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COHERENT RADIATION IN WHISPERING GALLERY MODES*

Robert Warnock, SLAC National Accelerator Laboratory, Stanford, CA 94309, U.S.A.†
John Bergstrom, Canadian Light Source, Saskatoon, SK S7N 0X4, Canada‡

Abstract

Theory predicts that Coherent Synchrotron Radiation in storage rings should appear in whispering gallery modes [1], which are resonances of the entire vacuum chamber and are characterized by their high frequencies and concentration of the field near the outer wall of the chamber. The theory assumes that the chamber is a smooth circular torus. We observe that a power spectrum from the NSLS-VUV ring [2], which has a vacuum chamber in bends like that of the model, shows a series of sharp peaks with frequencies close to those of the theory. Sharp peaks are also seen in highly resolved spectra at the Canadian Light Source[3], and those are invariant in position under large changes in the machine setup (energy, fill pattern, bunch length, CSR bursting or steady, optical beam line, etc.). Invariance of the spectrum suggests that it is due to resonances like whispering gallery modes, but they must be strongly perturbed from the circular case because of large outer wall excursions at the two IR ports.

INTRODUCTION

Coherent synchrotron radiation (CSR) from electron storage rings is now well established, in both bursting and continuous modes, following initial observations of the bursting mode reported in 2000-2002. To make a convincing theory of these phenomena one needs a good understanding of the radiation impedance in the presence of the metallic walls of the vacuum chamber. One often models the chamber by two infinite parallel plates, with the beam circulating in the midplane. This model accounts for the observed wavelength threshold of CSR in some machines, and perhaps some qualitative features of beam dynamics and bursting patterns. It is not clear that it is adequate for a fully realistic theory. A model closer to reality, that of a circular torus with rectangular cross section, gives a much different physical picture, in that the impedance has poles in the frequency plane, corresponding to resonances of the whole structure.[1]. We call these *whispering gallery modes*, since they are precisely analogous to acoustical modes in a cylindrical structure that were first explained theoretically by Lord Rayleigh [4]. As in the acoustical case, the wave function is concentrated near the outer wall and is described by Bessel functions of high order.

The resonances of the closed torus lead to a series of peaks in the power spectrum. If a similar spectrum should appear in a real machine it might be observed with a spectrometer of sufficient resolution, for instance one based on a Michelson interferometer. Unfortunately, the possibility of comparing data from such an instrument with theory was ignored until 2007, when the second named author noticed that results from the Brookhaven VUV light source, published in 2001, had a striking resemblance to theory. Here we review the comparison and comment on more recent data from the CLS.

THEORETICAL PREDICTION

One can solve analytically for the fields in a vacuum chamber consisting of a circular torus with perfectly conducting walls and rectangular cross section[1]. The inner wall of the torus has radius a, the outer wall radius b, and the height of the chamber is h. The field equations are solved in cylindrical coordinates (r, θ, z), with the vertical distance z measured along the symmetry axis, thus $a \le r \le b$, $0 \le \theta \le 2\pi, -h/2 \le z \le h/2$. The solution proceeds by Fourier expansions in θ and z, with corresponding wave numbers n and $\alpha_p = \pi p/h$, and a Laplace transform in time with the variable $-i\omega$ congugate to t. The boundary conditions on the planar surfaces $z = \pm h/2$ are met by choosing the proper Fourier expansions in z, while those on cylindrical surfaces are met by choosing the right linear combinations of Bessel functions of argument $\gamma_p r$, where $\gamma_p^2 = (\omega/c)^2 - \alpha_p^2$. The ensuing formula for the longitudinal impedance, given in Eq.(3.9) of Ref.[1], has poles in the ω-plane corresponding to whispering gallery resonances. The resonance conditions are

$$\begin{aligned} p_n(\gamma_p(\omega)b, \gamma_p(\omega)a) &= 0 \ \ (\mathrm{TM})\ , \\ s_n(\gamma_p(\omega)b, \gamma_p(\omega)a) &= 0 \ \ (\mathrm{TE})\ , \end{aligned} \tag{1}$$

in terms of thee Bessel function cross products $p_n(x, y) = J_n(x)Y_n(y) - Y_n(x)J_n(y)$, $s_n(x, y) = J_n'(x)Y_n'(y) - Y_n'(x)J_n'(y)$. The two types of resonances, labeled TM and TE, correspond to magnetic and electric fields transverse to the z-axis. These resonant frequencies differ very little from those of a pillbox chamber, obtained from the torus in the limit $a \to 0$, provided that the beam in the torus is not too close to the inner wall. This is because the fields are concentrated near the outer wall in both models.

This theory has been extended to include wall resistance [1]. Numerical results given here are all for the resistive model with realistic conductivity. The radiated power is computed as $P = 2(q\omega_0)^2 \sum_{n=1}^{\infty} \mathrm{Re}Z(n, n\omega_0)|\lambda_n|^2$, where λ_n is the Fourier transform of the longitudinal density λ. Fig.1 is a plot of $\mathrm{Re}Z(n, n\omega_0)$ with parameters for

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† warnock@slac.stanford.edu
‡ Jack.Bergstrom@lightsource.ca

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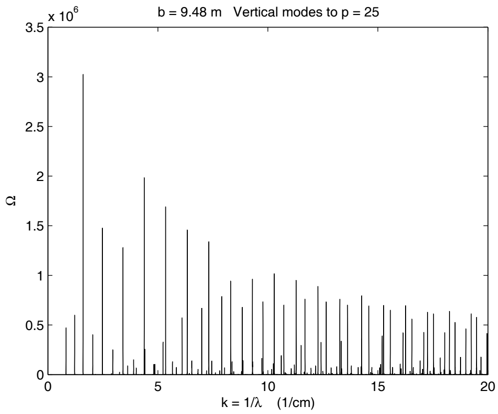

Figure 1: $\mathrm{Re}Z(n, n\omega_0)$ in ohms vs. $k = n/(2\pi R)$ in cm^{-1} for VUV parameters

the bends of the Brookhaven VUV ring ($R = 1.91\mathrm{m},\ w = b - a = 8\mathrm{cm},\ h = 4\mathrm{cm}$), conductivity of aluminum. The ordinate is $k = 1/\lambda = n/(2\pi R)$ in inverse centimeters. All vertical modes up to $p = 25$ have been included. It is assumed that the beam is at $r = R$ and $b = 1.948\mathrm{m}$; thus the beam is $2\mathrm{mm}$ off center, which gives a slightly better fit to the data than a centered beam.

COMPARISON TO VUV DATA

To compare with experiment we ignore straight sections, supposing that transitions from straights to bends have a minor effect on the placement of peaks in the power spectrum. Supposing then that the ring consists entirely of bends, ω_0 is to be understood as the angular velocity in bends, $\beta c/R$, not as the revolution frequency of the real ring with straights. The vacuum chamber in the bends of NSLS VUV is rather similar to the theoretical model, simply rectangular with circular wall.

Fig.2 shows a far IR spectrum taken at NSLS VUV using an interferometer with resolution down to $0.01\mathrm{cm}^{-1}$ [2]. To go to still longer wavelengths Carr *et al.* used microwave techniques involving a horn antenna, wave guides, and a frequency analyzer; see Kramer [2]. Fig.3 shows the long wavelength data, partly from microwave methods (open circles) and partly from the interferometer (black circles). The interferometer loses its value around $1.5\mathrm{cm}^{-1}$ due to insufficient signal. The spectrum of Fig2 appears in both coherent (bursting) and incoherent signals. The black circles in Fig.2 are from coherent radiation only, since the incoherent is too weak.

To compare with theory we first convolve the spectrum of Fig.1 with a narrow Gaussian to average over minor peaks and perhaps imitate experimental resolution to some extent. For the range of Fig.2 the Gaussian had a σ of $0.19\mathrm{cm}^{-1}$ whereas for the range of Fig.3 the σ was $0.037\mathrm{cm}^{-1}$ (since the experimental resolution is better there). The convolution gives well-defined peaks that are compared in Table 1 with peaks read from Figs.2, 3. The

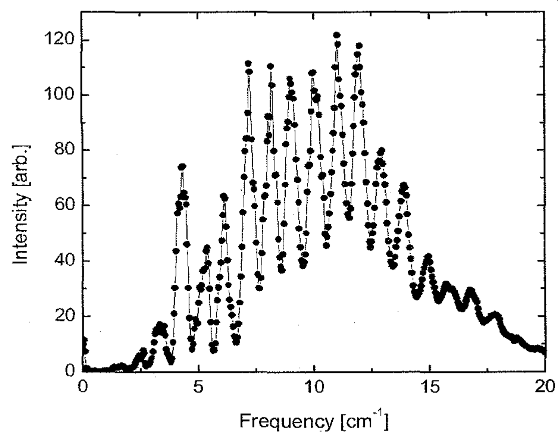

Figure 2: Far IR spectrum from interferometer at NSLS VUV.

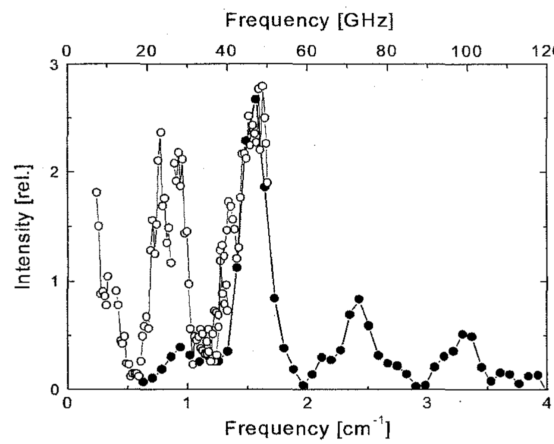

Figure 3: Long wavelength spectrum measured at NSLS VUV.

experimental entries marked with an asterisk correspond to somewhat doubtful small bumps or shoulders. The content of Table 1 for principal lines is shown graphically in Fig.4.

The structure in Fig.3 below $0.5\mathrm{cm}^{-1}$ is not due to CSR. Unlike the CSR part it is not polarized (Kramer,[2]), and it has been plausibly interpreted as a wake field effect from a bellows.

SPECTRA AT THE CANADIAN LIGHT SOURCE

A typical high resolution CSR spectrum from the Canadian Light Source (CLS) is shown in Fig.5. This came from a Bruker IFS 125 HR spectrometer, capable of resolution down to 0.001 cm^{-1}. In such spectra one can discern a nearly periodic fine structure with period of 0.074 cm^{-1} and a somewhat less definite coarse structure with period about 1.2 cm^{-1}. The locations of peaks in the fine structure were remarkably invariant under changes in the machine setup and in the IR optics. Over a three year period

Table 1: Theoretical frequencies compared to data of Fig.2 and Fig.3

Exp.	Thy.	Exp.	Thy.
0.80	0.827	6.10	6.31
0.93	—	7.25	7.32
1.32	1.21	9.00	8.32
1.57	1.60	10.0	9.29
2.10*	2.04	11.1	10.28
2.40	2.48	12.0	11.29
2.76*	2.94	12.8	12.33
3.10*	3.26	13.8	13.31
3.66*	3.62	15.0	14.3
3.88*	3.90	15.7-15.9	15.3
4.20	4.38	16.7	16.3
5.25	5.34	18.0	17.3
		18.8*	18.3

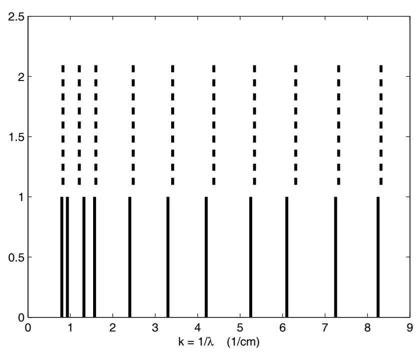

Figure 4: Comparison of experimental and theoretical spectra. Solid lines-experiment, dashed lines-theory.

(2007-2010) there were experiments with different beam energies (1.5 and 2.9 GeV), different tunes and fill patterns (1 and 210 bunches), and runs with both steady and bursting CSR. Moreover, there were major changes in the IR optics, including replacement of the front end optical system of the interferometer. Fig.6 shows two spectra taken one year apart under completely different machine setups.

The independence of spectrum and beam characteristics strongly suggests that the spectrum is determined by the vacuum chamber, in accord with our view of the VUV example. On the other hand, the close spacing of lines in the fine structure cannot be understood in terms of the smooth torus theory and the values of b and R in the normal dipole chambers of the ring. Rather, a much larger $b = R + 21$cm is required to give the spacing of 0.074 cm^{-1}. This larger effective b might arise from the special vacuum chambers at two IR ports, which flare outward a long distance as shown in Fig.7. Efforts are underway to model such abrupt changes in the outer wall radius, as well as to see the effect of straight sections on the CSR spectrum. The change in

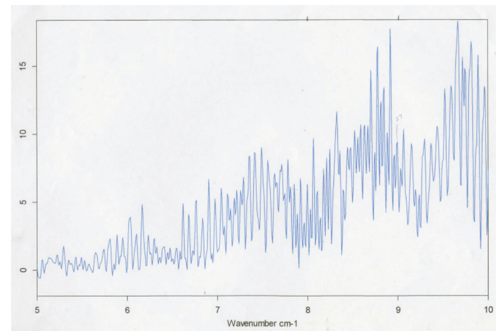

Figure 5: Typical CSR spectrum at CLS.

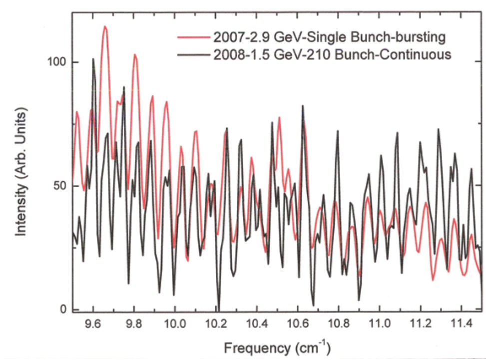

Figure 6: Spectra taken one year apart with different setups.

radius is studied by a “sidebox” model, with matching of sidebox modes to our standard modes in the main chamber.

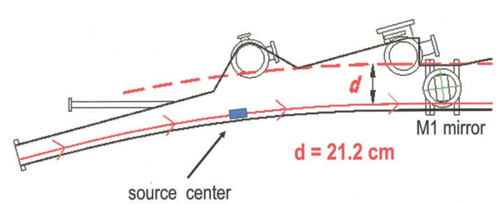

Figure 7: Flared vacuum chamber at IR ports of CLS.

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