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Schwinger-Dyson equation constraints on the gluon propagator

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Abstract

The gluon propagator plays a central role in determining the dynamics of QCD. In this work we use the Schwinger-Dyson equation to put analytic constraints on the structure of this propagator. In doing so, we find that the gluon spectral density contains an explicit massless component, but that the coefficient of this component vanishes in Landau gauge QCD, thus confirming the absence of massless gluons from the spectrum of the theory.

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1 Introduction

Understanding the nature of confinement in QCD is crucial for explaining why quarks and gluons are absent from the physical spectrum of the theory [1]. Although there remains much debate surrounding the precise confinement mechanism, it has been understood for many years that the non-perturbative structure of the gluon propagator plays an important role [2]. An issue that has received significant focus in the literature is what happens to the propagator in the low momentum *infrared* regime. Motivated by the issues surrounding gauge fixing, Gribov [3] and Zwanziger [4] proposed a form for the gluon propagator that vanishes in the limit $p^2 \to 0$. Similar forms have also been proposed which suggest that the gluon propagator has an effective mass [5]. In order to test both these and other hypotheses, a mixture of non-perturbative numerical and analytic techniques are often employed. In particular, the computation of the gluon propagator using lattice QCD and the Schwinger-Dyson equations remains a very active area of research [6, 7, 8, 9, 10, 11, 12]. Besides confinement, determining the structure of the gluon propagator is also important for describing other non-perturbative phenomena like the dynamics of quark-gluon plasma [13], a topic which is currently the focus of significant theoretical and experimental interest at facilities such as ALICE (CERN) and RHIC (Brookhaven).

Many of the approaches to analysing the structure of the gluon propagator involve using the Becchi-Rouet-Stora-Tyutin (BRST) quantisation of QCD to work in specific Lorentz covariant gauges. BRST quantisation involves the introduction of additional auxiliary gauge-fixing and ghost degrees of freedom in such a way that the equations of motion are no longer gauge invariant, but remain invariant under a residual BRST symmetry. The physical states are then defined to be those that are annihilated by the conserved charge associated with the BRST symmetry Q_B [14]. A key feature of BRST quantised QCD is that the space of states no longer possesses a positive-definite inner product, and thus negative norm states are permitted. This has the important implication that the momentum space correlation functions are no longer guaranteed to be non-negative [15]. Non-negativity violations of the gluon propagator are of particular relevance since this characteristic is often attributed to the absence of gluons from the physical spectrum [6, 16, 17], and recent numerical studies appear to indicate that these violations do indeed occur [7, 8, 10].

Although significant progress has been made in determining the structure of the gluon propagator, its behaviour remains far from understood. Part of the difficulty is that most of this progress has relied on numerical techniques such as lattice QCD and the solution of the Schwinger-Dyson equations, both of which have significant quantitative uncertainties. In Ref. [18], a more formal analytic approach was developed in order to determine the most general non-perturbative features of vector boson propagators. This approach involved the application of a rigorous quantum field theory framework, the construction of which is based on a series of physically motivated axioms [14, 15, 19, 20, 21]. Since these axioms are assumed to hold independently of the coupling regime, this allows genuine non-perturbative properties to be derived in a purely analytic manner. It turns out that because BRST quantised QCD involves a space of states with an indefinite inner product, this opens up the possiblity of the gluon vector propagator containing singular terms involving derivatives of $\delta(p)$ [18], a feature which can be indicative of confinement [22, 23, 24]. Nevertheless, it remains unclear whether the solutions of the gluon propagator derived using numerical techniques are actually sensitive to this type of singular behaviour, and so this further emphasises the need for a purely analytic approach[1]. In this paper we adopt such an approach. Instead of solving the Schwinger-Dyson equation explicitly, we use this equation to derive novel analytic constraints on the gluon propagator.

[1] Analytic approaches to constraining the non-perturbative structure of propagators have been pursued before, but have often relied on additional input such as the operator product expansion [25].

2 The gluon propagator in QCD

Before exploring the constraints that the Schwinger-Dyson equation imposes on the structure of the gluon propagator in BRST quantised QCD, it is important to first outline the dynamical characteristics of this theory, and the explicit form of the Schwinger-Dyson equation itself. The equations of motion of BRST quantised QCD are defined by

$$(D^\nu F_{\nu\mu})^a + \partial_\mu \Lambda^a = g j^a_\mu - igf^{abc}\partial_\mu \overline{C}^b C^c, \quad \partial^\mu A^a_\mu = \xi \Lambda^a, \tag{2.1}$$

$$\partial^\nu (D_\nu C)^a = 0, \qquad (D^\nu \partial_\nu \overline{C})^a = 0, \tag{2.2}$$

where C^a and $\overline{C}^a$ are the ghost and anti-ghost fields, Λ^a is an auxiliary field, and ξ is the renormalised gauge fixing parameter. It follows from Eq. (2.1) that the renormalised gluon field satisfies

$$\left[\partial^2 g_\mu{}^\alpha - \left(1 - \frac{1}{\xi_0}\right)\partial_\mu \partial^\alpha\right] A^a_\alpha = \mathcal{J}^a_\mu, \tag{2.3}$$

where ξ_0 is the *bare* gauge fixing parameter and $\mathcal{J}^a_\mu$ has the form

$$\mathcal{J}^a_\mu = g j^a_\mu - igf^{abc}\partial_\mu \overline{C}^b C^c + (Z_3^{-1} - 1)\partial_\mu \Lambda^a - igf^{abc}A^{b\nu}F^c_{\nu\mu} - gf^{abc}\partial^\nu (A^b_\nu A^c_\mu), \tag{2.4}$$

with Z_3 the gluon field renormalisation constant and j^a_μ the matter current. Furthermore, one assumes that the renormalised fields satisfy the following equal-time commutation relations:

$$\left[\Lambda^a(x), \Lambda^b(y)\right]_{x_0=y_0} = 0, \qquad \left[\Lambda^a(x), A^b_\nu(y)\right]_{x_0=y_0} = i\delta^{ab} g_{0\nu}\delta(\mathbf{x}-\mathbf{y}), \tag{2.5}$$

$$\left[A^a_\mu(x), A^b_\nu(y)\right]_{x_0=y_0} = 0, \qquad \left[F^a_{0i}(x), A^b_\nu(y)\right]_{x_0=y_0} = i\delta^{ab} g_{i\nu} Z_3^{-1}\delta(\mathbf{x}-\mathbf{y}). \tag{2.6}$$

Since the gluon propagator is defined by

$$\langle 0|T\{A^a_\mu(x)A^b_\nu(y)\}|0\rangle := \theta(x^0 - y^0)\langle 0|A^a_\mu(x)A^b_\nu(y)|0\rangle + \theta(y^0 - x^0)\langle 0|A^b_\nu(y)A^a_\mu(x)|0\rangle, \tag{2.7}$$

one can directly apply the dynamical conditions in Eqs. (2.3), (2.5) and (2.6) to this definition, and in doing so this implies the Schwinger-Dyson equation

$$\left[\partial^2 g_\mu{}^\alpha - \left(1 - \frac{1}{\xi_0}\right)\partial_\mu \partial^\alpha\right] \langle 0|T\{A^a_\alpha(x)A^b_\nu(y)\}|0\rangle = i\delta^{ab} g_{\mu\nu} Z_3^{-1}\delta(x-y) + \langle 0|T\{\mathcal{J}^a_\mu(x)A^b_\nu(y)\}|0\rangle, \tag{2.8}$$

which in momentum space has the form

$$-\left[p^2 g_\mu{}^\alpha - \left(1 - \frac{1}{\xi_0}\right)p_\mu p^\alpha\right] \widehat{D}^{ab\,F}_{\alpha\nu}(p) = i\delta^{ab} g_{\mu\nu} Z_3^{-1} + \widehat{J}^{ab}_{\mu\nu}(p), \tag{2.9}$$

where $\widehat{J}^{ab}_{\mu\nu}(p) := \mathcal{F}\left[\langle 0|T\{\mathcal{J}^a_\mu(x)A^b_\nu(y)\}|0\rangle\right]$. In what follows we will demonstrate that Eq. (2.9) imposes non-trivial analytic constraints on the structure of the gluon propagator.

In order to explicitly understand the constraints imposed on the gluon propagator $\widehat{D}^{ab\,F}_{\mu\nu}(p)$ by Eq. (2.9), one must consider the spectral representation of both $\widehat{D}^{ab\,F}_{\mu\nu}(p)$ and the current propagator $\widehat{J}^{ab}_{\mu\nu}(p)$ involving the non-conserved current $\mathcal{J}^a_\mu$. In Ref. [18] it was shown from Eqs. (2.1), (2.5) and (2.6) that the momentum space gluon propagator has the general form

$$\widehat{D}^{ab\,F}_{\mu\nu}(p) = i\int_0^\infty \frac{ds}{2\pi}\, \frac{\left[g_{\mu\nu}\rho^{ab}_1(s) + p_\mu p_\nu \rho^{ab}_2(s)\right]}{p^2 - s + i\epsilon} + \sum_{n=0}^{N}\left[c^{ab}_n\, g_{\mu\nu}(\partial^2)^n + d^{ab}_n \partial_\mu \partial_\nu (\partial^2)^{n-1}\right]\delta(p), \tag{2.10}$$

where c_n^{ab} and d_n^{ab} are complex coefficients which are linearly related[2] for $n \geq 1$. As previously discussed, the possibility of non-vanishing terms in Eq. (2.10) involving derivatives of $\delta(p)$ arises because the space of states in BRST quantised QCD has an indefinite inner product. As well as this general structural form, it turns out that the spectral densities $\rho_1^{ab}(s)$ and $\rho_2^{ab}(s)$ also satisfy the following conditions [18]

$$\rho_1^{ab}(s) + s\rho_2^{ab}(s) = -2\pi\xi\delta^{ab}\delta(s), \quad \int_0^\infty ds\, \rho_1^{ab}(s) = -2\pi\delta^{ab}Z_3^{-1}, \quad \int_0^\infty ds\, \rho_2^{ab}(s) = 0. \tag{2.11}$$

These constraints demonstrate that the gluon propagator contains only one independent spectral density, as expected. However, subtleties arise if one attempts to express the gluon propagator exclusively in terms of $\rho_1^{ab}(s)$ [18]. For this reason we will therefore keep both spectral densities explicit in the proceeding analysis.

Next, consider the structure of the propagator $\widehat{J}_{\mu\nu}^{ab}(p)$. The first constraint on this propagator arises from the fact that one can write the equations of motion for the gluon field as

$$\partial^\nu F_{\nu\mu}^a = gJ_\mu^a + \left\{Q_B, (D_\mu\overline{C})^a\right\}, \tag{2.12}$$

where $\partial^\mu J_\mu^a = 0$, and Q_B is the BRST operator [14]. By combining this equation with Eq. (2.1), the divergence of the current $\mathcal{J}_\mu^a$ can be written

$$\partial^\mu \mathcal{J}_\mu^a = \left\{Q_B, Z_3^{-1}\partial^2\overline{C}^a + (\partial^\mu D_\mu\overline{C})^a\right\}. \tag{2.13}$$

Using Eq. (2.2) together with the fact that $Q_B|0\rangle = 0$, it then follows that the correlator $\langle 0|\mathcal{J}_\mu^a(x)A_\nu^b(y)|0\rangle$ satisfies the condition

$$\partial_x^\mu \partial_y^\nu \langle 0|\mathcal{J}_\mu^a(x)A_\nu^b(y)|0\rangle = 0. \tag{2.14}$$

Using an analogous analysis as in the case of the gluon propagator [18], this condition implies that $\widehat{J}_{\mu\nu}^{ab}(p)$ has the same overall structural form

$$\widehat{J}_{\mu\nu}^{ab}(p) = i\int_0^\infty \frac{ds}{2\pi}\, \frac{\left[g_{\mu\nu}\widetilde{\rho}_1^{ab}(s) + p_\mu p_\nu \widetilde{\rho}_2^{ab}(s)\right]}{p^2 - s + i\epsilon} + \sum_{n=0}^{\widetilde{N}} \left[C_n^{ab}\, g_{\mu\nu}(\partial^2)^n + D_n^{ab}\partial_\mu\partial_\nu(\partial^2)^{n-1}\right]\delta(p), \tag{2.15}$$

where C_n^{ab} and D_n^{ab} are complex parameters which are related in the same manner as c_n^{ab} and d_n^{ab} in Eq. (2.10). Moreover, Eq. (2.14) implies that the spectral densities of this correlator are also not independent, and are in fact related as follows

$$\widetilde{\rho}_1^{ab}(s) + s\widetilde{\rho}_2^{ab}(s) = \widetilde{C}^{ab}\delta(s), \tag{2.16}$$

where $\widetilde{C}^{ab}$ is a constant coefficient. In order to determine $\widetilde{C}^{ab}$, one can consider the contracted propagator expression $p^\mu p^\nu \widehat{J}_{\mu\nu}^{ab}(p)$, which due to Eqs. (2.15) and (2.16) can be written

$$p^\mu p^\nu \widehat{J}_{\mu\nu}^{ab}(p) = \frac{i}{2\pi}p^2 \int_0^\infty ds\, \widetilde{\rho}_2^{ab}(s) + \frac{i}{2\pi}\widetilde{C}^{ab}. \tag{2.17}$$

Since $\widehat{J}_{\mu\nu}^{ab}(p)$ is defined by Eq. (2.9), contracting this equation with $p^\mu p^\nu$ gives an explicit expression for $p^\mu p^\nu \widehat{J}_{\mu\nu}^{ab}(p)$. In doing so, it follows from the Slavnov-Taylor identity[3] that

$$p^\mu p^\nu \widehat{J}_{\mu\nu}^{ab}(p) = 0, \tag{2.18}$$

[2] For $n = 0$, c_n^{ab} is unconstrained but d_n^{ab} vanishes [18].

[3] In this notation, the Slavnov-Taylor identity has the form $p^\mu p^\nu \widehat{D}_{\mu\nu}^{ab\,F}(p) = -i\xi\delta^{ab}$.

which in comparison with Eq. (2.17) therefore implies the spectral density constraints

$$\widetilde{\rho}_1^{ab}(s) + s\widetilde{\rho}_2^{ab}(s) = 0 \qquad (\widetilde{C}^{ab} = 0), \tag{2.19}$$

$$\int_0^\infty ds\, \widetilde{\rho}_2^{ab}(s) = 0. \tag{2.20}$$

Having derived the spectral structure of both the gluon and current propagators, one can now determine the explicit constraints imposed by Eq. (2.9). Inserting Eqs. (2.10) and (2.15) into Eq. (2.9), and separately equating[4] the purely singular terms involving derivatives of $\delta(p)$, one obtains

$$\left[-p^2 g_\mu^{\ \alpha} + \left(1 - \frac{1}{\xi_0}\right) p_\mu p^\alpha\right] \left[\sum_{n=0}^{N} \left[c_n^{ab}\, g_{\alpha\nu}(\partial^2)^n + d_n^{ab}\partial_\alpha\partial_\nu(\partial^2)^{n-1}\right]\delta(p)\right]$$
$$= \sum_{n=0}^{\widetilde{N}} \left[C_n^{ab}\, g_{\mu\nu}(\partial^2)^n + D_n^{ab}\partial_\mu\partial_\nu(\partial^2)^{n-1}\right]\delta(p), \tag{2.21}$$

$$\left[-p^2 g_\mu^{\ \alpha} + \left(1 - \frac{1}{\xi_0}\right) p_\mu p^\alpha\right] \left[i\int_0^\infty \frac{ds}{2\pi}\, \frac{\left[g_{\alpha\nu}\rho_1^{ab}(s) + p_\alpha p_\nu \rho_2^{ab}(s)\right]}{p^2 - s + i\epsilon}\right]$$
$$= i\delta^{ab} g_{\mu\nu} Z_3^{-1} + \left[i\int_0^\infty \frac{ds}{2\pi}\, \frac{\left[g_{\mu\nu}\widetilde{\rho}_1^{ab}(s) + p_\mu p_\nu \widetilde{\rho}_2^{ab}(s)\right]}{p^2 - s + i\epsilon}\right]. \tag{2.22}$$

Expanding out the left-hand-side of Eq. (2.21) it follows that the coefficients c_n^{ab} and d_n^{ab} are directly related to C_n^{ab} and D_n^{ab}. In particular, one has the relation

$$c_{n+1}^{ab} = -\frac{(2n+5)}{4(2n+3)(n+1)(n+3)} C_n^{ab}, \qquad n \geq 0 \tag{2.23}$$

Since both c_n^{ab}, d_n^{ab}, and C_n^{ab}, D_n^{ab} are separately linearly related, Eq. (2.23) implies that all of these parameters must be linearly related to one another. The significance of these relations is that they demonstrate that the coefficients of terms involving derivatives of $\delta(p)$ in the gluon propagator (c_n^{ab} and d_n^{ab} for $n \geq 1$) are proportional to the coefficients of $\delta(p)$ and derivatives of $\delta(p)$ in $\widehat{J}_{\mu\nu}^{ab}(p)$. In particular, for $n = 0$ Eq. (2.23) implies that if $\widehat{J}_{\mu\nu}^{ab}(p)$ has a non-vanishing $\delta(p)$ term, this is sufficient to prove that the gluon propagator contains a $\partial^2\delta(p)$ component. This characteristic is particularly relevant in the context of confinement, since the appearance of singular terms involving non-vanishing derivatives of $\delta(p)$ is related to the violation of the cluster decomposition property [22, 23, 24], which itself is a sufficient condition for confinement [14, 26]. Eq. (2.23) therefore demonstrates that the singular structure of the interaction current propagator $\widehat{J}_{\mu\nu}^{ab}(p)$ plays an important role in understanding this phenomenon.

In order to derive the constraints imposed by Eq. (2.22), one must expand this equation and then separately equate the terms on both sides which depend on $g_{\mu\nu}$ and $p_\mu p_\mu$. In doing so,

[4]Since the terms involving deriviatives of $\delta(p)$ have support only at $p = 0$, whereas the other terms are defined to have support outside $p = 0$ (in the closed forward light cone) [15], this justifies why these terms can be separately equated.

this implies the relations

$$-p^2\int_0^\infty \frac{ds}{2\pi}\,\frac{\rho_1^{ab}(s)}{p^2-s+i\epsilon} = \delta^{ab}Z_3^{-1} + \int_0^\infty \frac{ds}{2\pi}\,\frac{\widetilde{\rho}_1^{ab}(s)}{p^2-s+i\epsilon}, \tag{2.24}$$

$$\int_0^\infty \frac{ds}{2\pi}\,\frac{\left(1-\frac{1}{\xi_0}\right)\rho_1^{ab}(s) - \frac{1}{\xi_0}p^2\rho_2^{ab}(s)}{p^2-s+i\epsilon} = \int_0^\infty \frac{ds}{2\pi}\,\frac{\widetilde{\rho}_2^{ab}(s)}{p^2-s+i\epsilon}. \tag{2.25}$$

Using the constraints in Eq. (2.11), it follows from Eq. (2.24) that $\rho_1^{ab}(s)$ satisfies the equality

$$s\rho_1^{ab}(s) + \widetilde{\rho}_1^{ab}(s) = 0, \tag{2.26}$$

which in combination with Eq. (2.19) implies

$$s\left[\rho_1^{ab}(s) - \widetilde{\rho}_2^{ab}(s)\right] = 0. \tag{2.27}$$

In order to solve this equation it is important to recognise that because spectral densities are distributions, not functions, the solution is not necessarily continuous[5]. In fact, the general solution of Eq. (2.27) has the form: $\rho_1^{ab}(s) - \widetilde{\rho}_2^{ab}(s) = A^{ab}\delta(s)$, where A^{ab} is a constant coefficient [15]. By applying the integral constraints in Eqs. (2.11) and (2.20) this fixes the coefficient to: $A^{ab} = -2\pi\delta^{ab}Z_3^{-1}$, and hence

$$\rho_1^{ab}(s) = -2\pi\delta^{ab}Z_3^{-1}\delta(s) + \widetilde{\rho}_2^{ab}(s). \tag{2.28}$$

Applying an analogous approach to Eq. (2.25) subsequently leads to the following constraint

$$s\rho_2^{ab}(s) = 2\pi\delta^{ab}\left(Z_3^{-1} - \xi\right)\delta(s) - \widetilde{\rho}_2^{ab}(s). \tag{2.29}$$

In general, Eqs. (2.28) and (2.29) demonstrate that the behaviour of the gluon spectral densities is completely determined by the spectral densities of the current propagator $\widehat{J}_{\mu\nu}^{ab}(p)$. Moreover, Eq. (2.28) implies that $\rho_1^{ab}(s)$ contains an explicit massless contribution, which has an overall Z_3^{-1} coefficient. Since Z_3^{-1} is expected to vanish in Landau gauge QCD [6], massless gluons must therefore necessarily be absent from the spectrum of the theory[6]. This analytic result runs contrary to the hypothesis supported by numerical studies [7, 8, 10, 12], that a massless gluon is absent in Landau gauge because the spectral density violates non-negativity. Since the other component of the spectral density satisfies the sum rule $\int ds\, \widetilde{\rho}_2^{ab}(s) = 0$, it follows that $\widetilde{\rho}_2^{ab}(s)$ is in principle (continuously) negative over some range of s. It is therefore plausible that these studies are in fact probing this component of the spectral density. In other words, both lattice QCD and the numerical solutions of the Schwinger-Dyson equations are potentially insensitive to singular massless contributions, and hence the observation of non-negativity violation of the spectral density is actually a reflection of the sum rule for $\widetilde{\rho}_2^{ab}(s)$.

3 Conclusions

In this work we have demonstrated for the first time that the Schwinger-Dyson equation imposes non-trivial analytic constraints on the non-perturbative structure of the gluon propagator. These constraints imply that the gluon spectral density explicitly contains a massless component, but

[5] See Ref. [18] for a general discussion of this issue.

[6] Performing the same analytic procedure for the photon propagator would also result in a massless spectral density component with a Z_3^{-1} prefactor, where now Z_3 is the photon renormalisation constant. However, because Z_3 is expected to be finite in QED [6], and Z_3^{-1} non-vanishing, this instead guarantees that a massless photon *is* present in the spectrum.

that the coefficient of this component vanishes in Landau gauge QCD, hence ruling out the presence of massless gluons in the spectrum of the theory. As well as the purely theoretical relevance of this result, these constraints could provide important input for improving existing parametrisations of the propagator.

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