# Universal Effective Hadron Dynamics from Superconformal Algebra

Stanley J. Brodsky

SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA

Guy F. de Téramond

Universidad de Costa Rica, 11501 San Pedro de Montes de Oca, Costa Rica

Hans Günter Dosch

Institut für Theoretische Physik der Universität, Philosophenweg 16, D-69120 Heidelberg, Germany

Cédric Lorcé

Centre de Physique Théorique, École polytechnique, CNRS, Université Paris-Saclay, F-91128 Palaiseau, France

> sjbth@slac.stanford.edu gdt@asterix.crnet.cr dosch@thphys.uni-heidelberg.de cedric.lorce@polytechnique.edu

This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-76SF00515 and HEP.

#### Abstract

An effective supersymmetric QCD light-front Hamiltonian for hadrons composed of light quarks, which includes a spin-spin interaction between the hadronic constituents, is constructed by embedding superconformal quantum mechanics into AdS space. A specific breaking of conformal symmetry inside the graded algebra determines a unique effective quark-confining potential for light hadrons, as well as remarkable connections between the meson and baryon spectra. The results are consistent with the empirical features of the light-quark hadron spectra, including a universal mass scale for the slopes of the meson and baryon Regge trajectories and a zero-mass pion in the limit of massless quarks. Our analysis is consistently applied to the excitation spectra of the  $\pi$ ,  $\rho$ , K,  $K^*$  and  $\phi$  meson families as well as to the  $N, \Delta, \Lambda, \Sigma, \Sigma^*, \Xi$  and  $\Xi^*$  in the baryon sector. We also predict the existence of tetraquarks which are degenerate in mass with baryons with the same angular momentum. The mass of light hadrons is expressed in a universal and frameindependent decomposition in the semiclassical approximation described here.

# Contents

1	Intr	coduction	3
<b>2</b>	Supersymmetric quantum mechanics and hadron physics		<b>5</b>
	2.1	Construction of the Hamiltonian from the superconformal algebra $\ldots$ .	5
	2.2	Holographic embedding and the spin-spin interaction $\ldots \ldots \ldots \ldots \ldots$	9
	2.3	Tetraquarks	11
	2.4	Inclusion of quark masses and comparison with experiment $\ldots$ .	12
3 Conclusions		13	

# 1 Introduction

Semiclassical approximations for quantum field theory are particularly useful for hadron physics since the detailed dynamics of QCD is far from being fully understood [1, 2]. QCD is a strongly interacting relativistic quantum field theory where the presence of light quarks and the implementation of color confinement pose special problems [3, 4, 5]. The search for semiclassical bound-state equations in gauge theories has received a significant advance through the Maldacena conjecture [6] which states that, under stringent conditions, a strongly interacting relativistic Yang-Mills conformal quantum field theory is the holographic dual of a classical gravitational theory in a higher-dimensional anti-de Sitter (AdS) space. This conjecture, the AdS/CFT correspondence, has led to many successful applications in particle, heavy-ion, and solid-state physics [7].

A very successful phenomenological application of the gauge/gravity correspondence is light-front holographic QCD (LFHQCD) [8, 9, 10, 11]. In this case, the holographic variable z in the 5-dimensional classical gravity theory is identified with the boostinvariant transverse separation  $\zeta$  between constituents in the light-front quantization scheme [12, 13]. For a system of N constituents, the distance  $\zeta$  is that of the "active" constituent to the remaining cluster of the N-1 constituents. Such a cluster decomposition is necessary in order to describe baryons in LFHQCD, since there is only a single holographic variable. This identification does not necessarily imply that the cluster is a tightly bound system; it only requires that essential dynamical features can be described in terms of the holographic variable  $\zeta$ . This assumption is supported by the observed similarity between the baryon and meson spectra [14]. Qualitatively, the observed spectra of both light-quark baryons and mesons show approximately equally spaced parent and daughter trajectories with a common Regge slope [14]. This remarkable structure, especially the equal slopes of the meson and baryon trajectories, suggests the existence of a deeper underlying symmetry. In the AdS/CFT correspondence [6] the dual quantum field theory is, in fact, a superconformal gauge theory. Guided by these very general considerations, we will use a simple representation of superconformal algebra to construct semiclassical supersymmetric bound-state equations which are holographically mapped to relativistic Hamiltonian bound-state equations in the light front (LF) [15, 16, 17]. These wave equations satisfactorily reproduce the successful empirical results previously obtained from LFHQCD (see *e.g.* [11]), with the crucial advantage that additional constant terms in the confinement potential, which are essential for describing the observed phenomenology, are determined from the onset.

The supersymmetric approach to hadronic physics also provides unexpected connections across the heavy-light hadron spectra, a sector where one cannot start from a superconformal algebra because of the strong explicit breaking of conformal symmetry by heavy quark masses [18].

In our framework, the emerging dynamical supersymmetry between mesons and baryons is not a consequence of supersymmetric QCD at the level of fundamental fields, but the supersymmetry between the LF bound-state wave functions of mesons and baryons. This symmetry is consistent with an essential feature of color  $SU(N_C)$ : a cluster of  $N_C - 1$  constituents can be in the same color representation as the anti-constituent; for SU(3) this means  $\mathbf{\bar{3}} \in \mathbf{3} \times \mathbf{3}$  and  $\mathbf{3} \in \mathbf{\bar{3}} \times \mathbf{\bar{3}}^{-1}$ .

In  $AdS_5$  the positive and negative-chirality projections of the baryon wave functions, the upper and lower spinor components in the chiral representation of Dirac matrices, satisfy uncoupled second-order differential equations with degenerate eigenvalues. As we shall show in this letter, these component wave functions form, together with the boson wave functions, the supersymmetric multiplets.

The semiclassical LF effective theory based on superconformal quantum mechanics also captures other essential features of hadron physics that one expects from confined quarks in QCD. For example, a massless pseudoscalar  $q\bar{q}$  bound state –the pion– appears in the limit of zero-quark masses, and a mass scale emerges from a nominal conformal theory. Moreover, the eigenvalues of the light-front Hamiltonian predict the same slope for Regge trajectories in both n, the radial excitations, and L, the orbital excitations, as

<sup>&</sup>lt;sup>1</sup>This was the basis of earlier attempts [19, 20, 21] to combine mesons and baryons in supermultiplets.

approximately observed empirically. This nontrivial aspect of hadron physics [22, 23] – the observed equal slopes of the radial and angular Regge trajectories – is also a property of the Veneziano dual amplitude [24].

In this letter we will extend our previous analyses and derive a semiclassical lightfront relativistic Hamiltonian based on superconformal algebra and its holographic embedding, which includes a spin-spin interaction between the hadronic constituents. This extension of our previous results provides a remarkably simple, universal and consistent description of the light-hadron spectroscopy and their light-front wavefunctions. We also predict the existence of tetraquarks which are degenerate with baryons with the same angular momentum. The tetraquarks are bound states of the same confined colortriplet diquark and anti-diquark clusters which account for baryon spectroscopy; they are required to complete the supermultiplet structure predicted by the superconformal algebra.

# 2 Supersymmetric quantum mechanics and hadron physics

We start this section with a short recapitulation of our previous applications [16, 17] of superconformal quantum mechanics to light front holographic QCD following Ref. [25]. This novel approach to hadron physics not only allows a treatment of nucleons which is completely analogous to that of mesons, but it also captures the essential properties of the confinement dynamics of light hadrons and provides a theoretical foundation for the observed similarities between mesons and baryons. The superconformal algebraic approach is then extended to include the spin-spin interactions of the constituents and the contribution to the hadron spectrum from quark masses.

# 2.1 Construction of the Hamiltonian from the superconformal algebra

We briefly review the essential features of the simplest superconformal graded algebra [25] in one dimension,  $conf(\mathbb{R}^1)$ . It is based on the generators of translation, dilatation and the special conformal transformation H, D and K, respectively. By introducing the supercharges Q,  $Q^{\dagger}$ , S and  $S^{\dagger}$ , one constructs the extended algebraic structure [25, 26] with the relations

$$\frac{1}{2} \{Q, Q^{\dagger}\} = H, \qquad \qquad \frac{1}{2} \{S, S^{\dagger}\} = K, \{Q, S^{\dagger}\} = f \mathbf{I} - B + 2iD, \qquad \qquad \{Q^{\dagger}, S\} = f \mathbf{I} - B - 2iD, \qquad (1)$$

where f is a real number, **I** is the identity operator,  $B = \frac{1}{2}[\psi^{\dagger}, \psi]$  is a bosonic operator with  $\{\psi, \psi^{\dagger}\} = \mathbf{I}, S = x \psi^{\dagger}$  and  $S^{\dagger} = x \psi$ . The operators H, D and K satisfy the conformal algebra

$$[H, D] = iH, \qquad [H, K] = 2iD, \qquad [K, D] = -iK.$$
 (2)

The fermionic operators can be conveniently represented in a spinorial space  $S_2 = \mathcal{L}_2(\mathbb{R}^1) \otimes \mathbb{C}^2$  as  $2 \times 2$  matrices

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \quad Q^{\dagger} = \begin{pmatrix} 0 & 0 \\ q^{\dagger} & 0 \end{pmatrix}, \tag{3}$$

$$S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}, \quad S^{\dagger} = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}, \tag{4}$$

with

$$q = -\frac{\mathrm{d}}{\mathrm{d}x} + \frac{f}{x}, \qquad q^{\dagger} = \frac{\mathrm{d}}{\mathrm{d}x} + \frac{f}{x}.$$
(5)

Following the analysis of Fubini and Rabinovici [25], which extends the treatment of the conformal group by de Alfaro, Fubini and Furlan [27] to supersymmetry, we construct [16, 17] a generalized Hamiltonian from the supercharges

$$R_{\lambda} = Q + \lambda S = \begin{pmatrix} 0 & -\frac{\mathrm{d}}{\mathrm{d}x} + \frac{f}{x} + \lambda x \\ 0 & 0 \end{pmatrix}, \tag{6}$$

$$R_{\lambda}^{\dagger} = Q^{\dagger} + \lambda S^{\dagger} = \begin{pmatrix} 0 & 0 \\ \frac{\mathrm{d}}{\mathrm{d}x} + \frac{f}{x} + \lambda x & 0 \end{pmatrix}, \qquad (7)$$

namely

$$G = \{R_{\lambda}, R_{\lambda}^{\dagger}\}.$$
(8)

Since the dimensions of the generators Q and S are different, a scale  $\lambda$ , with dimensions of mass squared, is introduced in the Hamiltonian in analogy with the earlier treatment

of conformal quantum mechanics given in Ref. [27]. As shown by de Alfaro, Fubini and Furlan [27], the conformal symmetry of the action is retained despite the presence of a mass scale in the Hamiltonian.

The supercharges and the new Hamiltonian G satisfy, by construction, the relations:

$$\{R_{\lambda}^{\dagger}, R_{\lambda}^{\dagger}\} = \{R_{\lambda}, R_{\lambda}\} = 0, \qquad [R_{\lambda}, G] = [R_{\lambda}^{\dagger}, G] = 0, \tag{9}$$

which, together with Eq. (8), close a graded algebra sl(1/1), as in Witten's supersymmetric quantum mechanics [28]. Since the Hamiltonian G commutes with  $R^{\dagger}_{\lambda}$ , it follows that the states  $|\phi\rangle$  and  $R^{\dagger}|\phi\rangle$  have identical eigenvalues. Furthermore, it follows that if  $|\phi_0\rangle$  is an eigenvalue of G with zero eigenvalue, it is annihilated by the operator  $R^{\dagger}_{\lambda}$ :

$$R_{\lambda}^{\dagger}|\phi_{0}\rangle = 0. \tag{10}$$

In matrix representation Eq. (8) is given by

$$G = 2H + 2\lambda^2 K + 2\lambda \left( f \mathbf{I} - \sigma_3 \right), \quad \text{with} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{11}$$

The new Hamiltonian is diagonal, with elements:

$$G_{11} = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} + \lambda^2 x^2 + 2\lambda \left(f - \frac{1}{2}\right), \tag{12}$$

$$G_{22} = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} + \lambda^2 x^2 + 2\lambda \left(f + \frac{1}{2}\right).$$
(13)

These equations have the same structure as the second order wave equations in AdS space, which follow from a linear Dirac equation with a multiplet structure composed of positive and negative-chirality components [10, 16]. Mapping to light-front physics, one identifies the conformal variable x with  $\zeta$ , the boost-invariant LF separation of the constituents <sup>2,3</sup>.

<sup>&</sup>lt;sup>2</sup>In general, the invariant light-front variable of the *N*-quark bound state is  $\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{N-1} x_j \mathbf{b}_{\perp j} \right|$ , where *x* is the longitudinal momentum fraction of the active quark,  $x_j$  with  $j = 1, 2, \dots, N-1$ , the momentum fractions associated with the N-1 quarks in the cluster, and  $\mathbf{b}_{\perp j}$  is the transverse positions of the spectator quarks in the cluster relative to the active one [8]. For a two-constituent bound-state  $\zeta = \sqrt{x(1-x)} |\mathbf{b}_{\perp}|$ , which is conjugate to the invariant mass  $\frac{\mathbf{k}_{\perp}^2}{x(1-x)}$ .

<sup>&</sup>lt;sup>3</sup>These equations are analogous to the LFHQCD relation of Eqs. (5.28) and (5.29) to Eqs. (5.32) and (5.33) in Ref. [11]. In the case of fermions, the maximal symmetry of AdS was broken in LFHQCD by the introduction of an *ad hoc* Yukawa-like term in the AdS action [29]. This is unnecessary using superconformal algebra.

The operator  $G_{22}$  agrees with the LF Hamiltonian of the positive-chirality projection; similarly, the operator  $G_{11}$  acts on the negative-chirality component. The positivechirality component  $\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2)$  has orbital angular momentum  $L_B = f - \frac{1}{2}$  and it is the leading twist solution; the negative-chirality component  $\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$  has  $L_B + 1$ . The total nucleon wave function is the plane-wave superposition [10, 16]<sup>4</sup>

$$\Psi(x^{\mu},\zeta) = e^{iP \cdot x} \left[ \psi^{+}(\zeta) \frac{1}{2} \left(1 + \gamma_{5}\right) u(P) + \psi^{-}(\zeta) \frac{1}{2} \left(1 - \gamma_{5}\right) u(P) \right],$$
(14)

where u(P) is a Dirac spinor of a free nucleon with momentum P in four-dimensional Minkowski space [10, 11]. Both components have identical normalization [16], and thus the nucleon spin is carried by the LF orbital angular momentum [11] <sup>5</sup>.

The operator  $G_{11}$  is also the LF Hamiltonian of a meson with angular momentum  $J = L_M = f + \frac{1}{2}^6$ . The eigenfunctions of  $G_{11}$  and  $G_{22}$  are related by the fermionic operators  $R_{\lambda}$  and  $R_{\lambda}^{\dagger}$ , Eqs. (6) and (7); these supercharges can be interpreted as operators which transform baryon into meson wave functions and vice-versa [17]. The operator  $G_{11}$  is thus the Hamiltonian for mesons, and  $G_{22}$  is the Hamiltonian for the positive-chirality component, the leading-twist baryon wave function.

The eigenfunctions of the Hamiltonian  $G_{11}$  are  $\phi_{n,L}(z) \sim \zeta^{1/2+L} e^{-\lambda z^2/2} L_n^L(\lambda z^2)$ , with  $L = f + \frac{1}{2}$ . Using the relations  $nL_n^{\nu}(x) = (n+\nu)L_{n-1}^{\nu}(x) - xL_{n-1}^{\nu+1}(x)$  and  $L_n^{\nu-1}(x) = L_n^{\nu}(x) - L_{n-1}^{\nu}(x)$  between the associated Laguerre polynomials we find

$$R^{\dagger}_{\lambda}|\phi^{M}_{n,L}\rangle = 2\sqrt{\lambda}(n+L)^{1/2}|\phi^{B}_{n,L-1}\rangle,\tag{15}$$

where

$$|\phi_{n,L}^{M}\rangle = \begin{pmatrix} \phi_{n,L} \\ 0 \end{pmatrix}, \qquad |\phi_{n,L-1}^{B}\rangle = \begin{pmatrix} 0 \\ \phi_{n,L-1} \end{pmatrix}.$$
 (16)

This shows explicitly the remarkable relation  $L_M = L_B + 1$  which identifies the orbital angular momenta of the mesons with their baryon superpartners with identical mass [17].

The relation  $L_B = f - \frac{1}{2}$  shows that f must be positive for baryons, in accordance with the requirement that the superconformal potential  $\frac{f}{x}$  in (5) be bounded from be-

<sup>&</sup>lt;sup>4</sup>In AdS<sub>5</sub> the Dirac matrix corresponding to the fifth (the holographic) variable is proportional to the matrix  $\gamma_5$ ,  $\Gamma_z = i\gamma_5$ . Eq. (14) is, in general, the solution of the linear Rarita-Schwinger equation in AdS<sub>5</sub> space which determines the relative normalization of both components [10, 16].

<sup>&</sup>lt;sup>5</sup>The equality of the normalization of the L = 0 and L = 1 components is also predicted by the Skyrme model [30].

<sup>&</sup>lt;sup>6</sup>Compare Eq. (5.2) with Eq. (5.5) in Ref. [11].

low [25, 27]. However, for mesons, the negative value  $f = -\frac{1}{2}$  leads to angular momentum  $L_M = 0$ , which is allowed and is consistent with the Hamiltonian  $G_{11}$  for mesons <sup>7</sup>. We can therefore regard Eq. (12) as an extension of the supersymmetric theory with f > 0 to the negative value  $f = -\frac{1}{2}$  for mesons. It is clear from Eq. (15) that the fermion operator  $R^{\dagger}$  annihilates the lowest state corresponding to n = L = 0,  $R^{\dagger} |\phi_{n=0,L=0}\rangle = 0$ , in accordance with Eq. (10). Thus the pion has a special role in the superconformal approach to hadronic physics as a unique state of zero mass [17]. It also follows from Eq. (15) that meson states with n > 0 and L = 0, also corresponding to the marginal value  $f = -\frac{1}{2}$ , are not annihilated by  $R^{\dagger}$ . These states, however, are connected to unphysical fermion states with L = -1. These spurious states are eigenstates of the Hamiltonian G, Eq. (8), with positive eigenvalues and their presence seems unavoidable in the supersymmetric construction, since each state with eigenvalue different from zero should have a partner, as dictated by the index theorem [28].

The situation is completely analogous to the case where conformal symmetry is explicitly and strongly broken by heavy quark masses. In this case the superpotential is no longer constrained by conformal symmetry and it is basically unknown, but the meson-baryon supersymmetry still holds [18]. In particular, the L = 0 meson states have no supersymmetric baryon partner since they would correspond to unphysical L = -1states.

#### 2.2 Holographic embedding and the spin-spin interaction

The pion and nucleon trajectories can be consistently described by the superconformal algebraic structure mapped to the light front [17]. A fundamental prediction is a massless pion in the limit of zero quark masses. However, there remains a lingering question for the  $\rho$  and  $\Delta$  trajectories: in the case of LFHQCD, it was found necessary to introduce the concept of half-integer twist [11, 17] in order to describe the  $\Delta$  trajectory <sup>8</sup>. For states with  $J = L_M + s$ , where s is the total quark spin, the spin interaction follows from the holographic embedding of the bound-state equations [11]; it is not determined by the superconformal construction. This amounts to the modification of the meson Hamiltonian  $G_{11} \rightarrow G_{11} + 2\lambda s$ . In order to preserve supersymmetry, one must add the same term to the baryon Hamiltonian  $G_{22}$ . The resulting supersymmetric Hamiltonian

<sup>&</sup>lt;sup>7</sup>In LFHQCD the lowest possible value  $L_M = 0$  corresponds to the lowest possible value for the AdS<sub>5</sub> mass allowed by the Breitenlohner-Freedman stability bound [31].

<sup>&</sup>lt;sup>8</sup>The analogous problem does not arise for the meson trajectories since the resulting bound-state equation, from its holographic embedding, depends on the total and orbital angular momentum separately, see Eq. (5.2) with (5.5) in Ref. [11].

for mesons and baryons in the chiral limit is therefore

$$G_S = \{R_\lambda, R_\lambda^\dagger\} + 2\lambda \, s \,\mathbf{I},\tag{17}$$

For mesons s is the total internal quark spin of a meson. The identification of baryons as bound states of a quark and a diquark cluster provides a satisfactory interpretation of the supersymmetric implementation: in this case we can identify s with the spin of the diquark cluster. The spin of the diquark cluster of the  $\Delta$  trajectory and the nucleon family with total quark spin  $\frac{3}{2}$  must be s = 1: it is the natural superpartner of the  $\rho$  trajectory. For the nucleon family with total quark spin  $\frac{1}{2}$ , the cluster is, in general, a superposition of spin s = 0 and s = 1. Since the nucleon trajectory is the natural partner of the  $\pi$  trajectory, we have to choose the cluster spin s = 0 to maintain supersymmetry. In general, we take s as the smallest possible value compatible with the quantum numbers of the hadrons and the Pauli principle. This procedure reproduces the agreement with the empirical baryon spectrum obtained in our previous treatments without the unsatisfactory feature of introducing half-integer twist; all twists and orbital angular momenta are integers.

In the case of mesons, the lowest mass state of the vector meson family, the  $I = 1 \rho$ (or the  $I = 0 \omega$  meson) is annihilated by the fermion operator  $R^{\dagger}$ , and it has no baryon superpartner. This is possible, even though the  $\rho$  is a massive particle in the limit of zero quark masses, since the effect of the spin term  $2\lambda s$  in the new Hamiltonian is an overall shift of the mass scale without a modification of the LF wavefunction. The action of the fermion operator is thus the same as for the pseudoscalar meson family.

To summarize: The meson wave function  $\phi_M(L_M)$ , with LF orbital angular momentum  $L_M$  and quark spin s, and the positive-chirality (leading-twist) component wave function  $\psi_{B+}$  of a baryon, with cluster spin s and orbital angular momentum  $L_B = L_M - 1$ , are part of the supermultiplet

$$|\phi_H\rangle = \begin{pmatrix} \phi_M(L_M) \\ \psi_{B+}(L_B = L_M - 1) \end{pmatrix},$$
(18)

with equal mass. The supercharge  $R_{\lambda}^{\dagger}$  acts on the multiplet (18) and transforms the meson wave function into the corresponding baryon wave function. The meson and baryon mass spectra resulting from the Hamiltonian (17) are given by the simple formulæ:

Mesons 
$$M^2 = 4\lambda(n + L_M) + 2\lambda s,$$
 (19)

Baryons 
$$M^2 = 4\lambda(n + L_B + 1) + 2\lambda s.$$
 (20)

As discussed below Eq. (17), s is the internal quark spin for mesons and the lowest possible value of the cluster spin for baryons.

We are working in a semiclassical approximation; therefore hadron states are described by wave functions in the Hilbert space  $\mathcal{L}_2(R_1)$ , where the variable is the boostinvariant light-front transverse separation  $\zeta$ . The generators of the symmetries are operators in that Hilbert space. Since the wave functions (and spectra) are equal for hadrons and anti-hadrons, the superpartner of the meson is a baryon as well as the corresponding antibaryon.

In order to interpret these results for hadron physics, we shall assume that the constituents of mesons and baryons are quarks or antiquarks with the well-known quantum numbers. Thus the fermion operator  $R^{\dagger}_{\lambda}$  is interpreted as the transformation operator of a single constituent (quark or antiquark) into an anti-constituent cluster in the conjugate color representation.

#### 2.3 Tetraquarks

The supersymmetric states introduced in the previous section do not form a complete supermultiplet since the negative-chirality component wave function of the baryon has not been assigned as yet to its supersymmetric partner. We can complete the supersymmetric multiplet by applying the fermion operator  $R^{\dagger}_{\lambda}$  to the negative-chirality component baryon wave function and thus obtain a new bosonic state. As noted above, the operator  $R^{\dagger}_{\lambda}$  can be interpreted as transforming a constituent into a two-anticonstituent cluster. Therefore the operator  $R^{\dagger}_{\lambda}$  applied to the negative-chirality component of a baryon generates a tetraquark wave function,  $\phi_T = R^{\dagger}_{\lambda} \psi_{B-}$ , a bound state of a diquark and an anti-diquark cluster as depicted in Fig. 1.

The negative-chirality component of a baryon,  $\psi_{B-}$ , has LF angular momentum  $L_B + 1$  if its positive-chirality component partner has LF angular momentum  $L_B$ . Since  $R_{\lambda}^{\dagger}$  lowers the angular momentum by one unit, the angular momentum of the corresponding tetraquark is equal to that of the positive-chirality component of the baryon,  $L_T = L_B$ . The complete quadruplet supersymmetric representation thus consists of two fermion wave functions, namely the positive and negative-chirality components of the baryon



Figure 1: The supersymmetric quadruplet  $\{\phi_M, \psi_{B+}, \psi_{B-}, \phi_T\}$ . Open circles represent quarks, full circles antiquarks. The tetraquark has the same mass as its baryon partner in the multiplet. Notice that the LF angular momentum of the negative-chirality component wave function of a baryon  $\psi_{B-}$  is one unit higher than that of the positive-chirality (leading-twist) component  $\psi_{B+}$ .

spinor wavefunction  $\psi_{B+}$  and  $\psi_{B-}$ , plus two bosonic wave functions, namely the meson  $\phi_B$  and the tetraquark  $\phi_T$ . These states can be arranged as a 2 × 2 matrix:

$$\begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi_{B-}(L_B + 1) \\ \psi_{B+}(L_B) & \phi_T(L_T = L_B) \end{pmatrix},$$
(21)

on which the symmetry generators (1) and the Hamiltonian (17) operate <sup>9</sup>.

According to this analysis, the lowest-lying light-quark tetraquark is a partner of the  $b_1(1235)$  and the nucleon; it has quantum numbers  $I, J^P = 0, 0^+$ . The partners of the  $a_2(1320)$  and the  $\Delta(1233)$  have the quantum numbers  $I = 0, J^P = 1^+$ . Candidates for these states are the  $f_0(980)$  and  $a_1(1260)$ , respectively.

#### 2.4 Inclusion of quark masses and comparison with experiment

We have argued in [11] that the natural way to include light quark masses in the hadron mass spectrum is to leave the LF potential unchanged as a first approximation and add the additional term of the invariant mass  $\Delta m^2 = \sum_{i=1}^n \frac{m_i^2}{x_i}$  to the LF kinetic energy. The resulting LF wave function is then modified by the factor  $e^{-\frac{1}{2\lambda}\Delta m^2}$ , thus providing a relativistically invariant form for the hadronic wave functions. The effect of the nonzero quark masses for the squared hadron masses is then given by the expectation value of  $\Delta m^2$  evaluated using the modified wave functions. This prescription leads to

<sup>&</sup>lt;sup>9</sup>It is interesting to note that in Ref. [20] mesons, baryons and tetraquarks are also hadronic states within the same multiplet.

the quadratic mass correction

$$\Delta M^2[m_1, \cdots, m_n] = \frac{\lambda^2}{F} \frac{\mathrm{d}F}{\mathrm{d}\lambda},\tag{22}$$

with  $F[\lambda] = \int_0^1 \cdots \int \mathrm{d}x_1 \cdots \,\mathrm{d}x_n \, e^{-\frac{1}{\lambda} \left(\sum_{i=1}^n \frac{m_i^2}{x_i}\right)} \delta(\sum_{i=1}^n x_i - 1).$ 

The resulting expressions for the squared masses of all light mesons and baryons are:

Mesons 
$$M^2 = 4\lambda(n+L) + 2\lambda s + \Delta M^2[m_1, m_2],$$
 (23)

Baryons 
$$M^2 = 4\lambda(n+L+1) + 2\lambda s + \Delta M^2[m_1, m_2, m_3],$$
 (24)

where the different values of the mass corrections within the supermultiplet break supersymmetry explicitly. For the tetraquark the mass formula is the same as for the baryon except for the quark mass correction  $\Delta M^2[m_1, m_2, m_3, m_4]$  given by Eq. (22).

The pion mass of ~ 0.140 GeV is obtained if the non-strange light-quark mass is m = 0.045 GeV [11]. In the case of the K-meson, the resulting value for the strange quark mass is  $m_s = 0.357$  GeV [11]. The trajectories of K, K<sup>\*</sup> and  $\phi$ -mesons can then be readily calculated. (The predictions are compared with experiment in Ref. [11]. ) In Eq. (22) the values of  $x_i$  for the quarks are assumed to be uncorrelated. If one instead assumes maximal correlations in the cluster, i.e.  $x_2 = x_3$ , this affects the final result by less than 1 % for light quarks and less than 2 % for the  $\Omega^-$  which has three strange quarks. Therefore, the previously obtained agreement with the data [16] for the baryon spectra is hardly affected.

One can fit the value of the fundamental mass parameter  $\sqrt{\lambda}$  for each meson and baryon Regge trajectory separately using Eqs. (23) and (24). The results are displayed in Fig. 2. The best fit gives  $\sqrt{\lambda} = 0.52$  GeV as the characteristic mass scale of QCD.

### 3 Conclusions

Inspired by the correspondence of classical gravitational theory in 5-dimensional AdS space with superconformal quantum field theory in physical 4-dimensional space-time, as originally proposed by Maldacena, we have arrived at a novel holographic application of supersymmetric quantum mechanics to light-front quantized Hamiltonian theory in physical space-time. The resulting superconformal algebra, which is the basis of our semiclassical theory, not only determines the breaking of the maximal symmetry of the dual gravitational theory, but it also provides the form of the frame-independent color-



Figure 2: Best fit for the value of the hadronic scale  $\sqrt{\lambda}$  for the different Regge trajectories for baryons and mesons including all radial and orbital excitations using Eqs. (23) and (24). The dotted line is the average value  $\sqrt{\lambda} = 0.523$  GeV; it has the standard deviation  $\sigma = 0.024$  GeV. For the baryon sample alone the values are  $0.509 \pm 0.015$  GeV and for the mesons  $0.524 \pm 0.023$ GeV.

confining light-front potential in the semiclassical theory.

Remarkably, supersymmetric quantum mechanics together with light-front holography account for principal features of hadron physics, such as the approximatively linear Regge trajectories (including the daughter trajectories) with nearly equal slopes for all mesons and baryons in both L and n. One finds remarkable supersymmetric aspects of hadron physics, connecting the masses of mesons and their superpartner baryons which are related by  $L_M = L_B + 1$ . The predicted spectroscopy for the meson and baryon superpartners agree with the data up to an average absolute deviation of about 10 % <sup>10</sup>. The agreement with experiment is generally better for the trajectories with total (cluster) spin s = 1, such as the  $\rho - \Delta$  superpartner trajectory than for the trajectories with s = 0, the  $\pi - N$  trajectories. Features expected from spontaneous chiral symmetry breaking are obtained, such as the masslessness of the pion in the massless quark limit. The structure of the superconformal algebra also implies that the pion has no supersymmetric partner. The meson-baryon supersymmetry survives, even if the heavy quark

<sup>&</sup>lt;sup>10</sup>See also Figs. 5.3 and 5.4 in Ref. [11], Figs. 2 and 3 in Ref. [17], and Fig. 1 in Ref. [18].

masses strongly break the conformal symmetry [18].

The structure of the hadronic mass generation obtained from the supersymmetric Hamiltonian  $G_S$ , Eq. (17), provides a frame-independent decomposition of the quadratic masses for all four members of the supersymmetric multiplet. In the massless quark limit:

$$M_{H}^{2}/\lambda = \underbrace{\underbrace{(2n+L_{H}+1)}_{kinetic} + \underbrace{(2n+L_{H}+1)}_{potential}}_{contribution from AdS and superconformal algebra} + \underbrace{2(L_{H}+s) + 2\chi}_{2(L_{H}+s) + 2\chi} .$$
(25)

Here *n* is the radial excitation number and  $L_H$  the LF angular momentum of the hadron wave function; *s* is the total spin of the meson and the cluster respectively,  $\chi = -1$  for the meson and for the negative-chirality component of the baryon (the upper components in the susy-doublet) and  $\chi = +1$  for the positive-chirality component of baryon and for the tetraquark (the lower components). The contributions to the hadron masses squared from the light-front potential  $\lambda^2 \zeta^2$  and the light-front kinetic energy in the LF Hamiltonian, are identical because of the virial theorem.

We emphasize that the supersymmetric features of hadron physics derived here from superconformal quantum mechanics refers to the symmetry properties of the boundstate wave functions of hadrons and not to quantum fields; there is therefore no need to introduce new supersymmetric fields or particles such as squarks or gluinos.

We have argued that tetraquarks – which are degenerate with the baryons with the same (leading) orbital angular momentum– are required to complete the supermultiplets predicted by the superconformal algebra. The tetraquarks are the bound states of the same confined color-triplet diquarks and anti-diquarks which account for baryon spectroscopy.

The light-front cluster decomposition [32, 33] for a bound state of N constituents –as an "active" constituent interacting with the remaining cluster of N-1 constituents– also has implications for the holographic description of form factors. As a result, the form factor is written as the product of a two-body form factor multiplied by the form factor of the N-1 cluster evaluated at its characteristic scale. The form factor of the N-1 cluster is then expressed recursively in terms of the form factor of the N-2 cluster, and so forth, until the overall form factor is expressed as the N-1 product of two-body form factors evaluated at different characteristic scales. This cluster decomposition is in complete agreement with the QCD twist assignment which leads to counting-rule scaling laws [34, 35]. This solves a previous problem with the twist assignment for the nucleon [11]: The ground state solution to the Hamiltonian (13) for the nucleon corresponds to twist 2: the nucleon is effectively described by the wave function of a quark-diquark cluster. At short distances, however, all of the constituents in the proton are resolved, and therefore the falloff of the form factor at high  $Q^2$  is governed by the total number of constituents; *i.e.*, it is twist 3<sup>11</sup>. Also the twist assignment for the  $\Delta$  (and total quark spin- $\frac{3}{2}$  nucleons) deviates from the assignment introduced in our previous papers [10, 11, 16, 17]; the approach chosen here, dictated by supersymmetry, does not require the introduction of half-integer twist.

The emerging confinement mass scale  $\sqrt{\lambda}$  serves as the fundamental mass scale of QCD; it is directly related to physical observables such as hadron masses and radii; in addition, as discussed in Ref. [37], it can be related to the scheme-dependent perturbative QCD scales, such as the QCD renormalization parameter  $\Lambda_s$ .

# Acknowledgments

We thank Alexandre Deur for valuable discussions.

# References

- [1] A. M. Jaffe and E. Witten, Quantum Yang-Mills theory, http://www.claymath.org/sites/default/files/yangmills.pdf.
- [2] For a recent review see, N. Brambilla *et al.*, QCD and strongly coupled gauge theories: Challenges and perspectives, Eur. Phys. J. C 74, 2981 (2014) [arXiv:1404.3723 [hep-ph]].
- [3] V. Gribov, QCD at large and short distances (annotated version), Eur. Phys. J. C 10, 71 (1999) [arXiv:hep-ph/9807224].
- [4] V. N. Gribov, The theory of quark confinement, Eur. Phys. J. C 10, 91 (1999)
   [arXiv:hep-ph/9902279].
- [5] Y. L. Dokshitzer and D. E. Kharzeev, The Gribov conception of quantum chromodynamics, Ann. Rev. Nucl. Part. Sci. 54, 487 (2004) [arXiv:hep-ph/0404216].

<sup>&</sup>lt;sup>11</sup>A brief discussion of the LF cluster decomposition of form factors was given in Ref. [36] and a will be discussed in more detail elsewhere.

- [6] J. M. Maldacena, "The large-N limit of superconformal field theories and supergravity," Int. J. Theor. Phys. 38, 1113 (1999) [arXiv:hep-th/9711200].
- [7] For a recent review see, M. Ammon and J. Erdmenger, Gauge/gravity duality: Foundations and applications, Cambridge University Press (Cambridge (2015)).
- [8] Historically light-front holographic methods were introduced by matching the electromagnetic form factor in AdS space with the corresponding light-front expression in physical space-time. See: S. J. Brodsky and G. F. de Teramond, Hadronic spectra and light-front wave functions in holographic QCD, Phys. Rev. Lett. 96, 201601 (2006) [arXiv:hep-ph/0602252].
- [9] G. F. de Teramond and S. J. Brodsky, Light-front holography: A first approximation to QCD, Phys. Rev. Lett. 102, 081601 (2009) [arXiv:0809.4899 [hep-ph]].
- [10] G. F. de Teramond, H. G. Dosch and S. J. Brodsky, Kinematical and dynamical aspects of higher-spin bound-state equations in holographic QCD, Phys. Rev. D 87, 075005 (2013) [arXiv:1301.1651 [hep-ph]].
- [11] For a recent review of the light-front holographic approach to hadronic physics see: S. J. Brodsky, G. F. de Teramond, H. G. Dosch and J. Erlich, Lightfront holographic QCD and emerging confinement, Phys. Rept. 584, 1 (2015) [arXiv:1407.8131 [hep-ph]].
- [12] P. A. M. Dirac, Forms of relativistic dynamics, Rev. Mod. Phys. 21, 392 (1949).
- [13] For a review of light-front physics see: S. J. Brodsky, H. C. Pauli and S. S. Pinsky, Quantum chromodynamics and other field theories on the light cone, Phys. Rept. 301, 299 (1998) [arXiv:hep-ph/9705477].
- [14] E. Klempt and J. M. Richard, Baryon spectroscopy, Rev. Mod. Phys. 82, 1095 (2010) [arXiv:0901.205 [hep-ph]].
- [15] S. J. Brodsky, G. F. de Teramond and H. G. Dosch, Threefold complementary approach to holographic QCD, Phys. Lett. B 729, 3 (2014) [arXiv:1302.4105 [hep-th]].
- [16] G. F. de Teramond, H. G. Dosch and S. J. Brodsky, Baryon spectrum from superconformal quantum mechanics and its light-front holographic embedding, Phys. Rev. D 91, 045040 (2015) [arXiv:1411.5243 [hep-ph]].

- [17] H. G. Dosch, G. F. de Teramond and S. J. Brodsky, Superconformal baryonmeson symmetry and light-front holographic QCD, Phys. Rev. D 91, 085016 (2015)
   [arXiv:1501.00959 [hep-th]].
- [18] H. G. Dosch, G. F. de Teramond and S. J. Brodsky, Supersymmetry across the light and heavy-light hadronic spectrum, Phys. Rev. D 92, 074010 (2015) [arXiv:1504.05112 [hep-ph]].
- [19] Hadronic supersymmetry was introduced by Miyazawa. See, H. Miyazawa, Baryon number changing currents, Prog. Theor. Phys. 36, 1266 (1966).
- [20] S. Catto and F. Gursey, Algebraic treatment of effective supersymmetry, Nuovo Cim. A 86, 201 (1985).
- [21] D. B. Lichtenberg, Whither hadron supersymmetry?, arXiv:hep-ph/9912280.
- [22] L. Y. Glozman and A. V. Nefediev, Chiral symmetry and the string description of excited hadrons, Phys. Rev. D 76, 096004 (2007) [arXiv:0704.2673 [hep-ph]].
- [23] M. Shifman and A. Vainshtein, Highly excited mesons, linear Regge trajectories and the pattern of the chiral symmetry realization, Phys. Rev. D 77, 034002 (2008) [arXiv:0710.0863 [hep-ph]].
- [24] G. Veneziano, Construction of a crossing-symmetric, Regge-behaved amplitude for linearly rising trajectories, Nuovo Cim. A 57, 190 (1968).
- [25] S. Fubini and E. Rabinovici, Superconformal quantum mechanics, Nucl. Phys. B 245, 17 (1984).
- [26] R. Haag, J. T. Lopuszanski and M. Sohnius, All possible generators of supersymmetries of the S-matrix, Nucl. Phys. B 88, 257 (1975).
- [27] V. de Alfaro, S. Fubini and G. Furlan, Conformal invariance in quantum mechanics, Nuovo Cim. A 34, 569 (1976).
- [28] E. Witten, Dynamical breaking of supersymmetry, Nucl. Phys. B 188, 513 (1981).
- [29] Z. Abidin and C. E. Carlson, Nucleon electromagnetic and gravitational form factors from holography, Phys. Rev. D 79, 115003 (2009) [arXiv:0903.4818 [hep-ph]].

- [30] S. J. Brodsky, J. R. Ellis and M. Karliner, Chiral symmetry and the spin of the proton, Phys. Lett. B 206, 309 (1988).
- [31] P. Breitenlohner and D. Z. Freedman, Stability in gauged extended supergravity, Annals Phys. **144**, 249 (1982).
- [32] S. J. Brodsky, C. R. Ji and G. P. Lepage, Quantum chromodynamic predictions for the deuteron form factor, Phys. Rev. Lett. 51, 83 (1983).
- [33] S. J. Brodsky and C. R. Ji, Factorization property of the deuteron, Phys. Rev. D 33, 2653 (1986).
- [34] S. J. Brodsky and G. R. Farrar, Scaling laws at large transverse momentum, Phys. Rev. Lett. **31**, 1153 (1973); Scaling laws for large momentum transfer processes, Phys. Rev. D **11** (1975) 1309.
- [35] J. Polchinski and M. J. Strassler, Hard scattering and gauge/string duality, Phys. Rev. Lett. 88, 031601 (2002) [arXiv:hep-th/0109174].
- [36] G. F. de Teramond, The spectroscopy and form factors of nucleon resonances from superconformal quantum mechanics and holographic QCD, arXiv:1602.04899 [hep-ph].
- [37] A. Deur, S. J. Brodsky and G. F. de Teramond, Connecting the hadron mass scale to the fundamental mass scale of quantum chromodynamics, Phys. Lett. B 750, 528 (2015) [arXiv:1409.5488 [hep-ph]]; On the interface between perturbative and nonperturbative QCD, Phys. Lett. B 757, 275 (2016) [arXiv:1601.06568 [hep-ph]].