SLAC-PUB-16085

Dark energy from the string axiverse

Marc Kamionkowski,[1] Josef Pradler,[2] and Devin G. E. Walker[3]

[1] *Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles St., Baltimore, MD 21218, USA*

[2] *Institute of High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, 1050 Vienna, Austria*

[3] *SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025, USA*

String theories suggest the existence of a plethora of axion-like fields with masses spread over a huge number of decades. Here we show that these ideas lend themselves to a model of quintessence with no super-Planckian field excursions and in which all dimensionless numbers are order unity. The scenario addresses the "why now" problem—i.e., why has accelerated expansion begun only recently—by suggesting that the onset of dark-energy domination occurs randomly with a slowly decreasing probability per unit logarithmic interval in cosmic time. The standard axion potential requires us to postulate a rapid decay of heavy axions to Standard Model fields. The need for these decays is averted, though, with the introduction of a slightly modified axion potential. In either case, a Universe like ours arises in roughly 1 in 500 universes. The scenario may have a host of observable consequences.

arXiv:1409.0549v1 [hep-ph] 1 Sep 2014

We still lack a well-established explanation for the origin of the accelerated cosmic expansion observed in the Universe today [1]. The simplest guess, Einstein's cosmological constant, works fine but requires a new fundamental parameter with the unpalatable dimensionless amplitude of 10^{-120}. Quintessence circumvents this problem by suggesting that the apparent cosmological constant is simply the vacuum energy associated with the displacement of a scalar field from the minimum of its potential [2]. Still, quintessence does not solve the "why now" problem; i.e., why the Universe transitions from decelerated expansion to accelerated expansion only fairly recently, after the Universe has cooled 30 orders of magnitudes below the Planck temperature. Ideas that involve alternative gravity or large extra dimensions also generally require, ultimately, the tuning of some parameter to be extremely small [3]. Ideas based upon the string landscape [4] and/or anthropic arguments [5] suggest that the value of the cosmological constant in our Universe just happens to be the one, of $\sim 10^{120}$, that allows intelligent observers.

Here we show that ideas from string theory lend themselves to a quintessence explanation for cosmic acceleration that addresses the "why now" problem. It has long been understood [6, 7] that an axion-like field provides a natural candidate for a quintessence field, as the shift symmetry can protect the extraordinary flatness required of the quintessence potential. This solution, though, requires either that the axion decay constant have a super-Planckian value, or that the initial axion misalignment angle is extremely close to the value π that maximizes the potential, an option that is considered fine-tuned, if not overlooked (but, as we will see, will be essential in our scenario). The hypothesis that quintessence is an axion-like field also requires that the dark-energy density today must still be put in by hand.

String theory may give rise to a "string axiverse" [8, 9], a family of $O(100)$ axion-like fields with masses that span a huge number of decades. In each decade of the cosmic expansion, one of these axions becomes dynamical and has some small chance to drive an accelerated expansion. These chances are determined by the initial value, assumed to be selected at random, of the axion misalignment angle. There is thus some chance that the Universe will expand by ~ 30 decades in scale factor before it undergoes accelerated expansion. As we will see, this probability turns out to be $\sim 1/500$ given the distribution of axion masses and symmetry-breaking scales suggested by the string axiverse [9], We thus have an explanation for dark energy that involves no parameters that differ from unity by more than an order of magnitude. Although there is still some element of chance or anthropic selection required, a Universe that looks like ours arises as a $\sim$one-in-500 occurrence, rather than a one-in-10^{120} event. The model also specifies precisely the set of initial conditions post-inflation as the set of randomly chosen misalignment angles.

In the remainder of the paper we describe the scenario and clarify the assumptions made. As we will see, some additional mechanism must be postulated to account for the effects of the fields that do *not* become dark energy. One possibility is that the heaviest axions in the scenario, which would otherwise contribute an unacceptably large density at the time of big-bang-nucleosynthesis (BBN), undergo rapid decay. Another possibility that evade the overclosure problem involve a slight modification to the usual axion potential.

We postulate a collection of axion fields each labeled by an integer $a = 1, 2, 3, \ldots$, and we define the misalignment angle $\theta_a \equiv \phi_a/f_a$, where f_a is the axion decay constant for the ath field ϕ_a. The Lagrangian for this axion field is then,

$$\mathcal{L}_a = -\frac{f_a^2}{2}(\partial_\mu\theta_a)^2 - \Lambda_a^4 U(\theta_a), \quad U(\theta) = 1 - \cos\theta, \quad (1)$$

and the angle θ_a resides in the interval $\theta_a \in [-\pi, \pi]$. Here

This work was supported at SLAC by the US Department of Energy under contract DE-AC02-76SF00515 and HEP.

$2\Lambda_a^4$ is the maximum vacuum energy associated with the ath axion field.

There are a variety of ways in which axion fields may be populated in string theory. Here, to be concrete, we specify a particular realization in which the parameters for the ath field are [9],

$$\Lambda_a^4 = \mu^4 e^{-S_a}, \qquad f_a = \frac{\alpha M_P}{S_a}, \tag{2}$$

where $\alpha \sim 1$ is an order-unity constant, in line with the theoretical prejudice that f_a should be close to some fundamental scale (Planck or GUT) where global symmetries are expected to be broken. The choice of α is also compatible with the conjecture that gravity is the weakest force [10]. Here, $M_P = (8\pi G)^{-1/2} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass, and μ is a mass parameter related to the geometric mean of the supersymmetry-breaking scale and the Planck mass, which we take to be $\mu = \mu_{12}\, 10^{12}$ GeV, with μ_{12} a dimensionless constant. Finally, S_a is the action of the string instanton that generates the axion potential. We will take it to be $S_a = \beta a$ with β a dimensionless constant of order unity.

The mass of the ath axion is

$$m_a = \frac{\Lambda_a^2}{f_a} \simeq H_0 \frac{\mu_{12}^2}{\alpha} \left(\frac{S_a}{229.4}\right) e^{-(S_a - 229.4)/2}, \tag{3}$$

where $H_0 \simeq 10^{-33}\,\text{eV}$ is the present-day Hubble rate. Note that $f_a \lesssim M_P$, and so field excursions in the model are never super-Planckian. In this model, the distribution of axion masses is nearly (up to logarithmic corrections) constant per logarithmic interval in axion mass; i.e., $dN/d(\log_{10} m) \simeq 4.6/\beta$. Thus, if we take $\beta \sim 5$, there is roughly one axion per decade of axion mass.

Now consider the time evolution of these axion fields. The equation of motion for each scalar field is

$$\ddot{\theta}_a + 3H\dot{\theta}_a + m_a^2 \sin(\theta_a) = 0, \tag{4}$$

where the dot denotes the derivative with respect to cosmic time t. The Hubble parameter is determined from the Friedmann equation,

$$H^2 = \frac{1}{3M_p^2}\left[\sum_a \rho_a + \rho_m R^{-3} + \rho_r R^{-4}\right], \tag{5}$$

where $R(t)$ is the scale factor normalized to $R = 1$ today, and ρ_m and ρ_r are respectively the matter and radiation energy density today. The energy density in the ath axion field is given by,

$$\rho_a = \frac{1}{2} f_a^2 \dot{\theta}_a^2 + \Lambda_a^4 U(\theta_a). \tag{6}$$

The pressure p_a of the ath field is given by the same expression but with the sign of second term (the potential) reversed, and the equation-of-state parameter is $w_a = p_a/\rho_a$.

The axion equation of motion is integrated from $t = 0$ with an initial field value $\theta_{a,I}$ and $\dot{\theta}_a(t = 0) = 0$. We surmise that each $\theta_{a,I}$ is selected at random from a uniform distribution in the range $-\pi < \theta_{a,I} < \pi$. At sufficiently early times that $m_a \lesssim 2H$ the axion field is frozen because of Hubble friction and its vacuum energy is negligible compared with the matter/radiation density.

There are then two possibilities for the subsequent evolution after the Universe cools sufficiently so that $m_a \simeq 2H$, when the scale factor is $\sim R_a$, determined by $2H(R_a) \simeq m_a$. The first possibility, which occurs if $|\theta_{a,I}|$ is not too close to π, is that the axion field begins to oscillate and behaves as nonrelativistic matter with an energy density that decreases as $\rho_a \sim \Lambda_a^4[1 - \cos(\theta_{a,I})](R_a/R)^3$.

The second possibility, which occurs if $|\theta_{a,I}|$ is close enough to π, is that the axion field rolls slowly towards its minimum, with an equation-of-state parameter $w_a < -1/3$. In this case, the energy density of this axion field may come to dominate the cosmic energy budget and drive a period of accelerated expansion. The condition for the field to roll slowly is $\epsilon = (M_P^2/2)(V'/V)^2 < 1$, which requires the initial misalignment angle to be in the range

$$\pi > |\theta_{a,I}| \gtrsim \pi - 2\sqrt{2} f_a/M_p = \pi - 2\sqrt{2}\alpha/(\beta a). \tag{7}$$

The probability that the ath field will drive accelerated expansion is thus $\sim 2\sqrt{2}\alpha/(\beta a \pi) \simeq (\alpha/\beta)/a$.

We now take an initial time near the onset of radiation domination, after inflation and reheating, when the Universe has a temperature $T_{\rm re}$ which we suppose is $T_{\rm re} \gtrsim \mu$. We suppose that at this time, the initial field values $\theta_{a,I}$ for the fields that enter the horizon after reheating (those with $m_a \lesssim T_{\rm re}$) have been fixed during inflation. The successful dark-energy model that we seek is then one in which there is no accelerated expansion from $T_{\rm re}$ until very recent times, redshift $z \sim 1$, at which point the Universe enters a period of dark-energy domination.

Recall that the axion mass decreases monotonically (for $a \gtrsim$ few) with a and that the Hubble parameter decreases with time. Therefore, the different axion fields become dynamical ($m_a \simeq 2H$) in a sequence of increasing a. There is some small chance, $\sim (\alpha/\beta)$, that the first axion ($a = 1$) would drive accelerated expansion, and if so, that cannot describe our Universe. Suppose, though, that it does not drive accelerated expansion. There is then some smaller chance, $\sim (\alpha/\beta)/2$, that the second axion field will drive accelerated expansion. If it does, then that is not our Universe. Cumulatively, the chance that the first $a - 1$ fields do not drive accelerated expansion but that ath field does is

$$P(a) = \frac{\alpha}{\beta a}\Pi_{b=1}^{a-1}\left(1 - \frac{\alpha}{\beta b}\right) \simeq \frac{(\alpha/\beta)}{\Gamma(1 - \alpha/\beta)}\frac{1}{a^{1+\alpha/\beta}}, \tag{8}$$

where the approximation is valid for $a \gtrsim \beta/\alpha$. This equation encapsulates the heart of this model for dark energy.

The important point is that there is for the relevant values of a a slowly falling probability per unit logarithmic interval in axion mass, for a given axion to act as dark energy. From this it follows that *there is a slowly decreasing probability per logarithmic interval in cosmic time (or scale factor or redshift or cosmic temperature) for the Universe to become dark-energy dominated.*

The axion field that describes cosmic acceleration in our Universe is one which has a density $2m_a^2 f_a^2 \simeq \Omega_\Lambda \rho_c \simeq 0.7\,(3\,H_0^2 M_P^2)$, where Ω_Λ is the fraction of the critical density $\rho_c = 3\,H_0^2 M_p^2$ in dark energy today. From this it follows that $(f_a/M_p)^2 (m_a/H_0)^2 \simeq 1$ and thus that the axion field responsible for cosmic acceleration must have $m_a \simeq (a\beta/\alpha) H_0$. Combining this with Eq. (3), we find that if $\mu_{12} = \alpha = 1$ and $\beta = 5$, for example, then the field that becomes dark energy has index $a \simeq 44$ (and $m_a = H(R_a)$ is met at redshift $z \simeq 55$). From Eq. (8), the probability that a given Universe will have a cosmological constant like that we observe, with $a = 44$ is $\sim 1/500$. *We conclude that if we were to look at 500 post-inflation universes with different randomly selected sets of initial field values $\theta_{a,I}$, we would expect one of them to wind up looking like our Universe.* We thus have an explanation for cosmic acceleration that invokes no dimensionless parameters that differ from unity by more than one order of magnitude. The results depend only logarithmically on μ and do not differ considerably for order-unity changes to α and β.

The next step in the consideration of these models is to understand the effects of the ~ 50 axion fields that do not become dark energy. A typical such field will have $\theta_{a,I} \sim 1$ and will, when it begins to oscillate when $m_a \sim H$, have an energy density $\sim m_a^2 f_a^2$ that is smaller than the critical density $\sim H^2 M_P^2$ by a factor $\sim S_a^{-2}$. However, the energy density in the coherent field oscillations that ensue scales with the scale factor as R^{-3}, as opposed to the radiation density, which scales as R^{-4}. The energy density of these fields, especially those that enter the horizon first, thus comes to dominate the energy density of the Universe. If these axions do not decay, they overclose the Universe by a huge amount; for example, the first field ($a = 1$), with $m_a \sim 10^4$ GeV, would have an energy density today $\sim 10^{12}$ times the current critical density.

If, however, the axions with masses $m_a \gtrsim H_{\rm bbn} \sim 10^{-17}$ eV—those with $a < a_{\rm bbn} \simeq 29$ decay on a timescale less than ∼sec (when BBN begins) to Standard Model particles, then they will simply heat the Universe, without any observational consequences (apart from the dilution of any pre-existing dark-matter density or baryon number). Such decays, though, are unlikely to be sufficiently rapid, for axions at the lower-mass end of this mass range, for a parametric decay rate $\Gamma \sim m_a^3/f_a^2$, or even $\Gamma \sim m_a$. Another possibility, though, is that only a handful of the heaviest axions decay prior to BBN. If the ath axion did not decay, it would contribute, following the reasoning above, a density $(\rho_a/\rho_R)_{\rm bbn} \propto S_a^{-3/2} e^{-S_a/4}$. From this it follows that the undecayed-axion energy density at BBN is dominated by only a handful of the heaviest axions, those with the smallest values of a. With reasonable shifts to the values of the parameters we have chosen (e.g., a slightly larger μ), decay rates greater $\Gamma \gtrsim H_{\rm bbn}$ are not implausible for fields with $a \sim$ few. If these heaviest axions then decay to SM particles before BBN, they can dilute the contributions of the lighter axion fields to the cosmic energy budget at BBN.

Next consider the axions that enter the horizon after BBN but before CMB decoupling ($a_{\rm cmb} \simeq 41$). The energy density of these axions must not exceed the bound on the dark-matter density at CMB decoupling inferred from the CMB. Again, the problem is dominated by the most massive axion, that with $m_a \sim 10^{-17}$ eV, with $a \simeq 29$, that enters the horizon at BBN. The contribution of this axion to the critical density at that time is $\sim S_a^{-2} \simeq 5 \times 10^{-5}$. Since its density scales as R^{-3}, it has a density $\Omega_a \sim 0.1$ today. Depending on the details, therefore, it (and other axions with $44 > a > 29$ may need to decay, or it may actually make up the dark matter! The axions that enter the horizon later—those with $a > 29$—may contribute a small fraction of the dark-matter density.

To summarize, the heaviest axions, those with $a \sim$ few must somehow decay to SM particles before BBN. Those that enter at later times are consistent with current observations. It may also be that the dark matter is comprised primarily of one of these fields, presumably that with $a \simeq 29$ and $m_a \sim 10^{-17}$ eV.

Some thought should be given to isocurvature perturbations, since the axions that decay or that may make up the dark matter (or part of it) have fluctuations that are not correlated with the curvature perturbations induced during inflation. However, if the primordial plasma of SM particles is due primarily to the decays of the heaviest axions, then the resulting perturbations are likely to be mostly adiabatic. If the dark matter turns out to be the $m_a \sim 10^{-17}$ eV axion, then there may be some worry that the perturbations in the axion–dark-matter density may be isocurvature. However, the scenario does not require these axions to make up the dark matter. Even if they are the dark matter, there may be mechanisms (e.g., Ref. [11]) to avoid problems with isocurvature perturbations. If the scenario proceeds via the $(1-\cos\theta)^3$ potential, then there are no isocurvature perturbations. In summary, it will be important to insure that isocurvature perturbations are not a problem in any detailed implementation of the scenario that we outline, but isocurvature perturbations are not necessarily a showstopper.

The principal objection one might have to the scenario above is the rapid decays to Standard Model radiation required of the heaviest axion fields. We now propose a slightly revised scenario in which there are no problems with overclosure, and no field decays required.

Suppose that the axion potential function $U(\theta) = 1 - \cos\theta$ is replaced by $U(\theta) = (1-\cos\theta)^3$. The broad outline of the scenario described above remains the same. The principal difference, though, is that once the field begins to oscillate, it oscillates about a minimum that is $V(\phi) \propto \phi^6$, rather than ϕ^2. Such oscillations behave as matter with equation-of-state parameter $w = 1/2$ and have an energy density that decays with scale factor as $R^{-9/2}$ [12, 13], more rapidly than radiation, which decays as R^{-4}. Thus, the energy density in the fields that do not become dark energy always remains negligible compared with the dominant radiation and matter densities.

The slow-roll condition $\epsilon = (M_p^2/2)(V'/V)^2 < 1$ for this altered potential is $\pi > \theta_{a,I} > \pi - (2\sqrt{2}/3)\alpha/(\beta a)$, more restrictive than in the first scenario (for the same α and β) by a factor of three. The probability for the ath field to have $w < -1/3$ is obtained from Eq. (8) with the replacement $(\alpha/\beta) \to (\alpha/\beta)/3$.

Although there is no mass associated with the field now (the curvature about the minimum of the potential is zero), there is an oscillation frequency $\omega_a(\phi_0)$ that depends on the amplitude ϕ_0 of the oscillation. This frequency is small as the misalignment-angle amplitude $\theta_0 = \phi_0/f_a \to \pi$ and then increases as θ_0 decreases to $\theta_0 \sim 2.38$ at which point the oscillation frequency is $\omega_a \simeq 0.69\,\Lambda_a^2/f_a$. The subsequent decrease $\omega_a \propto R^{-3/2}$ of the oscillation frequency is slower than the decrease $H \propto a^{-2}$ of the Hubble parameter. Thus, once the Hubble parameter has decreased below ω_a, and the field begins to oscillate, it continues to oscillate thereafter. The energy density thus becomes negligible compared with radiation, as claimed above.

We now recapitulate and then make closing remarks. We suppose that there are several hundred axion fields with masses and decay constants distributed as in Eqs. (2) and (3). With the mass parameter μ chosen to be $\sim 10^{12}$ GeV, the highest-mass axion (that with $a = 1$) has a mass $\sim \mu \sim 10^{12}$ GeV, and the axion with mass $m_a \sim H_0 \sim 10^{-33}$ eV comparable to the Hubble parameter today has $S_a \simeq 229$, or $a \simeq 46$ if $\beta = 5$. We then surmise that after inflation, which presumably reheats the Universe to a temperature $T \gtrsim \mu$, the initial misalignment angles $\theta_{a,I}$ for all the fields are selected at random. We then argue that there is, with the canonical parameters chosen, a roughly 1 in 500 chance that the Universe will undergo radiation- and matter-dominated expansion phases until a redshift $z \sim 1$ when the dark energy associated with the field with $a \simeq 44$ will take over. Our Universe turns out to be this one-in-500 occurrence either because of the luck of the draw and/or from some anthropic selection. In this regard, the scenario may require anthropic elements similar to those in landscape scenarios. The one-in-500 coincidence, though, does not make as stringent demands on the imagination as a one-in-10^{120} coincidence.

The ~ 0.002 probability we arrive at depends only weakly to order-unity variations in the parameters α and β and only logarithmically on μ. This probability, Eq. (8), is obtained by requiring that none of the axion fields that enter the horizon before that (with $a = 44$) that gives rise to the observed accelerated expansion lead to accelerated expansion. This requirement, though, may be too restrictive as some of these earlier dark-energy-dominated phases may be relatively short and have little observable impact. If so, then the probability that a Universe like the one we observe arises may be larger, closer to $\sim (\alpha/\beta)a \sim 1/200$. Conversely, though, measurements that constrain the equation-of-state parameter w to be relatively close to -1 suggest that the $a = 44$th field that drives the observed dark energy must have ϵ small compared with unity, and thus an initial misalignment angle closer to π than the full range allowed by Eq. (7). This will then decrease the probability accordingly. It may also be interesting to explore how the scenario is modified if fields are populated with a distribution different from that, in which S_a is populated uniformly in a, assumed here. The scenario requires that we postulate either a rapid decay of the highest-mass fields that enter the horizon before BBN, or a non-standard axion potential in which the effects of these other fields quickly redshift away. It will be interesting to consider more complete models in which these requirements arise [14].

There may also be, depending on the detailed implementation of the ideas presented here, observational consequences of this scenario. First of all, the model predicts a quintessence-dominated Universe today, with a value of the observed equation-of-state parameter w that differs from -1—i.e., the dark energy is not a cosmological constant;[1] this should be tested with forthcoming measurements of the expansion history [17]. In this scenario, cosmic acceleration is due to a quintessence field, and *not* a modification to gravity; cosmic modified-gravity searches [18] should therefore all turn up null results. If the field has an axion-like coupling to electromagnetism, there may be cosmic birefringence observed [7, 19]. Residual decays of the fields that enter the horizon before BBN or shortly after BBN may give rise to spectral distortions in the CMB (see, e.g., Ref. [20] and references therein) or the spectrum of primordial perturbations on very small scales [21]. There may be some component of primordial perturbations that is isocurvature. There may be observable consequences for astrophysical black holes [22]. And the dark matter may be composed, at least in part, of a $\sim 10^{-17}$ eV axion, something that may be sought with future experiments [23]. It will be interesting to study these possibilities in more detailed implementations of

[1] The scenario we present is thus distinct from that envisionsed in Ref. [15], that is similarly in spirit to N-flation [16], in which there is a cosmological constant that is due to the displacement of the many axion fields with $m_a < H_0$ from their minimum.

the ideas presented here.

We thank A. Arvanitaki for useful discussions. MK thanks the Aspen Center for Physics, supported by NSF Grant No. PHY-1066293, for hospitality. MK acknowledges the support at JHU of NSF Grant No. 0244990, the John Templeton Foundation, and the Simons Foundation. JP is supported by the New Frontiers program of the Austrian Academy of Sciences. DW is supported in part by a grant from the Ford Foundation via the National Academies of the Sciences as well as the US Department of Energy, contract DE-ACO2-76SF00515.

[1] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998) [arXiv:astro-ph/9805201]; S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999) [arXiv:astro-ph/9812133].

[2] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988); C. Wetterich, Astron. Astrophys. **301**, 321 (1995) [arXiv:hep-th/9408025]; K. Coble, S. Dodelson and J. A. Frieman, Phys. Rev. D **55**, 1851 (1997) [arXiv:astro-ph/9608122]; M. S. Turner and M. J. White, Phys. Rev. D **56**, 4439 (1997) [arXiv:astro-ph/9701138]; R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998) [arXiv:astro-ph/9708069].

[3] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006) [arXiv:hep-th/0603057]; R. R. Caldwell and M. Kamionkowski, Ann. Rev. Nucl. Part. Sci. **59**, 397 (2009) [arXiv:0903.0866 [astro-ph.CO]]; A. Silvestri and M. Trodden, Rept. Prog. Phys. **72**, 096901 (2009) [arXiv:0904.0024 [astro-ph.CO]].

[4] R. Bousso and J. Polchinski, JHEP **0006**, 006 (2000) [hep-th/0004134].

[5] S. Weinberg, Phys. Rev. Lett. **59**, 2607 (1987).

[6] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990); J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995) [astro-ph/9505060];

[7] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998) [arXiv:astro-ph/9806099].

[8] P. Svrcek and E. Witten, JHEP **0606**, 051 (2006) [hep-th/0605206].

[9] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, Phys. Rev. D **81**, 123530 (2010) [arXiv:0905.4720 [hep-th]].

[10] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, JHEP **0706**, 060 (2007) [hep-th/0601001].

[11] T. Higaki, K. S. Jeong and F. Takahashi, Phys. Lett. B **734**, 21 (2014) [arXiv:1403.4186 [hep-ph]].

[12] M. S. Turner, Phys. Rev. D **28**, 1243 (1983).

[13] M. C. Johnson and M. Kamionkowski, Phys. Rev. D **78**, 063010 (2008) [arXiv:0805.1748 [astro-ph]].

[14] J. Pradler and M. Kamionkowski, in preparation

[15] P. Svrcek, [hep-th/0607086].

[16] S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, JCAP **0808**, 003 (2008) [hep-th/0507205].

[17] A. J. Albrecht *et al.*, arXiv:astro-ph/0609591; E. V. Linder, Rept. Prog. Phys. **71**, 056901 (2008) [arXiv:0801.2968 [astro-ph]]; J. Frieman, M. Turner and D. Huterer, Ann. Rev. Astron. Astrophys. **46**, 385 (2008) [arXiv:0803.0982 [astro-ph]]; D. H. Weinberg, M. J. Mortonson, D. J. Eisenstein, C. Hirata, A. G. Riess and E. Rozo, Phys. Rept. **530**, 87 (2013) [arXiv:1201.2434 [astro-ph.CO]].

[18] T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. **513**, 1 (2012) [arXiv:1106.2476 [astro-ph.CO]]; A. Joyce, B. Jain, J. Khoury and M. Trodden, arXiv:1407.0059 [astro-ph.CO].

[19] A. Lue, L. M. Wang and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999) [arXiv:astro-ph/9812088]; N. F. Lepora, arXiv:gr-qc/9812077.

[20] J. Chluba, arXiv:1405.6938 [astro-ph.CO].

[21] D. Jeong, J. Pradler, J. Chluba and M. Kamionkowski, Phys. Rev. Lett. **113**, 061301 (2014) [arXiv:1403.3697 [astro-ph.CO]]; T. Nakama, T. Suyama and J. Yokoyama, Phys. Rev. Lett. **113**, 061302 (2014) [arXiv:1403.5407 [astro-ph.CO]].

[22] A. Arvanitaki and S. Dubovsky, Phys. Rev. D **83**, 044026 (2011) [arXiv:1004.3558 [hep-th]].

[23] P. W. Graham and S. Rajendran, Phys. Rev. D **88**, 035023 (2013) [arXiv:1306.6088 [hep-ph]]; D. Budker, P. W. Graham, M. Ledbetter, S. Rajendran and A. Sushkov, Phys. Rev. X **4**, 021030 (2014) [arXiv:1306.6089 [hep-ph]].