SLAC–PUB-16315
June, 2015

On the Trail of the Higgs Boson

Michael E. Peskin[1]

SLAC, Stanford University, Menlo Park, California 94025 USA

ABSTRACT

I review theoretical issues associated with the Higgs boson and the mystery of spontaneous breaking of the electroweak gauge symmetry. This essay is intended as an introduction to the special issue of *Annalen der Physik*, "Particle Physics after the Higgs".

Submitted to *Annalen der Physik*

[1]Work supported by the US Department of Energy, contract DE–AC02–76SF00515.

Contents

1 Introduction **1**

2 Why do we need the Higgs boson? **2**

2.1 Electroweak quantum numbers 2

2.2 The Higgs mechanism 4

2.3 Goldstone boson equivalence 6

3 The Standard Model of the Higgs field **8**

3.1 Formulation of the model 9

3.2 Natural flavor conservation 9

3.3 The end of the universe 11

4 What is wrong with the Standard Model? **12**

4.1 "Naturalness" 12

4.2 Ordering in condensed matter 14

5 Alternatives to the Minimal Standard Model **15**

5.1 Orientation 15

5.2 Supersymmetry 16

5.3 Higgs as a gauge boson 19

5.4 Higgs as a Goldstone boson 21

6 Where is the new physics? **22**

1 Introduction

The discovery of the Higgs boson in July 2012 [1,2] brought particle physics to a significant milestone. Since the 1970's, we have had a "Standard Model" of the strong, weak, and electromagnetic interactions that has seemed to describe all elementary particle phenomena observed at high-energy accelerators. Over forty years, this model has passed a series of increasingly stringent tests. As the parameters of the model became better defined and its predictions tested more incisively, points of disagreement between theory and experiment have faded away. Now the last elementary particle predicted by this model has been observed.

This discovery has cast the field of particle physics into a high state of tension. It is possible that our understanding of nature's particles and forces is complete, at least for the foreseeable future. It is equally well possible that the trail we have been following will veer off to reveal a completely new set of particles and interactions. The arguments on both sides rely on properties of the Higgs boson.

Thus, to introduce a volume on future experiments and facilities in high-energy physics, it is valuable to review what is known about the Higgs boson and what is expected from it. To what extent does the Standard Model provide a beautiful and simple theory that solves the problem for which the Higgs boson was invented? To what extent does this solution still leave mysteries? To what extent is this theory inadequate and in need of replacement? I will address these questions in this review.

There are many arguments outside the domain of the Higgs boson that the Standard Model is incomplete as a description of nature. The Standard Model does not include gravity. The Standard Model does not explain the small size of the cosmological constant or provide an alternative explanation for the accelerating expansion of the universe. The Standard Model does not have a place for the dark matter that makes up 85% of the mass in the unverse. The Standard Model does not have a place for neutrino masses (though these are readily accounted for by adding right-handed neutrinos). The Standard Model does not explain the preponderance of matter over antimatter in the universe. Solutions to these questions might imply new phenomena that will be observed in the near future. The review [3] describes these possibilities. On the other hand, all of these questions have possible explanations for which the observation of new physics is far out of reach. In this review, I will put all of these issues aside and speak only about the implications that we can draw from the physics of the Higgs boson.

2 Why do we need the Higgs boson?

To discuss whether the the Standard Model (SM) gives an adequate theory of the Higgs boson, we first need to review the reasons that the Higgs boson is needed. Our long wait for the Higgs discovery encouraged the idea that this particle might not actually exist. It was always possible that the Higgs boson could have been very heavy and difficult to discover. However, we could not have lived without it. The Higgs field, the field for which the boson is a quantum excitation, fills two essential purposes in the SM. It is necessary to provide the masses of quarks and leptons, and it is necessary to provide the masses of the W and Z bosons. In this section, I will review the evidence for these statements.

2.1 Electroweak quantum numbers

It is natural to think that there is no difficulty in generating masses for elementary particles. Particles should just *have* mass. However, in relativistic quantum field theory, this point of view is naive and incorrect.

To build a wave equation for a relativistic field, we start from fields that transform under the Poincaré group according to basic irreducible representations. The simplest such representations, and the ones that serve as the building-blocks for all others, are representations for massless fields. The wave equations for these fields describe two particle states—a massless particle with helicity J (an integer or half-integer) and a massless antiparticle with helicity $-J$ [4].

A massles particle always travels at the speed of light. We can never stop it and manipulate it at rest. This is the raeson that the restriction to a single helicity state is consistent. If a particle has mass, we can bring it to rest by a Lorentz transformation and then turn it in an arbitrary direction. This will rotate the state of helicity J into any linear combation of the $(2J+1)$ states of a spin J representatioin of the rotation group. If the particle is not its own antiparticle, another $(2J+1)$ antiparticle states are also required. Thus, special relativity places an essential barrier to promoting a zero-mass particle to nonzero mass. This promotion can only be done if we can identify the $(2J+1)$ elementary Poincaré representations that will form the massive particle and if we can mix these states together quantum-mechanically. In quantum mechanics, states can mix only if they have the same values of conserved quantum numbers. We must watch out for this restriction whenever we try to give mass to a massless particle.

We can understand the required mixing in a very explicit way for particles of spin $\frac{1}{2}$. A massive spin-$\frac{1}{2}$ particle is described by the Dirac equation. The Dirac field is a four-component field. In the basis in which the chirality operator γ^5 is diagonal, the spinors decompose as

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad \text{with} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \ , \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{1}$$

In this notation, the familar Lagrangian for the Dirac equation

$$\mathcal{L} = \overline{\Psi}(i \not{\partial} - m)\Psi \tag{2}$$

takes the form

$$\mathcal{L} = \psi_L^\dagger i\overline{\sigma}^\mu \partial_\mu \psi_L + \psi_R^\dagger i\sigma^\mu \partial_\mu \psi_R - m(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) \ . \tag{3}$$

For $m = 0$, this formula makes explicit that the massless Lagrangian contains the left-handed ($J = -\frac{1}{2}$) and right-handed ($J = +\frac{1}{2}$) fermions completely separately. This remains true when we couple to gauge fields, since this coupling is done by replacing the derivative by the covariant derivative

$$\partial_\mu \to D_\mu = (\partial_\mu - i \sum_a g_a A_\mu^a) \ , \tag{4}$$

which acts separately on ψ_L and ψ_R. The fields ψ_L and ψ_R are said to have definite *chirality*. The mass term in (3) mixes the these definite chirality states to form the four states of a massive spin-$\frac{1}{2}$ particle and its antiparticle. In order for this mixing to be permitted, these two fields must have the same quantum numbers [5].

However, we know that the left- and right-handed quarks and leptons do not have the same quantum numbers. The essence of parity violation in the weak interactions is that the W bosons couple to the left-handed chirality states and not to the right-handed chirality states. In the SM, the W and Z bosons are gauge fields. Then the couplings of each quark or lepton reflects its quantum numbers under the $SU(2) \times U(1)$. The left-handed fields have $I = \frac{1}{2}$ under the weak interaction $SU(2)$ gauge group; the right-handed fields have $I = 0$, and the two fields also differ in their values of the hypercharge Y.

We can escape this problem if the W and Z bosons are not gauge fields. However, the evidence for gauge invariance in the weak interactions is overwhelming. Gauge invariance implies the equality of the $SU(2) \times U(1)$ couplings g and g' in the couplings of each species of quarks and leptons. Thus, the quality of the overall precision electroweak fit [6,7] is a consequence of gauge invariance. The Yang-Mills structure of the $SU(2)$ vector boson interactions specifies the structure of the $WW\gamma$ and WWZ couplings, which are measured to be in agreement with this predicton to percent accuracy [8].

The only other way to escape this problem is to assume that $SU(2) \times U(1)$ is a symmetry of the W and Z boson equations of motion that is not respected by the ground state of the electroweak interactions. That is, the symmetry must be spontaneously broken. If this is true, there must be an agent that is responsible for the symmetry breaking. This would be an elementary or composite field that transforms under the $SU(2) \times U(1)$ gauge group and nevertheless acquires a nonzero value throughout space. To preserve translation and Lorentz invariance, the field must be a scalar with a constant value, independent of position. Quarks, leptons, and vector bosons could make symmetry-violating transitions through their coupling to this nonzero field value.

For electroweak spontaneous symmetry breaking (henceforth, EWSB), we call this field the *Higgs field*. Any such field must have quantum excitations, corresponding to perturbations that change the vacuum value of the field as a function of space and time. The lowest mass excitation is called the *Higgs boson*.

2.2 The Higgs mechanism

Generating mass for spin-1 particles such as the W and Z bosons brings in additional difficulties. Even before we add the mass term, the reduction of a 4-vector field $A_\mu(x)$ to the Poincaré representation with helicity 1 and -1 states is quite delicate. The simplest idea is to quantize the vector field as one would a scalar field. This leads to a set of creation and annihilation operators with the Lorentz-invariant commutation relations

$$[a_p^\mu, a_k^{\nu\dagger}] = (-g^{\mu\nu})(2\pi)^3 \delta(\vec{p} - \vec{k}) \ . \tag{5}$$

The metric has $(-g^{00}) = -1$, and so the operator $a_k^{0\dagger}$ creates states with negative norm, implying negative probability. The interactions of the theory must be set up so that these negative probability particles can never be emitted. Quantum Electrodynamics, and, more generally, Yang-Mills gauge theories, achieve this. The proof makes strong use of local gauge invariance [9–11].

To give mass to a spin-1 particle, we must add to the helicity ± 1 states a helicity 0 state that will complete the needed complement of $3 = (2J+1)$ spin states. We must do this without disrupting the cancellation of negative probability states described in the previous paragraph.

The achievement of Higgs, Brout and Englert, and Guralnik, Hagen, and Kibble [12–14] was to show that spontaneous breaking of the gauge symmetry achieves this in a Lorentz-invariant way. Spontaneous breaking of a continuous symmetry automatically generates a massless particle, the Goldstone boson [15]. This boson is created and destroyed by the current associated with the symmetry. In a gauge theory, this same current defines the coupling of the vector boson. So, the vector boson and the Goldstone boson naturally mix. The fact that this mixing preserves the gauge invariance of the equations of motion and the natural cancellation of negative

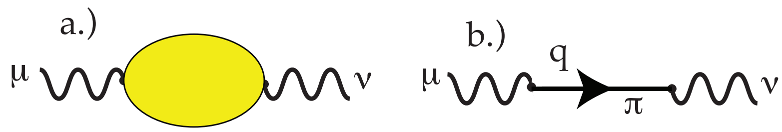

Figure 1: (a) Vector boson self-energy diagram; (b) contribution to this diagram from a Goldstone boson intermediate state.

probability states was already known from studies of superconductivity [16,17]. It is a nice bonus that the mixing is also fully relativistic.

It is not so difficult to describe the process of mass generation more explicitly. At zeroth order, a gauge field obeys Maxwell's equations and thus must be massless. A mass can potentially be generated by the self-energy diagram shown in Fig. 1(a). For a scalar field, the analogous diagram would directly shift the mass to a nonzero value. For a gauge field, however, the currents must be conserved. The diagram must, then, have the form

$$-ig^2\Big(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\Big)\Pi(q) \ . \tag{6}$$

The vector boson obtains a mass if

$$\Pi(q^2) \neq 0 \text{ at } q^2 = 0 \ . \tag{7}$$

However, if all states created by the current are massive, this diagram cannot have a singularity at $q^2 = 0$. We must find $\Pi(q^2) \sim q^2$, and so the vector boson mass cannot be moved away from zero. This is the standard argument in QED that quantum corrections do not generate a mass for the photon.

Spontaneous symmetry breaking alters this conclusion. In a system with spontaneous breaking of a continuous symmetry, there is a Goldstone boson. The Goldstone boson is created and destroyed by the current that couples to the gauge field. This is the same current whose 2-point function gives the vacuum polarization diagram in (6). The matrix element for the current to destroy the Goldstone boson is written

$$\langle \pi(q)| \, j^\mu(x) \, |0\rangle = iFq^\mu \ ; \tag{8}$$

Lorentz invariance fixes the form of the matrix element, requiring a parameter F with the dimensions of mass. From the form of (8), we find a contribution to the vector boson self-energy shown in Fig. 1(b),

$$(-igFq^\mu)\frac{i}{q^2}(igFq^\nu) \ . \tag{9}$$

This is compatible with (6), the complete self-energy, only if that expression reduces, as $q^2 \to 0$, to

$$-i(gF)^2\Big(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\Big) \ . \tag{10}$$

We see that the presence of the Goldstone boson requires a mass for the vector boson

$$m^2 = (gF)^2 \ . \tag{11}$$

This is the Higgs mechanism.

2.3 Goldstone boson equivalence

There are many formal questions that one might ask about the simple argument given in the previous section. Of these, there is one that is particularly important to discuss: What is the eventual spectrum of particles left by the mechanism? Does it successfully remove all massless states, and all negative-metric states? The short answer to this question is that the combination of the vector field and the Goldstone boson field leads to precisely three physical states, the three polarization states of a massive vector boson. Depending on the gauge chosen to represent the gauge field, there may be additional unphysical states, of zero or nonzero mass. However, these states are merely an artifact of the method of calculation. They make no contribution to the scattering amplitudes of the physical vector boson states and associated matter particles. For a careful proof of this statement, see [9–11].

However, the Goldstone boson does not quite disappear. There are definite phenomenological consequences of its role in the vector boson mass generation. Recall that the role of the Goldstone boson is to provide a helicity 0 state that combines with the helicity ± 1 gauge boson states to provide the 3 states required for a massive spin 1 particle. If the final massive boson is near rest, its states will be a mixture of gauge boson and Goldstone boson states that might be difficult to untangle. However, for a highly boosted massive vector boson, there shoud be a clear distinction between the helicity ± 1 states, which should have properties similar to massless transversely polarized gauge bosons, and the helicity 0 state, which should have properties similar to those of the original Goldstone boson.

This intuition is made precise by the Goldstone Boson Equivalence Theorem (GBET) of Cornwall, Levin, Tiktopoulos, and Vayonakis [18,19]. The theorem states that the amplitude for producing the helicity 0 state of a massive gauge boson, considered in the limit of large momentum, becomes equal to the amplitude for producing the original Goldstone boson. For example, for the W boson of the SM,

$$\mathcal{M}(X \to Y + W_0^+) \approx \mathcal{M}(X \to Y + \pi^+) + \mathcal{O}(\frac{m_W^2}{E_W^2}) \ . \tag{12}$$

where π^+ is the Goldstone boson associated with the spontaneous breaking of the $SU(2)$ gauge symmetry. Chanowitz and Galliard have given a careful statement and proof of this theorem, valid for any number of vector bosons [20].

A surprising consequence of the GBET is seen in top quark decay. The top quark decays by the simple process $t \to W^+ b$, to an on-shell W boson. Naively, one might expect that the rate of this decay would be an expression of the form $\Gamma_t \sim g^2 m_t$, where g is the weak interaction coupling constant. Really, though, the formula for the decay rate is

$$\Gamma_t = \frac{g^2 m_t}{32\pi} \, (\frac{m_t^2}{2m_W^2} + 1) \, (1 - \frac{m_W^2}{m_t^2})^2 \ , \tag{13}$$

ignoring the bottom quark mass and higher-order quantum corrections. One term in the formula for the rate is enhanced by a factor $m_t^2/2m_W^2$.

The formula (13) can be derived without any reference to gauge invariance. One simply uses the standard form of the V–A weak interaction coupling

$$\Delta\mathcal{L} = \frac{g}{\sqrt{2}} W_\mu \, \bar{b}_L \gamma^\mu t_L \tag{14}$$

and sums over the three polarization states of the massive W boson in the final state. The enhancement can be seen to be associated with the decay to a helicity 0 W boson. However, the result only makes sense when viewed from the perspective of the GBET. According to the GBET, the top quark couples to the helicity 0 component of the W boson with the strength of the top quark coupling to the Higgs field, which is larger than the coupling of the t to the weak interactions. More explicitly, if v is the vacuum value of the HIggs field, these couplings have the ratio

$$\frac{y_t^2}{g^2} = \frac{y_t^2 v^2/2}{g^2 v^2/2} = \frac{m_t^2}{2m_W^2} \tag{15}$$

This is exactly the enhancement factor seen in (13).

A corollary to this argument is that the W bosons in top decays should be polarized, with

$$\frac{\Gamma(t \to bW_0)}{\Gamma(t \to bW_T)} = \frac{m_t^2}{2m_W^2} \approx 2.3 \ . \tag{16}$$

This prediction is confirmed by measurements at the Tevatron and the LHC [21–23]. Apparently, the W boson secretly knows that its mass comes from the Higgs mechanism even in processes in which the Higgs boson is not involved directly.

The GBET controls many other features of the high-energy dynamics of W and Z bosons. One of the most striking predictions occurs in the cross section for $e^+e^- \to W^+W^-$ at high energy. At leading order, there are three Feynman diagrams that contribute to this process, shown in Fig 2(a). Figure 2(b) shows the measurement of this cross section by the LEP experiments, together with the prediction of the SM [8]. The figure also shows the effect of omitting the Z and photon diagrams. The individual diagrams have a different energy-dependence than the final answer, growing more strongly with energy by a factor of s/m_W^2. Remarkably, the $WW\gamma$ and

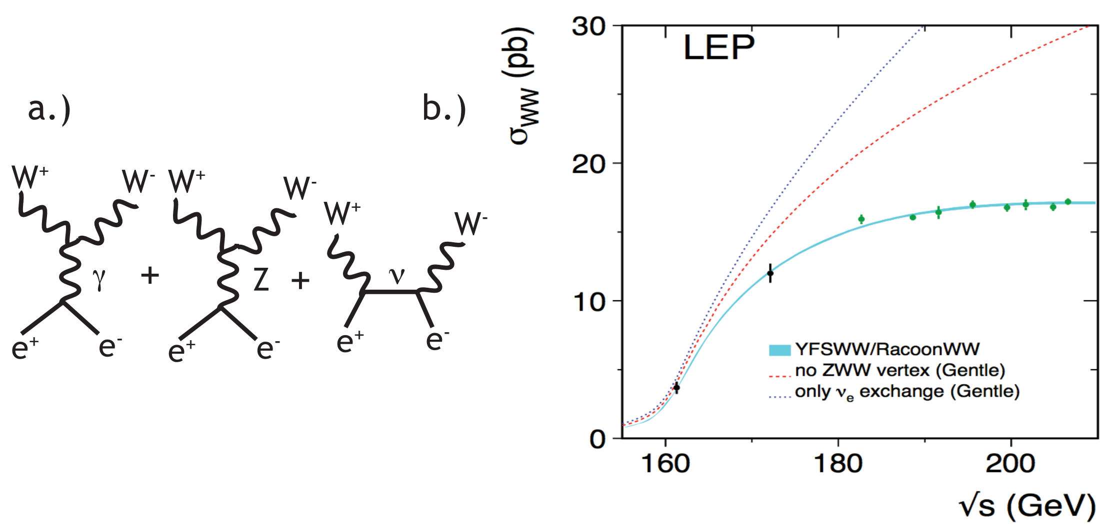

Figure 2: (a) Leading-order Feynman diagrams contributing to the cross section for $e^+e^- \to W^+W^-$; (b) measured values of this cross section from the LEP experiments, from [8].

WWZ vertices are correctly structured to give an almost complete cancellation of the three diagrams at high energy [24]. This cancellation is difficult to understand from the point of view of the diagrams in Fig. 2(a). It makes more sense from the point of view of Goldstone boson equivalence. The cross section for the production of two longitudinally polarized W bosons, $e^+e^- \to W_0^+W_0^-$ can equal the cross section for the production of a pair of Goldstone bosons, $e^+e^- \to \pi^+\pi^-$, only if the terms with an extra power of s/m_W^2 cancel in the full calculation.

Thus, there are many aspects of elementary particle behavior that do not involve the Higgs boson directly but nevertheless require the Higgs mechanism for their explanation.

3 The Standard Model of the Higgs field

We have now seen that the Higgs mechanism and EWSB are essential parts of the gauge theory of weak interactions. How can this symmetry breaking be implemented? The simplest choice is to have one $SU(2)$ doublet of scalar fields, with a potential that causes this field to acquire a vacuum value. The full structure of the SM is given by this one doublet scalar field, together with the $SU(3) \times SU(2) \times U(1)$ gauge bosons and the quarks and leptons. In this section, I will review some properties of this model as it relates to the Higgs field.

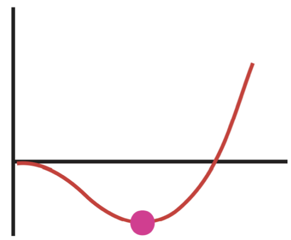

Figure 3: Potential energy of the Higgs field in the Standard Model.

3.1 Formulation of the model

The symmetry-breaking sector of the SM is built from a scalar field $\varphi_a(x)$, $a = 1, 2$, that transforms under $SU(2) \times U(1)$ with $I = \frac{1}{2}$ and $Y = \frac{1}{2}$. The renormalizable Lagrangian for this field takes the form

$$\mathcal{L} = |D_\mu \varphi|^2 - V(\varphi) \tag{17}$$

with

$$V(\varphi) = \mu^2 |\varphi|^2 + \lambda |\varphi|^4 \ . \tag{18}$$

The choice

$$\mu^2 < 0 \tag{19}$$

leads to a potential of the form shown in Fig. 3. The minimum occurs at

$$v = \sqrt{-\mu^2/\lambda} \ . \tag{20}$$

From the relation between v and the W boson mass

$$m_W = gv/2 \tag{21}$$

and our excellent knowledge of the value of the $SU(2)$ coupling g from precision electroweak measurements, we have

$$v = 246 \text{ GeV} \ . \tag{22}$$

The related parameters v and μ^2 are the only dimensionful parameters in the SM. So, within the SM, (22) sets the scale for all quark, lepton, and vector boson masses.

3.2 Natural flavor conservation

In the SM, the masses of vector bosons are generated by the Higgs mechanism, as I have already described. The masses of quarks and leptons are generated in a more *ad hoc* way. The SM simply postulates a coupling of the fermion and Higgs multiplets that respects the $SU(2) \times U(1)$ gauge symmetry. Then, when the Higgs field acquires its vacuum value, the fermions receive mass.

A remarkable feature of the SM is that no special structure is needed to produce mass terms of a very simple form. The most general Higgs-fermion couplingx, or *Yukawa couplingx*, consistent with renormalizability and $SU(2) \times U(1)$ are

$$\Delta\mathcal{L} = Y_e^{ij}\overline{L}^i \cdot \varphi e_R^j \; + \; Y_d^{ij}\overline{Q}^i \cdot d_R^j \; + \; Y_u^{ij}\overline{Q}^i * \varphi^* u_R^j \; + \; h.c. \tag{23}$$

where $i, j = 1, 2, 3$ run over the quark and lepton generations, L^i, Q^i are the left-handed lepton and quark fields, and the contractions of these $SU(2)$ doublet fields with the Higgs field are

$$\overline{L} \cdot \varphi = \overline{L}_a \varphi_a \;, \qquad \overline{Q} \cdot \varphi = \overline{Q}_a \varphi_a \;, \qquad \overline{Q} * \varphi^* = \epsilon_{ab} \overline{Q}_a \varphi_b^* \;. \tag{24}$$

The matrices Y_f^{ij} may be arbitrary 3×3 complex matrices. Thus, apparently, they permit arbitrarily strong flavor and CP mixing.

This mixing, however, can be removed by simple field redefinitions. Any 3×3 complex matrix can be represented as

$$Y_f = V_{fL}^\dagger \cdot y_f \cdot V_{fR} \tag{25}$$

where y_f is real, positive, and diagonal and V_{fL} and V_{fR} are independent unitary matrices. Using this decomposition of the Yukawa matrices, define

$$e_R'^j = V_{eR}^{jk} e_R^k \;, \qquad d_R'^j = V_{dR}^{jk} d_R^k \;, \qquad u_R'^j = V_{uR}^{jk} u_R^k \;. \tag{26}$$

and

$$L'^j = V_{eL}^{jk} L_R^k \;, \qquad d_L'^j = V_{dL}^{jk} d_L^k \;, \qquad u_L'^j = V_{uL}^{jk} u_L^k \;. \tag{27}$$

This change of variables removes factors of the $V_{f\,L,R}$ from the Yukawa terms, but it potentially reintroduces these factors into the kinetic terms and gauge coupling terms of the fermion fields. However, it may be shown that all factors of the $V_{f\,L,R}$ that might appear in these terms cancel out, except for a modification of the W boson coupling to quarks,

$$\frac{g}{\sqrt{2}} W_\mu^+ \overline{u}_L \gamma^\mu d_L \;\; \to \;\; \frac{g}{\sqrt{2}} W_\mu^+ \overline{u}_L \gamma^\mu \, V_{CKM} \, d_L \;, \tag{28}$$

where $V_{CKM} = V_{uL} V_{dL}^\dagger$. The Yukawa couplings of the Higgs field become completely diagonal,

$$\Delta L = \frac{y_{ei} h}{\sqrt{2}} \overline{e}_L^i e_R^i \; + \; \frac{y_{di} h}{\sqrt{2}} \overline{d}_L^i d_R^i \; + \; \frac{y_{ui} h}{\sqrt{2}} \overline{u}_L^i u_R^i \; + \; h.c. \tag{29}$$

In the full Yukawa Lagrangian, the matrices $V_{f\,L,R}$ appear only in the combination V_{CKM}.

Finally, the flavor and CP mixings in the original Yukawa term (23) remain only in three physical couplings. First, they appear in V_{CKM}. This unitary matrix, the *Cabibbo-Kobayashi-Maskawa matrix*, is required as the source for all observed flavor mixing and CP violation in the weak interactions [25]. Second, the matrix V_{eL} appears in the neutrino mass matrix, another place where there is known flavor mixing and the possibility of CP violation [26]. These two residual appearances should be considered sucesses of the model, giving flavor and CP violation precisely in the ways that it is observed in experiments. Finally, the sum of the overall phases of the $V_{f\,L,R}$ matrices of quarks shifts the QCD θ parameter. If this parameter is nonzero, it leads to P and T violation in the strong interactions, in particular, to a nonzero neutron electric dipole moment [27]. This is a problem for the SM whose solution requires the introduction of a new particle, the *axion*, or additional symmetries [28].

It is fair to consider that the successes of this analysis outweighs its problems. The outcome is referred to as *natural flavor and CP conservation* in the Higgs interactions. It is important to note that this property is typically lost in models of the Higgs sector that generalize the one in the SM.

3.3 The end of the universe

The Higgs sector of the SM has one more, quite unexpected, property. If the SM is exact up to energies much higher than those currently probed by accelerators, we can extrapolate its behavior using the renormalization group. The most important effect of this is that the Higgs field self-coupling, which determines the form of the Higgs potential, evolves with energy scale.

The dominant terms in the renormalization group equation for the Higgs self-coupling are

$$\frac{d\lambda}{d\log Q} = \frac{3}{2\pi^2}\left[\lambda^2 - \frac{y_t^4}{32} + \cdots\right] \tag{30}$$

where λ is the Higgs field self-coupling and y_t is the top quark Yukawa coupling. The correction due to the top quark turns out to be larger and fixes the sign of the right-hand side. Then λ becomes smaller at higher energy scales, eventually becoming negative. With the current value of the top quark mass, $m_t = 173$ GeV [29], this analysis predicts that λ will become negative at about 10^{11} GeV. This means that, if the SM is exact at energies beyond 10^{11} GeV, the conventional vacuum of this model is unstable. With a very long half-life, estimated to be 10^{600} yr, the vacuum expectation value of the Higgs field should tunnel to a very large value, near the Planck scale [30].

It is not clear that this instability is a problem with the Standard Model, or a feature of it. It has been argued that our universe must be unstable, to avoid the prediction that most intelligent beings in the universe are "Boltzmann brains", isolated conscious entities produced spontaneously by quantum processes [31]. It is also possible, within the current uncertainties, that the top quark has the lighter value, 171.1 GeV, that brings the Higgs potential just to zero at the Planck scale. This conjecture is the basis for the idea of asymptotically vanishing Higgs interaction and the Higgs field as the inflaton [32].

4 What is wrong with the Standard Model?

So, couldn't the Standard Model with one elementary Higgs field describe everything? I have already pointed out the existence of phenomena such as dark matter and dark energy whose explanation certainly lies outside the Standard Model. The model must also be extended with effective operators to generate neutrino masses. But one could pose the question more narrowly: Couldn't the Standard Model provide a complete explanation for the phenomena of elementary particle physics up to the currently conceived limits of accelerator energies?

As far as accelerator-based experiments are concerned—aside from some much-discussed discrepancies such as the value of the muon $(g-2)$ [33]—the SM does an excellent job of explaining the wide variety of elementary particle phenomena. The main objection to the idea that the SM is a complete explanation comes from theory.

4.1 "Naturalness"

The SM is a compact description of elementary forces, but, still, it contains a large number of parameters. These include the $SU(3) \times SU(2) \times U(1)$ gauge couplings—g_s, g, and g'—the quark and lepton masses, the four CKM mixing angles, and two parameters from the Higgs field potential, or, equivalently, the Higgs mass and vacuum expectation value. This is already 18 free parameters. A complete specification of the model starting from the most general renormalizable Lagrangian with the SM gauge symmetry would also require specification of the three θ angles and the full 3 complex Yukawa matrices, a total of 62 parameters.

It is the faith of physicists that, eventually, we will be able to predict these parameters from an underlying theory. However, this simply cannot be done within the SM. In the Standard Model, the higher-order corrections to all of these parameters are infinite and require renormalization. That is, we can only make sense of rhe model by fixing *a priori* the parameters that we would ideally like to compute.

This problem is particularly galling for the parameters of the potential energy of the Higgs field. The renormalizability of the theory requires that the Higgs potential takes the simple form

$$V(\varphi) = \mu^2|\varphi|^2 + \lambda|\varphi|^4 \ , \tag{31}$$

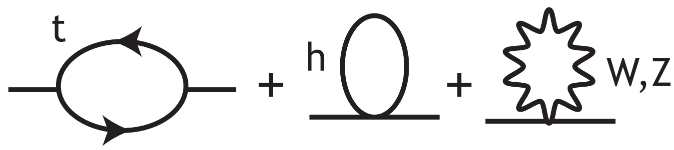

Figure 4: One-loop corrections to the Higgs field mass term in the Standard Model.

up to radiative corrections. For spontaneous symmetry breaking, the renormalized value of the parameter μ^2 should be negative. But not even the qualitative prediction that the symmetry is broken is a prediction of the model. The μ^2 parameter could have either sign; there is no logic that prefers one sign to the other. Though the potential (31) is relatively simple, we have to take its parameter values as given. We cannot ask where these values come from.

Thus, the hypothesis that the SM is the complete description of elementary particle interactions is a statement about the ultimate limits of physics explanation. This hypothesis implies that we cannot predict the Higgs boson mass and the quark and lepton masses, or even the qualitative form of the Higgs potential, from deeper principles. Any such predictions require a model with more structure than is present in the in the SM.

The form of the first corrections to the μ^2 parameter give a hint as to where that additional structure might lie. At the level of one-loop corrections, the formula for the renormalized μ^2 parameter reads

$$\mu^2 = \mu^2_{bare} - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \frac{\lambda}{8\pi}\Lambda^2 + \frac{9\alpha_w + 3\alpha'}{4\pi}\Lambda^2 + \cdots \ , \tag{32}$$

where the corrections written out come from the loop diagrams shown in Fig. 4, with top quarks, Higgs bosons, and W and Z bosons, respectively. All of these diagrams depend quadratically on the ultraviolet cutoff, as shown. The diagrams have different signs, so even the final sign of μ^2—which determines whether the symmetry is broken or not—depends on how these diagrams are regularized.

Setting Λ equal to the Planck scale, this equation implies an absurd cancellations of the first 30 decimal places of the terms on the right-hand side of (32) This difficulty is called the “gauge hierarchy problem” and is often presented in itself as a motivation for new physics [34]. I have stressed already that, for me, this problem is part of a larger problem—that the mass and couplings of the Higgs boson should be predicted, and that the SM is inadequate to that task.

We can, however, use (32) as a rough way to estimate the masses of new particles required in a theory that predicts the value of μ. The first thing these particles must do is to cancel the quadratic divergences in the diagrams of Fig. 4. Let us assume naively that the calculation of μ^2 will not entail a cancellation in more than the first decimal place. Then we expect new particles of mass less than 2 TeV to cancel the top quark loop correction, new particles of mass less than 3 TeV to cancel the Higgs loop correction, and new particles of mass less than 5 TeV to cancel the W and Z loop corrections.

This is not in itself a proof that new particles must be present in the energy regime of the LHC and other planned accelerators. But, it indicates a tremendous opportunity for discovery.

4.2 Ordering in condensed matter

Many particle physicists do not consider the spontaneous symmetry breaking of the weak interaction as a problem in itself. They feel that this phenomenon is adequately explained by the Higgs scalar field, perhaps up to the question raised above of the sign of μ^2.

In other areas of physics, there are many examples of spontaneous symmetry breaking. I feel that it is important for particle physicists to make a close study of these systems. They teach us that the presence of spontaneous symmetry breaking is not a random choice taken by a physical system but, rather, is always the result of a comprehensible, and often fascinating, mechanism. In this section, I will briefly discuss some examples.

The best understood example of spontaneous symmetry breaking is that of superconductivity. The fact that most metals are superconducting in their ground states was one of the most puzzling mysteries of condensed matter physics in the first half of the 20th century. The problem was finally solved by Bardeen, Cooper, and Schrieffer [35], who noticed that the sharpness of the Fermi surface at low temperature amplifies the effect of any small attractive interaction of electrons, such as that due to exchange of phonons. This leads to a ground state containing a thermodynamic number of electron pairs in an ordered condensate. This model leads to a successful quantitative description of superconductivity [36].

The idea of condensation of fermion pairs stimulated by an attractive interaction has been applied to other, quite different, physical systems. It describes the array of superfluid ground states of He^3 [37]. Pairing of protons and neutrons is seen in nuclear spectra (although rigorously, because nuclei have finite size, there is not true spontaneous symmetry breaking) [38]. Nambu and Jona-Lasinio used a model with pair condensation to explain the spontaneous breaking of chiral symmetry in the strong interactions [39]. Unfortunately, in this case, there is no Fermi surface at a nonzero momentum, so pairing requires that the attractive interaction between nucleons or quarks be above some critical strength. (If the analogy with superconductivity were more exact, we would understand the generation of the constituent quark masses better, and Nambu would have received his Nobel Prize decades earlier.) Finally, the theory of the BCS ground state and its effect on the electromagnetic field directly stimulated the original papers on the Higgs mechanism [12–14,16,17].

Magnets provide other examples of condensed matter sysems with spontaneous symmetry breaking. Magnetism is confined to a limited region of the periodic table with almost but not completely filled d orbitals. This produces an ordering of electron spins through a principle called *Hund's rule*: Because the d electrons repel one another electrostatically, they tend to favor antisymmetric orbital configurations and therefore symmetric spin configurations [40]. When a thermodynamic number of electrons are involved, the spins of these electrons take on a common classical direction, and the ground state violates rotational symmetry.

Other examples of condensed matter systems with spontaneous symmetry breaking depend in even more detail on the atomic or molecular forces involved [41]. Assemblages of large long or flat molecules lead to the ordering of liquid crystals [42]. Crystal lattices with soft directions of distortion allow displacive transitions that lower the crystal symmetry [43]. It is wonderful how many different types of ordering are seen in condensed matter physics and how, in each case, the nature of the ordering has a direct and intuitive physical explanation.

Shouldn't this be true also for the spontaneous breaking of the weak interaction symmetry? If we fail to search for this explanation, it will be an opportunity lost.

5 Alternatives to the Minimal Standard Model

What kind of elementary particle theory could provide the explanation for EWSB? This question has received a great deal of thought since Steven Weinberg first discussed it 40 years ago [44]. The answers that have been proposed would fill a book. Here I will briefly introduce the main types of theories now under consideration.

5.1 Orientation

Before I discuss specific models of EWSB, I should emphasize that these models, though based on very different physical mechanisms, share many general features. Here I would like to highlight three of these.

First, theories of EWSB are not simple or minimal in structure. The reason for this is that one of the problems a theory of electroweak symmetry breaking must solve is to render the corrections to the Higgs mass term shown in Fig. 4 finite and calculable. To do this, some symmetry or other principle must prohibit the Higgs mass term μ^2 from obtaining divergent corrections from quantum fluctuations of very high momentum. This is not straightforward. The Higgs mass term

$$\mu^2|\varphi|^2 \tag{33}$$

is a scalar under the Lorentz group, $SU(2) \times U(1)$. It respects all other symmetries encountered in a first course on quantum field theory. To forbid this term, we need to invoke more advanced symmetry principles, for which examples will be given below. Theories with these structures are not mere extensions of the SM but have their own profound implications.

Second, models of EWSB contain new particles that contribute to the radiative corrections to the Higgs mass parameter μ^2. The higher symmetry of the model might make the Higgs mass parameter finite in principle, but this mass term must also be finite in practice when its 1-loop radiative corrections are calculated. This requires new particles, with masses suggested to be of TeV size, to cancel the divergences due to the heavy SM particles t, h, W, and Z.

Finally, many theories of EWSB involve the top quark in an essential way. Since the top quark is the heaviest particle of the SM, and therefore the one most strongly coupled to the Higgs field, the top quark contribution in Fig. 4 is the term with the largest coefficient. And, this coefficient is negative. Even after this term is made finite by adding extra particles in the loop, it is quite plausible that the result remains negative. If so, it can be the strongest effect driving EWSB. In the specific models that I discuss below, we will see specific physics explanations for why the contribution of the top quark and its partner particles has the correct sign to drive EWSB.

5.2 Supersymmetry

Although the mass term of a scalar field is not obviously restricted by symmetry, the mass term of a spinor typically violates some global symmetry such as a chiral symmetry. A relation between the Higgs scalar field and a spinor field then might have the power to prohibit corrections to the Higgs mass term. To implement this, we would postulate a symmetry

$$\delta\varphi = \bar{\epsilon}\,\psi \ , \tag{34}$$

where ψ is a spin-$\frac{1}{2}$ field and ϵ is a spinor parameter. A symmetry that connects fields with spin differing by $\frac{1}{2}$ unit is called a *supersymmetry.*

It turns out that the combination of supersymmetry and Lorentz invariance has very strong implications [45]. In theories with both symmetries, there must be a conserved spin-$\frac{1}{2}$ charge Q whose square is the Hamiltonian. More precisely, the charge Q satisfies

$$\{Q_\alpha, Q_\beta\} = 2\gamma^\mu_{\alpha\beta} P_\mu \ , \tag{35}$$

where P_μ is the total energy-momentum of the theory. Then every particle in the theory must participate in the supersymmetry. In a supersymmetric extension of the SM, not only do the Higgs bosons have spin-$\frac{1}{2}$ partners, but also the quarks and leptons have spin-0 partners with the same $SU(2) \times U(1)$ quantum numbers, and the gauge bosons have spin-$\frac{1}{2}$ partners.

There is a large literature on the spectrum of particles predicted by supersymmetry and the expectations for the properties of supersymmetric particles that might be found at colliders [46–49]. In the remainder of this section, I will focus tightly on the connection between supersymmetry and EWSB.

Supersymmetry has a number of specific implications for the Higgs field. First, it motivates the presence of scalar fields in the theory. In the SM, the Higgs field is the one and only scalar field. In the supersymmetric extension of the SM, there is a scalar field for each left- or right-handed fermion. In addition, the constraints of supersymmetry imply that it is not possible for a single Higgs field to give mass to both u and d quarks. At the minimum, two Higgs fields, H_u and H_d, are needed. H_d can also give mass to the charged leptons.

The large number of scalar fields brings a new problem. The vacuum expectation value of the Higgs field breaks the weak interaction symmetry $SU(2) \times U(1)$ to electromagnetism, giving the pattern of symmetry breaking that we see in nature. But at first sight, it seems equally possible that one of the other scalar fields in the theory will obtain a vacuum expectation value. This would always be a disaster. For example, if the scalar partner of the right-handed top quark were to obtain a vacuum expectation value, that would leave $SU(2)$ invariant while breaking $U(1)$ and also the $SU(3)$ color symmetry of QCD. An explanation for electroweak symmetry breaking in supersymmetry must also include an explanation of why the other scalar fields do not acquire vacuum values.

In a theory with exact supersymmetry, the mass parameters for the scalar fields are highly restricted. Since the quarks and leptons cannot have mass terms in the absence of electroweak symmetry breaking, then also the associated scalar fields cannot have mass terms. The only allowed mass term is one involving the two Higgs fields H_u and H_d,

$$\mu^2(|H_u|^2 + |H_d|^2) \tag{36}$$

This is a positive $(\text{mass})^2$. It can be shown that the radiative corrections to this term vanish to all orders in perturubation theory. Essentially, loop diagrams involving quarks, leptons, and gauge bosons are cancelled exactly by loop diagrams involving their spin 0 and spin-$\frac{1}{2}$ partners.

A realistic model of supersymmetry must be more complex. Exact supersymmetry would imply a charged scalar particle with the same mass as the electron, but no such particle exists. The most straightforward way to resolve this problem is to assume that supersymmetry is spontaneously broken. It can be shown that spontaneous supersymmetry breaking among any particles in nature—even unknown particles with

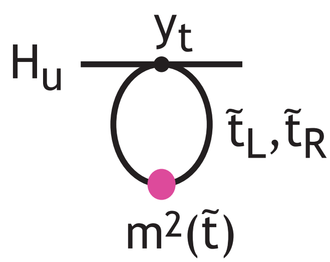

Figure 5: A diagram by which the supersymmetry-breaking mass terms of $\widetilde{t}_L$, $\widetilde{t}_R$ renormalize the mass term of the Higgs field H_u.

very large masses—will eventually feed down to the particles of the SM and produce masses for the partners of the quarks, leptons, and gauge bosons. We can then make a model of EWSB along the following lines: Spontaneous breaking of supersymmetry gives mass to some new particles at very short distances, and this in turn gives mass to the supersymmetric partners of SM particles. Radiative corrections involving those mass terms can then induce a potential energy function that can favor a nonzero vacuum value for the Higgs field.

Specifically, this can work as follows: Three scalar fields are coupled by the top quark Yukawa coupling—the scalar partners of the left- and right-handed top quarks, $\widetilde{t}_L$ and $\widetilde{t}_R$, and the Higgs field H_u. All three fields receive mass from supersymmetry breaking. Arrange that these $(\text{mass})^2$ terms are all positive, approximatey equal, and of TeV size. Then compute the 1-loop corrections to these mass terms, which come from diagrams of the form of Fig. 5. This correction is negative by explicit calculation [50–52]. All three $(\text{mass})^2$ terms receive these negative contributions, but the correction to the Higgs mass is largest, because of the factor of 3 from QCD color flowing around the loop. (The $\widetilde{t}_L$ and $\widetilde{t}_R$ mass terms also receive positive corrections from diagrams involving the supersymmetric partner of the gluon.) This calculation creates a potential energy function with a negative $(\text{mass})^2$ for the H_u. It explains why this scalar field—and no other—obtains a vacuum expectation value.

Ultimately, this model of electroweak symmetry breaking is testable. The masses of the top quark partners and the Higgs boson spin-$\frac{1}{2}$ partners should not be too far above the 1 TeV mass scale. The Higgs partners, which are very difficult to discover at the LHC, could still be as light as 100 GeV [53,54]. If we could discover these particles and measure their masses and decay products, it will be possible to extract all of the parameters that enter the calculation of the Higgs potential [55]. If all of the pieces fit together, we could then claim to understand EWSB at the same level at which we understand the appearance of superconductivity in metals.

5.3 Higgs as a gauge boson

Supersymmetry produces a calculable theory of the Higgs mass term in the following way: At the level at which the symmetry is exact, this symmetry strongly constrains the Higgs potential. If the symmetry is then softly broken, new terms appear as calculable radiative corrections. These latter terms give the negative $(\text{mass})^2$ that drives electroweak symmetry breaking. Other types of models can implement this same philosophy in different ways.

Another familiar principle that can keep a particle mass at zero is local gauge invariance. We can use this in the Higgs story through the idea of *gauge-Higgs unification* [56,57]. In this approach, we assume that the universe is 5-dimensional, with the 5th dimension compact and small. A gauge field A_M in 5 dimensions has 5 components, $M = 0, 1, 2, 3, 5$. In the compactified geometry, the first four components make up a 4-dimensional gauge field. The last component A_5 transforms as a scalar field in 4 dimensions. If the full gauge group of the 5-dimensional theory is larger than $SU(2) \times U(1)$ and contains fields that transforms like the doublet of $SU(2) \times U(1)$, we can interpret the A_5 component of those extra gauge fields as a Higgs boson multiplet.

A simple example is given by assuming the gauge group of the 5-dimensional theory to be $SU(3)$. Let t^A be the 3×3 matrices that generate $SU(3)$. The gauge fields of $SU(3)$ in 5 dimensions take the form

$$A_M^A t^A = \begin{pmatrix} A_M^a \sigma^a/2 + \frac{1}{2} B_M & \Phi_M \\ \Phi_M^\dagger & -B_M \end{pmatrix} \tag{37}$$

where σ^a are the usual 2×2 Pauli sigma matrices and Φ_M is 2×1.

The $SU(3)$ symmetry can be broken by boundary conditions in the 5th dimension. After appropriate symmetry breaking, the fields A_μ^a will be the gauge fields of the weak interaction $SU(2)$, and B_μ will combine with another $U(1)$ to provide the weak interaction $U(1)$ gauge field. The components Φ_M have the quantum numbers $(I, Y) = (\frac{1}{2}, \frac{1}{2})$, and so the doublet Φ_5 has just the quantum numbers of the Higgs scalar doublet. Similar constructions can be made using larger gauge groups for the 5-dimensional theory. An attractive choice is to take the 5-dimensional gauge symmetry to be $SO(5)$. This group has a subgroup $SO(4) = SU(2) \times SU(2)$, such that one can interpret one $SU(2)$ as the weak interaction isospin gauge group. The $SO(5)$ bosons not contained in $SO(4)$ have the quantum numbers of Higgs fields [58]. A vecuum expectation value of one of these $SO(5)$ bosons breaks $SU(2) \times SU(2)$ to a diagonal $SU(2)$ group. That unbroken symmetry can be nterpreted as the *custodial* symmetry that protects the relation $m_W^2 = m_Z^2 \cos^2\theta_w$ from receiving large radiative corrections [59].

Electroweak symmetry breaking will take place if there is an energetic reason why the 5th component of the 5-dimensional gauge field should take on a nonzero value.

In a 5-dimensional theory with a periodic 5th dimension, there is in fact a good reason for this, the *Hosotani-Toms mechanism* [60,61]. If A_M is a 5-dimensional gauge field, a particle travelling around the 5th dimension acquires a phase

$$W = \exp[ig \oint dx^5 A_5 Q] \tag{38}$$

where Q is the charge of the field under the gauge symmetry associated with A_M. For bosons, the energy is typically minimized when $W = 1$ and maximized when $W = -1$. However, for fermions, it is the reverse: the energy is minimized for $W = -1$. A somewhat formal way to understand this is to recall that the functional integral representation of the thermodynamic partition function for fermions uses fermion fields that are antiperiodic around a compact Euclidean direction. Though this mechanism, a (t, b) quark doublet in the 5-dimensioaal space can force the quantity

$$W = \exp[ig \oint dx^5 \Phi_5] \tag{39}$$

to be nonzero, where Φ_M is the off-diagonal gauge field multiplet indicated in (37). From a 4-dimensional point of view, this is the induction of a negative (mass)^2 for the Higgs field by radiative corrections due to a heavy top quark and associated heavy quarks. The same physics appears in other examples of compact 5-dimensional geometries. In particular, in the Randall-Sundrum warped 5-dimensional spacetime [62], a very similar computation shows that radiative corrections due to a heavy top quark can drive some A_5 with the quantum numbers of the Higgs field to acquire a vacuum expectation value [63]

The 5-dimensional picture for the creation of the Higgs potential implies that this potential is free of ultraviolet divergences. The reason for this is that the phase factor (38) is nonlocal over the 5th dimension. Quantum fluctuations smaller than the full size of the 5th dimension see only a part of the integral in (38) and cannot distinguish this from a local gauge transformation. From a 4-dimensional point of view, though, the elimination of divergences seems quite surprising. The states of a (t, b) multiplet in 5 dimensions can be represented in terms of their momentum in 4 dimensions and their momentum around the 5th dimension. The momentum in the 5th dimension contributes to the mass term in the Dirac equation. Thus, the 5-dimensional (t, b) multiplet is seen in 4-dimensions as a 4-dimensional (t, b) multiplet plus an infinite number of more massive states, called *Kaluza-Klein states*, with the same quantum numbers. Each of these states gives a quadratically divergent contribution to the Higgs boson (mass)^2. But, by what seems to be a miracle, the sum of all of the contributions is finite.

If these 5-dimensional theories are correct, we will discover the heavy partners of t and b one by one as we search for new particles at higher energies. By measuring the properties of these particles, we could in principle extract their couplings to the Higgs field and directly verify the cancellation of divergences and the generation of a finite, negative Higgs (mass)^2.

5.4 Higgs as a Goldstone boson

There is a third way to construct a model in which the Higgs boson begins as a massless particle and acquires negative $(\text{mass})^2$ by radiative corrections. This is to begin with a new set of strong interactions at an energy scale well above the electroweak scale, say, 10 TeV, that breaks a global symmetry to a subgroup that contains $SU(2) \times U(1)$. This produces Goldstone bosons, one for each broken global symmetry direction. It is easy to arrange that some of these Goldstone bosons form a multiplet that transforms as $(I, Y) = (\frac{1}{2}, \frac{1}{2})$ under $SU(2) \times U(1)$. We can identify this multiplet with the Higgs scalar doublet. We can now add $SU(2) \times U(1)$ gauge interactions and other weak couplings. These will lead to radiative corrections that will generate a nonzero potential function for the Goldstone boson fields and drive these fields to acquire nonzero vacuum expectation values. Models of this type are known as *Little Higgs* models [64,65].

A theory of Goldstone bosons is described by a Lagrangian that is invariant under the original global symmetry. The global symmetry may be nonlinearly realized, but still there are significant constraints that come from this structure. In particular, these models also require new heavy quarks with charge $+\frac{2}{3}$, with vectorlike couplings to the weak interactions. As we saw in the case of 5-dimensional models, these heavy quarks can partially cancel the radiate correction to the Higgs $(\text{mass})^2$ from the top quark, making this correction ultraviolet-finite but still negative [64]. The cancellation of ultraviolet divergences implies relations between the Higgs couplings of the new heavy quarks and that of the top quark, and these could eventually be tested experimentally [66].

There is a connection between Little Higgs models and the 5-dimensional models discussed in the previous section. According to the AdS/CFT correspondence of Maldacena [67,68], a scale-invariant quantum field theory model in 4 dimensions has an equivalent representation as a 5-dimensional model in which global symmetries of the first theory become local gauge symmetries of the second theory. This idea allows the 5-dimensional models in warped backgrounds discussed in the previous section to be reinterpreted as 4-dimensional models. The Kaluza-Klein states of the 5-dimensional theory are reinterpreted as the spectrum of bound states of the strongly coupled 4-dimensional system. The gauge field Higgs multiplet in the previous case becomes the Goldstone boson Higgs multiplet that we have discussed here [69].

The three types of models described in this section illustrate in different ways the possibility of dynamical explanations of the state of spontaneously broken symmetry required for the $SU(2) \times U(1)$ theory of the weak interactions. These models are not simple modifications of the SM. They require large numbers of new particles that must eventually be discovered by accelerator experiments at high energies.

6 Where is the new physics?

If the arguments for physics beyond the SM are so compelling, and if the particle spectra expected are so rich, then why haven't we found evidence for these particles? I think that every theorist who puts forward arguments similar to those above is troubled by this question. In all three models above, the new particles introduced to explain the Higgs potential would be expected to have masses at the scale of hundreds of GeV. For top quark partners and other particle with QCD interactions, LHC searches exclude most scenarios with new colored particles in this mass range [70]. In addition, we have not yet seen signs of indirect effects of new particles shifting low-energy observables from their SM values.

Nevertheless, the situation is different now than in earlier eras of particle physics. Quantum field theories with only vectorlike coupling obey a general property called the *Appelquist-Carrazone Decoupling Theorem*. This is the statement that, if heavy particles are added to the theory, any new interactions due to those particles are suppressed by a factor $1/M^2$, where M is the mass of the heavy particle [71]. For example, if we add a new heavy quark of mass M to QCD and measure its effects at a scale $Q \ll M$, any new terms are suppressed by Q^2/M^2. This follows from the fact that the QCD Lagrangian is already the most general renormalizable Lagrangian one can write that contains the known quarks and has the QCD gauge symmetry. Corrections to the Lagrangian induced by effects of the heavy quark can only change the quark masses and the QCD coupling, and add higher-dimension operators with coefficients that explicitly contain $1/M^2$. The shifts of quark masses and α_s are visible only if we can independently measure these parameters at energies above the heavy quark mass. I we cannot, we cannot know that these shifts have taken place. Then the only new and observable terms are of order $1/M^2$,

For a theory with chiral interactions and spontaneously broken symmetry, the situation can be quite different. The strong statement in the previous paragraph requires gauge invariance. At previous stages of our knowledge, our description of particle physics included some members of $SU(2) \times U(1)$ gauge multiplets but not others, for example, s but not c, b but not t, or the longitudinally polarized W boson but not the Higgs boson. In this situation, corrections involving the missing states could contribute large terms to appropriately chosen amplitudes [72]. Examples are provided by the c and t contributions to K–$\overline{K}$ and B–$\overline{B}$ mixing, the top quark loop contribution to precision electroweak observables such as m_W, and the top quark loop contribution to the $h \to gg$ decay amplitude. We have seen already in Section 2.3 that the large top quark Yukawa coupling can lead to effects that are much larger than expected naively from perturbaation theory. But now that the full SM particle content has been discovered, we have returned to the situation in which any new particles added to the model must have vectorlike couplings and virtual effects suppressed as $1/M^2$.

Thus, it is quite plausible that new particles outside the SM might be present in nature but have only minor effects on observations at currently explored energies. When we finally reach the new particles thresholds, we turn a corner, and a new realm of physics will come into view. Large multiplets of new particles will suddenly appear. The reality of these particles will become obvious. Later, papers will be written about the surprisingly small effects of these particles on low-energy observables.

The mot powerful way to search for physics beyond the Standard Model, now more than ever, is to search for new thresholds at the highest energy accelerators. It is exciting that, this year, the LHC will finally be running close to its design energy. Over the next fifteen years, the LHC will open up a territory in which to search for strongly interacting particles about 3 times greater than that currently explored, and a territory for particles with only electroweak interactions—and signatures appropriate to hadron colliders—about 4 times larger than the current one

Still, the direct search for new particles at high-energy accelerators is ultimately limited by the collider energy. Each step to higher energy is now a major technical, social, and political endeavor. So it is important that it is also possible to search for these particles through new high-precision probes for the $1/M^2$ effects that they induce.

In particular, the next e^+e^- collider will be able to carry out high-precision studies of rhe couplings of the Higgs boson and top quark. I have emphasized in this review that these two particles stand at the very center of the mystery of electroweak spontaneous symmetry breaking. New particles that would provide an explanation for this question must necessarily couple to the Higgs boson. These new particles are also very likely to couple to or mix with the top quark. Today, the Higgs boson and the top quark are incompletely understood experimentally. The couplings of the Higgs boson and the electroweak couplings of the top quark are measured only to the 20% level. The International Linear Collider, a 500 GeV e^+e^- collider based on superconducting RF cavities, is now designed and ready for construction. Experiments at that accelerator would bring these measurements below the percent level of accuracy and would be sensitive to the effects of the new particle scenarios discussed in this review [73,74].

Above all, we need to keep our faith in the basic tenet of physics—that the phenomena of nature have explanations, and that those explanations can be found by probing nature at successively deeper levels. New forces and interactions are out there. In time, we will find them.

ACKNOWLEDGEMENTS

I am grateful to Halina Abramowicz, Allen Caldwell, and Brian Foster for their invitation to write for this volume, to Raymond Brock and Beate Heinemann for their encouragement, and to many colleagues at SLAC and elsewere for discussions of the issues put forward here. This work was supported by the U.S. Department of Energy under contract DE–AC02–76SF00515.

References

[1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[3] J. L. Rosner, *et al.*, in the Proceedings of the APS DPF Community Summer Study (Snowmass 2013), N. Graf, J. L. Rosner, and M. E. Peskin, eds., `http://www.slac.stanford.edu/econf/C1307292/`, arXiv:1401.6075 [hep-ex].

[4] S. Weinberg, *The Quantum Theory of Fields*, vol. 1, Chapter 2. (Cambrige University Press, 1995).

[5] There is an alternative form of a mass term for spin-$\frac{1}{2}$ particles, in which the mixing is between massless particle states with chirality $-\frac{1}{2}$ and antiparticle states with chirality $+\frac{1}{2}$. This is called a *Majorana mass*. The mixing is permitted only if a particle number symmetry (for example, lepton number) is violated.

[6] S. Schael *et al.* [ALEPH and DELPHI and L3 and OPAL and SLD and LEP Electroweak Working Group and SLD Electroweak Group and SLD Heavy Flavour Group Collaborations], Phys. Rept. **427**, 257 (2006) [hep-ex/0509008].

[7] LEP Electroweak Working Group [ALEPH and CDF and D0 and DELPHI and L3 and OPAL and SLD and LEP Electroweak Working Group and Tevatron Electroweak Working Group and SLD Electroweak and Heavy Flavour Groups Collaborations], arXiv:1012.2367 [hep-ex].

[8] S. Schael *et al.* [ALEPH and DELPHI and L3 and OPAL and LEP Electroweak Collaborations], Phys. Rept. **532**, 119 (2013) [arXiv:1302.3415 [hep-ex]].

[9] G. 't Hooft and M. J. G. Veltman, Nucl. Phys. B **50**, 318 (1972).

[10] B. W. Lee and J. Zinn-Justin, Phys. Rev. D **5**, 3155 (1972).

[11] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979).

[12] P. W. Higgs, Phys. Lett. **12**, 132 (1964), Phys. Rev. Lett. **13**, 508 (1964).

[13] F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964).

[14] G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. **13**, 585 (1964).

[15] J. Goldstone, Nuovo Cim. **19**, 154 (1961).

[16] Y. Nambu, Phys. Rev. **117**, 648 (1960).

[17] P. W. Anderson, Phys. Rev. **130**, 439 (1963).

[18] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974) [Phys. Rev. D **11**, 972 (1975)].

[19] C. E. Vayonakis, Lett. Nuovo Cim. **17**, 383 (1976).

[20] M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. B **261**, 379 (1985).

[21] T. Aaltonen *et al.* [CDF and D0 Collaborations], Phys. Rev. D **85**, 071106 (2012) [arXiv:1202.5272 [hep-ex]].

[22] G. Aad *et al.* [ATLAS Collaboration], JHEP **1206**, 088 (2012) [arXiv:1205.2484 [hep-ex]].

[23] S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1310**, 167 (2013) [arXiv:1308.3879 [hep-ex]].

[24] W. Alles, C. Boyer and A. J. Buras, Nucl. Phys. B **119**, 125 (1977).

[25] A. J. Bevan *et al.* [BaBar and Belle Collaborations], Eur. Phys. J. C **74**, no. 11, 3026 (2014) [arXiv:1406.6311 [hep-ex]].

[26] M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP **1411**, 052 (2014) [arXiv:1409.5439 [hep-ph]].

[27] R. J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys. Lett. B **88**, 123 (1979) [Phys. Lett. B **91**, 487 (1980)].

[28] M. Dine, in *Flavor Physics for the Millennium, Proceedings of the 2000 TASI School*, J. L. Rosner, ed. (World Scientific, 2001), hep-ph/0011376.

[29] L. Brigliadori, EPJ Web Conf. **90**, 05001 (2015).

[30] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, JHEP **1312**, 089 (2013) [arXiv:1307.3536 [hep-ph]].

[31] R. Bousso and B. Freivogel, JHEP **0706**, 018 (2007) [hep-th/0610132].

[32] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008) [arXiv:0710.3755 [hep-th]].

[33] J. P. Miller, E. d. Rafael, B. L. Roberts and D. Stckinger, Ann. Rev. Nucl. Part. Sci. **62**, 237 (2012).

[34] M. Dine, arXiv:1501.01035 [hep-ph], to appear in Ann. Rev. Nucl. Part. Sci. **65** (2015).

[35] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[36] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, 1975).

[37] A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1976).

[38] A. Bohr and B. L. Mottelson, *Nuclear Structure* (Benjamin, 1975).

[39] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961), Phys. Rev. **124**, 246 (1961).

[40] D. C Mattis, *The Theory of Magnetism Made Simple* (World Scientific, 2006).

[41] P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics.* (Cambridge University Press, 1995).

[42] P. G. de Gennes, *The Physics of Liquid Crystals.* (Clarendon Press, 1974).

[43] M. Imada, A. Fujimori, and Y. Tokura, Rev. Mod. Phys. **70**, 1039 (1998).

[44] S. Weinberg, Phys. Rev. D **13**, 974 (1976).

[45] R. Haag, J. T. Lopuszanski and M. Sohnius, Nucl. Phys. B **88**, 257 (1975).

[46] S. Martin, in *Perspectives on Supersymmetry II*, G. L. Kane, ed. (World Scientific, 2010) [hep-ph/9709356].

[47] M. Drees, R. Godbole, and P. Roy, *Theory and Phenomenology of Sparticles.* (World Scientific, 2004).

[48] H. Baer and X. Tata, *Weak Scale Supersymmetry* (World Scientific, 2006).

[49] M. E. Peskin, in *Exploring New Frontiers Using Colliders and Neutrinos (proceedings of TASI 2006)*, S. Dawson and R. N. Mohapatra, eds. (World Scientific, 2008) [arXiv:0801.1928 [hep-ph]].

[50] L. E. Ibanez and G. G. Ross, Phys. Lett. B **110**, 215 (1982).

[51] L. Alvarez-Gaume, M. Claudson and M. B. Wise, Nucl. Phys. B **207**, 96 (1982).

[52] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. **68**, 927 (1982) [Prog. Theor. Phys. **70**, 330 (1983)].

[53] G. Aad *et al.* [ATLAS Collaboration], JHEP **1404**, 169 (2014) [arXiv:1402.7029 [hep-ex]].

[54] V. Khachatryan *et al.* [CMS Collaboration], Phys. Rev. D **90**, no. 9, 092007 (2014) [arXiv:1409.3168 [hep-ex]].

[55] M. E. Peskin, Prog. Theor. Phys. Suppl. **123**, 507 (1996) [hep-ph/9604339].

[56] L. J. Hall, Y. Nomura and D. Tucker-Smith, Nucl. Phys. B **639**, 307 (2002) [hep-ph/0107331].

[57] M. Kubo, C. S. Lim and H. Yamashita, Mod. Phys. Lett. A **17**, 2249 (2002) [hep-ph/0111327].

[58] R. Contino, L. Da Rold and A. Pomarol, Phys. Rev. D **75**, 055014 (2007) [hep-ph/0612048].

[59] P. Sikivie, L. Susskind, M. B. Voloshin and V. I. Zakharov, Nucl. Phys. B **173**, 189 (1980).

[60] Y. Hosotani, Phys. Lett. B **126**, 309 (1983).

[61] D. J. Toms, Phys. Lett. B **126**, 445 (1983).

[62] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].

[63] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B **671**, 148 (2003) [hep-ph/0306259].

[64] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP **0208**, 021 (2002) [hep-ph/0206020]; N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP **0207**, 034 (2002) [hep-ph/0206021].

[65] M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. **55**, 229 (2005) [hep-ph/0502182].

[66] M. Perelstein, M. E. Peskin and A. Pierce, Phys. Rev. D **69**, 075002 (2004) [hep-ph/0310039].

[67] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999) [Adv. Theor. Math. Phys. **2**, 231 (1998)] [hep-th/9711200].

[68] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000) [hep-th/9905111].

[69] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP **0108**, 017 (2001) [hep-th/0012148].

[70] For examples of models that are exceptions to this statement, seee J. Fan, M. Reece and J. T. Ruderman, JHEP **1111**, 012 (2011) [arXiv:1105.5135 [hep-ph]], JHEP **1207**, 196 (2012) [arXiv:1201.4875 [hep-ph]].

[71] T. Appelquist and J. Carazzone, Phys. Rev. D **11**, 2856 (1975).

[72] M. S. Chanowitz, M. A. Furman and I. Hinchliffe, Phys. Lett. B **78**, 285 (1978), Nucl. Phys. B **153**, 402 (1979).

[73] H. Baer, *et al.*, "The International Linear Collider Technical Design Report - Volume 2: Physics," `http://www.linearcollider.org/ILC/Publications/Technical-Design-Report`, arXiv:1306.6352 [hep-ph].

[74] K. Fujii, *et al.*, arXiv:1506.05992 [hep-ex].