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# THEORETICAL ANALYSIS OF A LASER UNDULATOR-BASED HIGH GAIN FEL

Panagiotis Baxevanis and Ronald Ruth
SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

*Abstract*

The use of laser (or RF) undulators is nowadays considered attractive for FEL applications, particularly those that aim to utilize relatively low-energy electron beams. In the context of the standard theoretical analysis, the counter-propagating laser pulse is usually treated in the plane-wave approximation, neglecting amplitude and phase variation. In this paper, we develop a three-dimensional, analytical theory of a high-gain FEL based on a laser or RF undulator, taking into account the longitudinal variation of the undulator field amplitude, the laser Gouy phase and the effects of emittance and energy spread in the electron beam. Working in the framework of the Vlasov-Maxwell formalism, we derive a self-consistent equation for the radiation amplitude in the linear regime, which is then solved to good approximation by means of an orthogonal expansion technique. Numerical results obtained from our analysis are used in the study of an example of a compact, laser undulator-based, X-ray FEL.

## INTRODUCTION

In recent years, the free-electron laser (FEL) has emerged as one of the leading methods for producing bright, coherent radiation up to the X-ray region. The push for higher photon energy is facilitated by the use of higher electron energy and/or shorter undulator period. For magnetic undulators, implementing the latter option is generally limited by technical considerations to periods of a few mm at best. On the other hand, the wide availability of lasers with very high peak power (in the TW level) often raises the possibility of generating X-rays through the head-on collision of an electron beam with a counter-propagating laser pulse, where the field of the pulse acts as an undulator with period equal to half the laser wavelength. Though this process is usually studied in the framework of incoherent, inverse Compton scattering sources, one can also consider the production of coherent radiation from an FEL that uses such a laser undulator, provided a high-brightness electron beam is available [1]. Here, we present a theoretical description of such a device, paying particular attention to the effects introduced by variations in the amplitude and phase of the effective undulator field. A specific numerical example for a soft X-ray, laser undulator-based FEL is included in order to illustrate the main points of our analysis and also to highlight some the main challenges involved in the realization of this concept.

## THEORY

### *FEL Configuration and Single Particle Motion*

To begin with, let us assume that the laser radiation is monochromatic, linearly polarized along the $x$ direction and has a Gaussian transverse profile. The electric field of the laser pulse - which is propagating along the negative $z$ direction - can then be written as $\mathbf{E}_L = E_L(r, z)\hat{x}$, where

$$E_L = -E_0 \frac{w_0}{w} \exp\left(-\frac{r^2}{w^2}\right) \sin[k_L(z+ct) + \frac{k_L r^2}{2R} - u] \,. \tag{1}$$

Here, $r^2 = x^2 + y^2$, $k_L = 2\pi/\lambda_L$ ($\lambda_L$ is the laser wavelength), $w = w(z) = w_0(1 + \bar{z}^2/z_R^2)^{1/2}$ is the laser spot size, $R = R(z) = \bar{z} + z_R^2/\bar{z}$ is the radius of curvature and $u = u(z) = \tan^{-1}(\bar{z}/z_R)$ is the Gouy phase. In these relations, $\bar{z} = z - z_w$, where $z = z_w$ is the position of the laser waist, $z_R$ is the Rayleigh length, $E_0$ is the field amplitude at the waist and $w_0 = (2z_R/k_L)^{1/2}$ is the minimum spot size. Moreover, the counter-propagating electron beam is also assumed to be round and Gaussian, with transverse size $\sigma_e(z) = (\sigma^2 + \sigma'^2\bar{z}^2)^{1/2}$, where $\sigma$ and $\sigma'$ are the rms beam size and divergence at the location of the electron beam waist, which we take to be the same as that of the laser. Both the laser pulse and the electron beam are characterized by a uniform longitudinal profile, with temporal durations $t_L$ and $t_e$ respectively ($t_L \gg t_e$). For simplicity, we can also assume that their front ends collide at $z = 0$, when $t = 0$. Each electron in the beam interacts with the laser field for a time $t \approx L_I/c = t_L/2$, where $L_I = ct_L/2$ is the corresponding interaction length. For the configuration under consideration, we assume that the interaction region is centered around the common waist, so that $L_I = 2z_w < z_R$. The laser power $P_L$ is given by the relation $P_L = \pi c\varepsilon_0 E_0^2 w_0^2/4$ - where $\varepsilon_0$ is the vacuum permittivity - while the total energy in the laser pulse is $U_L = P_L t_L$.

Next, we consider the motion of electrons in the combined field of the laser and the emitted FEL radiation. As is the case with standard FEL schemes, the transverse motion is predominantly determined by the undulator field. The vertical magnetic field of the laser is $B_L \approx -E_L/c$ so the total force in the $x$ direction is $F_x \approx -e(E_L - v_z B_L) \approx -2eE_L$ (the electric and magnetic force contributions are equal and add up). This force gives rise to an oscillatory motion similar to that in a conventional magnetic wiggler. To establish this connection in a way that takes into account the dominant effects of the undulator field inhomogeneities, we consider the equation of motion in the hori-

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zontal direction, i.e. $x'' = d^2x/dz^2 \approx F_x/\gamma m_0 c^2$ ($\gamma m_0 c^2$ is the electron energy) or

$$x'' \approx \frac{2eE_0}{\gamma m_0 c^2}\frac{w_0}{w}\exp\left(-\frac{r^2}{w^2}\right)\sin[k_L(z+ct)+\frac{k_L r^2}{2R}-u]\,. \tag{2}$$

To proceed, we first note that both the transverse dependence of the laser amplitude and the quadratic phase $\propto 1/R$ can be neglected as long as $\sigma_e \ll w$, which we assume to be true. The next step is to eliminate the time variable $t$ by introducing the ponderomotive electron phase $\psi = k_L(z + ct) + k_r(z - ct)$, where $k_r = 2\pi/\lambda_r$ and $\lambda_r$ is the resonant wavelength of the FEL radiation. In view of this definition, we have

$$k_L(z+ct) = \frac{2k_L z}{1-k_L/k_r} - \frac{k_L}{k_r - k_L}\psi \approx k_u z\,, \tag{3}$$

where $k_u = 2k_L/(1 - k_L/k_r) \approx 2k_L$ and we have anticipated the fact that $k_r \gg k_L$. The horizontal electron motion can be decomposed into a slowly varying part $x_\beta$ and a fast, oscillatory part $x_w$, i.e. $x = x_w + x_\beta$. Taking into account that $w$ and $u$ both change very little over the scale of an undulator period $\lambda_u = 2\pi/k_u \approx \lambda_L/2$ (since $z_R \gg \lambda_u$), Eq. (2) can be integrated to give the wiggle velocity

$$x'_w \approx -\frac{K(z)}{\gamma}\cos[k_u z - u(z)]\,, \tag{4}$$

where $K(z) = K_0(w_0/w) = K_0(1 + \bar{z}^2/z_R^2)^{-1/2}$ is a $z$-dependent effective undulator parameter and $K_0 = 2eE_0/(m_0c^2k_u) \approx eE_0/(m_0c^2k_L)$ is its value at the waist. For $(L_I/z_R)^2 \ll 1$, we only need to consider the effect of the $z$-dependence of $K$ on the phase equation, as in the case of a tapered magnetic undulator. From the definition of the phase, we obtain $d\psi/dz = k_L + k_r + (k_L - k_r)\beta_z^{-1}$. Here, the scaled longitudinal velocity $\beta_z$ can be approximated by $\beta_z^{-1} \approx 1 + \beta_\perp^2/2 + 1/(2\gamma^2)$, where the transverse normalized velocity is in turn given by $\beta_\perp^2 \approx x'^2 + y'^2 = (x'_w + x'_\beta)^2 + y'^2$. Combining the above yields

$$\begin{aligned}\frac{d\psi}{dz} &= 2k_u\delta - \frac{k_r}{2}({x'_\beta}^2 + {y'}^2 + 2x'_w x'_\beta)\\ &- \frac{k_r K_0^2}{4\gamma_0^2}\cos(2k_u z - 2u) - \frac{k_r K_0^2}{4\gamma_0^2}\left[\left(\frac{w_0}{w}\right)^2 - 1\right]\,,\end{aligned} \tag{5}$$

where $\delta = \gamma/\gamma_0 - 1$ is the deviation from the average electron energy $\gamma_0 m_0 c^2$ and we have used the resonance condition

$$k_u = \frac{k_r}{2\gamma_0^2}(1 + \frac{K_0^2}{2})\,. \tag{6}$$

Switching to the slowly varying phase variable $\theta = \psi + Q_0\sin(2k_u z - 2u)$ - where $Q_0 = k_r K_0^2/(8k_u\gamma_0^2) = K_0^2/(4 + 2K_0^2)$ - we obtain the equation

$$\frac{d\theta}{dz} = \theta' = 2k_u\delta - \frac{k_r}{2}\mathbf{p}^2 + F(z)\,, \tag{7}$$

where $\mathbf{p} = (x'_\beta, y')$ and

$$\begin{aligned}F(z) &= -\,2k_u Q_0\left[\left(\frac{w_0}{w}\right)^2 - 1\right]\\ &= -\,2k_u Q_0\left[\left(1 + \frac{\bar{z}^2}{z_R^2}\right)^{-1} - 1\right]\,.\end{aligned} \tag{8}$$

Thus, we conclude that the diffraction of the laser leads to an inhomogeneous driving term on the RHS of the phase equation (Eq. (7)). Turning to the energy exchange equation, we start from $m_0c^2 d\gamma/dt = -ev_xE_r \approx -ecx'_wE_r$. Moreover, we express the electric field $E_r$ of the linearly polarized FEL radiation as

$$E_r = \frac{1}{2}\int_0^\infty d\nu E_\nu(\mathbf{x}, z)e^{i\nu k_r(z-ct)} + c.c. \tag{9}$$

where $\nu = \omega/\omega_r$, $E_\nu$ is the radiation amplitude, $\mathbf{x} = (x, y)$ and $\omega_r = 2\gamma_0^2 c k_u/(1 + K_0^2/2)$ is the resonant frequency (while *c.c.* stands for complex conjugate). Using the above equation and the definition of $\theta$, we obtain (upon averaging over the fast wiggle motion)

$$\frac{d\delta}{dz} = \kappa_1\int_0^\infty d\nu E_\nu(\mathbf{x}, z)e^{-i[u(z)+\Delta\nu k_u z]}e^{i\nu\theta} + c.c.\,, \tag{10}$$

where $\Delta\nu = \nu - 1$ is the detuning and $\kappa_1 = eK_0[JJ]/(4\gamma_0^2 m_0c^2)$, with $[JJ] = J_0(Q_0) - J_1(Q_0)$. We note that the laser electric field has not been included in the above derivation as it is 90 degrees out of phase with the wiggle velocity and does not contribute to the averaging. Eqs. (7) and (10) are the pendulum equation for the laser undulator. As was implicitly assumed when we considered an electron beam with a single waist, the wiggle-averaged transverse motion is approximated by a drift. This assumption is valid as long as the interaction length $L_I$ is smaller than the exponentiation length $\beta_p = \gamma_0 w_0/K_0$ for the ponderomotive laser defocusing effect.

## *Radiation Field Equation*

In view of the single particle equations of motion and following the standard perturbation approach [2, 3], we find that, up to the onset of saturation effects, the operation of the laser undulator FEL can be described through the following set of linearized, frequency-domain, Vlasov-Maxwell equations:

$$\frac{\partial f_\nu}{\partial z} + \mathbf{p}\frac{\partial f_\nu}{\partial \mathbf{x}} + i\nu\theta' f_\nu = -\kappa_1\frac{\partial f_0}{\partial\delta}E_\nu e^{-i[u(z)+\Delta\nu k_u z]} \tag{11}$$

and

$$\frac{\partial E_\nu}{\partial z} + \frac{\nabla_\perp^2 E_\nu}{2ik_r} = -\kappa_2 e^{i[u(z)+\Delta\nu k_u z]}\int d^2\mathbf{p}\int d\delta f_\nu\,, \tag{12}$$

where $\kappa_2 = eK_0[JJ]/(2\varepsilon_0\gamma_0)$ and $f_\nu$ is the Fourier amplitude of the perturbation $f_1$ to the beam distribution function. The background distribution $f_0$ is given by

$$f_0 = \frac{I/(ec)}{(2\pi)^{5/2}\sigma^2\sigma'^2\sigma_\delta} \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right) \times \exp\left[-\frac{(\mathbf{x}-\mathbf{p}\bar{z})^2}{2\sigma^2} - \frac{\mathbf{p}^2}{2\sigma'^2}\right], \tag{13}$$

where $I$ is the peak current and $\sigma_\delta$ is the rms energy spread of the electron beam. For an initially unmodulated beam, Eqs. (11) and (12) can combined into a single equation for the radiation amplitude $E_\nu$ :

$$\frac{\partial E_\nu}{\partial z} + \frac{\nabla_\perp^2 E_\nu}{2ik_r} = \int_0^z d\zeta \int d^2\bar{\mathbf{x}} \Lambda(\mathbf{x}, \bar{\mathbf{x}}, z, \zeta) E_\nu(\bar{\mathbf{x}}, \zeta), \tag{14}$$

where

$$\Lambda(\mathbf{x}, \bar{\mathbf{x}}, z, \zeta) = -\frac{4i\rho^3 k_u^3}{\pi\sigma'^2\xi} e^{i[G(\zeta)-G(z)-u(\zeta)+u(z)-\Delta\nu k_u \xi]} \times e^{-2\sigma_\delta^2 k_u^2 \xi^2} \exp\left(-\frac{(\bar{\zeta}\mathbf{x}-\bar{z}\bar{\mathbf{x}})^2}{2\sigma^2\xi^2} - \frac{1+ik_r\sigma'^2\xi}{2\sigma'^2\xi^2}(\bar{\mathbf{x}}-\mathbf{x})^2\right). \tag{15}$$

Here, $\xi = \zeta - z$, $\bar{\zeta} = \zeta - z_w = \xi + \bar{z}$, $\rho$ is the FEL parameter, given by

$$\rho = \left(\frac{I}{16 I_A} \frac{K_0^2 [JJ]^2}{\gamma_0^3 \sigma^2 k_u^2}\right)^{1/3} \tag{16}$$

($I_A \approx 17$ kA is the Alfven current) and we have defined

$$G(z) = -2Q_0 z_R k_u \left[\tan^{-1}\left(\frac{\bar{z}}{z_R}\right) - \frac{\bar{z}}{z_R}\right] \tag{17}$$

so that $dG(z)/dz = F(z)$. The derivation of Eqs. (14) and (15) is entirely analogous to the one given in [3], the only new terms being those which contain $G$ and $u$. Though the above equations are our basic result, we can obtain additional insight into the physics of the system under study by considering the transformation $a_\nu(\mathbf{x}, z) = E_\nu(\mathbf{x}, z) e^{i[G(z)-u(z)-\Delta\nu k_u z]}$. The field equation then becomes

$$\left(\frac{\partial}{\partial z} + i\Delta\nu_{ef} k_u + \frac{\nabla_\perp^2}{2ik_r}\right) a_\nu(\mathbf{x}, z) = \int_0^z d\zeta \int d^2\bar{\mathbf{x}} \Omega(\mathbf{x}, \bar{\mathbf{x}}, z, \zeta) a_\nu(\bar{\mathbf{x}}, \zeta), \tag{18}$$

where

$$\Omega(\mathbf{x}, \bar{\mathbf{x}}, z, \zeta) = -\frac{4i\rho^3 k_u^3}{\pi\sigma'^2\xi} e^{-2\sigma_\delta^2 k_u^2 \xi^2} \times \exp\left(-\frac{(\bar{\zeta}\mathbf{x}-\bar{z}\bar{\mathbf{x}})^2}{2\sigma^2\xi^2} - \frac{1+ik_r\sigma'^2\xi}{2\sigma'^2\xi^2}(\bar{\mathbf{x}}-\mathbf{x})^2\right) \tag{19}$$

is the new integral kernel and and, more importantly,

$$\Delta\nu_{ef} = \Delta\nu + \frac{1}{k_u}\left[\frac{du}{dz} - F(z)\right] \tag{20}$$

$$= \Delta\nu + \frac{1}{k_u z_R}\left(1 + \frac{\bar{z}^2}{z_R^2}\right)^{-1} + 2Q_0\left[\left(1 + \frac{\bar{z}^2}{z_R^2}\right)^{-1} - 1\right]$$

is an effective detuning containing the contributions of the Gouy phase and the tapering-like effect due to diffraction. Since the FEL bandwidth is of the order of $\rho$ and we usually have $\rho k_u z_R \gg 1$, the former is typically very small and can be ignored.

In order to solve the the FEL equation (Eq. (14)) for a specified initial amplitude $E_\nu(\mathbf{x}, 0)$, we can expand $E_\nu$ in terms of a complete set of orthogonal transverse modes. For full details on the expansion technique, we refer to [3]. Here, we merely present the main points. In particular, we have

$$E_\nu(\mathbf{x}, z) = \bar{\varepsilon}_\nu \sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} C_{pq}(z) \psi_{pq}(\mathbf{x}, z), \tag{21}$$

where the coefficients $C_{pq}$ are chosen to be dimensionless and $\bar{\varepsilon}_\nu$ is a constant that can be related to the initial amplitude. The transverse basis we employ consists of Gauss-Laguerre modes

$$\psi_{pq}(\mathbf{x}, z) = \left(\frac{p!}{(p+|q|)!}\right)^{1/2} \left(\frac{\sqrt{2}r}{w_r}\right)^{|q|} \times L_p^{|q|}\left(\frac{2r^2}{w_r^2}\right) \psi_{00}(\mathbf{x}, z) e^{iq\phi} e^{-i(2p+|q|)u_r}, \tag{22}$$

where $(r, \phi)$ are polar coordinates in the transverse plane, $p$ and $q$ are integers with $p \geq 0$ and $L_p^{|q|}$ are the associated Laguerre polynomials. Here,

$$\psi_{00}(\mathbf{x}, z) = \left(\frac{k_r \beta_1}{\pi}\right)^{1/2} \frac{1}{z - i\beta} \exp\left(\frac{ik_r r^2}{2(z - i\beta)}\right) \tag{23}$$

is the fundamental basis mode, defined through a complex-valued function $\beta = \beta_1 + i\beta_2 = \beta(z)$, while

$$w_r = \left(\frac{2}{k_r \beta_1}\right)^{1/2} |z - i\beta| \tag{24}$$

and

$$u_r = \arctan\left(\frac{z + \beta_2}{\beta_1}\right) \tag{25}$$

are, respectively, the spot size and the Gouy phase associated with it. The basis functions described above satisfy the orthonormality condition $\int d^2\mathbf{x} \psi_{nm}^* \psi_{pq} = \delta_{pn}\delta_{qm}$. Projecting Eq. (14) onto our Gauss-Laguerre basis, we ultimately obtain a set of coupled integro-differential equations for the expansion coefficients:

$$\frac{dC_{nm}}{dz} = (2n + |m| + 1)\frac{iC_{nm}}{2\beta_1}\frac{d\beta_2}{dz} + \sqrt{n(n+|m|)}\frac{C_{n-1,m}}{2\beta_1}\frac{d\beta}{dz} - \sqrt{(n+1)(n+|m|+1)}\frac{C_{n+1,m}}{2\beta_1}\frac{d\beta^*}{dz} + \int_0^z d\zeta \sum_{p=0}^{\infty} C_{pm}(\zeta) \Lambda_{pm}^{nm}(z, \zeta, \beta, \beta_\zeta), \tag{26}$$

where

$$\Lambda_{pm}^{nm}(z,\zeta,\beta,\beta_\zeta)$$
$$= \int d^2\mathbf{x}\psi_{nm}^*(\mathbf{x},z)\int d^2\bar{\mathbf{x}}\,\psi_{pm}(\bar{\mathbf{x}},\zeta)\Lambda(\mathbf{x},\bar{\mathbf{x}},z,\zeta)$$
$$= \frac{8i\rho^3k_u^3}{D}\frac{(-1)^{p+n+1}(p+n+|m|)!}{(n!p!)^{1/2}[(p+|m|)!(n+|m|)!]^{1/2}}$$
$$\times\left(\frac{\beta_{1\zeta}}{\beta_1}\right)^{\frac{|m|+1}{2}}\frac{(z-i\beta)^{n+|m|}}{(\zeta-i\beta_\zeta)^{p+|m|}}\frac{(\zeta+i\beta_\zeta^*)^p}{(z+i\beta^*)^n}$$
$$\times\,\xi e^{-2\sigma_\delta^2k_u^2\xi^2}e^{i[G(\zeta)-G(z)-u(\zeta)+u(z)-\Delta\nu k_u\xi]}$$
$$\times\frac{(X-Y)^p(X-1)^n}{X^{p+n+|m|}}\frac{d^pb^{|m|}}{a^{p+|m|}}$$
$$\times\,{}_2F_1\left(-p,-n;-p-n-|m|;J\right)\,. \tag{27}$$

In the equation given above, $\beta_\zeta$ is shorthand for $\beta(\zeta)$, ${}_2F_1$ is a Gaussian hypergeometric function and

$$a = 1+\frac{\sigma'^2\bar{z}^2}{\sigma^2}+ik_r\sigma'^2\xi\frac{z-i\beta_\zeta}{\zeta-i\beta_\zeta}\,, \tag{28}$$
$$d = a-\frac{2k_r\sigma'^2\beta_{1\zeta}\xi^2}{|\zeta-i\beta_\zeta|^2}\,, \tag{29}$$
$$b = 1+\frac{\sigma'^2\bar{z}\bar{\zeta}}{\sigma^2}+ik_r\sigma'^2\xi\,, \tag{30}$$
$$Y = \frac{\beta_{1\zeta}}{\beta_1}\frac{|z-i\beta|^2}{|\zeta-i\beta_\zeta|^2}\frac{b^2}{ad}\,, \tag{31}$$

with

$$D_1 = \left(1+\frac{\sigma'^2\bar{z}^2}{\sigma^2}+ik_r\sigma'^2\xi\right)(\zeta-i\beta_\zeta)-ik_r\sigma'^2\xi^2\,, \tag{32}$$
$$D_2 = k_r\sigma'^2\xi-i\left(1+\frac{\sigma'^2\bar{\zeta}^2}{\sigma^2}\right)+(\frac{1}{k_r}+i\sigma'^2\xi)\frac{\zeta-i\beta_\zeta}{\sigma^2}\,, \tag{33}$$
$$D_3 = \frac{D_1}{z-i\beta}\ ,\ \ D = \frac{iD_1+(z+i\beta^*)D_2}{2\beta_1}\,, \tag{34}$$
$$X = \frac{D}{D_3}\ ,\ \ J = 1-\frac{Y}{(X-Y)(X-1)}\,. \tag{35}$$

Since basis modes with different angular indices are uncoupled, we usually concentrate on a particular value of $m$. Generally speaking, after specifying the basis function $\beta(z)$, a truncated version of the set of Eq. (26) is solved numerically. This yields information on the evolution of the FEL radiation through the linear, high-gain regime. Specifically, the radiation power and transverse size are, respectively, given by

$$P(z) = P_0\sum_{n=0}^{\infty}|C_{nm}(z)|^2 \tag{36}$$

and

$$\sigma_r^2(z) = (w_r^2/4)\times \tag{37}$$
$$\frac{1}{\sum_{n=0}^{\infty}|C_{nm}(z)|^2}\left\{\sum_{n=0}^{\infty}(2n+|m|+1)|C_{nm}(z)|^2-2\,\mathrm{Re}[e^{2iu_r}\sum_{n=1}^{\infty}\sqrt{n(n+|m|)}C_{n-1,m}(z)C_{nm}^*(z)]\right\}\,,$$

where $P_0$ is the input power. In this paper, we will only consider seeding with a Gaussian mode so we may take $m=0$. In this case, it can be shown that selecting the basis function according to the equation

$$\frac{d\beta}{dz} = -\frac{2\beta_1}{C_{00}}\int_0^z d\zeta C_{00}(\zeta)\Lambda_{00}^{10}(z,\zeta,\beta,\beta_\zeta) \tag{38}$$

leads to reliable results with a fewer number of modes.

Table 1: Laser and Electron Beam Parameters

| Parameter | |
|---|---|
| Laser wavelength $\lambda_L$ | 10 $\mu$m |
| Laser power $P_L$ | 1 TW |
| Pulse energy $U_L$ | 70 J |
| Minimum spot size $w_0$ | 330 $\mu$m |
| Rayleigh length $z_R$ | 3 cm |
| beam energy $\gamma_0 mc^2$ | 12.9 MeV |
| Undulator parameter $K_0$ | 0.2 |
| Resonant wavelength $\lambda_r$ | 4 nm |
| Peak current $I$ | 1 kA |
| Energy spread $\sigma_\delta$ | $10^{-4}$ |
| Normalized emittance $\gamma_0\epsilon=\gamma_0\sigma\sigma'$ | 0.1 $\mu$m |
| Minimum beta (e-beam) $\beta_0=\sigma/\sigma'$ | 5 mm |
| Interaction length $L_I$ | 1 cm |
| FEL parameter $\rho$ | $6.7\times10^{-4}$ |
| Saturation power $P_S$ | 5 MW |

## NUMERICAL RESULTS

To illustrate our theoretical analysis, we have considered the parameters shown in Table 1. They refer to a laser undulator-based, soft X-ray FEL using a 10 $\mu$m laser. We emphasize that this parameter set is not the result of a full-blown, rigorous optimization. Rather, it was obtained in the following simplified way: given the desired wavelength for the FEL radiation, the laser wavelength and power and the e-beam brightness parameters (normalized emittance, energy spread and peak current), we determined the energy and the rms size of the electron beam that would a) minimize the the length of the interaction region while keeping it smaller than the Rayleigh length b) ensure that the detuning due to the diffraction of the laser is smaller than the FEL bandwidth. In view of Eq. (20), the latter is quantified by stipulating that $2Q_0(\bar{z}^2/z_R^2)\leq(Q_0/2)(L_I^2/z_R^2)<\rho$. Thus, we expect to limit the effect of laser diffraction on

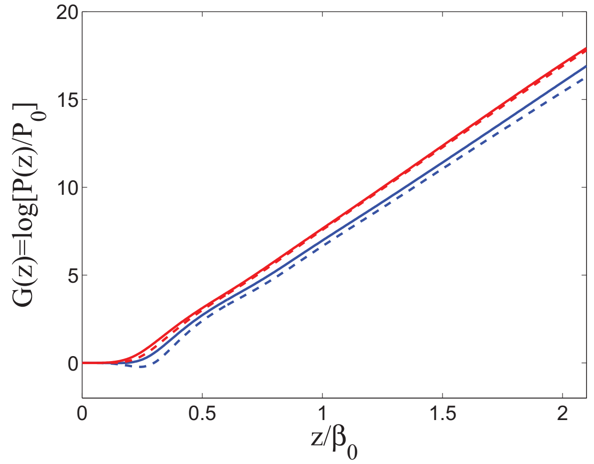


Figure 1: FEL gain $G(z) = \log[P(z)/P_0]$ for $\Delta\nu/(2\rho) = 0.0/-0.5$ (blue/red curves - the same legend holds for the next two figures). Here, the solid lines refer to calculations that include the variation of the undulator parameter with $z$ in contrast to the dashed lines, for which a uniform undulator has been assumed.

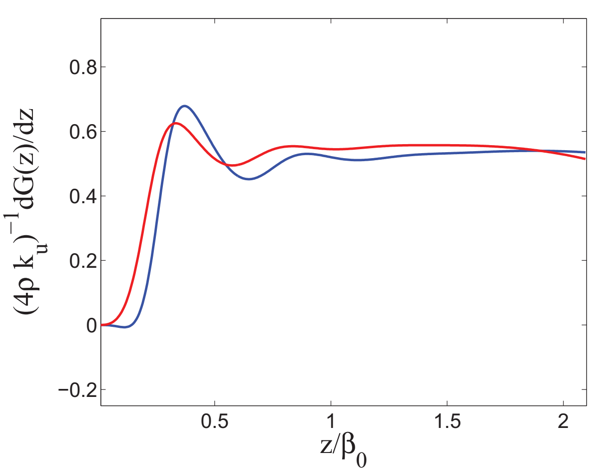


Figure 2: Power growth rate $(1/P)dP/dz$ (in units of $4\rho k_u$) as a function of the undulator distance $z$ (in units of $\beta_0$).

the FEL gain and also to minimize the required energy of the pulse. For our parameter study, we approximate the interaction length by a fixed number (usually 20) of power gain lengths, calculated either through the 1D expression $L_{1D} = \lambda_u/(4\pi\sqrt{3}\rho)$ or by making use of Ming Xie's fitting formulas [4], on which the saturation power estimate is also based.

Having established the basic operating parameters, we perform a more rigorous calculation taking into account the amplitude and phase inhomogeneities of the undulator field, as well as the variation of the transverse electron beam size with $z$. Our linear analysis is based on the expansion technique discussed earlier and its results are

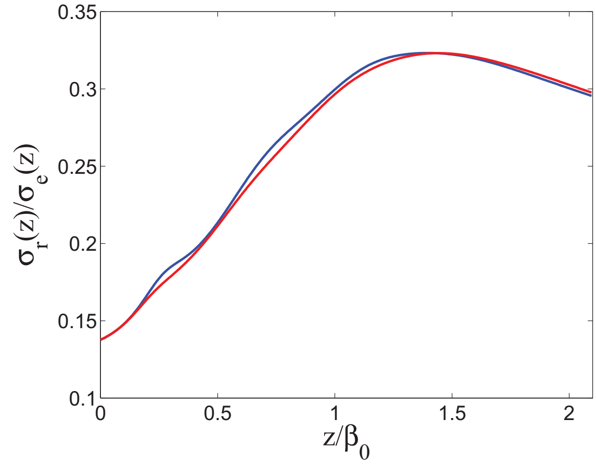


Figure 3: Evolution of the radiation to electron beam size ratio. This ratio is maximum close to the waist.

shown in Figs. 1-3. Plotted are results derived using a single expansion mode, which have been shown to be in good agreement with multi-mode calculations. All calculations use a Gaussian mode as an external seed field. We observe significant gain (15-17 power exponentiations) over the interaction length of 1 cm while we also verify that the diffraction-induced tapering effect only has a small impact on the gain. Finally, we note that this analysis does not include longitudinal space charge effects, which can become important for FELs with small $K$ parameters. In fact, for our case, the typical space charge parameter $\Omega_p/(2\rho k_u)$ is about 30%, which means that we are operating in a regime where space charge can be of concern.

## CONCLUSIONS

We have studied the operation of a laser undulator-based, high gain FEL in the linear regime using a formalism that takes into account three-dimensional effects in the e-beam as well as amplitude and phase variations in the undulator field due to the diffraction of the laser. Using our theoretical analysis, we have explored some parameters for a soft X-ray FEL using a 10 $\mu$m laser. In general, it would appear that such devices may be feasible, though they would require a very high-brightness e-beam and relatively long laser pulses, with a rather large pulse energy.

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