

A Possible Solution to the $B \rightarrow \pi\pi$ Puzzle Using the Principle of Maximum Conformality

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The measured $B_d \rightarrow \pi^0\pi^0$ branching fraction deviates significantly from conventional QCD predictions, a puzzle which has persisted for more than 10 years. This may be a hint of new physics beyond the Standard Model; however, as we shall show in this paper, the pQCD prediction is highly sensitive to the choice of the renormalization scales which enter the decay amplitude. It is conventional to choose a typical momentum transfer as the renormalization scale and to take an arbitrary range to estimate the theory uncertainty. However, the prediction using this procedure depends on the renormalization scheme, leaves a non-convergent renormalon series, and gives an arbitrary estimate of the systematic error. In contrast, if one fixes the renormalization scale using the Principle of Maximum Conformality (PMC), all non-conformal $\{\beta_i\}$ -terms in the perturbative expansion series are resummed into the running coupling, one then obtains a unique, scale-fixed, scheme-independent prediction at any finite order. The PMC is a generalization of the BLM procedure, and it reduces in the Abelian limit to the standard Gell Mann-Low procedure used for precision tests of QED. We show that the renormalization scale uncertainties for $B \rightarrow \pi\pi$ can be greatly reduced by applying the PMC. Combining the errors in quadrature, we obtain the CP averaged branching ratio $\mathcal{B}(B_d \rightarrow \pi^0\pi^0)|_{\text{PMC}} = (0.98^{+0.28}_{-0.32}) \times 10^{-6}$ by using the light-front holographic low-energy model for the running coupling. All of the CP-averaged $B \rightarrow \pi\pi$ branching fractions predicted by the PMC are consistent with the Particle Data Group average values and the newly Belle data.

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B -meson hadronic two-body decays contain a wealth of information on the physics underlying the charge-parity (CP) violation. Measurements of the B -meson two-body branching ratios and their CP asymmetries provide key information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. One challenge that has puzzled the theoretical physics community for more than 10 years is that the measured branching ratio for the decay of the B meson to neutral pion pairs $B_d \rightarrow \pi^0\pi^0$ is significantly larger than the theoretical predictions based on the QCD factorization approach [1–4] and the perturbative QCD approach [5].

Beneke *et al.* (BBNS) [6] have developed a systematic QCD analysis of $B \rightarrow \pi\pi$ based on the factorization of long-distance and short distance dynamics. The BBNS predictions for the branching ratios of $B_d \rightarrow \pi^+\pi^-$ and $B^\pm \rightarrow \pi^\pm\pi^0$ are consistent with CLEO, BaBar, and Belle data. However, the BBNS prediction for the $B_d \rightarrow \pi^0\pi^0$ branching ratio deviates significantly from the measurements [3]. There have been suggestions on how to resolve this puzzle and to obtain a consistent explanation of all $B \rightarrow \pi\pi$ channels within the same framework. In particular, Beneke *et al.* [7] have noted that the next-to-leading order (NLO) QCD corrections to the color-suppressed hard spectator scattering amplitude

$\alpha_2(\pi\pi)$ could be important, as seen from their calculation of vertex corrections up to next-to-next-to-leading order (NNLO) level [4]. However, even after including those higher-order QCD corrections, the discrepancy remains. There is also the concern that the large K factor implied by the higher-order corrections to the branching ratio of $B_d \rightarrow \pi^0\pi^0$, as well as the large renormalization scale uncertainties, may make pQCD calculations questionable.

According to *renormalization group invariance*, a valid prediction for a physical observable should be independent of theoretical conventions, such as the choice of the renormalization scheme and the value of the initial renormalization scale. This important principle is satisfied by the Principle of Maximum Conformality (PMC) [8, 9]. The running behavior of the coupling constant is determined by its $\{\beta_i\}$ -function via the renormalization group equation. Conversely, the knowledge of the $\{\beta_i\}$ -terms can be used to determine the optimal scale of a particular process; this is the main goal of the PMC. If one fixes the renormalization scale of the pQCD series using the PMC, all non-conformal $\{\beta_i\}$ -terms in the perturbative expansion series are resummed into the running coupling, and one obtains a unique, scale-fixed, scheme-independent prediction at any finite order. The resulting $\{\beta_i\}$ -free pQCD series is thus identical to the conformal

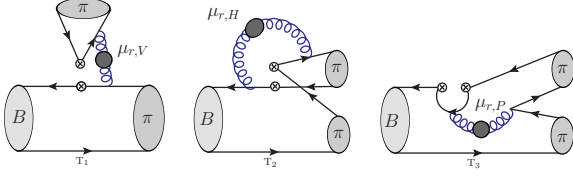


FIG. 1: Typical Feynman diagrams for the $B \rightarrow \pi\pi$ decays, which are sizable and correspond to α_1 , α_2 , α_4 (or α_6), respectively. $\mu_{r,V}$, $\mu_{r,H}$ and $\mu_{r,P}$ are renormalization scales for these diagrams; they are different in general. Other Feynman diagrams can be obtained by shifting one of the gluon end-points to different quark lines. The vertex “ $\otimes\otimes$ ” denotes the insertion of a 4-fermion operator Q_i . And the big dot stands for the renormalized gluon propagator whose light-quark loop determines the β_0 -terms and hence the optimal scale for the running behavior of the QCD coupling constant.

series and is renormalization-scheme independent. The PMC prediction satisfies all self-consistency conditions of the renormalization group, such as reflectivity, symmetry, and transitivity [10]. The PMC is a generalization of the BLM procedure [11], and it reduces in the Abelian limit to the standard Gell Mann-Low procedure [12] used for precision tests of QED.

$$\begin{aligned} \mathcal{T}_p = & \alpha_1^p (\pi\pi) (\bar{u}b)_{V-A} \otimes (\bar{d}u)_{V-A} + \alpha_2^p (\pi\pi) (\bar{d}b)_{V-A} \otimes (\bar{u}u)_{V-A} + \alpha_3 (\pi\pi) (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V-A} + \\ & \alpha_4^p (\pi\pi) (\bar{q}b)_{V-A} \otimes (\bar{d}q)_{V-A} + \alpha_5 (\pi\pi) (\bar{d}b)_{V-A} \otimes (\bar{q}q)_{V+A} + \alpha_6^p (\pi\pi) (-2) (\bar{q}b)_{S-P} \otimes (\bar{d}q)_{S+P}. \end{aligned} \quad (3)$$

A summation over $q = u, d$ is implied in this equation, and the required currents are $(\bar{q}q')_{V\pm A} = \bar{q}\gamma^\mu(1 \pm \gamma_5)q'$ and $(\bar{q}q')_{S\pm P} = \bar{q}(1 \pm \gamma_5)q'$. The branching ratio for $B \rightarrow \pi\pi$ is given by $\mathcal{B}(\bar{B} \rightarrow \pi\pi) = \tau_B |\mathcal{A}(\bar{B} \rightarrow \pi\pi)|^2 S / (16\pi m_B)$, where the symmetry parameter $S = 1/2!$ for $\pi^0\pi^0$, and $S = 1$ for $\pi^+\pi^-$ or $\pi^\pm\pi^0$, respectively.

$$\begin{aligned} \mathcal{A}(\bar{B}_0 \rightarrow \pi^+\pi^-) &= i \frac{G_F}{\sqrt{2}} m_B^2 f_+^{B \rightarrow \pi}(0) f_\pi |\lambda_c| \{ R_b e^{-i\gamma} [\alpha_1^u + \alpha_4^u + \alpha_6^u r_\chi] - [\alpha_4^c + \alpha_6^c r_\chi] \}, \\ \mathcal{A}(\bar{B}_0 \rightarrow \pi^0\pi^0) &= i \frac{G_F}{\sqrt{2}} m_B^2 f_+^{B \rightarrow \pi}(0) f_\pi |\lambda_c| \{ R_b e^{-i\gamma} [-\alpha_2^u + \alpha_4^u + \alpha_6^u r_\chi] - [\alpha_4^c + \alpha_6^c r_\chi] \}, \\ \mathcal{A}(B^- \rightarrow \pi^-\pi^0) &= i \frac{G_F}{\sqrt{2}} m_B^2 f_+^{B \rightarrow \pi}(0) f_\pi |\lambda_c| (R_b / \sqrt{2}) e^{-i\gamma} [\alpha_1^u + \alpha_2^u], \end{aligned} \quad (4)$$

where $R_b = |V_{ub}V_{ud}^*|/|V_{cb}V_{cd}^*|$, and γ is the V_{ub}^* phase. The coefficient $r_\chi = 2m_\pi^2 / [\bar{m}_b(\mu_r)(\bar{m}_u(\mu_r) + \bar{m}_d(\mu_r))]$, which equals to 1.18 when setting the scale $\mu_r = m_b$.

In the following, we will apply the PMC procedure to the BBNS analysis with the goal of eliminating the renormalization scale ambiguity and achieving an accurate pQCD prediction that is independent of theoretical conventions. In fact, as we shall show, the PMC can provide a solution to the $B \rightarrow \pi\pi$ puzzle.

The amplitude for $B \rightarrow \pi\pi$ decay, assuming the dominance of valence Fock states for both the B meson and the final-state pions, can be expressed as

$$\langle \pi\pi | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \pi\pi | \mathcal{T}_p | \bar{B} \rangle. \quad (1)$$

The effective weak Hamiltonian [13]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3\dots 6} C_i Q_i \right], \quad (2)$$

where the $\lambda_p = V_{pd}^* V_{pb}$, $Q_i(\mu_f)$ are local four-fermion interaction operators, and the $C_i(\mu_f)$ are the corresponding short-distance Wilson coefficients at the factorization scale $\mu_f \simeq m_b$. The operator that creates the weak transition in the Standard Model is

Typical Feynman diagrams which provide non-zero contributions to the $B \rightarrow \pi\pi$ decays and correspond to α_1 , α_2 , α_4 and α_6 , respectively, are illustrated in Fig.(1). The resulting amplitudes under the $\overline{\text{MS}}$ -scheme for $B \rightarrow \pi\pi$ can be written as [6]

Here $f_\pi(f_B)$ is the pion (B -meson) decay constant, and $f_+^{B \rightarrow \pi}(0)$ is the $B \rightarrow \pi$ transition form factor at the zero momentum transfer. The CP conjugate amplitudes are

obtained from the above by replacing $e^{-i\gamma}$ to $e^{+i\gamma}$. The topological tree amplitude α_1 expresses the contribution when the final $(\bar{u}d)$ -pair (produced from the virtual W^-) forms the pion directly. The tree amplitude α_2 expresses the contribution obtained when the final $(\bar{u}d)$ -pair from W^- separates and the \bar{u} -quark forms a pion by coalescing with the spectator u -quark. The amplitudes α_i ($i=3,6$) are topological penguin amplitudes. Note that when the spectator quark combines with one of the quarks from W^- to form a pion, a color-suppressed factor $\sim 1/N_c$ emerges. Thus, the amplitude α_1 provides the domi-

nant contributions relative to the color-suppressed $\alpha_{2,4,6}$. However this color suppression can effectively disappear when one includes higher-order gluonic interactions to $\alpha_{2,4,6}$; their contributions thus can be sizable. At present, consistent pQCD calculations of the tree amplitudes $\alpha_{1,2}$ and their vertex corrections have been evaluated up to NNLO level. The QCD correction to the hard spectator scattering interaction has been calculated up to NLO level [4].

We rewrite the contributions in the following convenient form:

$$\alpha_1^p = C_1 + \frac{1}{N_c} \left[C_2 + C_F C_2 \left\{ \frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} V_1 + \left(\frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \right)^2 \beta_0 \tilde{V}_1 \right\} + \left(\frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \right)^2 V_2 + \frac{4C_F C_2 \pi^2}{N_c} \left\{ \frac{\alpha_s(\mu_{r,H}^{\text{init}})}{4\pi} H_1 + \left(\frac{\alpha_s(\mu_{r,H}^{\text{init}})}{4\pi} \right)^2 \beta_0 \tilde{H}_1 \right\} + \left(\frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \right)^2 H_2 \right], \quad (5)$$

$$\alpha_2^p = C_2 + \frac{1}{N_c} \left[C_1 + C_F C_1 \left\{ \frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} V_1 + \left(\frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \right)^2 \beta_0 \tilde{V}_1 \right\} + \left(\frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \right)^2 V_3 + \frac{4C_F C_1 \pi^2}{N_c} \left\{ \frac{\alpha_s(\mu_{r,H}^{\text{init}})}{4\pi} H_1 + \left(\frac{\alpha_s(\mu_{r,H}^{\text{init}})}{4\pi} \right)^2 \beta_0 \tilde{H}_1 \right\} + \left(\frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \right)^2 H_3 \right]. \quad (6)$$

The penguin diagrams provide small contributions to the amplitudes. They are written as

$$\alpha_4^p = C_4 + \frac{C_3}{N_c} \left[1 + \frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} C_F V_1 + \frac{\alpha_s(\mu_{r,H}^{\text{init}})}{4\pi} \frac{4C_F \pi^2}{N_c} H_1 + \frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \frac{C_F}{N_c} P_{\pi,2}^p \right], \quad (7)$$

$$\alpha_6^p = C_6 + \frac{C_5}{N_c} \left[1 + \frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} C_F (-6) + \frac{\alpha_s(\mu_{r,V}^{\text{init}})}{4\pi} \frac{C_F}{N_c} P_{\pi,3}^p \right]. \quad (8)$$

Here $\beta_0 = (11N_c - 2n_f)/3$, V_i (\tilde{V}_i) denotes the vertex corrections, and H_i (\tilde{H}_i) denotes the hard spectator scattering contributions. The corresponding expressions for the functions V_i , \tilde{V}_i , H_i , \tilde{H}_i and $P_{\pi,n}^p$ can be obtained from Refs. [4, 14]. The initial scales are set to $\mu_{r,P}^{\text{init}} = \mu_{r,V}^{\text{init}}$. The quantity $P_{\pi,n}^p$ refers to the contribution from the pion twist- n light-cone distribution amplitude. In the calculation both twist-2 and twist-3 terms are taken into consideration. Note that the Wilson coefficients C_1 and C_2 are different from the definition of Ref.[13], where the labels 1 and 2 are interchanged.

In order to apply PMC scale-setting, we have divided

the amplitudes into β_0 -dependent nonconformal and β_0 -independent conformal parts, respectively. There are two typical momentum flows for the process; thus, we have assigned two arbitrary initial scales $\mu_{r,V}^{\text{init}}$ and $\mu_{r,H}^{\text{init}}$ for the vertex contributions and hard spectator scattering contributions. In the case of conventional scale setting, the scales are fixed to be their typical momentum transfers, i.e. $\mu_{r,V} \equiv \mu_{r,V}^{\text{init}} \sim m_b$ and $\mu_{r,H} \equiv \mu_{r,H}^{\text{init}} \sim \sqrt{\Lambda_{\text{QCD}} m_b}$.

After applying PMC scale setting, all non-conformal β_0 -terms are resummed into the effective running coupling, and the amplitudes become

$$\alpha_1^{p,\text{PMC}} = C_1 + \frac{1}{N_c} \left[C_2 + C_2 C_F \frac{\alpha_s(Q_1^V)}{4\pi} V_1 + \left(\frac{\alpha_s(Q_1^V)}{4\pi} \right)^2 V_2' + \frac{4C_2 C_F \pi^2}{N_c} \frac{\alpha_s(Q_1^H)}{4\pi} H_1 + \left(\frac{\alpha_s(Q_1^H)}{4\pi} \right)^2 H_2 \right], \quad (9)$$

$$\alpha_2^{p,\text{PMC}} = C_2 + \frac{1}{N_c} \left[C_1 + C_1 C_F \frac{\alpha_s(Q_1^V)}{4\pi} V_1 + \left(\frac{\alpha_s(Q_1^V)}{4\pi} \right)^2 V_3' + \frac{4C_1 C_F \pi^2}{N_c} \frac{\alpha_s(Q_1^H)}{4\pi} H_1 + \left(\frac{\alpha_s(Q_1^H)}{4\pi} \right)^2 H_3 \right], \quad (10)$$

$$\alpha_4^{p,\text{PMC}} = C_4 + \frac{C_3}{N_c} \left[1 + \frac{\alpha_s(Q_1^V)}{4\pi} C_F V_1 + \frac{\alpha_s(Q_1^V)}{4\pi} \frac{4C_F \pi^2}{N_c} H_1 + \frac{\alpha_s(Q_1^V)}{4\pi} \frac{C_F}{N_c} P_{\pi,2}^p \right], \quad (11)$$

$$\alpha_6^{p,\text{PMC}} = C_6 + \frac{C_5}{N_c} \left[1 + \frac{\alpha_s(Q_1^V)}{4\pi} C_F (-6) + \frac{\alpha_s(Q_1^V)}{4\pi} \frac{C_F}{N_c} P_{\pi,3}^p \right], \quad (12)$$

where

$$Q_1^V = \mu_{r,V}^{\text{init}} \exp \left[-\frac{\tilde{V}_1}{2V_1} \right], \quad Q_1^H = \mu_{r,H}^{\text{init}} \exp \left[-\frac{\tilde{H}_1}{2H_1} \right]$$

denote the separate PMC scales for the vertex contribution and the hard spectator scattering contribution, respectively. For the penguin amplitude, there is no β -terms to determine its PMC scale, we take it as Q_1^V , the same as the scale of the vertex amplitude, since both types of diagrams have similar space-like momentum transfers. There is a residual scale dependence due to unknown higher-order $\{\beta_i\}$ -terms, which however is highly suppressed [8, 9]. Both V_1 and \tilde{V}_1 have an imaginary part. We use the real part to set the PMC scale Q_1^V . The values of the resulting PMC scales are $Q_1^V \simeq 1.59$ GeV and $Q_1^H \simeq 0.75$ GeV; they are nearly independent of the initial scales $\mu_{r,V}^{\text{init}}$ and $\mu_{r,H}^{\text{init}}$. A major problem is that the PMC scale Q_1^H is close to Λ_{QCD} in the $\overline{\text{MS}}$ scheme. To avoid this low-scale problem, we utilize the commensurate scale relation [15, 16] to transform the $\overline{\text{MS}}$ running coupling to an effective charge defined from a measured physical process. In particular the coupling $\alpha_s^{g^1}(Q)$ defined from the Bjorken sum rule is very well measured. The leading order commensurate scale relation gives $\alpha_s^{\overline{\text{MS}}}(0.75\text{GeV}) = \alpha_s^{g^1}(2.04\text{GeV})$. We adopt the light-front holography model proposed in Ref. [18] as an estimate of the running behavior of $\alpha_s^{g^1}(Q)$. This model is based on the light-front holographic mapping of classical gravity in anti-de Sitter space, modified by a positive sign dilaton background and leads to a reasonable nonperturbative effective coupling. The confinement potential and light-front Schrödinger equation derived from this approach accounts well for the spectroscopy and dynamics of light-quark hadrons.

The input parameters are chosen as [1]: the B -meson lifetime $\tau_{B^+} = 1.641\text{ps}$ and $\tau_{B_d} = 1.519\text{ps}$; $f_B = 0.194$ GeV and $f_{\pi} = 0.130$ GeV; for the CKM parameters, we use $\gamma = 68.6^\circ$, $|V_{cb}| = 0.041$, $|V_{cd}^*| = 0.230$ and

$|V_{ub}| = 4.15 \times 10^{-3}$. The b -quark pole mass $m_b = 4.8$ GeV, and the c -quark pole mass $m_c = 1.5$ GeV. The $B \rightarrow \pi$ form factor at zero momentum transfer is taken as $f_+^{B \rightarrow \pi}(0) = 0.25_{-0.03}^{+0.03}$, which is estimated by a NLO light-cone sum rules calculation [19]. The n -th moment of the B meson's light-front distribution amplitude is adopted as $\lambda_B = 0.20_{-0.02}^{+0.04}$, $\lambda_1 = -2.2$ and $\lambda_2 = 11$ [4]. As usual, we set $\mu_f = \mu_{r,H}^{\text{init}}$ or $\mu_f = \mu_{r,V}^{\text{init}}$, and vary $\mu_{r,V}^{\text{init}} \in [1/m_b, 2m_b]$ and $\mu_{r,H}^{\text{init}} \in [1\text{GeV}, 2\text{GeV}]$ for analyzing the scale uncertainty. In general the factorization and the renormalization scales are different, thus one has to determine the full factorization and renormalization scale dependent expressions for all of the amplitudes; these can be derived using Eqs.(9,10,11,12) via a general scale translation [16].

We present our predictions for the CP-averaged $B \rightarrow \pi\pi$ in Tables I and II. The CP conjugate branching ratios are obtained from the CP conjugate amplitudes following the same procedures. An increased branching ratio is observed after PMC scale setting. This indicates that the resummation of the non-conformal series is important.

If one assumes conventional scale setting, there is large renormalization-scale uncertainties, especially for the color-suppressed topologically-dominated progresses. In contrast, the ambiguity of the renormalization scale has been greatly suppressed by using the PMC.

Are shown by Table II, after applying PMC scale setting, the renormalization scale uncertainty has been greatly suppressed as required. Table II shows that all the CP-averaged branching ratios of $B \rightarrow \pi\pi$ are consistent with the data after PMC scale-setting. By adding the mentioned errors in quadrature, we obtain $\mathcal{B}(B_d \rightarrow \pi^0 \pi^0)|_{\text{Conv.}} = (0.39_{-0.09}^{+0.11}) \times 10^{-6}$ and $\mathcal{B}(B_d \rightarrow \pi^0 \pi^0)|_{\text{PMC}} = (0.98_{-0.32}^{+0.28}) \times 10^{-6}$, where ‘Conv.’ means calculated using conventional scale setting. After PMC scale setting, the central value for $\mathcal{B}(B_d \rightarrow \pi^0 \pi^0)$ increases by $\sim 100\%$ even when we choose the conventional result $(0.47_{-0.15}^{+0.08}) \times 10^{-6}$. If we had more accurate non-perturbative parameters such as the $B \rightarrow$

TABLE I: Dependence on the renormalization scale of the CP-averaged branching ratio $\mathcal{B}(B \rightarrow \pi\pi)$ (in unit 10^{-6}) assuming conventional scale setting and PMC scale setting, where three typical (initial) scales are adopted. The first column uncertainty are from $B \rightarrow \pi$ form factor and the second column are from the B -meson moment.

$\mu_{r,V}^{\text{init}}; \mu_{r,H}^{\text{init}}$	Conventional			PMC		
	$m_b/2; 1 \text{ GeV}$	$m_b; 1.5 \text{ GeV}$	$2m_b; 2 \text{ GeV}$	$m_b/2; 1 \text{ GeV}$	$m_b; 1.5 \text{ GeV}$	$2m_b; 2 \text{ GeV}$
$B^- \rightarrow \pi^- \pi^0$	$5.32^{+1.12+0.21}_{-1.00-0.29}$	$5.26^{+1.11+0.19}_{-1.00-0.28}$	$5.25^{+1.12+0.18}_{-1.01-0.27}$	$5.82^{+1.07+0.34}_{-0.99-0.50}$	$5.82^{+1.07+0.34}_{-0.99-0.50}$	$5.82^{+1.07+0.34}_{-0.99-0.50}$
$B_d \rightarrow \pi^+ \pi^-$	$6.10^{+1.72+0.20}_{-1.50-0.13}$	$5.93^{+1.65+0.18}_{-1.46-0.13}$	$5.82^{+1.62+0.17}_{-1.41-0.11}$	$5.60^{+1.82+0.50}_{-1.57-0.33}$	$5.60^{+1.82+0.50}_{-1.57-0.33}$	$5.60^{+1.82+0.50}_{-1.57-0.33}$
$B_d \rightarrow \pi^0 \pi^0$	$0.47^{+0.04+0.07}_{-0.04-0.10}$	$0.39^{+0.04+0.07}_{-0.03-0.08}$	$0.36^{+0.03+0.06}_{-0.03-0.08}$	$0.98^{+0.06+0.18}_{-0.05-0.23}$	$0.98^{+0.06+0.18}_{-0.05-0.23}$	$0.98^{+0.06+0.18}_{-0.05-0.23}$

TABLE II: The CP-averaged $\mathcal{B}(B \rightarrow \pi\pi)$ (in units of 10^{-6}). The predicted errors are squared averages of those from the $B \rightarrow \pi$ form factor, the B -meson moment, and the factorization scale. For the factorization scale error, we take $\mu_{f,H} = 4.8 \pm 0.8 \text{ GeV}$ and $\mu_{f,V} = 1.5 \pm 0.3 \text{ GeV}$. The PDG and Belle data are presented as a comparison.

$\mathcal{B}r (10^{-6})$	Data	Conv.	PMC
$B^- \rightarrow \pi^- \pi^0$	5.5 ± 0.4 [1]	$5.26^{+1.13}_{-1.04}$	$5.82^{+1.18}_{-1.15}$
$B_d \rightarrow \pi^+ \pi^-$	5.12 ± 0.19 [1]	$5.93^{+1.67}_{-1.47}$	$5.60^{+1.97}_{-1.66}$
$B_d \rightarrow \pi^0 \pi^0$	$0.90 \pm 0.12 \pm 0.10$ [20]	$0.39^{+0.11}_{-0.09}$	$0.98^{+0.28}_{-0.32}$

π form factor etc., we could achieve a more precise pQCD prediction. One can define a ratio $\mathcal{R}_\pi(\pi^- \pi^0) = \Gamma(B^- \rightarrow \pi^- \pi^0)/(d\Gamma(B_d \rightarrow \pi^+ \ell^- \bar{\nu}_\ell)/dq^2|_{q^2=0})$ to cut off the uncertainty from the $B \rightarrow \pi$ form factors. In the QCD factorization framework, we have $\mathcal{R}_\pi(\pi^- \pi^0) = 3\pi^2 f_\pi^2 |V_{ud}|^2 |\alpha_1 + \alpha_2|^2$, which leads to $\mathcal{R}_\pi(\pi^- \pi^0)|_{\text{PMC}} = 0.86^{+0.09}_{-0.09}$. It is consistent with the heavy flavor averaging group prediction 0.81 ± 0.14 [2] within errors.

In summary, the PMC provides a systematic and unambiguous way to set the renormalization scale for high-energy QCD processes, thus greatly improving the precision of tests of the Standard Model. We have applied the PMC with the goal of solving the $B_d \rightarrow \pi^0 \pi^0$ puzzle. When one applies PMC scale setting, the non-conformal β_0 -dependent terms are resummed into the running coupling, and we obtain the optimal scales $Q_1^V \simeq 1.59 \text{ GeV}$ and $Q_1^H \simeq 0.75 \text{ GeV}$ for those channels.

The PMC results for $B^- \rightarrow \pi^- \pi^0$ and $B_d \rightarrow \pi^+ \pi^-$ are not very different compared with traditional predictions: for $B^- \rightarrow \pi^- \pi^0$, the difference is about 10%; for $B_d \rightarrow \pi^+ \pi^-$, the difference is less than 10%. However, the $B_d \rightarrow \pi^0 \pi^0$ channel is dominated by the color-suppressed vertex and power-suppressed penguin diagrams. And the difference becomes about 100%. Our prediction agrees with the recent preliminary Belle result $\mathcal{B}(B_d \rightarrow \pi^0 \pi^0) = (0.90 \pm 0.12 \pm 0.10) \times 10^{-6} (6.7\sigma)$ [20]. The PMC prediction will become more precise when the nonconformal terms are determined to higher order in the strong coupling α_s . Thus, the PMC provides a possible solution for the $B_d \rightarrow \pi^0 \pi^0$ puzzle.

As a final remark, we have found that the factorization scale uncertainty brings an additional 5% – 10% error into the pQCD prediction. The factorization scale un-

certainty occurs even for a conformal theory; thus, how to set the factorization scale reliably is another important problem.

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