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Comment on "A limit on the variation of the speed of light arising from quantum gravity effects" aka "Testing Einstein's special relativity with Fermi's short hard gamma-ray burst GRB090510"

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Recently the Fermi GBM and LAT Collaborations reported their new observational data disfavoring quite a number of the quantum gravity theories, including the one suggesting the nonlinear (logarithmic) modification of a quantum wave equation. We show that the latter is still far from being ruled out: it is not only able to explain the new data but also its phenomenological implications turn out to be more vast (and more interesting) than one expected before.

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One of the outcomes of theory [1] is that under some conditions the LIV corrections to propagation speed of a particle are given by

$$v/c = 1 \pm \frac{E}{E_{\text{QG}}} + \mathcal{O}(E^2/E_{\text{QG}}^2), \qquad (1)$$

which, after integrating $\partial E/\partial p = v(E)$, leads to the vacuum dispersion relation of the form

$$\left| \frac{p^2 c^2}{E^2} - 1 \right| = \frac{E}{E_{\text{QG}}} + \mathcal{O}(E^2/E_{\text{QG}}^2), \qquad (2)$$

and the agreement with the existing observational data (at that time) was established.

In more recent Ref. [2] it was suggested to rule out such dispersions for $E_{\text{QG}} = M_{\text{Planck}} c^2$ on experimental grounds - essentially, because the predicted absolute value of the series coefficient for a linear term, unity, seems to be significantly larger than the one suggested by observations.

After that, more thorough analysis of our theory's predictions has been done. It turns out that for the extremely ultrarelativistic particles, such as the high-energy photons from the not-very-distant GRB's, the above-mentioned dispersion relations are unlikely applicable because for that physical situation our theory suggests different ones. Let us see it. To begin, the *primary* outcome of our theory is not the dispersion relations themselves but the expression for the invariant, $d\tau^2/(E - E_0)^2$, where τ is the proper time, E_0 is the energy of the vacuum of a theory, in our case $E_0 = \pm E_{\text{QG}}$. From this one concludes that for any two particles

$$\frac{d\tau_2}{d\tau_1} = \frac{E_2 - E_0}{E_1 - E_0}, \qquad (3)$$

where τ_i and E_i are the proper time and energy for the ith particle. When neglecting the cosmological effects it simplifies to

$$\frac{v_1}{v_2} \sqrt{\frac{c^2 - v_2^2}{c^2 - v_1^2}} = \frac{E_2 - E_0}{E_1 - E_0}, \qquad (4)$$

where $v_i = dx_i/dt_i$, by t we denote the time coordinate measured by a distant observer.

This equation reveals the following subtlety: if the particles are essentially relativistic or even ultrarelativistic and their velocities are nearly the same, then the value of a square root in the equation above crucially depends on whether the ratio v_1/c tends to unity "stronger" than v_1/v_2. Thus, there exist two limit regimes: *linear* or standard relativistic - when the ratio v_1/v_2 approaches one "stronger" than v_1/c does, and *non-perturbative* or extreme ultrarelativistic - when it is other way around. In the former case one can approximate the square root in Eq. (4) by unity, then eventually one arrives at the dispersions (1) and (2). In the latter regime such approximation is not valid, instead, one should perform the non-perturbative calculation to eventually obtain:

$$\frac{v^{(s)}}{c^{(s)}} = 1 + \frac{\chi^2 - 1}{\chi^2} \frac{E}{E_0} + \mathcal{O}(E^2/E_{\text{QG}}^2), \qquad (5)$$

where $c^{(s)} \equiv c/\chi$ is the "renormalized" speed of light, χ is the emerging parameter representing the ratio of the "bare" speed of light and the "renormalized" one. Without specifying a concrete model of quantum gravity, our theory can not provide the exact value of χ, it gives only

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the approximate range for subluminal particles: $\chi^2 \gtrapprox 1$. This dispersion mode is not ruled out by the Fermi's data - those can only put further bounds for the parameter. More details as well as some new predictions can be found in the revised version of Ref. [1] (v3+).

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[1] K. G. Zloshchastiev, "Logarithmic nonlinearity in generally covariant quantum theories: Origin of time and observational consequences," arXiv:0906.4282 [hep-th].

[2] A. A. Abdo *et al.* [Fermi LAT/GBM Collaborations], "A limit on the variation of the speed of light arising from quantum gravity effects," Nature **462** (2009) 331-334; A. A. Abdo *et al.*, "Testing Einstein's special relativity with Fermi's short hard gamma-ray burst GRB090510," arXiv:0908.1832 [astro-ph.HE].