There are two talks in the beam-beam session. But beam-beam is an issue that permeates in several other sessions. So in this summary I have taken the liberty to include some materials extracted also from other sessions.

FLIP-FLOP

The first talk was “Flip-flop instability in FCC-ee at low energies” by Dmitry Shatilov.

The old flip-flop as we know it is a 1D effect. A new intriguing 3D flip-flop is now discovered for strong-weak cases when the beam intensity asymmetry exceeds ~10%. The instability mechanism is rather involved, requiring several ingredients. Missing one of them removes the instability. Ingredients include:

1. asymmetry in beam intensities
2. beamstrahlung
3. crossing beams
4. x-y coupling

This flip-flop instability has a beam intensity threshold. Below a certain threshold, even asymmetric beams do not become unstable. The threshold can be increased by lowering $\beta_x^*$ (and raising $\varepsilon_x$ holding luminosity fixed).

A slide from Dmitry Shatilov:

The flip-flop threshold depends on asymmetry in the bunch currents. But even in symmetrical case the flip-flop was observed (with larger bunch population).
SIMULATIONS

The second talk was “FCC-ee beam-beam strong-strong simulations for all working and mitigation” by Kazuhito Ohmi.

For FCCee at Higgs energy, it was found that the beam-beam limit behaves rather differently for a strong-weak case and a strong-strong case. For a strong-weak case, it was found that the beam-beam limit $\xi$ depends sensitively on the choice of the working point. For one working point, it can be as high as $\xi = 0.6$, while for another working point it is 0.2. Two observations can be made:

- The fact that a strong-weak case can have large beam-beam limit is in sharp contrast with the prediction by 3D flip-flop (as in the previous talk), where it was observed that a small asymmetry in beam intensities leads to a strong instability. The present-day beam-beam is a subtle subject involving multiple parameters and multiple physical mechanisms. Careful and complete considerations are necessary to draw final conclusions.
- The sensitivity to working point apparently appears when the working point is in the proximity to $\frac{1}{2}$ tunes.

In contrast to the strong-weak case, the strong-strong cases seem to converge to a beam-beam limit $\xi = 0.2$ at the Higgs energy, insensitive to the choice of working point. Very interesting is the observation that in the FCCee case with crossing beams, there is a strong beam-beam-induced high-mode coherent x-z oscillation, while the lowest x-z mode is stable. This oscillation becomes more serious at the Z energies, when the beam-beam limit is reduced to 0.06. It was further observed that these x-z oscillations can be removed by substantially lowering $\beta_x^*$ and raising $\varepsilon_x$, curiously the same trick to cure the 3D flip-flop instability.
Two slides from Kazuhito Ohmi showing the beam-beam limits:

**ξ** limit for FCC-ee H

No clear difference for (0.54,0.61) from weak-strong ξ_{lim} = 0.2. Big difference for (0.51,0.55), the limit in weak-strong is extremely high ξ_{lim} = 0.6 (ws), ξ_{lim} = 0.2 (ss).

ξ limit is weakly dependent of tune in Strong-strong simulation.

It is possible to achieve ξ_L = 0.15 for tlep-H.

---

**ξ** limit in Z factory

- Error bar is fluctuation due to coherent motion
- The beam-beam parameter is limited 0.06, and coherent oscillation
- Big difference from the weak-strong results ξ_{lim} = 0.2 at (0.51,0.55).
- No big difference ξ_{lim} = 0.06(ss), 0.1(ws) in (0.54,0.61).

ξ_s = 0.13
ξ_{target} = 0.17
L_{target} = 2.2 × 10^{36} cm^{-2}s^{-1}
One more slide from Kazuhito Ohmi showing the beam-beam induced high-mode x-z oscillation:

**Simulated Beam distribution using simple model II**

![Simulated Beam distribution](image)

Lab frame (not collision frame)

**BEAM-BEAM LIMIT, WHICH FORMULA TO USE?**

The beam-beam limit formulae used in the designs of FCCee and CEPC are different! (And you think a basic formula like this should have long been settled?) The formula used for the FCCee design is

$$\xi_y = \frac{r_e N_b}{2 \pi} \frac{\beta_y^*}{\sigma_x^\star \sigma_y^\star \sqrt{1 + \phi_{piw}^2}}$$

This formula is based on a physical model that treats the beam-beam effect as nonlinear resonances. It predicts a beam-beam limit of $\xi_y = 0.16$ for FCCee.

The beam-beam limit formula used in the CEPC design is

$$\xi_y = \frac{2845}{2\pi} \sqrt{\frac{U_0}{2\gamma E_0 N_{IP}}} \times F_l$$

where $F_l$ is the beam-beam limit enhancement factor by crab waist scheme and so far it is assumed to be 1.6 for Higgs and 2.6 for Z by the CEPC design [Reference: J. Gao, Nucl. Instrum. Methods A 533, 270 (2004)]. This formula is based on a diffusion model treating beam-beam kicks as random noise. It predicts $\xi_y = 0.11$ for CEPC. It notably has a dependence on the number of interaction points $N_{IP}$, and it does not depend on the tunes.
The two formulae have completely different parameter dependences and completely different scalings. Past experience seemed to declare beam-beam limit values closer to the CEPC prediction. On the other hand, latest simulations seem to confirm the FCCee prediction. The two models assume two extreme opposite physical pictures. The nonlinear resonances picture assumes perfect correlation from one beam-beam kick to the next (e.g., perfect correlation is assumed at least for the number of turns in a simulation), while the diffusion picture assumes a complete loss of phase correlation between kicks even in the same turn. Which is correct? One must feel widely unsatisfying when the two most prominent (and costly) colliders of today have used formulae so different as their most basic and the very first design equation!

LONG RANGE BEAM-BEAM EFFECTS

This is no longer a critical issue with a new partial-double-ring design at CEPC.

BEAMSTRAHLUNG

Beamstrahlung is a new issue, but is now well accepted as it should. The need of a flat beam at the collision point and the need of a very large energy aperture are taken into design considerations, affecting the design very seriously.
beamstrahlung – potential limit at 175 GeV

synchrotron radiation in the field of opposing bunch at the IPs, ‘beamstrahlung’, can become lifetime limit for large bunch populations, small horizontal beam size & short bunches

\[
\tau_{\text{ip}} \approx \frac{p^{3/2}}{\gamma^2 \sigma_z} \sqrt{\eta} \exp(A \eta p / \gamma^2), \quad \frac{1}{\rho} \text{sr} \frac{N_{e}}{\gamma \sigma_{x} \sigma_{z}}
\]

\(\eta\): ring energy acceptance

lifetime expression by V. Telnov, modified version by A. Bogomyagkov et al.

for acceptable lifetime, \(\rho \times \eta\) must be sufficiently large

- flat beams (large \(\sigma_{x}\))
- bunch length
- large momentum acceptance: aiming for \(\geq 1.5\%\) at 175 GeV
  - LEP: <1% acceptance, SuperKEKB \(\sim 1.5\%\)

V. Telnov, A. Bogomyagkov,
E. Levichev, D. Shatilov, et al.

Another slide from Frank Zimmermann:

beamstrahlung effect at lower energy

IP photon emission increases equilibrium energy spread & bunch length

\[
A \equiv 1.4 \frac{n_{ip} \tau_{SR} \epsilon}{4 T_{\text{rev}}} \frac{N_{e}^{2} \gamma^{2}}{\alpha \sigma_{x}^{2}}
\]

\[
\sigma_{\delta,\text{tot}}^{2} - \sigma_{\delta,\text{SR}}^{2} = A \left( \frac{\sigma_{\delta,\text{SR}}}{\sigma_{\delta,\text{tot}}^{2}} \frac{1}{\sigma_{\delta,\text{SR}}^{2}} \right)^{2}
\]

\[
\sigma_{\delta,\text{tot}}^{2} = \left[ \frac{1}{2} \sigma_{\delta,\text{SR}}^{2} + \left( \frac{1}{4} \sigma_{\delta,\text{SR}}^{4} + A \frac{\sigma_{\delta,\text{SR}}^{2}}{\sigma_{\delta,\text{SR}}^{2}} \right)^{1/2} \right]^{1/2}
\]

huge effect: factor 2-5 increase possible! included in the design parameters

On the other hand, beamstrahlung induced background is not considered too serious for FCCee and CEPC.

Slide from Qinglei Xiu:

### Beamstrahlung

<table>
<thead>
<tr>
<th>Symbol</th>
<th>LEP2</th>
<th>CEPC</th>
<th>ILC 250GeV</th>
<th>ILC 500GeV</th>
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<tbody>
<tr>
<td>$E_{cm}$ [GeV]</td>
<td>209</td>
<td>240</td>
<td>250</td>
<td>500</td>
</tr>
<tr>
<td>$N \times 10^{10}$</td>
<td>58</td>
<td>37.1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_x / \sigma_y$ [mm]</td>
<td>270000/350</td>
<td>73700 / 160</td>
<td>729 / 7.7</td>
<td>474 / 5.9</td>
</tr>
<tr>
<td>$\sigma_y$ [$\mu m$]</td>
<td>16000</td>
<td>2260</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>$\beta_x / \beta_y$ [mm]</td>
<td>1500/50</td>
<td>800 / 1.2</td>
<td>13 / 0.41</td>
<td>11 / 0.48</td>
</tr>
<tr>
<td>$\gamma_x / \gamma_y$ [mm$\cdot$mrad]</td>
<td>9.81/0.051</td>
<td>1594.5 / 4.79</td>
<td>10 / 0.035</td>
<td>10 / 0.035</td>
</tr>
<tr>
<td>($\gamma$)</td>
<td>2.5e-5</td>
<td>4.7e-4</td>
<td>0.02</td>
<td>0.06</td>
</tr>
</tbody>
</table>

- Characterized by: $\langle \gamma \rangle = \frac{5}{6} \frac{N \gamma^2}{\alpha(\sigma_x+\sigma_y)\sigma_x}$
- The effects of beamstrahlung at CEPC should be much smaller than ILC

### ROUND BEAMS

Beam-beam effect is expected to become weaker for round beams because the system becomes effectively 1D and the nonlinear dynamical effects become weaker. This is particularly suggested for low energy colliders. For the FCCee and CEPC, however, round beams are ruled out due to beamstrahlung.

### INTERPLAY OF BEAM-BEAM AND LATTICE NONLINEARITIES

This issue was mentioned a few times at the workshop. One example is that the IR nonlinearities (there is no shortage of them!) plus the nonlinearities of the crab waist sextupoles mess up the ingenious and delicate crab waist function. Cancellation technique is needed to further improve the crab waist scheme.
A slide from Yukiyoshi Ohnishi:

- Cancellation between crab-waist sextupoles does not work because the transfer map is not linear between them.
- The sources of nonlinear terms are fringe field of the final focus quadrupole and kinematic terms from the drift space in the vicinity of IP.

Another slide from Qing Qin:

- Cottalk between beam-beam effects and lattice nonlinearities
- \(|C_{11}| < 200\)
SUMMARY

- Important progress is being made. New beam-beam effects are discovered and studied.

- With performance being pushed so hard, more subtleties that were unimportant in the past now arise. New effects keep being discovered.
  (a) the requirement of crab waist
  (b) effect of residual nonlinearities after the crab waist cancellation
  (c) beamstrahlung
  (d) 3D flip-flop instability
  (e) coherent x-z oscillation
  (f) interplay with lattice nonlinearities
  (g) interplay with collective effects (no discussion at this workshop)
  (h) etc.

  Beam-beam issue is more critical than ever.

- But not all pieces have been settled, including the most basic design formula of the beam-beam limit. As we explore deeper, it is expected that more serious learning is still ahead. It is suggested that there should be at least 10 talks in the beam-beam session in the next 2018 workshop.

- SUPERKEKB and HL-LHC should play important roles in the learning process.

This work was supported by U.S. DOE Contract No. DE-AC02-76SF00515.