provided by Dr. David Politzer since I began to work on this problem. I also acknowledge discussions with Dr. R. P. Feynman. Rajan Gupta and Barton Zwiebach read the manuscript.

References

[1] D. A. Kirzhnitz and A. D. Linde, Phys. Lett. 42B (1972) 471, Ann. Phys. 101 (1976) 195; L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320; S. Weinberg, Phys. Rev. D9 (1974) 3357.

[2] M. B. Volosin, I. Yu. Kobzarev and L. B. Okun, Yad. Riz. 20 (1974) 1229 [Sov. J. Nucl. Phys. 20 (1975) 644].

[3] S. Coleman, Phys. Rev. D15 (1977) 2929.

[4] G. C. Callan and S. Coleman, Phys. Rev. D16 (1977) 1762.

[5] A. D. Linde, Phys. Lett. 100B (1981) 37.

[6] A. H. Guth and S. H. Tye, Phys. Rev. Lett. 44 (1980) 631, 963; A. H. Guth, Phys. Rev. D23 (1981) 347; A. H. Guth and E. J. Weinberg, Phys. Rev. D23 (1981) 876.

[7] S. W. Hawking, I. G. Moss and J. M. Steward, Bubble Collisions in the Very Early Universe, Cambridge (PAMTP) preprint.

[8] A. D. Linde, Phys. Lett. 108B (1982) 389; A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett 48 (1982) 1220.

[9] S. W. Hawking and I. G. Moss, Phys. Lett. 110B (1982) 35.

[10] R. F. Sawyer, Perturbations of the Expanding Bubble Solution in Scalar Field Theory: Four Dimensional Case, UCSB preprint TH-53 (1982).

[11] M. Horibe and A. Hosoya, Prog. Theor. Phys. 67 (1982) 816.

[12] A. Erdelyi et al., Table of Integral Transforms (Bateman Manuscript Project) vol. 1, (McGraw-Hill, New York, 1954) 30.

Figure Captions

Fig. 1. The potential $U(\varphi)$ that has a stable state φ_- and a meta-stable state φ_+.

Fig. 2. The self-energy graphs with cuts that give the lowest nontrivial contributions to the pair creation amplitude and thus the energy E^p. The dashed line represents the Higgs propagator with mass μ, and the solid line the massless fermion propagator.

Fig. 3. The behavior of the created Higgs particles with $k^2 = 5\mu^2$ and $k_r = \mu T/\pi b$. The cones A, B and C with dots show regions where the particles created at the apexes A, B and C of each cone go through.

Fig. 4. The behaviour of the power of T in (3.24).

Fig. 5. The radial velocity β_r (4.6) of the c.m. system of the massive fermion pairs relative to the wall. The shaded region gives the probability (4.7) of the pairs going out of the bubble.

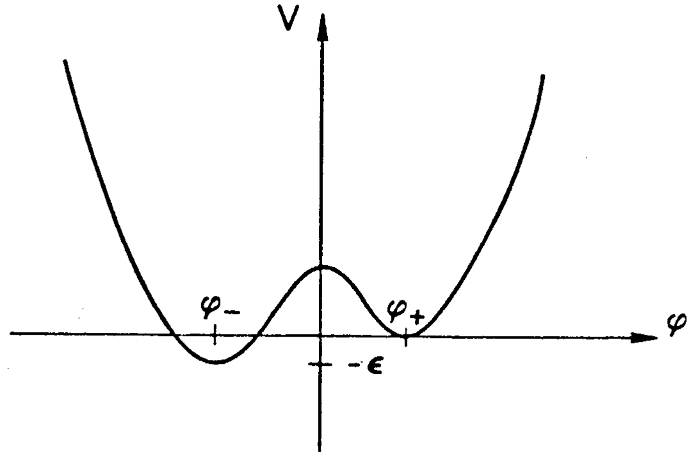

Fig.1

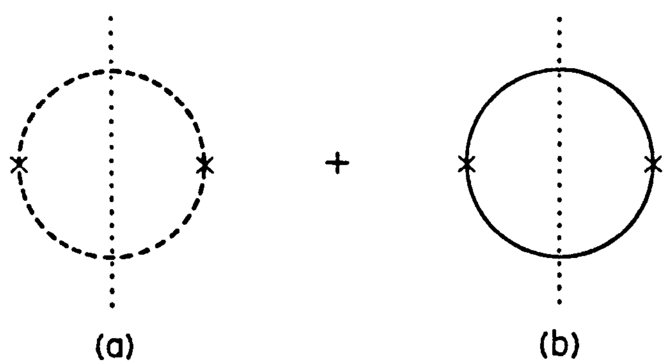

Fig.2

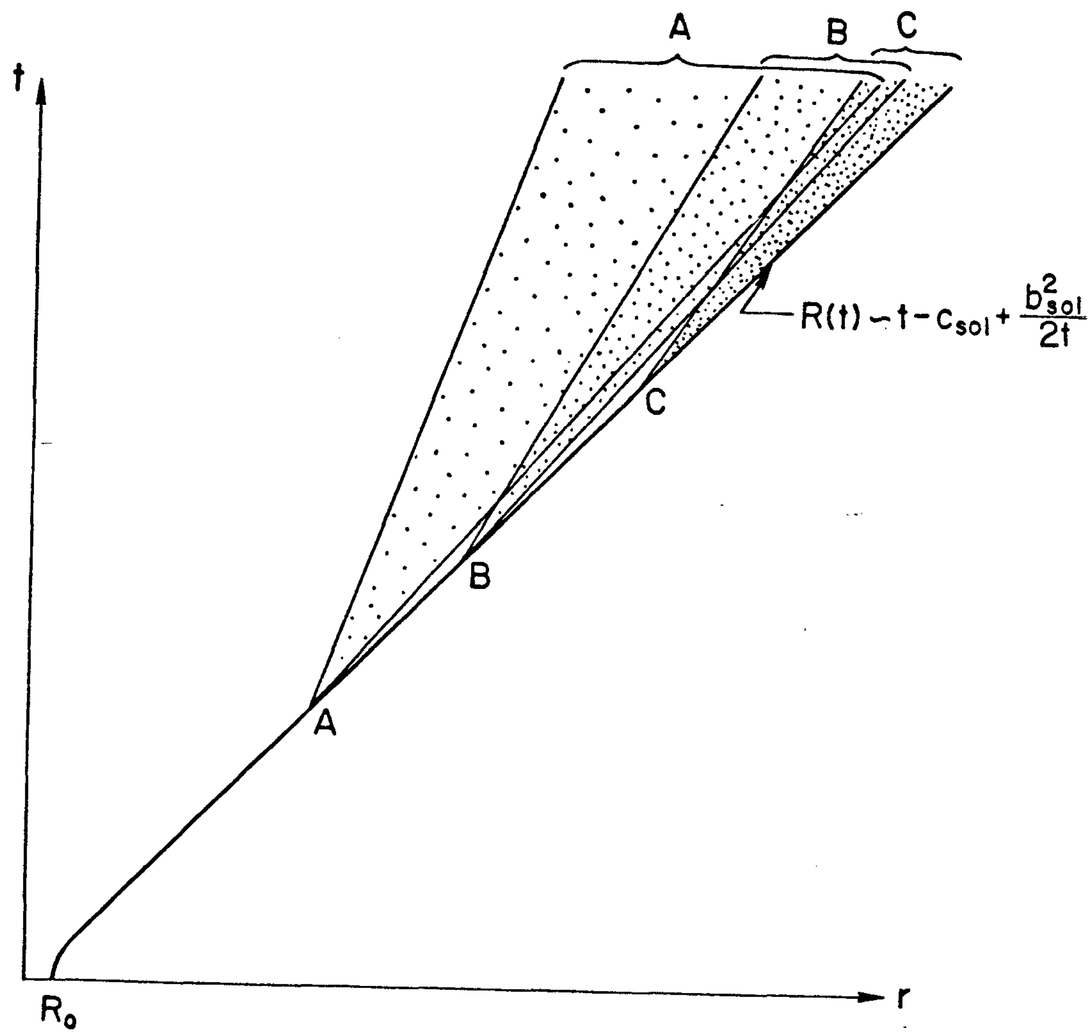

Fig.3

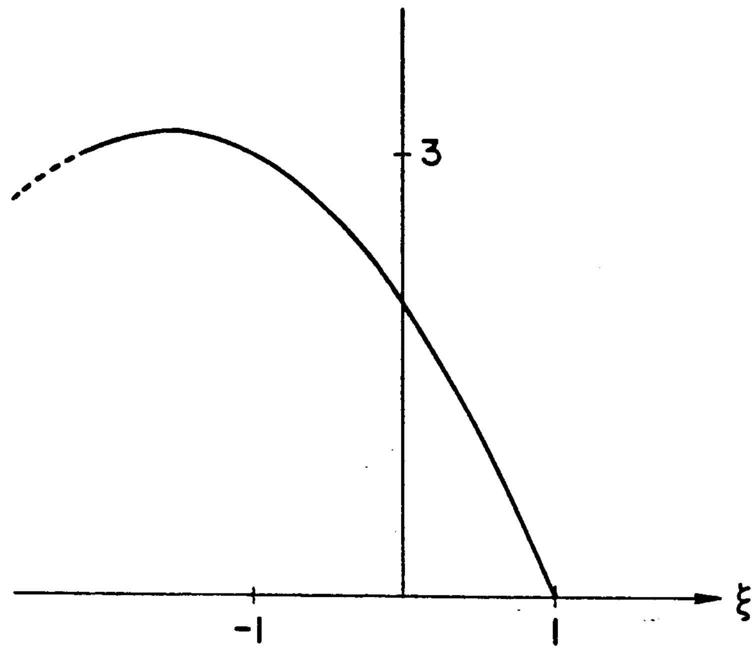

Fig. 4

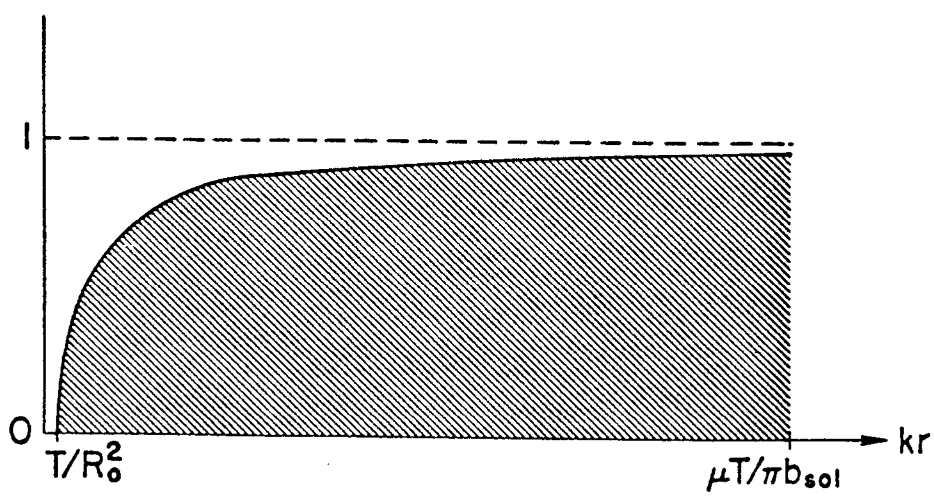

Fig. 5