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DISPERSIVE PROPERTY OF THE PULSE FRONT TILT OF A SHORT PULSE OPTICAL UNDULATOR*

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Abstract

A short pulse laser can be used as an optical undulator to achieve a high-gain and high-brightness X-ray free electron laser (FEL) [1]. To extend the interaction duration of electron and laser field, the electron and laser will propagate toward each other with an small angle. In addition, to maintain the FEL lasing resonant condition, the laser pulse shape need be flattened and the pulse front will be titled. Due to the short pulse duration, the laser pulse has a broad bandwidth. In this paper, we will first describe the method of generalized Gaussian beam propagation using ray matrix. By applying the Gaussian beam ray matrix, we can study the dispersive property after the pulse front of the short laser is tilted. The results of the optics design for the proposal of SLAC Compton scattering FEL are shown as an example in this paper.

INTRODUCTION

With rapid progress in generating table-top terawatt laser pulse and fiber optics, the optical undulator can provide effective magnetic field B_u on the order of kilo-Tesla, which can provide strong enough effective undulator strength K for lasing. To fulfil an optical undulator for FEL, the interaction range of electron and laser pulse should be with 10-20 FEL gain length, and the equivalent undulator strength K should be kept constant for a given radiation γ. In order to increase the electron and laser pulse interaction range, the laser and electron need to co-propagate synchronously.

It is known that the angular dispersion (AD) will generate pulse front tilt (PFT). Gratings are ideal for this purpose as they can introduce large linear angular chirps. Nevertheless, besides the PFT, AD will also increase spatial dispersion (SD). As the pulse propagates, different frequency in the pulse becomes increasingly separated from each other. AD will also introduce negative group-delay dispersion (GDD). Both SD and GDD will lead temporal broadening of the laser and degrade the performance of the optical undulator in FEL. Therefore it is important to investigate the dispersive property before the PFT laser is sent to interact with an electron bunch.

The geometrical optics uses ray transfer matrix (also called ABCD matrix) to trace the light ray path through space and optical devices. Take one dimension ray trace for example:

$$\begin{pmatrix} x \\ \theta \end{pmatrix}_{out} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}_n \cdots \begin{pmatrix} A & B \\ C & D \end{pmatrix}_1 \begin{pmatrix} x \\ \theta \end{pmatrix}_{in} \tag{1}$$

where x is position, θ is the slope and matrices 1 to n represent different optical components or spaces. The radius of curvature of the ray is $q = \frac{x}{\theta}$. The ABCD law for the radius of curvature is:

$$q_{out} = \frac{Aq_{in}+B}{Cq_{in}+D} \tag{2}$$

It is easy to extend the ray transfer matrix to two dimensions. Where A, B, C, and D become 2 by 2 matrices.

The ABCD matrix could be extended to "ray-pulse" matrices which takes account of dispersive effects in both spatial coordinates (as in the usual paraxial ABCD matrix approach) and in the temporal domain [2]. These matrices could be applied to write a space-time integral analogous to a generalized Huygens integral. By using both ray-pulse matrices and the propagation laws for Gaussian ray pulses which are space and time varying, the conventional results for Gaussian beams through various optical components could be derived. In this paper, we will first describe the method of generalized Gaussian beam propagation using ray matrix. By applying this method, we will investigate the dispersive properties of the optics design for the proposal of SLAC Compton scattering FEL.

GENERALIZED GAUSSIAN BEAM PROPAGATION USING RAY MATRIX

A short pulse laser is a finite size Gaussian beam. The electric field of the laser propagating along the z-axis in (x, ω) space can be expressed as:

$$E(x,\omega) = E_0 \exp\left(-\frac{\omega^2\tau_0^2}{4}\right)\exp\left(-i\frac{k_0x^2}{2q}\right) \tag{3}$$

where k_0 is the nominal wave-number, ω is the offset from the centre angular frequency, τ_0 is the pulse length and q is the complex q parameter of a Gaussian beam:

$$\frac{1}{q(z)} = \frac{1}{z+iZ_R} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi W^2(z)} \tag{4}$$

where Z_R is the Rayleigh range, $R(z)$ is the radius of curvature of the wave front, λ_0 the nominal wave length and $w(z)$ is the spot size $w(z) = w_0\sqrt{1+(\frac{z}{z_R})^2}$ with w_0 being the waist size.

A comprehensive matrix method for propagating Gaussian ultrashort pulses with Gaussian spatial profiles having spatio-temporal couplings was given by Kostenbauder [2]. The ray-pulse matrix for an optical system that introduces couplings can be written as [3]:

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$$K = \begin{bmatrix} \frac{\partial x_{out}}{\partial x_{in}} & \frac{\partial x_{out}}{\partial \theta_{in}} & 0 & \frac{\partial x_{out}}{\partial \nu_{in}} \\ \frac{\partial \theta_{out}}{\partial x_{in}} & \frac{\partial \theta_{out}}{\partial \theta_{in}} & 0 & \frac{\partial \theta_{out}}{\partial \nu_{in}} \\ \frac{\partial t_{out}}{\partial x_{in}} & \frac{\partial t_{out}}{\partial \theta_{in}} & 1 & \frac{\partial t_{out}}{\partial \nu_{in}} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A & B & 0 & E \\ C & D & 0 & F \\ G & H & 1 & I \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

This Kostenbauder matrix can be used to model optical elements and their effects in transforming input pulse parameters (expressed as a vector) to output ones. The electric field of a finite size Gaussian beam in ray-pulse matrix formula is [3]:

$$E(x,t) = \exp\left\{-i\frac{\pi}{\lambda_0}\begin{pmatrix} x \\ -t\end{pmatrix}^T Q^{-1}\begin{pmatrix} x \\ t\end{pmatrix}\right\}$$
$$= exp\left\{-i\frac{\pi}{\lambda_0}[(Q^{-1})_{11}x^2 + (Q^{-1})_{12}xt - (Q^{-1})_{21}xt - (Q^{-1})_{22}t^2]\right\} \quad (6)$$

$$(Q^{-1})_{11} = \frac{1}{q} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi W^2(z)}, (Q^{-1})_{22} = i\frac{\lambda_0}{\pi\tau^2} \quad (7)$$

$$PFT = \frac{Im((Q^{-1})_{12} - (Q^{-1})_{21})}{2Im((Q^{-1})_{22})} \quad (8)$$

The slice beam size $\Delta x_l(t)$ and the correlated beam size $\Delta x_G(t)$ are [3]:

$$\Delta x_l(t) = \frac{1}{2}\left(\frac{\lambda_0}{\pi}\frac{1}{Im(Q^{-1})_{11}}\right)^{1/2}$$
$$\Delta x_G(t) = \frac{1}{2}\left(\frac{\lambda_0}{\pi}\frac{Im(Q^{-1})_{22}}{Im(Q^{-1})_{11}Im(Q^{-1})_{22} + Im(Q^{-1})_{12}^2}\right)^{1/2} \quad (9)$$

The slice pulse width Δt_l and the correlated pulse width Δt_G are:

$$\Delta t_l = \frac{1}{2}\left(\frac{\lambda_0}{\pi}\frac{1}{Im(Q^{-1})_{22}}\right)^{1/2}$$
$$\Delta t_G = \frac{1}{2}\left(\frac{\lambda_0}{\pi}\frac{Im(Q^{-1})_{11}}{Im(Q^{-1})_{11}Im(Q^{-1})_{22} + Im(Q^{-1})_{12}^2}\right)^{1/2} \quad (10)$$

For an input pulse with no spatio-temporal distortions and flat phase

$$Q_{in} = \begin{bmatrix} q_0 & 0 \\ 0 & -i\frac{\pi\tau_0^2}{\lambda_0}\end{bmatrix}, q_0 = i\frac{\pi w_0^2}{\lambda_0} \quad (11)$$

The output Q matrix is:

$$Q_{out} = \frac{\begin{bmatrix} A & 0 \\ G & 1\end{bmatrix} Q_{in} + \begin{bmatrix} B & \frac{E}{\lambda_0} \\ H & \frac{I}{\lambda_0}\end{bmatrix}}{\begin{bmatrix} C & 0 \\ 0 & 1\end{bmatrix} Q_{in} + \begin{bmatrix} D & \frac{F}{\lambda_0} \\ 0 & 1\end{bmatrix}} = \frac{\bar{A}\, Q_{in} + \bar{B}}{\bar{C} Q_{in} + \bar{D}} \quad (12)$$

Using the output Q-matrix, virtually all the properties of the output pulse through the optical components can be studied. The Q-matrix in x-t domain can be transformed to x-ω, k-ω and k-t domains using analytical relations derived in [3]. In this paper, we investigate the SD after the PFT. Therefore, we also like to investigate the x-ω domain. From [3]:

$$E(x,\omega) \propto \exp\{R_{xx}x^2 + R_{x\omega}x\omega - R_{\omega\omega}\omega^2\} \quad (13)$$

The corresponding Fourier transformed matrix components are:

$$R_{xx} = -i\frac{\pi}{\lambda_0}\left((Q^{-1})_{11} + \frac{(Q^{-1})_{12}^2}{(Q^{-1})_{22}}\right)$$
$$R_{x\omega} = -\frac{\pi}{2\lambda_0}\frac{(Q^{-1})_{12}}{(Q^{-1})_{22}} \quad (14)$$
$$R_{\omega\omega} = -i\frac{\pi}{4\lambda_0}\frac{1}{(Q^{-1})_{22}}$$

The slice beam size $\Delta x_l(\omega)$ and the correlated beam size $\Delta x_G(\omega)$ are [3]:

$$\Delta x_l(\omega) = \frac{1}{2}\left(-\frac{1}{Re(R_{xx})}\right)^{1/2}$$
$$\Delta x_G(\omega) = \frac{1}{2}\left(\frac{Re(R_{\omega\omega})}{Re(R_{xx})Re(R_{\omega\omega}) + Re(R_{x\omega})^2}\right)^{1/2} \quad (15)$$

The slice frequency spread $\Delta\omega_l$ and the correlated frequency spread $\Delta\omega_G$ are:

$$\Delta\omega_l = \frac{1}{2}\left(-\frac{1}{Re(R_{\omega\omega})}\right)^{1/2}$$
$$\Delta\omega_G = \frac{1}{2}\left(\frac{Re(R_{xx})}{Re(R_{xx})Re(R_{\omega\omega}) + Re(R_{x\omega})^2}\right)^{1/2} \quad (16)$$

The physical meaning of the coupling terms $Re(R_{x\omega})$ is related to SD $\frac{\partial x_0}{\partial\omega} = -\frac{Re(R_{x\omega})}{Re(R_{xx})}$ and the $Im(R_{x\omega})$ is wave-front-tilt dispersion (WFD). The couplings correlation coefficient is defined:

$$\rho_{x\omega} = \frac{Re(R_{x\omega})}{\sqrt{-Re(R_{xx})Re(R_{\omega\omega})}}$$
$$\Delta x_l(\omega) = \Delta x_G(\omega)\sqrt{1-\rho_{x\omega}^2},$$
$$\Delta\omega_l = \Delta\omega_G\sqrt{1-\rho_{x\omega}^2} \quad (17)$$

INVESTIGATION OF THE DISPERSIVE PROPERTY OF OPTICS DESIGN

With the knowledge of pulse ray matrix propagation, we are able to investigate the dispersive property of the optics design. Figure 1 is a schematic layout of optics design for the proposal of SLAC Compton scattering FEL. The purpose of the design is to flat the Gaussian laser pulse and provide PFT to sheer the beam so that the interaction region can be extended [3]. In this paper, we concentrate on the study of dispersive properties; therefore the component digital micromirror device (DMD) in Fig. 1 is treated as grating in the analysis. The incident angle into grating G is 3° and the angle of reflection is 24°. The following 4f system recombines all

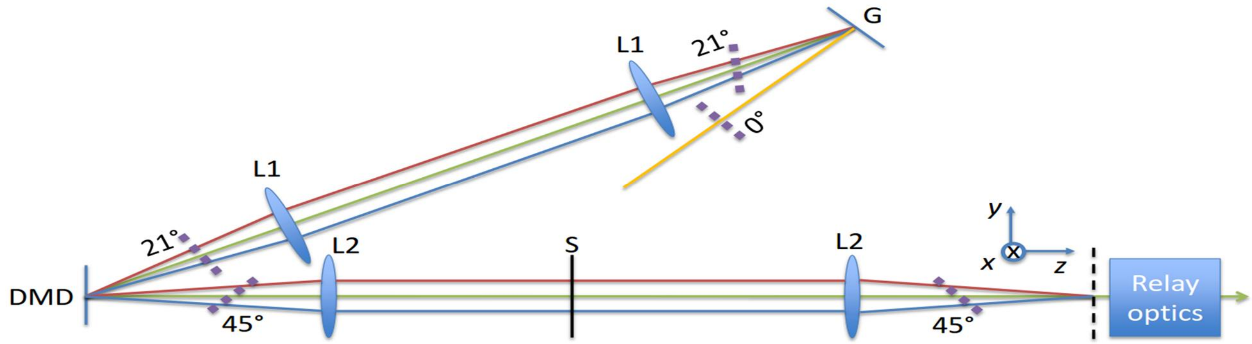

Figure 1: Schematic layout of optic design for the proposal of SLAC Compton scattering FEL. Where *G* means grating, *L*1, *L*2: lenses, DMD: digital micromirror device, *S*: slit. The yellow line represents an input fs pulse, red line chief ray of the longest wavelength λ_l, green line chief ray of the central wavelength λ_c, blue line chief ray of the shortest wavelength λ_s. The black dash line represents the image plane and the purple dash line the pulse front.

spectral components at the DMD plane. The diffraction grating *G* and the DMD are placed symmetrically to ensure a correct imaging relationship. The incident angle into DMD is 24° and the reflect angle is 0°. Another 4*f* system recombines the spectral at the Relay optics. The Relay optics is chromatic and is designed to compress the vertical beam from mm to μm. At this moment the detail design of Relay optics has not been finished yet. Since it is chromatic it is not contributed to the dispersive effect. We will skip it at this investigation. An 800 nm titanium-sapphire laser with Δx_G = 20 mm, Δt_G= 20 fs is used as the initial pulse. The amplitude of the initial pulse electrical field distributions in *x*-*t* and *x*-*ω* domain are shown in Fig. 2.

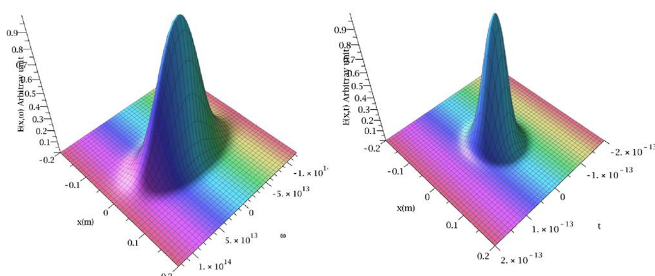

Figure 2 The initial laser pulse in *x*-*ω* (left) and *x*-*t* (right) domains. The color map in *x*-*ω* plot represents *ω* and in *x*-*t* represents *t*.

The *x*-*ω* and *x*-*t* plots of amplitude of pulse electrical field before Relay optics are shown in Fig. 3. Comparing the *x*-*t* plot to Fig. 2, the pulse is sheered along the *x* direction. The scale of *x* axis is one tenth and the *t* scale is 10 times of the *x*, *t* scales in Fig. 2. The SD is kept zero as shown in Fig. 3. The slice *x* size is smaller. However comparing the *x*-*ω* plot in Fig. 2, the correlated *x* size is increased. The increase is from the imaginary part of the term $R_{x\omega}$ which is contributed by the wave-front-tilt dispersion. The parameters in Eqs. 8, 9, 10, 15 and 16 of the initial beam and the final beam are listed in Table 1. The units in the table are in MKS.

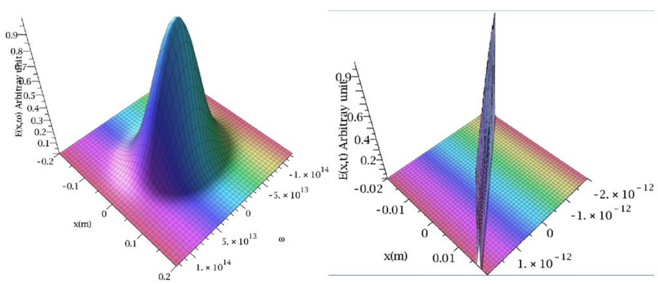

Figure 3 The laser pulse before the Relay optics in *x*-*ω* (left) and *x*-*t* (right) domains.

Table 1: Ray parameters

	Initial	Final
PFT	0	1.16E-10
$\Delta x_l(t)$, $\Delta x_G(t)$	0.02, 0.02	1.7E-4, 0.03
Δt_l , Δt_G	2E-14, 2E-14	2E-14, 3.5E-12
$\Delta x_l(\omega)$, $\Delta x_G(\omega)$	0.02, 0.02	0.03, 0.03
$\Delta\omega_l$, $\Delta\omega_G$	2.5E13, 2.5E13	2.5E13, 2.5E13

CONCLUSION

By using ray-pulse matrices, we are able to investigate the properties of short pulse laser after propagated through a series of optic components. From the investigation we prove we can design an optics system without SD after the PFT. However there is $Im(R_{x\omega})$ term contribution from wave-front-tilt dispersion remained.

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