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STUDY OF ULTRA-LOW EMITTANCE DESIGN FOR SPEAR3

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Abstract

Since its 2003 construction, the SPEAR3 synchrotron light source at SLAC has continuously improved its performance by raising beam current, top-off injection, and smaller emittance. This makes SPEAR3 one of the most productive light sources in the world. Now to further enhance the performance of SPEAR3, we are looking into the possibility of converting SPEAR3 to an ultra-low emittance storage ring within its site constraint.

INTRODUCTION

The X-ray brightness of ID beamlines is the major figure-of-merit of 3rd generation light source. The brightness *B* of an up-right phase ellipse is:

$$B \propto \frac{N_\gamma}{(\Delta\lambda/\lambda)\Delta t \sum_x \sum_{x'} \sum_y \sum_{y'}}, \tag{1}$$

where Nγ is the number of photons in the central radiation cone, (Δλ/λ) is the radiation bandwidth; Δt is the time scale of interest, and Σx, Σx', etc., are the convolution of the electron beam phase space and the single electron radiation distribution given by

$$\sum_x = \sqrt{\sigma_x^2 + \sigma_\gamma^2} \quad \sum_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{\gamma'}^2}\ , \tag{2}$$

where σ_x and $\sigma_{x'}$ are the transverse rms size and divergence of the electron beam for horizontal plane. A similar equation holds for the vertical plane. σ_γ and $\sigma_{\gamma'}$ are the transverse beam sizes and divergences of the photon beam for a zero emittance electron beam. For an undulator of length L, it is approximately described by a radiation source at the center of the undulator given by

$$\sigma_{r'} \approx \sqrt{\frac{\lambda}{2L}} \quad \sigma_r \approx \frac{1}{2\pi}\sqrt{2\lambda L}\ . \tag{3}$$

The product $\sigma_\gamma\ \sigma_\gamma' = \lambda/(2\pi)$ sets the minimum possible emittance, which is achieved in the limit of zero electron beam emittance.

To maximize the brightness, we must minimize the effect of convolution with the electron distribution. That is the electron phase space has to be well matched to the radiation. For a given radiation $\sigma_\gamma\ \sigma_\gamma'$ the minimization is when:

$$\varepsilon_{x,y} \le \frac{\lambda}{2\pi}, \quad \frac{\sigma_{x,y}}{\sigma_{x',y'}} = \frac{\sigma_r}{\sigma_{r'}} \approx \frac{L}{\pi}\ . \tag{4}$$

The equilibrium electron emittance, ε_0, results from the balance between quantum excitation and radiation damping [1]

$$\varepsilon_0 \approx F(\nu, lattice)\frac{C_q\gamma^2}{J_x}\theta_B^3\ , \tag{5}$$

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where θ_B is the bending angle per dipole, $C_q = 3.84\times10^{-13}$ m, J_x is horizontal damping partition. Here we assume it's a horizontal bend. J_x is in the range of 1~3 depending on the gradient in dipole and γ is the Lorentz factor. F is a function of lattice design. It involves the integral of H function in the dipoles:

$$H(s) = \beta_x(s)\eta'_x(s)^2 + 2\alpha_x(s)\eta_x(s)\eta'_x(s) + \frac{1+\alpha_x(s)}{\beta_x(s)}\eta_x(s)^2 , \tag{6}$$

where $\beta_x(s)$, $\alpha_x(s)$ are the amplitude functions of electron motion and $\eta_x(s)$ and $\eta_x'(s)$ are the dispersion function and its derivative. It has been proved that with matched lattice functions to the dipole length and θ_B the minimum emittance are [2]:

$$\varepsilon_{MEDBA} = \frac{C_q\gamma^2}{4\sqrt{15}J_x}\theta_B^3,\ \varepsilon_{METME} = \frac{C_q\gamma^2}{12\sqrt{15}J_x}\theta_B^3 . \tag{7}$$

Here DBA means the dipole is dispersion free at one side and two such dipoles can make the cell dispersion free outside the dipoles. The TME (theoretical minimum emittance) dipole has dispersion at both ends. The minimum emittance of TME dipole is one third of the DBA dipole.

Today's light sources deliver radiation from 100 eV to 100 keV. The corresponding electron emittance for diffraction-limited source is from 2 nm to 2 pm. The horizontal emittance of SPEAR3 now is 9.8 nm and vertical emittance is 10 pm with 0.1% coupling. The emittance of SPEAR3 has to be reduced in order to match the hard x-ray radiation.

It's clear that an effective way to reduce the emittance is to put as many small angle TME dipoles as possible in a cell. This is the concept of multi-bend achromatic (MBA) cell. Figure 1 shows the theoretical minimum emittance of a 3 GeV, 18 period storage ring with horizontal damping partition $J_x = 1$. The horizontal axis is the number of dipoles per unit cell. The blue curve is the emittance of MBA cell with achromatic match. The emittance of minimum MBA achromatic cell is [2]:

$$\varepsilon_{MEMBA} = \frac{C_q\gamma^2}{4\sqrt{15}J_x}\theta_1^3 \text{(outer dipole angle)}. \tag{8}$$

In order to match the integral of H function between the TME and DBA dipoles, the length of the TME dipole is $3^{1/3}$ times the length of the DBA dipole in an isomagnetic cell. θ_1 in Eq. 8 is the bend angle of the DBA type dipole in the MBA cell. The emittance of an MBA cell is always larger than the pure TME cell because two TME dipoles are replaced by two DBA type dipoles. A 3 GeV, 18 cells, four-bend MBA lattice gives the emittance of 300 pm. These are the theoretical minimum values. Taking into consideration realistic constraints such as cell length and magnetic field strength, the design value usually is a few times larger than the theoretic minimum values.

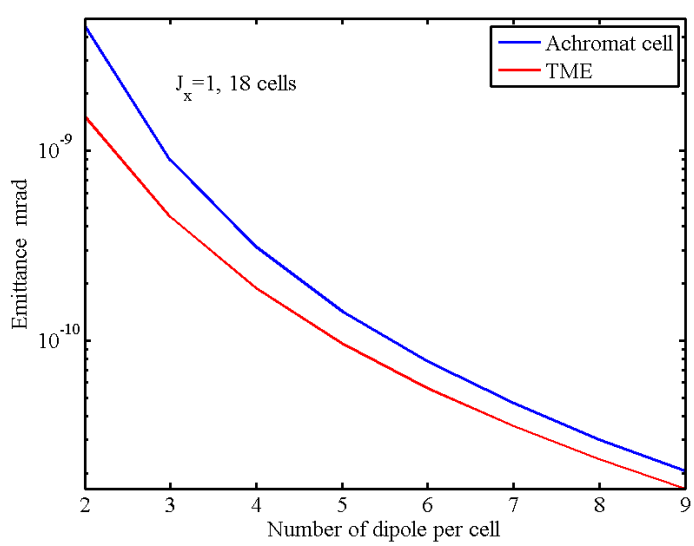

Figure 1: Theoretic minimum emittance (TME) of a 3 GeV, 18 cell ring with different number of

bends per cell. The blue curve is the minimum emittance of MBA cell with achromatic match.

MBA is the trend for the design of synchrotron storage rings under construction or for upgrade proposals. Table 1 summarizes the parameters of a few storage ring designs using MBA cells. The column M1 (= ε_0 C^3 /E^2) in Table 1 is one way to evaluate the effect of linear optics to emittance. It is approximately constant for machines of various energies and circumferences with similar optics [3]. The smaller value of M1 means the optics design is toward the optimized emittance. It's noticeable that ALS II has significantly smaller M1 than the rest of the MBA designs in Table 1. The reason is when calculating M1, the circumference is assumed linearly proportional to the number of dipole N_B (=$2\pi/\theta_B$). It does not take into consideration of energy dependence of bending radius. If the dipole field is fixed, N_B scales according to C/E. The column M2 (=$\varepsilon_0 C^3/E^5$) is a modification of the assessment. A more straight forward way is to calculate the effect of linear optics according to Eq. 8 with M3 (=$\varepsilon_0/E^2/\theta_1^3$). For an upgrade machine M3 is more realistic evaluation of the optics design because it reflects the constraint of cell length and super period of the existing storage to make small emittance.

Table 1: Summary of Various MBA Storage Ring Light Source Designs without Intrabeam Scattering. M1=$\varepsilon_0 C^3/E^2$ is given in units of pm km^3 /GeV^2. M2 =$\varepsilon_0 C^3/E^5$ is given in units of pm km^3 /GeV^5 and M3=$\varepsilon_0/E^2/\theta_1^3$/1E6 is given in units of pm/GeV^2

Name	**Energy (GeV)**	**Circumference (km)**	**Emittance (pm)**	**Structure NBA/period**	**M1**	**M2**	**M3**
MAX IV [4]	3	0.528	263	7-BA/20	4.301	0.159	0.672
Sirius [5]	3	0.518	280	5-BA/20	4.324	0.160	0.254
SPRING8-II [6]	6	1.43595	149	6-BA/48	5.510	0.026	0.389
ESRF upgrade [7]	6	0.8444	150	7-HBA/32	2.509	0.012	0.430
APS-U [8]	6	1.104	60	7-HBA/40	2.243	0.010	0.336
ALS II [9]	2	0.2	100	9-BA/12	0.200	0.025	0.308
SPEAR3* upgrade	3	0.234	640	4-BA/18	0.911	0.034	0.195

*The emittance shown here is for a double-4BA cell.

SPEAR3 MBA RING DESIGN

GRADIENT DIPOLE

Most of the MBA designs take advantage of dipoles with a defocussing gradient. This has several advantages [4]: the number of magnets per achromat is reduced; the partition number J_x is larger than 1 which reduces the emittance at the price of smaller longitudinal partition number; the length of the cell is small, therefore reducing the total circumference. Some of these MBAs require strong dipole gradient which is beyond the capability of conventional dipole with sloped pole face. A shifted quadrupole as dipole which provides 0.78 T dipole field and 50 T/m gradients is proposed by ALS II team [10]. Table 2 summarizes the parameters of gradient dipoles used in different projects [11]. It falls into two categories: the first three, MAX IV, Sirius, and Diamond, use combined function dipoles providing low to moderate gradient in dipoles; the rest use shifted strong quadrupoles providing strong gradient.

Table 2: Summary of Dipole Quadrupole [11]

	B at beam [T]	**Gradient [T/m]**	**Aperture [mm]**
MAX IV	0.52	8.6	28
Sirius	0.584	7.8	28
Diamond DDBA	0.8	14.4	30
ESRF2 DQ1	0.54	37	38
ESRF2 DQ2	0.42	48	38

APS	0.5	38	26
ALS II	0.78	50	24

QUADRUPOLE AND SEXTUPOLE

In general, small emittance is achieved with small β_x and η_x in the dipoles, which require strong focussing. Another reason for strong gradient quadrupoles is to make the length of quadrupoles short to make the MBA compact in order to fit in the existing cell length. The aperture for quadrupoles is reduced, which allows stronger gradient. Taking the ALS II project for example the quadrupole aperture is reduced from ALS 35 mm bore radius to ALS II 12 mm bore radius. The maximum gradient can reach 100 T/m [10]. Because of the strong focussing the natural chromaticity are large. The small dispersion makes the strength of sextupoles for linear chromaticity correction even stronger. This tends to reduce the dynamic aperture and momentum aperture. The maximum sextupole gradient proposed by ESRF is 4900 T/m^2 (nominal 3200 T/m^2) [11]. These quadrupole and sextupole strength limitations will be used in SPEAR3 MBA design study.

SPEAR3 MBA CELL

SPEAR3 is a two-fold symmetric storage ring of circumference 234.14 m. It has 14 normal DBA cells with cell length 11.69 m and arc length 6.4 m (where arc length is defined as the trajectory distance from the entrance of the first dipole to the exit of the last dipole in a cell) and four matching DBA cells, two per half ring. The matching cell has bending angle three quarter of the normal cell with cell length 18.76 m and arc length 6.79 m. SPEAR3 has the shortest arc length among all the designs listed in Table 1. The upgrade requires keeping the same number of ID beamlines and keeping the ID beamlines at same locations. These make it a challenge to put large number bend MBA inside the DBA arc. The highest dipole field of 0.8 T from Table 2 is chosen. This reduces the total dipole length as much as possible. The gradient in dipoles can be high, using ALS II 50 T/m shift quadrupole, or medium using Diamond 14.4 T/m combined function dipole. Different bend MBA from three to five bends have been studied without magnet field constraints. With the above magnet field constraints, a four bend MBA is the best that can be used to fit into the 6.4 m arc length. Further study shows no good location to put orthogonal sextupoles inside the achromat. This makes the sextupole strength needed for chromaticity correction extremely high. In order to solve this problem two QBA cells are combined with a dispersive middle straight to add a pair of chromatic sextupoles SD/SF for chromaticity correction. Fig. 2 shows the linear optics of the Double QBA (DQBA). The emittance of this DQBA cell is 664 pm.rad. The tunes per DQBA cell are $\Delta\nu_x/\Delta\nu_y$ 3.678/2.151. The β_x/β_y at two ID straights are: 2.3 m/2.85 m, and 2.5 m/5.0 m. The dispersion at the middle straight is 0.07 m. The maximum gradient in the dipole is smaller than 23 T/m. The Diamond type combined function dipole may be used. All the gradients of independent quadrupoles are smaller than 100 T/m. The length of sextupoles is 20 cm. It is inserted in the split quadrupole where beta function is large. This is similar to MAX IV. The strength of SD/SF for linear chromaticity correction is -4000/4000 T/m^2. The strength may increase when considering the correction of linear chromaticity of whole ring.

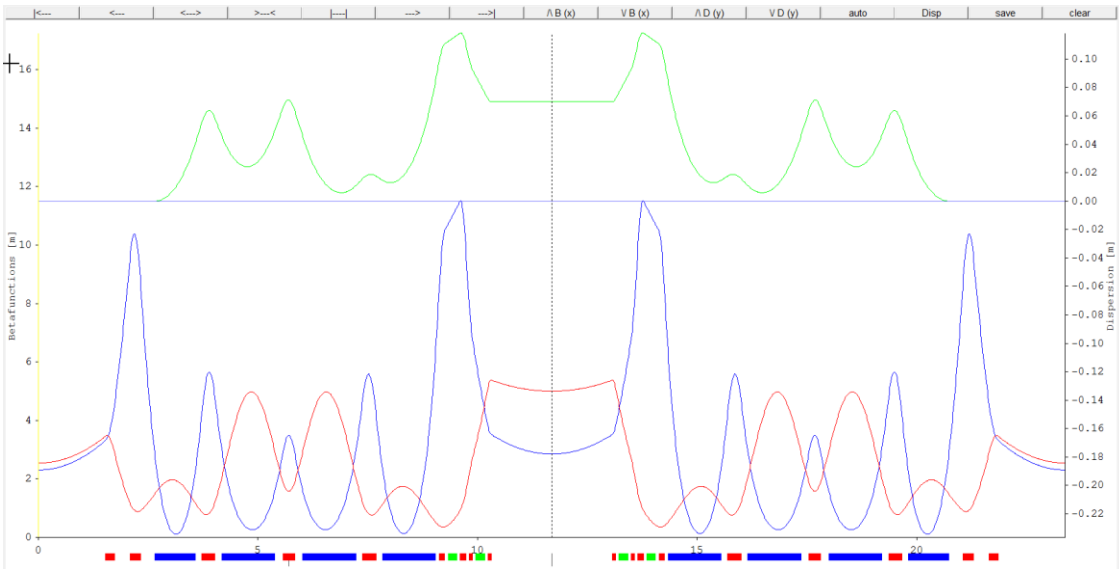

Figure 2: Optics functions of SPEAR3 DQBA cell. The blue line is beta x, the red line is beta y and the green line is dispersion. At the bottom of the plot the blue block represents dipole, the red is quadrupole and the green is sextupole.

MATCHING THE RING

In addition to the 14 normal DBA cells in SPEAR3, there are 4 longer matching DBA cells which can be converted into two matching DQBA cells. The whole ring will be composed of 2 matching DQBA, 6 normal DQBA and two QBA cells extracted from the normal DQBA. All the straight sections and ID beam lines are kept at the same positions as they are. The parameters of ID beam line are shown in table 3. There are four wiggler beam lines located at 5S, 7S, 15S and 16S where the dispersions are preferred to be zero in order to keep emittance low. The sextupole gradient is kept below 5000 T/m^2. The emittance of all the cells are managed to be less than 0.75 nmrad, in order to keep the ring emittance less than 0.75 nmrad.

Table 3: Summary of SPEAR3 ID beam line. The wiggler beam lines are located at 5S, 7S, 15S and 16S.

Beamline	Nperiods	period [cm]	B [T]	L [m]
4 (16S)	10	23	2	2.3
5 (12S)	10	18.3	0.5	1.83
6 (11S)	27	7	1	1.89
7 (5S)	10	23	2	2.3
9 (7S)	8	26	1.95	2.08
10 (6S)	15	12.85	1.33	1.9275
11 (15S)	13	17.5	2	2.275
12 (9S)	66	2.2	1.01	1.452
13 (17S)	65	5.69	0.85	3.6985
15 (13S)	86	2.2	1	1.89

DQBA matching cell

The optics functions of the DQBA matching cell is shown in figure 3. The half-cell length is 17.616 m. The emittance of this DQBA matching cell is 378 pm.rad. The tunes per QDBA cell are $\Delta\nu_x/\Delta\nu_y$ 4.097/1.98. The β_x/β_y at two ID straights are: 2.85 m/4.47 m, and 5.0 m/1.6 m. The dispersions at the middle straight are 0.07/0.04 m. The

maximum gradient in the dipole is smaller than 22 T/m. The strength of SD/SF for local linear chromaticity correction is -4000/4000 T/m^2.

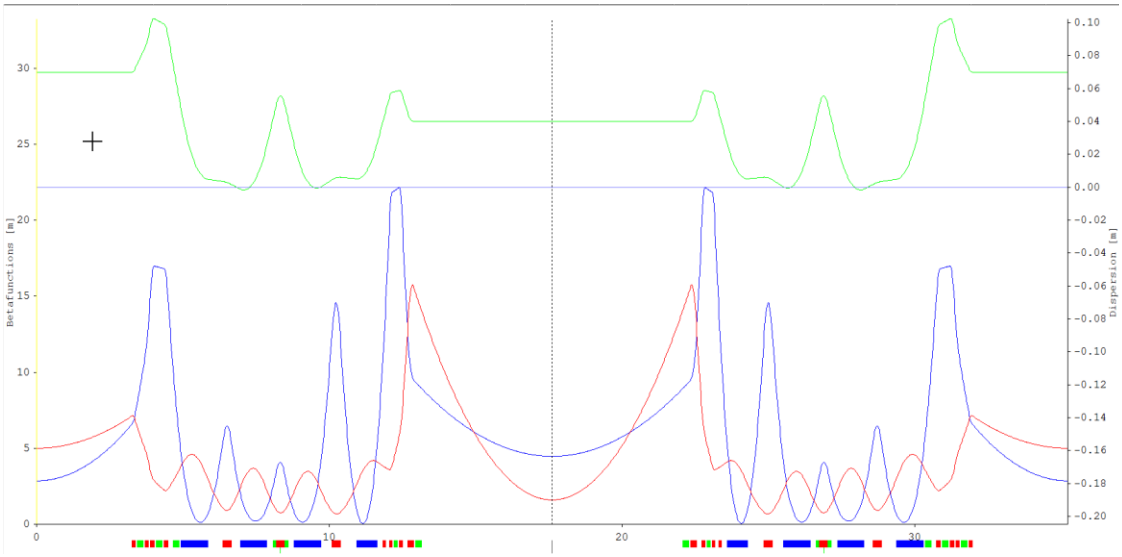

Figure 3: Optics functions of SPEAR3 DQBA matching cell.

QBA matching cell

This QBA cell is used to insert between two DQBA cells. The optics functions of the QBA cell is shown in figure 4. The cell length is 11.691m. The emittance of this QBA cell is 720 pm.rad. The tunes per QDBA cell are $\Delta\nu_x/\Delta\nu_y$ 1.753/1.126. The β_x/β_y at two ID straights are: 2.3 m/2.5 m. This is an achromatic cell. The maximum gradient in the dipole is smaller than 25 T/m. There are no orthogonal locations for SD and SF. Therefore there is no design for local chromaticity correction.

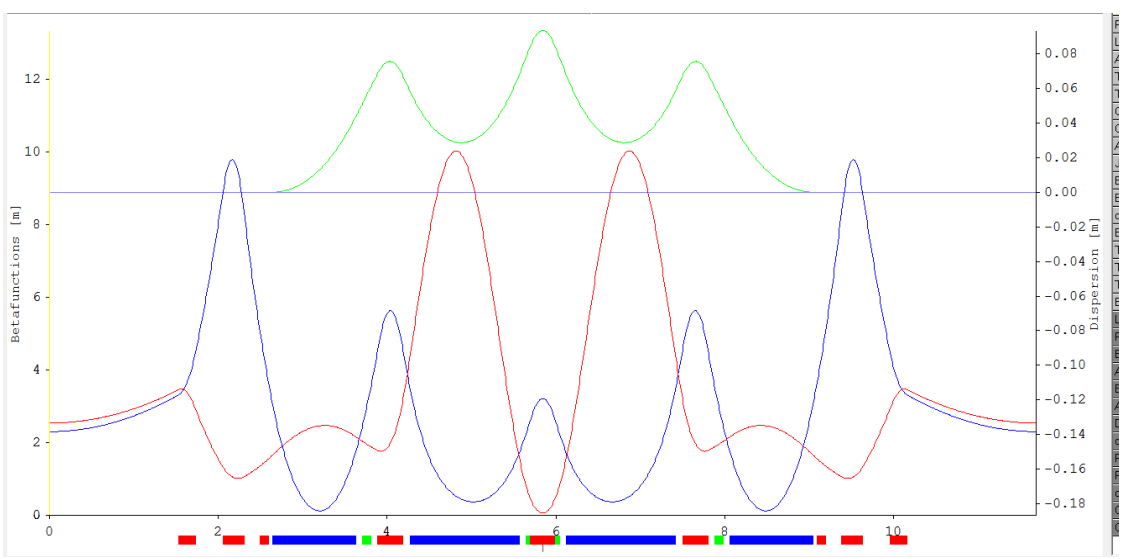

Figure 4: Optics functions of QBA matching cell.

SPEAR3 ring without insertion device

The optics functions of the SPEAR3 ring without insertion devices is shown in figure 5. The arrangement of the cells is to keep the ring symmetry and as many wiggler straights dispersion free as possible. The emittance of the ring is 637 pm.rad. The tunes of the ring are $\Delta\nu_x/\Delta\nu_y$ 33.767/19.128. The horizontal damping partition is 2.06. The energy loss per turn is 0.58 MeV. The sextupole strengths for linear chromaticity correction are less than 5000 T/m^2. However the dynamic aperture is less than 1 mm. Both the nonlinear chromatic and geometric effects need to be minimized. Adding harmonic sextupoles will help to correct the nonlinear geometric effect. The main parameters of the ring design are summarized in Table 4.

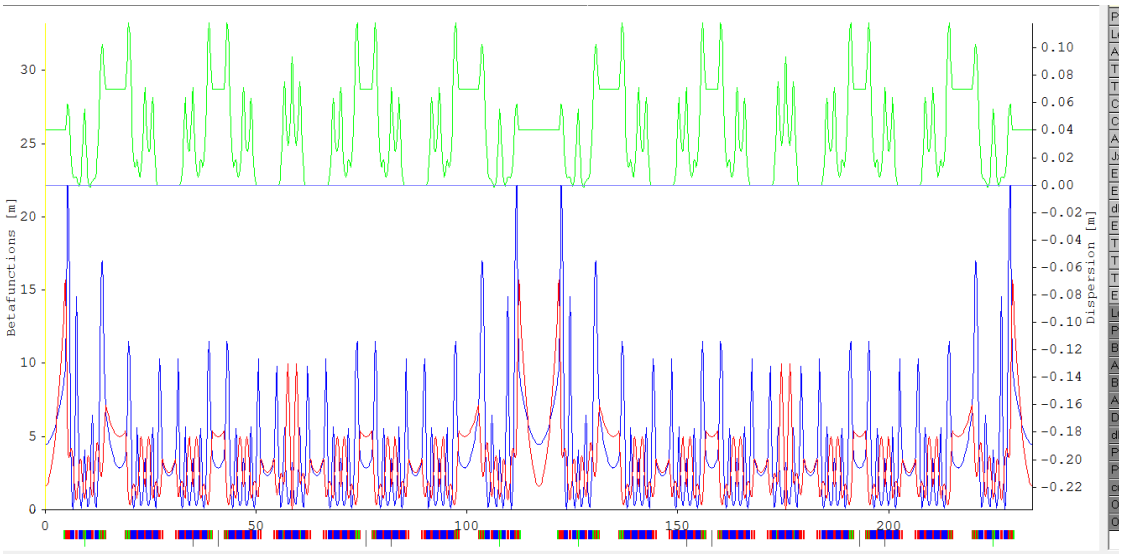

Figure 5: Optics functions of SPEAR3 ring without insertion devices.

SPEAR3 ring 1 with insertion devices

The optics functions of the SPEAR3 ring with insertion devices is shown in figure 6. The dispersion free straights are: 2, 4, 5, 7, 11, 13, 14 and 16. The wiggler straights are: 5, 7, 15 and 16. The wiggler straight 15 is not dispersion free. The emittance of the ring is 700 pm.rad. The emittance is slightly increased due to one wiggler straight is not dispersion free. The tunes of the ring are $\Delta\nu_x/\Delta\nu_y$ 33.767/19.165. The vertical tune is increased due to the insertion devices. The horizontal damping partition is 1.82. The energy loss per turn is 0.756 MeV.

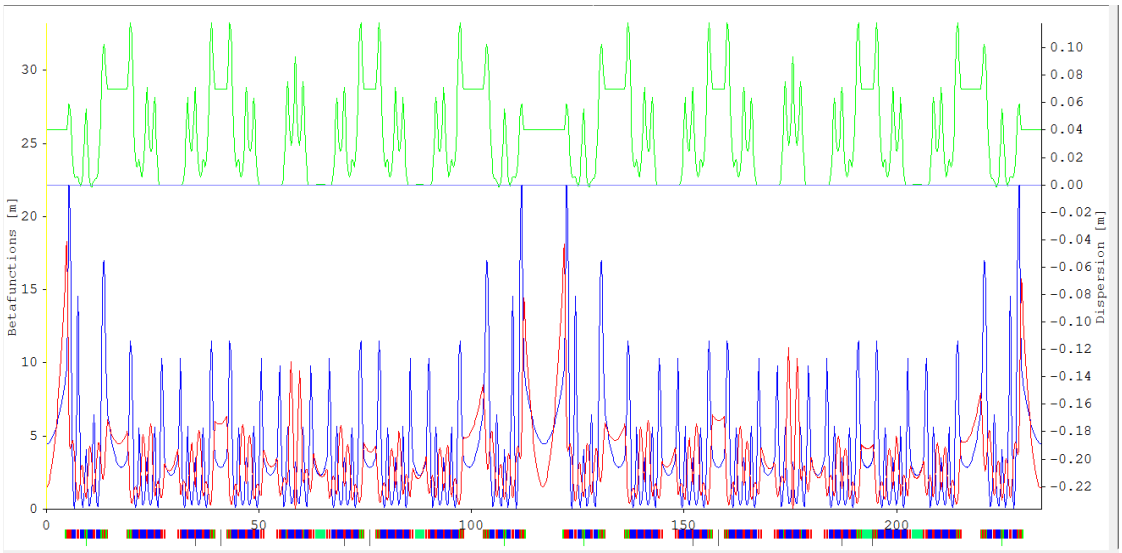

Figure 6: Optics functions of SPEAR3 ring with insertion devices. The dispersion free straights are: 2, 4, 5, 7, 11, 13, 14 and 16. The wiggler beam lines are: 5, 7, 15 and 16. The wiggler straight 15 is not dispersion free.

SPEAR3 ring 2 with insertion devices

To further reduce the emittance, the non-dispersion free wiggler straight in the previous section can become dispersion free by rearranging the QBA cell at right hands side as shown in figure 7. The dispersion free straights are: 2, 4, 5, 7, 11, 13, 15 and 16. The wiggler straights are: 5, 7, 15 and 16. The optics functions of the ring with insertion devices is shown in figure 7. Now all the wiggler straights are dispersion free and all the wigglers are damping wigglers. The emittance of the ring is 567 pm.rad. The tunes of the ring are $\Delta\nu_x/\Delta\nu_y$ 33.767/19.159. The horizontal damping partition is 1.82. The energy loss per turn is 0.756 MeV.

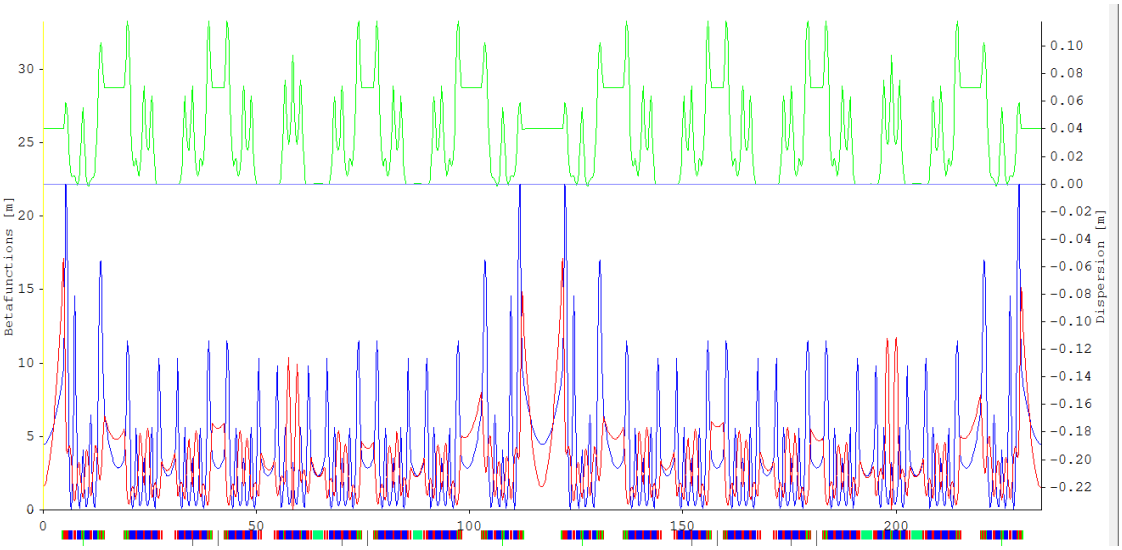

Figure 7: Optics functions of SPEAR3 ring with insertion devices. The dispersion free straights are: 2, 4, 5, 7, 11, 13, 15 and 16. The wiggler beam lines are: 5, 7, 15 and 16. All the wiggler straights are dispersion free.

Table 4: Main parameters of SPEAR3 Ring

	Ring w/o ID	Ring 1	Ring 2
Energy [GeV]	3	3	3
Circumference [m]	234.144	234.144	234.144
Emittance [nm.rad]	0.637	0.696	0.567
Betatron tune, x/y	33.77/19.13	33.77/19.17	33.77/19.16
Momentum compaction(E-3)	0.589	0.588	0.588
RMS momentum spread(E-3)	1.069	1.059	1.059
Jx	2.06	1.82	1.82
Natural chromaticity, x/y	-104/-34.6	-104/-35.1	-104/-35.3
Energy loss [MeV/turn]	0.58	0.756	0.756
Number of ID straights	18	18	18
Number of dispersion free ID straights	8	8	8
βx/βy at ID center [m]	2.3 /2.85 2.5 /5.0	2.3 /2.85 2.5 /5.0	2.3 /2.85 2.5 /5.0

MINIMIZATION OF EMITTANCE WITH LONGITUDINAL GRADIENT DIPOLE

It was first suggested by A. Wrulich in 1992[12] that introducing dipoles with a longitudinal field variation may provide an emittance significantly below the minimum emittance of a homogeneous TME bending magnet. The emittance can be reduced further without increasing the number of dipoles. The minimization is limited by the ratio of maximum and average field in the dipole [13]. A detailed study [14] for applying longitudinal gradient dipole to SPEAR3 shows that although the theoretic minimum emittance could be reduced further with longitudinal gradient dipole, it is hard to apply to SPEAR3 design due to the space and magnet strength limitations. By relaxing the matching requirements of minimum emittance of longitudinal gradient dipole, the best emittance achieved is approximately equal to the DQBA design in this report, while the strengths of some quadrupoles are much higher [14].

CONCLUSION

The theoretic minimum emittance when replacing the SPEAR3 DBA cell with a QBA cell is 330 pm.rad with damping partition$J_x = 1$. With the limited space and the number of quadrupoles, it's not possible to match to the minimum emittance for the middle dipole and the end dipole simultaneously. To effectively correct the linear chromaticity, two QBA cells are combined as a DQBA cell with a dispersive middle straight in which a pair of chromatic sextupoles, SD/SF, are added for chromaticity correction. The best emittance with DQBA cells and matching cells of SPEAR3 with $J_x = 2$ is 0.637 nm.rad. It is an order of magnitude smaller than the low emittance of SPEAR3 today. By arranging the dispersion free sections to the wiggler beamlines the wigglers become damping wiggler and the emittance can be reduced to 0.567 nm.rad. The gradient of sextupole for linear chromaticity correction is less than 5000 T/m^2. However the dynamic aperture is less than 1 mm. It's a challenge to optimize the nonlinear effects to increase the dynamic aperture and the limitation of sextupole magnet strength makes the optimization harder.

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