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Fermionic Corrections to the Heavy-Quark Pair Production in the Quark-Antiquark Channel

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We describe the analytic calculation of the fermionic two-loop QCD corrections to the heavy-quark pair production process in the quark-antiquark channel.

1. INTRODUCTION

The top quark is the heaviest fermion of the Standard Model. Since its discovery at the Fermilab Tevatron [1], its mass has been measured to within a few percent, while its production cross section and couplings are currently known with larger uncertainty. With the large number of top quarks expected to be produced at the LHC, the study of its properties will become precision physics. To interpret these upcoming precision data, equally precise theoretical predictions are mandatory. These demand foremost the calculation of higher order corrections in perturbative QCD.

At present, the top quark pair production cross section is known to next-to-leading order (NLO) in the QCD coupling constant [2]. For this process, the resummation of next-to-leading logarithmically enhanced corrections (NLL) improves upon the fixed-order NLO prediction [3]. Electroweak one-loop corrections to $t\bar{t}$ production are equally available [4]. For the top quark pair production cross section, which is expected to be measured to within a few percent accuracy, the currently available theoretical prediction is not sufficiently precise. Recent studies [5] indicate a scale uncertainty on these predictions of 7%, and a parton distribution uncertainty of 6%. While the latter may be improved upon by more precise determinations of the parton distribution functions at HERA and LHC, the former requires the calculation of perturbative corrections at next-to-next-to-leading order (NNLO) in QCD.

The calculation of the full NNLO corrections to the top quark pair production cross section requires three types of ingredients: two-loop matrix elements for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$, one-loop matrix elements for hadronic production of $t\bar{t}$+(1 parton) and tree-level matrix elements for hadronic production of $t\bar{t}$+(2 partons). The latter two ingredients were computed previously in the context of the NLO corrections to $t\bar{t}$+jet production [6]. They contribute to the $t\bar{t}$ production cross section through configurations where up to two final state partons can be unresolved (collinear or soft), and their implementation thus may require further developments of subtraction techniques at NNLO.

Both two-loop matrix elements were computed analytically in the small-mass expansion limit $s, |t|, |u| \gg m^2$ in [7], starting from the previously known massless two-loop matrix elements for $q\bar{q} \to q'\bar{q}'$ [8] and $gg \to q\bar{q}$ [9]. An exact numerical representation of the two-loop matrix element $q\bar{q} \to t\bar{t}$ has been obtained very recently [10]. In [11] we computed all two-loop contributions to $q\bar{q} \to t\bar{t}$ arising from closed fermion loops in a compact analytic form, providing also a first independent validation of the results of [7, 10]. Our results allow for a fast numerical evaluation and permit the analytical study of the cross section near threshold. In the rest of this proceeding we briefly discuss the structure of the two-loop fermionic corrections and the calculational techniques employed to evaluate them.

2. STRUCTURE

The scattering process we consider is $q(p_1)+\overline{q}(p_2) \to t(p_3)+\bar{t}(p_4)$ in Euclidean kinematics, where $p_i^2 = 0$ for $i = 1, 2$ and $p_j^2 = -m^2$ for $i = 3, 4$. The Mandelstam variables are defined as follows: $s = -(p_1+p_2)^2$, $t = -(p_1-p_3)^2$, $u = -(p_1-p_4)^2$. Conservation of momentum implies that $s+t+u = 2m^2$. The squared matrix element (averaged over the spin and color of the incoming quarks and summed over the spin of the outgoing ones), calculated in

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$d = 4 - 2\varepsilon$ dimensions, can be expanded in powers of the strong coupling constant α_S as follows:

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_S^2}{N_c^2}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right] . \tag{1}$$

The tree-level amplitude involves a single diagram and its contribution to Eq. (1) is given by

$$\mathcal{A}_0 = 4N_c C_F\left[\frac{(t-m^2)^2 + (u-m^2)^2}{s^2} + \frac{2m^2}{s} - \varepsilon\right] , \tag{2}$$

where N_c is the number of colors and $C_F = (N_c^2-1)/2N_c$.

The NLO term $\mathcal{A}_1$ in Eq. (1) arises from the interference of one-loop diagrams with the tree-level amplitude [2]. The NNLO term $\mathcal{A}_2$ consists of two parts, the interference of two-loop diagrams with the Born amplitude and the interference of one-loop diagrams among themselves: $\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$. The latter term $\mathcal{A}_2^{(1\times 1)}$ was studied extensively in [12]. $\mathcal{A}_2^{(2\times 0)}$ can be decomposed according to color and flavor structures as follows:

$$\mathcal{A}_2^{(2\times 0)} = N_c C_F\left[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right) + N_h\left(N_c D_l + \frac{E_l}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\right] , \tag{3}$$

where N_l and N_h are the number of light- and heavy-quark flavors, respectively. The coefficients $A, B, \ldots, F_h$ in Eq. (3) are functions of s, t, m, and ε. These quantities were calculated in [7] in the approximation $s, |t|, |u| \gg m^2$. For a fully differential description of top quark pair production at NNLO, the complete mass dependence of $\mathcal{A}_2^{(2\times 0)}$ is required. An exact numerical expression for it has been obtained in [10]. In [11], we derived exact analytic expressions for all the terms in Eq. (3) arising from two-loop diagrams involving at least a fermion loop (i.e. the coefficients D_i, E_i, F_j with $i = l, h$ and $j = l, h, lh$), providing also an independent confirmation of the results of [7, 10].

3. CALCULATION

The two-loop Feynman diagrams for $q\bar{q} \to t\bar{t}$ were generated with QGRAF [13]. The interference with the tree-level amplitude, as well as the color and Dirac algebra, were simplified by using a FORM [14] code. Out of the $\sim$ 200 two-loop diagrams contributing to the amplitude, about 60 are proportional to N_l and/or N_h. There is only one two-loop box topology contributing to the N_l part of the squared amplitude, and a single other two-loop box topology proportional to N_h. These two box topologies are very similar to the ones encountered in the evaluation of the two-loop QED corrections to Bhabha scattering [15, 16], and can be evaluated with the same techniques.

All two-loop integrals appearing in these amplitudes are reduced to a set of master integrals (MIs) using two independent implementations of the Laporta algorithm [17]. Only part of these MIs were available in the literature [18] from previous two-loop calculations of the heavy quark form factors [19] and amplitudes for Bhabha scattering [15, 16, 20]. The remaining MIs were evaluated in [11] by employing the differential equation method [21].

All the MIs were calculated in the non-physical region $s < 0$. The transcendental functions appearing in the MIs are one- and two-dimensional harmonic polylogarithms (HPLs) [22] of maximum weight four and three, respectively. Both sets of functions can be rewritten in terms of conventional Nielsen's polylogarithms.

Following the procedure outlined in the present section, it was possible to obtain the expression of the bare squared matrix elements involving diagrams proportional to N_l and/or N_h. The UV divergencies were renormalized in a mixed scheme described in detail in [11]. In order to cross check our analytical results, we expanded them in the $s, |t|, |u| \gg m^2$ limit. The first term in the expansion agrees with the results published in [7]; the second order term agrees with the results found in the `Mathematica` files included in the arXiv version of [10]. We also find complete agreement with the numerical result of Table 3 in [10], corresponding to a phase space point in which the $s, |t|, |u| \gg m^2$ approximation cannot be applied.

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