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A STUDY OF RESONANT EXCITATION OF LONGITUDINAL HOMS IN THE CRYOMODULES OF LCLS-II[*]

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INTRODUCTION

The Linac Coherent Light Source (LCLS) at SLAC, the world's first hard X-ray FEL, is being upgraded to the LCLS-II. The major new feature will be the installation of 35 cryomodules (CMs) of TESLA-type, superconducting accelerating structures, to allow for high rep-rate operation. It is envisioned that eventually the LCLS-II will be able to deliver 300 pC, 1 kA pulses of beam at a rate of 1 MHz.

At a cavity temperature of 2 K, any heat generated (even on the level of a few watts) is expensive to remove. In the last linac of LCLS-II, L3—where the peak current is highest—the power radiated by the bunches in the CMs is estimated at 13.8 W (charge 300 pC option, rep rate 1 MHz) [1, 2]. But this calculation ignores resonances that can be excited between the bunch frequency and higher order mode (HOM) frequencies in the CMs, which in principle can greatly increase this number. A. Sukhanov, et al, have addressed this question for the LCLS-II in a calculation—where they make assumptions, including a cavity-to-cavity mode frequency variation (with rms 1 MHz)—to end up with a conservative estimate of 10^{-3} probability of the beam losing an extra watt (beyond the non-resonant 13.8 W) in a CM [3].

In the present work we look at the problem in a different way, and calculate the multi-bunch wakefields excited in a CM of LCLS-II, in order to estimate the probability of the beam losing a given amount of power. Along the way, we find some interesting properties of the resonant interaction. In detail, we begin this report by finding the wakes experienced by bunches far back in the bunch train. Then we present a complementary approach that calculates the field amplitude excited in steady-state by a train of bunches, and show that the two approaches agree. Next we obtain the properties of the 450 longitudinal HOMs that cover the range 3–5 GHz in the CMs of LCLS-II, where we include the effects of the inter-CM ceramic dampers. At the end we apply our method using these modes. More details can be found in Ref. [4].

Selected beam and machine properties in L3 of LCLS-II, some of which we use in calculations, are given in Table 1.

MULTI-BUNCH WAKE

Consider a train of equally spaced bunches, each with charge q, moving at the speed of light c and exciting a cavity longitudinal HOM defined by wavenumber k and loss factor $\varkappa$ [$= \frac{1}{4}ck(R/Q)$]. If the quality factor Q is infinitely large,

Table 1: Selected beam and machine properties in the LCLS-II. Nominally the bunch charge is 100 pC, and the maximum charge is 300 pC (but with the same peak current).

Parameter name	Value	Unit
Charge per bunch, q	300	pC
Beam current, I	1	kA
Full bunch length, ℓ	90	μm
Repetition rate, f_0	1	MHz
Fundamental mode frequency, f_{rf}	1.3	GHz
TM0 cut-off frequency, f_{co}	2.94	GHz
Non-resonant HOM power loss, P_{sb}	13.8	W
Single bunch total loss factor, $\varkappa_{sb}$	154.	V/pC

the voltage loss of bunch n to the mode is given by

$$\Delta V_n = 2q\varkappa \left(\frac{1}{2} + \sum_{n'=1}^{n-1} \cos k(s_n - s_{n'}) \right) , \quad (1)$$

where s_n is position of bunch n within the train, with a larger number representing a position further toward the back.

If the bunch spacing is an integer multiple of the mode wavelength, then $\Delta V_1 = q\varkappa$, $\Delta V_2 = 3q\varkappa$, ..., $\Delta V_n = 2q\varkappa(n - \frac{1}{2})$, which means that, on resonance, the loss grows linearly with bunch number.

With finite (but large) Q, the loss at bunch n becomes

$$\Delta V_n = 2q\varkappa \left(\frac{1}{2} + \sum_{n'=1}^{n-1} \cos k(s_n - s_{n'}) e^{-\frac{k(s_n - s_{n'})}{2Q}} \right) . \quad (2)$$

The terms in parenthesis—which we call the *normalized loss*—can be written as

$$h(n) \equiv \frac{1}{2} + \mathcal{R}e \left[e^{-\alpha} \frac{1 - e^{-(n-1)\alpha}}{1 - e^{-\alpha}} \right] , \quad (3)$$

where $\mathcal{R}e(z)$ means take the real part of z; and $\alpha = (\frac{1}{Q} - 2i)\pi\nu$, with the tune $\nu \equiv f/f_0 = ck/(2\pi f_0)$ and f_0 is the bunch frequency. For example, with $f \approx 4$ GHz and $f_0 = 1$ MHz, the tune $\nu \approx 4000$. Note that h is normalized so that for the case of no multi-bunch wake effect (the single bunch case), $h = \frac{1}{2}$.

The LCLS-II bunch trains are very long (at $f_0 = 1$ MHz a million bunches pass by per second). For any mode with finite Q the wake eventually settles to a steady-state solution. [This happens for $n \gg 1/\mathcal{R}e(\alpha)$; with *e.g.* $f = 4$ GHz, $f_0 = 1$ MHz, $Q = 10^7$, this requires only that $n \gg 800$.] Thus we are here really interested in the limiting loss $h_{lim} \equiv \lim_{n\to\infty} h(n)$. Letting $n \to \infty$ in Eq. 3, the normalized loss can be written as (we drop the subscript)

$$h = \frac{1}{2} \left(\frac{\sinh x}{\cosh x - \cos y} \right) , \quad (4)$$

with $x = \pi\nu/Q$ and $y = 2\pi\nu$. We see that the loss is given by a simple analytical expression. Also, note that $\cos y$ in Eq. 4 only depends on the fractional part of the tune, ν_f, so we can let $y = 2\pi\nu_f$.

From the result, Eq. 4, we see that there are only two parameters that give our limiting loss, h: (ν/Q) and ν_f. The fractional part of the tune gives unique results only for an interval of 0.5, so let us choose it to be in the interval $0 \leq \nu_f \leq \frac{1}{2}$. We note that when Q is small, $h \approx \frac{1}{2}$ and we are left with the single bunch wake effect. However, for $Q \to \infty$ (and $\nu_f \neq 0$), $h \to 0$. Thus, in this case, the multi-bunch wake effect acts as a kind of Landau damping, where the wake loss experienced by the bunches is less than the single bunch wake, and is, in fact, near zero!

In Fig. 1 we plot h as given by Eq. 4 as function of x for several values of y. The small argument approximation $h = \frac{x}{x^2+y^2}$ is also given by dashes, and we see that it agrees well. The peak of $h(x)$ is at $x = y$, and the peak value is $1/(2x)$. The limit of all curves for $x \to \infty$ is $\frac{1}{2}$.
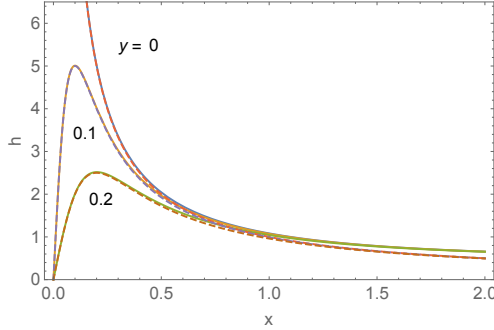

Figure 1: Limiting loss h vs. x for values of $y = 0, 0.1, 0.2$.

STEADY-STATE FIELDS

An alternate approach for finding the resonant effect is to consider the steady-state fields left in a mode in a CM by the bunch train. Let us begin with the case $Q = \infty$. Let the field for time $t < 0$ be given by

$$W(t < 0) = W_0 \cos(\omega t + \phi) , \qquad (5)$$

where $\omega = kc$ is the mode frequency. At $t = 0$ a bunch passes by. For $t > 0$ the wake becomes

$$W(t > 0) = W_0 \cos(\omega t + \phi) - W_1 \cos(\omega t) , \quad (6)$$

where $W_1 (> 0)$ is the excitation by the passing bunch.

In steady state, the wake should repeat itself with a time period T_s, the time spacing between bunches. This means

$$W_0 \cos[\omega(t - T_s) + \phi] = W_0 \cos(\omega t + \phi) - W_1 \cos(\omega t) , \qquad (7)$$

for time t in the interval $[0 < t < T_s]$. Expanding into $\cos(\omega t + \phi)$ and $\sin(\omega t + \phi)$ terms, and requiring each term to vanish, we obtain the steady state conditions

$$\begin{aligned} W_0 \cos \omega T_s &= W_0 - W_1 \cos \phi , \\ W_0 \sin \omega T_s &= -W_1 \sin \phi . \end{aligned} \qquad (8)$$

We are given W_1, ω, T_s, and want to solve for W_0, ϕ. The solution is

$$\begin{aligned} W_0 &= -\frac{W_1}{2 \sin \frac{\omega T_s}{2}} , \\ \phi &= \frac{\pi}{2} + \frac{\omega T_s}{2} . \end{aligned} \qquad (9)$$

Note the resonance when $\omega T_s = 2\pi n$.

The bunch change in energy at each passage is given by

$$\Delta E = W_0 \cos \phi - \frac{W_1}{2} , \qquad (10)$$

where a postive value indicates energy gain. Substituting the solution, Eqs. 9, we obtain

$$\Delta E = 0 . \qquad (11)$$

All bunches lose the same amount of energy which is zero. This is not a surprising result because when $Q = \infty$ there is no energy loss anywhere.

Now let us consider the case of finite Q. When Q is finite, the field, before the bunch arrives (for $t < 0$), becomes

$$W(t) = W_0 e^{-(\omega t + \phi)/2Q} \cos(\omega t + \phi) . \qquad (12)$$

At $t = 0$, a bunch passes by, and the fields become (for $t > 0$)

$$W(t) = W_0 e^{-(\omega t + \phi)/2Q} \cos(\omega t + \phi) - W_1 e^{-\omega t/2Q} \cos \omega t . \qquad (13)$$

At steady state the wake repeats itself with time period T_s. This means that

$$\begin{aligned} W_0 e^{-[\omega(t - T_s) + \phi]/2Q} &\cos[\omega(t - T_s) + \phi] \\ = W_0 e^{-(\omega t + \phi)/2Q} &\cos(\omega t + \phi) - W_1 e^{-\omega t/2Q} \cos \omega t \end{aligned} \quad (14)$$

for t in the interval $[0 < t < T_s]$.

Expanding into $\cos(\omega t + \phi)$ and $\sin(\omega t + \phi)$ terms, and requiring each term to vanish, we obtain the steady state conditions

$$\begin{aligned} z e^{\xi w} \cos w &= z - e^{\xi \phi} \cos \phi , \\ z e^{\xi w} \sin w &= -e^{\xi \phi} \sin \phi , \end{aligned} \qquad (15)$$

where $\xi = 1/(2Q)$, $w = \omega T_s$, and $z = W_0/W_1$. We need to solve for ϕ and z. The solution is

$$\begin{aligned} \tan \phi &= \frac{e^{\xi w} \sin w}{e^{\xi w} \cos w - 1} , \\ z &= -\frac{e^{\xi \phi}}{\sqrt{e^{2\xi w} + 1 - 2e^{\xi w} \cos w}} . \end{aligned} \qquad (16)$$

Finally, we obtain the bunch energy change as it passes through the cavity by computing

$$\Delta E = W_0 e^{-\phi/2Q} \cos \phi - \frac{W_1}{2} . \qquad (17)$$

Now if we substitute from Eqs. 16 and then make the correspondence, $-\Delta E/W_1 = h$, we arrive again at Eq. 4. Thus, we see that both approaches—finding the wake far back in the train or finding the steady-state field—yield the same result, as they should.

HOMS IN THE LCLS-II CMS

The computer code ACE3P [5, 6] was used to calculate 450 longitudinal modes in the range of 3–5 GHz, obtaining frequency f, loss factor $\varkappa = (\omega/4)(R/Q)$, and Q. The model was 2D, beginning with the second half of a CM, then a ceramic absorber, and finally the first half of the following CM (see Fig. 2).
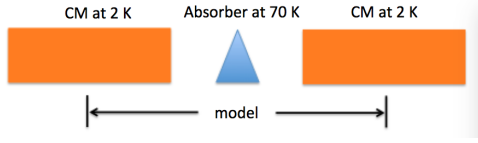

Figure 2: Schematic of model used for the mode calculations

The ceramic absorber is meant to remove power in modes above cut-off at 70 K, where it is cheaper to do so. The ceramic is Ceradyne CA-137, with $\epsilon = 15 - 4i$ [7]. Other lossy materials in the model are copper-plated bellows and a drift pipe, both at 2 K. The boundary conditions were E type (perfectly conducting walls); note, however, that changing boundary conditions from E type to H type shifted the frequencies slightly, but did not otherwise significantly change the results. The beam pipe radius of the cavities is 3.9 cm, which gives a TM0 cut-off frequency of 2.94 GHz. Thus the modes that we calculate will tend not to be trapped.

The numerically obtained mode properties are plotted in Fig. 3. Shown are frequencies f_m as function of mode number, loss factors $\varkappa_m$ vs. frequency, and quality factor Q_m vs. frequency (in GHz), on a semi-log scale. With a CM containing eight 9-cell cavities we expect to see bands of approximately 72 mode length; from the top plot it appears we see about 5 such bands. From the middle plot we note that the bands with the strongest modes are one near 3.8 GHz and another near 4.2 GHz. Note that the sum of all the loss factors in our frequency range, $\varkappa_{sum} = 6.0$ V/pC, representing 3.9% of the total single bunch loss, $\varkappa_{sb}$. As for the Q's, they reach from very small numbers up to $\sim 10^8$. We find that the average $\langle \log_{10}(Q_n) \rangle \approx 5$. The mode with the largest loss factor—which we will call the *strongest mode*—has $f = 3.86$ GHz, $\varkappa = 1.34$ V/pC, and $Q = 1 \times 10^7$ (indicated by orange plotting symbols in the figures).

In Fig. 4 we present statistics of the modes in histograms: density of modes, dn/df [GHz^{-1}] vs. f [GHz] (top left), loss factors $\varkappa$ [mV/(pC*CM)] (top right), Q's (bottom left), and real part of impedance $\mathcal{R}e(Z)$ [kΩ/CM] vs. f [GHz] (bottom right). Lines give expected density of modes and $\mathcal{R}e(Z)$. The density of modes estimate $dn/df \approx (2\pi/c^2)N_cAf$, with number of cells in a CM, $N_c = 72$, and cell cross-sectional area $A = 92.9$ cm^2, agrees reasonably well with the numerical results. The impedance curve was obtained from a time-domain, single bunch calculation using a bunch with rms length of 50 μm [1, 2]; although the areas under the curves of the impedance plot roughly agree, we see that the time-domain calculation does not capture the (relatively) low frequency impedance behavior.
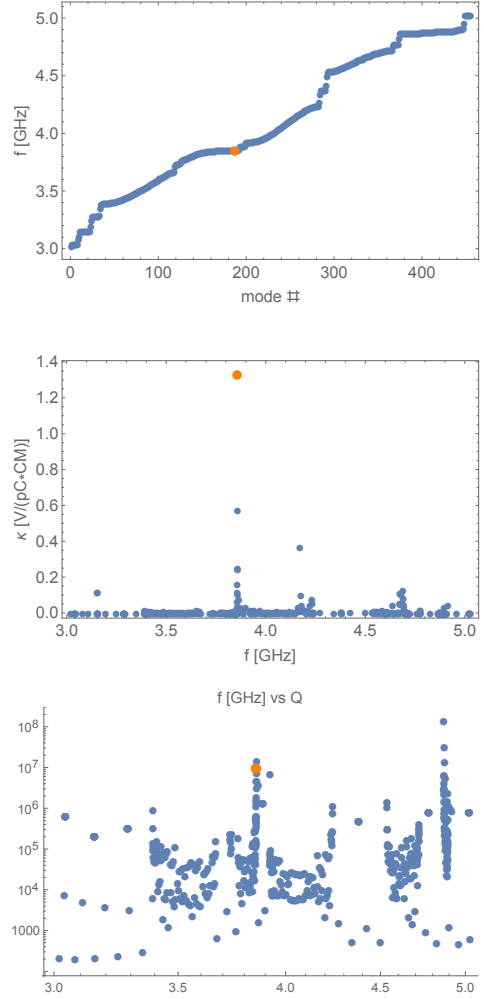

Figure 3: Mode properties: frequencies f_m (top), loss factors $\varkappa_m$ vs. f_m (middle), and quality factors Q_m vs. f_m in [GHz] (bottom). The strongest mode is shown in orange.
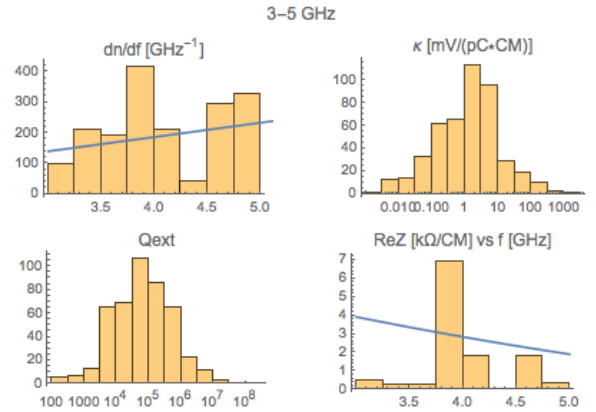

Figure 4: Statistics of modes: density of modes, dn/df [GHz^{-1}] vs. f [GHz] (top left), histogram of loss factors $\varkappa$ [mV/(pC*CM)] (top right), histogram of Q's (bottom left), and $\mathcal{R}e(Z)$ [kΩ/CM] vs. f [GHz]. Lines gives expected density of modes and $\mathcal{R}e(Z)$.

STRONGEST MODE

Let us begin by studying the wake effect of the strongest mode. The frequency $f \approx 3.85694$ GHz, the tune $\nu \approx 3900$ (assuming $f_0 = 1$ MHz), and the fractional tune $\nu_f = 0.0579$, yielding $h = 0.009$. Nominally, the wake effect of this mode is damped. However, we know the HOM frequency of the real CM only to an accuracy > 0.5 MHz (see [8]). This means that we have no information about the value of ν_f! This further implies that we can only calculate the resonance wake effect statistically, as a probability.

In Fig. 5 we plot $h(\nu_f)$ for the strong mode ($Q = 10^7$); we plot it on a semi-log plot since it is extremely spiked. The red dot gives the nominal value, *i.e.* the one obtained assuming that f, as given by ACE3P, is correct and exact; it can be approximated by $h = \frac{\pi\nu}{2Q}(1 - \cos 2\pi\nu_f)^{-1}$. The peak value $\hat{h} \equiv h(0) \approx Q/(\pi\nu) = 820$.
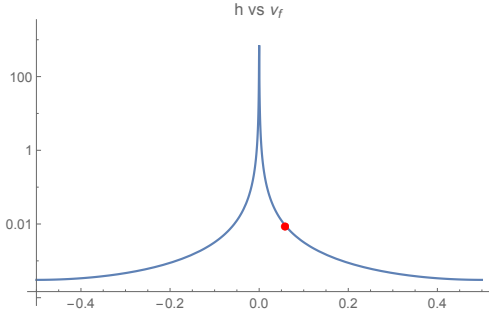

Figure 5: For the strongest mode, the normalized loss h as function of fractional tune ν_f (assuming $f_0 = 1$ MHz). The red dot gives the nominal value.

It is interesting to study how the loss in the strongest mode varies with Q, while keeping ν_f fixed at 0.0579. The plot is shown in Fig. 6, with the nominal case given in red. For low Q ($Q \lesssim \pi\nu/2$) the bunches communicate weakly, and $h \approx \frac{1}{2}$; for large Q, such as the nominal case, $h \approx 0$. The peak is located at $Q \approx \frac{1}{2}\nu/\nu_f = 3.3 \times 10^4$, and has a value of $h \approx (4\pi\nu_f)^{-1} = 1.37$. We see again that a higher Q can lead to less loss.
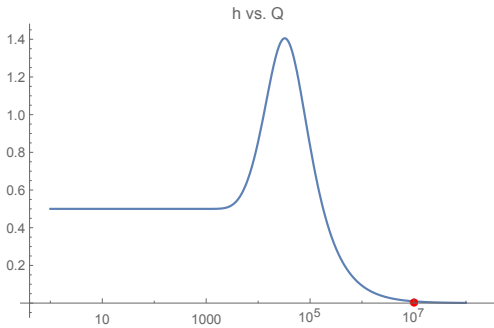

Figure 6: For strongest mode, where $\nu_f = 0.0579$: the growth factor $h(Q)$ if we vary Q. The peak is at $Q = 3.3 \times 10^4$. The nominal point is indicated in red.

Probabilities

Since we do not know the mode frequency accurately enough, ν_f can be taken from a uniform distribution of random numbers, and we can then find the probability of having a loss of a certain size. We are interested in two types of probabilities: the probability of likely outcomes, say the 50th, 75th, or 90th quantile, and the probability of hitting a large, relatively unlikely resonant peak.

First, it is important to note that the average of $h(\nu_f)$ as given by Eq. 4 equals $\frac{1}{2}$. This means that the average effect for many structures (or seeds in a Monte Carlo simulation) is equal to the non-resonant wake effect. We will perform such a Monte Carlo simulation in the next section, where we consider the effect of many modes. However, to find the probability of relative rare resonant growth it is worthwhile to develop an analytical approach, too.

The probability of ν_f landing between ν_f and $\nu_f + d\nu_f$ can be written as

$$\mathcal{P}(\nu_f)\,d\nu_f = \mathcal{P}(h)\,dh = \frac{\mathcal{P}(\nu_f)}{|dh/d\nu_f|}\,dh\,. \quad (18)$$

Here $\mathcal{P}(\nu_f) = 2H(\nu_f)H(\frac{1}{2} - \nu_f)$, the uniform distribution [$H(s)$ is the unit step function], and from Eq. 4 we find that

$$|dh/d\nu_f|^{-1} = \frac{(-\cos y + \cosh x)^2}{\pi \sin y \sinh x}\,, \quad (19)$$

with $x = \pi\nu/Q$ and $y = 2\pi\nu_f$. For the strongest mode, we plot in Fig. 7 the probability $\mathcal{P}(h)$. Note that the abscissa only reaches to a maximum of $\hat{h} = 820$. The dashed curve is $\mathcal{P} = \frac{1}{2\pi}(\pi\nu/Q)^{1/2}h^{-3/2} = (0.0056)h^{-3/2}$, which is seen to be a good approximation away from the peak. Finally, to find the (complimentary) cumulative probability of losing h or more to this mode, we need to perform the integral

$$\mathcal{S}(h) = \int_h^{\hat{h}} \mathcal{P}(h')\,dh'\,. \quad (20)$$

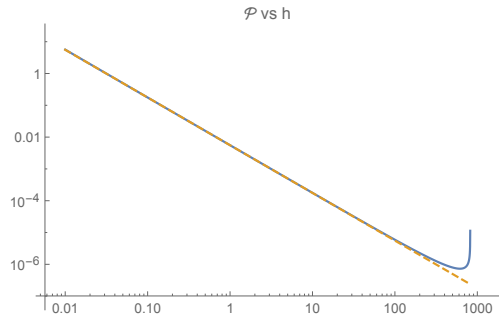

Figure 7: Probability $\mathcal{P}(h)$ for the strongest mode. The dashed curve is the approximation $\mathcal{P} = (0.0056)h^{-3/2}$.

We saw that the total non-resonant loss of the beam in a CM is 13.8 W. One question one might ask is, What is the probability of losing a watt or more to the strong mode? For 1 W, the effective h in mode m is given by

$$h = \frac{1}{2}\frac{1}{13.8}\frac{\varkappa_{sb}}{\varkappa_m}\,, \quad (21)$$

where $\varkappa_{sb}$ is total single bunch loss factor (see Table 1). Here $h = 4.1$. Performing the integral in Eq. 20, we find that the probability of losing ≥ 1 W to the strong mode is $\mathcal{S}(h \geq 4.1) = 0.55\%$.

ALL THE MODES

We can repeat the calculation of the probability of losing one watt in a CM, but now including all 450 modes. For a total 1 W power loss with many modes, we, in principle, need to consider correlated probabilities between the modes. However, since all the probabilities for the individual modes to lose a watt are small, the correlated probabilities have even smaller contributions. Thus, the total probability of losing one watt when considering all the modes is just the sum of the individual probabilities $\mathcal{S}(h)$ as obtained by Eq. 20, using h according to Eq. 21.

For a mode to contribute to the probability of losing a watt requires that $\hat{h} > h$, where h is given in Eq. 21; *i.e.* that $Q_m \varkappa_m / (\nu_m \varkappa_{sb}) > \pi / (2 \cdot 13.8)$. Of all our modes only 10 satisfy this condition, and all of these are in the band near 3.8 GHz. The probability contribution of the 10 modes and their frequencies are shown in Fig. 8; the strongest mode is indicated by the orange dot. The total probability of losing power $P \geq 1$ W to all the modes is the sum of the contributions, or 2.9%.

Figure 9: Results for 10k cases (seeds), histogram of $w_{tot} = 2 \sum_m h_m \varkappa_m / \varkappa_{sum}$ for all $m = 450$ modes, with ν_f generated from random numbers, normalized so that for no resonance effect, one obtains $w_{tot} = 1$. Note: 600 values are off-scale to the right.

is a 50% chance that a power equal to half the non-resonant loss (or less) is lost, a 75% chance that $\frac{2}{3}$ (or less) is lost, and a 90% chance that an amount equal to the non-resonant loss (or less) is lost. We see that it is much more likely that the multi-bunch wake will reduce the power loss, compared to the non-resonant wake effect, than it is to make it larger. More results, including the study of the effects of errors in bunch timing, bunch charge, and cavity frequencies will be given in Ref. [4].
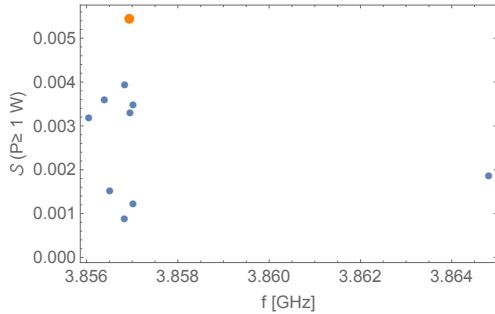

Figure 8: (Cumulative) probability $\mathcal{S}$ that loss is greater than 1 W, for the different modes. The orange dot indicates the strongest mode, *i.e.* the mode with the largest loss factor $\varkappa$.

Figure 10: Cumulative probability of losing power P or less to the modes in the range 3–5 GHz. The non-resonant effect alone is 0.55 W (the dashed line).

Another way of obtaining probabilities is by performing Monte Carlo simulations. We have performed such a simulation, calculating $w_{tot} = 2 \sum_m h_m \varkappa_m / \varkappa_{sum}$ for all $m = 450$ modes, with ν_f generated from a uniform distribution of random numbers (see Fig. 9). Note that w_{tot} is normalized so that, for no resonance effect, one obtains $w_{tot} = 1$. The calculations were performed for 10k cases (seeds). This calculation finds that $\mathcal{S}(P \geq 1 \text{ W}) = 3.5\%$; considering that we are in the tail of the distribution, this result is in reasonable agreement to the analytical result.

Finally, in Fig. 10, using the Monte Carlo results, we plot the cumulative probability $(1 - \mathcal{S})$ of losing power P or less to the modes in the range 3–5 GHz. The non-resonant loss to these modes is 0.55 W (the dashed line). We find that there
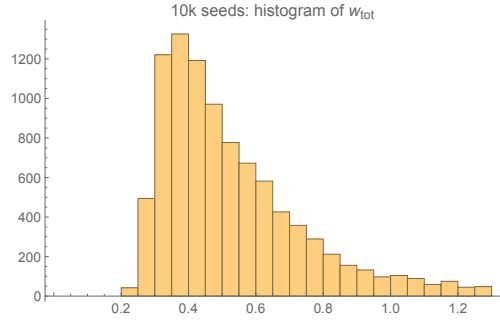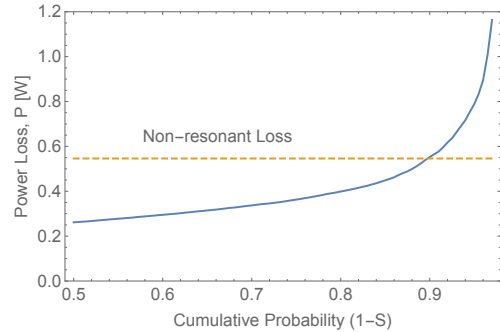
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