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The Relation between the Fundamental Scale Controlling High-Energy Interactions of Quarks and the Proton Mass

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Quantum Chromodynamics (QCD) provides a fundamental description of the physics binding quarks into protons, neutrons, and other hadrons. QCD is well understood at short distances where perturbative calculations are feasible. Establishing an explicit relation between this regime and the large-distance physics of quark confinement has been a long-sought goal. A major challenge is to relate the parameter Λ_s, which controls the predictions of perturbative QCD (pQCD) at short distances, to the masses of hadrons. Here we show how new theoretical insights into QCD's behavior at large and small distances lead to an analytical relation between hadronic masses and Λ_s. The resulting prediction, $\Lambda_s = 0.341 \pm 0.024$ GeV agrees well with the experimental value 0.339 ± 0.016 GeV. Conversely, the experimental value of Λ_s can be used to predict the masses of hadrons, a task which has so far only been accomplished through intensive numerical lattice calculations, requiring several phenomenological input parameters.

The masses of hadrons, the bound states of quarks, originate from the energy of the confining interactions of QCD. It is unclear why the typical hadron mass scale is of order 1 GeV.

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Naively, one would expect this mass scale to be explicitly present in the QCD Lagrangian, the "master equation" of the theory. However, the only scale appearing in the QCD Lagrangian corresponds to quark masses, and they are too small to be relevant: $m_q \sim 10^{-3}$ GeV. An important mass scale, Λ_s, however, does exist. This parameter controls the strength of the coupling of quarks when they interact at short distances. Its precise definition emerges during QCD's renormalization, the standard procedure in quantum field theory which eliminates infinities that would otherwise render the theory senseless. Our results can be implemented for any choice of renormalization procedure, but we will use here the value of Λ_s defined by the $\overline{MS}$ (modified minimal subtraction) renormalization scheme. The parameter $\Lambda_s = \Lambda_{\overline{MS}}$ can be determined to high precision from experimental measurements of high-energy, short-distance processes where the strength of QCD is small because of asymptotic freedom [1, 2], and pQCD is thus applicable.

This paper presents the first analytic relation linking Λ_s to hadron masses. It allows us to precisely predict the value of Λ_s taking a hadronic mass as input, or, conversely, to calculate the hadron masses using Λ_s. To establish this relation between mass scales we use an approximation to QCD in its large-distance regime called " light-front holographic QCD" (for brevity AdS/QCD) [3, 4, 5]. AdS/QCD originates from a connection of QCD calculated on Dirac's light-front dynamics in physical space-time to Einstein's gravity calculated in a 5-dimensional Anti-de Sitter (AdS) space (AdS space is, loosely speaking, a space with a constant negative curvature). The connection between gravity projected on the 4-dimensional AdS space boundary with QCD in physical space-time explains the terminology "holographic". On the other hand, the small-distance physics, such as the violent collisions of quarks occurring at the Large Hadron Collider at CERN, is well-described by pQCD. The two regimes overlap at intermediate distances, a phenomenon called "quark-hadron duality" [6]. This permits us to match the two descriptions and obtain an analytical relation between Λ_s and hadron masses. This relation is a pure calculation, not relying on fit or free parameters.

In AdS/QCD the forces that bind quarks are related to a large-distance modification of the AdS space curvature which encodes confinement dynamics [5]. This specific modification of the AdS geometry traces to a basic mechanism due to de Alfaro, Fubini and Furlan (dAFF) [7], which allows for the emergence of a mass scale κ in the theory. The mechanism also specifies the form of the quark-confining light-front potential. The result is an excellent description of hadrons of arbitrary spin [8], incorporating many of their observed spectroscopic and dynamical features [9]. The strict correspondence between QCD on the light front and AdS/QCD implies that the latter has its foundations well rooted in QCD. AdS/QCD also prescribes the form of the QCD coupling α_s, which dictates the strength of the force between quarks in the confinement regime [10].

In AdS/QCD, the scale κ controlling quark confinement also predicts hadron masses. For example, κ can be determined from the ρ hadron mass: $\kappa = M_\rho/\sqrt{2} = 0.548$ GeV [9]. We shall relate κ to $\Lambda_{\overline{MS}}$ by matching AdS/QCD results, which explicitly contain κ, to pQCD results, which explicitly depend on $\Lambda_{\overline{MS}}$. The matched quantity is α_s. In pQCD, the space-time dependence of α_s originates from short-distance quantum effects which are folded into its definition. The scale Λ_s controls this space-time dependence [1, 2]. Equivalently, the space-time dependence of the AdS/QCD coupling stems from the effects of the AdS space curvature [10].

The coupling α_s can be defined from the Bjorken sum rule for spin-dependent electron-nucleon inelastic scattering [11, 12]. The resulting coupling, $\alpha_{g_1}(Q)$, is approximately the QCD-analog of the Gell-Mann-Low coupling $\alpha(Q)$ of Quantum Electrodynamics [10]. Here Q is the absolute value of the 4-momentum transferred by the scattered electron to the nucleon. It sets the scale at which a process is observed. By the uncertainty principle, the 4-momentum Q is small (large) at large (short) space-time distances.

AdS/QCD predicts the behavior of $\alpha_{g_1}(Q)$ at large distance i.e., small values of Q [10]:

$$\alpha_{g_1}^{AdS}(Q) = \pi \exp\left(-Q^2/4\kappa^2\right), \tag{1}$$

thereby explicitly connecting the small-Q dependence of $\alpha_{g_1}(Q)$ to κ, and thus to hadronic masses. Eq. 1 is valid only at small Q where QCD is a strongly coupled theory and thus where the AdS/QCD methods are applicable.

The large Q-dependence of α_s is computed from the pQCD β series:

$$Q^2 d\alpha_s/dQ^2 = \beta(Q) = -(\beta_0\alpha_s^2 + \beta_1\alpha_s^3 + \beta_2\alpha_s^4 + \cdots), \tag{2}$$

where the β_i coefficients are known up to β_3 in the $\overline{MS}$ scheme [13]. Furthermore, $\alpha_{g_1}^{pQCD}(Q)$ can be itself expressed as a perturbative expansion in $\alpha_{\overline{MS}}(Q)$ [11, 12]. Hence, pQCD predicts the form of $\alpha_{g_1}(Q)$ at large Q:

$$\alpha_{g_1}^{pQCD}(Q) = \pi\Big[\alpha_{\overline{MS}}/\pi + a_1\left(\alpha_{\overline{MS}}/\pi\right)^2 + a_2\left(\alpha_{\overline{MS}}/\pi\right)^3 + a_3\left(\alpha_{\overline{MS}}/\pi\right)^4 + \cdots\Big]. \tag{3}$$

The coefficients a_i are known up to order a_3 [14].

The complete dependence of α_{g_1} in Q must be analytic. The existence at moderate values of Q of a dual description of QCD in terms of either quarks and gluons or hadrons (the "quark-hadron duality" [6]) allows us to match the AdS/QCD and pQCD forms, Eqs. 1 and 3 respectively. This is done by imposing continuity of both $\alpha_{g_1}(Q)$ and its derivative, as shown in Fig. 1. The resulting two equalities then provide a unique value of Λ_s from the scheme-independent scale κ, and fix the scale Q_0 characterizing the transition between the large and short-distance regimes of QCD.

We have solved this two-equation system analytically at leading order of Eqs. 2 and 3, and numerically otherwise. The leading-order analytical relation between $M_\rho = \sqrt{2}\kappa$ and $\Lambda_{\overline{MS}}$ is:

$$\Lambda_{\overline{MS}} = M_\rho e^{-(a+1)}/\sqrt{a}, \tag{4}$$

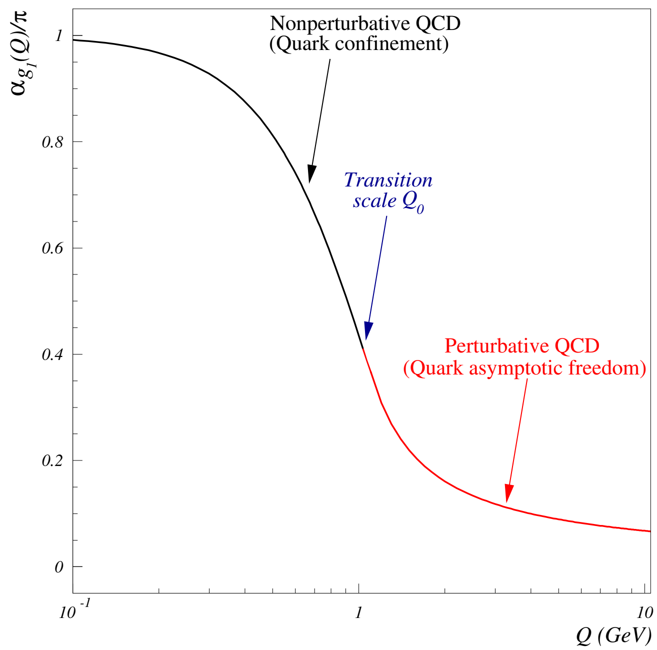

Figure 1: Unified strong coupling from the analytic matching of perturbative and nonperturbative QCD regimes. The analytic matching determines the relation between $\Lambda_{\overline{MS}}$ and hadron masses as well as the transition scale between the large and short-distance regimes of QCD.

with $a = 4\big(\sqrt{ln(2)^2 + 1 + \beta_0/4} - ln(2)\big)/\beta_0$. For $n_f = 3$ quark flavors, $a \simeq 0.55$.

In Fig. 2 the AdS/QCD prediction of $\alpha_{g_1}^{AdS}(Q)$ (1) is plotted together with data [15, 16]. Even though it has no adjustable parameters, the predicted Gaussian shape of $\alpha_{g_1}^{AdS}(Q)$ agrees well with data [10]. We also show in this figure how $\alpha_{g_1}^{pQCD}(Q)$ depends on the β_n and $\alpha_{\overline{MS}}$ orders used in Eqs. 2 and 3, respectively. The curves converge quickly to a universal shape independent of the perturbation orders; at orders β_n or $\alpha_{\overline{MS}}^n$, $n > 1$, the $\alpha_{g_1}^{pQCD}(Q)$ are nearly identical. Our result at β_3, the same order to which the experimental value of $\Lambda_{\overline{MS}}$ is extracted, is $\Lambda_{\overline{MS}} = 0.341 \pm 0.024$ GeV for $n_f = 3$. The uncertainty stems from the extraction of κ from the ρ or proton mass and a small contribution from ignoring the quark masses. Our uncertainty is similar or better than that of the individual experimental determinations, which combine to $\Lambda_{\overline{MS}} = 0.339 \pm 0.016$ GeV [13].

Numerical lattice techniques have thus far provided the most accurate determination of $\Lambda_{\overline{MS}}$.

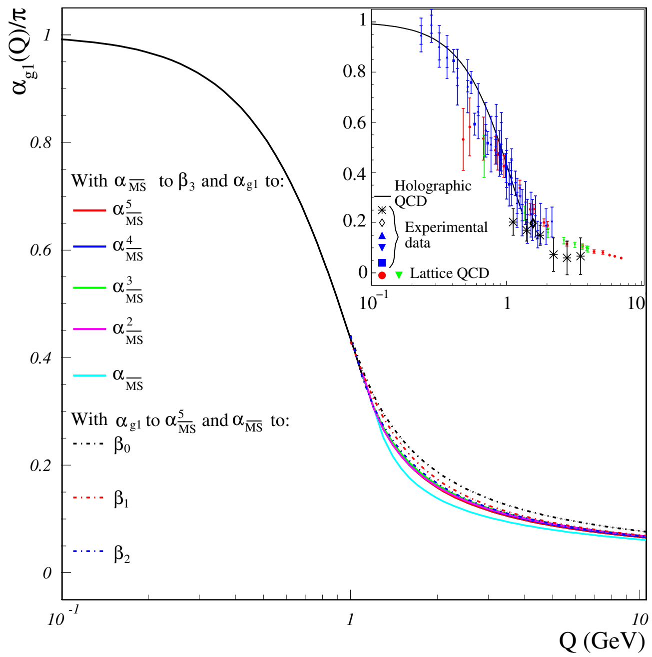

Figure 2: The dependence of α_{g_1} on the orders of the β and $\alpha_{\overline{MS}}$ series. The continuous black line is the AdS coupling. The continuous colored lines are the matched pQCD couplings for all available orders in the $\alpha_{\overline{MS}}$ series (the order of the β series was kept at β_3). The dash-dotted colored lines are the matched couplings at different orders in the β series (the order of the series was kept at $\alpha^5_{\overline{MS}}$). The curves beyond the leading order are observed to be remarkably close. The comparison between the AdS coupling and the data is shown in the embedded figure. This comparison is shown within the range of validity of AdS/QCD.

The combined world average yields 0.340 ± 0.008 GeV [13]. The accuracy of the individual Lattice calculations is similar to ours [17]. Although Lattice QCD numerically relates confinement dynamics to the value of $\Lambda_{\overline{MS}}$, our approach has several advantages: Lattice results are "black-box", numerically-intensive computations offering limited insights into the underlying processes. Analytical calculations, e.g. Chiral Perturbation Theory [18] or the Schwinger-Dyson formalism [19], provide a complementary understanding. Furthermore, lattice QCD requires typically five input parameters obtained from experiments [20], while in our approach, knowing only one parameter (the mass of the ρ meson, for example) is sufficient. Finally, the approximations used in Lattice QCD (discretized finite space-time, large quark masses, quenching, etc.,) make its systematics quite different from ours.

Our AdS/QCD approach also determines the transition scale Q_0. At order β_0,

$$Q_0 = M_\rho/\sqrt{a}. \tag{5}$$

At order β_3, $Q_0^2 \simeq 1.25 \pm 0.19$ GeV^2. This value is similar to the traditional lower limit $Q^2 > 1$ GeV^2 used for pQCD. An approximate value similar to ours was found in Ref. [21], which terminates the evolution of $\alpha_s(Q)$ near $Q \simeq 1$ GeV in order to enforce quark-hadron duality for the proton structure function $F_2(x, Q^2)$ measured in deep-inelastic experiments.

Conversely, we can use the ratio between $\Lambda_{\overline{MS}}$ and κ to predict the hadron spectrum. For example, starting with the experimental value of $\Lambda_{\overline{MS}}$, one obtains $M_\rho = 0.777 \pm 0.091$ GeV, in near perfect agreement with the measurement $M_\rho = 0.77525 \pm 0.00025$ GeV [13]. Our computed proton or neutron mass, $M_N = 1.099 \pm 0.129$ GeV, agrees within 1.2 σ with the averaged experimental values, 0.939 ± 0.000 GeV. Other hadron masses are calculated as orbital and radial excitations [8], [9], similarly to the computation of the energy levels of the hydrogen atom. Thus, using $\Lambda_{\overline{MS}}$, the hadron mass spectrum is calculated self-consistently within the framework described here. Note that QCD has no knowledge of conventional units of mass such as GeV; only ratios are predicted.

In summary, starting from first principles, and without additional parameters, we have obtained an explicit relation between the quark-confining nonperturbative dynamics of QCD at large-distances and the short-distance dynamics of pQCD; we thus link the pQCD scale Λ_s to the observed hadron masses. The analytic form of the QCD running coupling at all energy scales is also determined. The predicted value $\Lambda_{\overline{MS}} = 0.341 \pm 0.024$ GeV agrees well with the experimental average 0.339 ± 0.016 GeV and the lattice prediction 0.340 ± 0.008 GeV, and is of similar accuracy to the best individual experimental or lattice results. Moreover, we can identify a scale Q_0 which defines the transition point between pQCD and nonperturbative

QCD. Its value, $Q_0 \simeq 1$ GeV, agrees with observations. Conversely, starting with $\Lambda_{\overline{MS}}$ we can self-consistently compute the hadron mass spectrum within about 1-σ accuracy. The energy of quark confinement provides most of the ordinary hadron masses, and it can now be precisely related to the pQCD scale Λ_s. Since ordinary hadrons constitute about 99.95% of the mass of the visible matter of the universe, this helps to better understand most of the matter around us.

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