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Consistency Test of Dark Energy Models

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Abstract

Recently we proposed a new approach to the testing of dark energy models based on the observational data. In that work we focused particularly on quintessence models for demonstration and invoked a widely used parametrization of the dark energy equation of state. In this paper we take the more recent SN Ia, CMB and BAO data, invoke the same parametrization, and apply this method of consistency test to five categories of dark energy models, including the ΛCDM model, the generalized Chaplygin gas, and three quintessence models: exponential, power-law and inverse-exponential potentials. We find that the exponential potential of quintessence is ruled out at the 95.4% confidence level, while the other four models are consistent with data. This consistency test can be efficiently performed since for all models it requires the constraint of only a single parameter space that by choice can be easily accessed.

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I. INTRODUCTION

Compelling evidences from Type Ia supernovae (SN Ia) and other cosmological observations show that the expansion of the universe is undergoing an accelerating stage at late times (see Ref. 1 for reviews). Within the framework of general relativity and assuming homogeneity and isotropy, this indicates that there should exist an energy source, termed dark energy, which provides a significant negative pressure to cause this acceleration. Thus far the nature of dark energy remains unresolved and is generally regarded as one of the most tantalizing problems in cosmology. While a positive cosmological constant remains the simplest realization of dark energy, current observational data have not ruled out the possibility of a time-evolving dark energy.[2] As one awaits more information from the future observations, the constraining power of the next-generation observations and new analysis methods are being pursued.[2]–[10]

Many dark energy models have been proposed and studied. For the cosmological constant, its value has been constrained by observations (see Ref. 11, for example). The quintessence model, which invokes a time-varying scalar field,[12]–[14] generally allows its energy density and equation of state to evolve with time. There are various quintessence models with different potential forms (see, Refs. 15–18, 5, for example) that have been proposed. Studies of the classification[19, 20] and the general dynamical behavior[21] of quintessence have been carried out. There have been also works on the reconstruction of quintessence potentials[21]–[28],[8] and the investigations on how future observational data can constrain individual models of quintessence[3]–[5]. The generalized Chaplygin gas (see Ref. 29 and references therein) has been proposed to either unify dark matter and dark energy[30] or to simply play the role of dark energy[31, 32]. The constraint of the generalized Chaplygin gas has been obtained.[32]–[34] While these works have helped us study the possible nature of dark energy and obtain constraints of the parameters of an individual dark energy model, it should be desirable to explore possible means to determine whether a particular dark energy model can be ruled out by the observational data.

Recently, we introduced a new approach to testing the consistency between a dark energy model and the observational data.[8] We tested the exponential (see Ref. 5 and references therein) and the power-law potentials[17, 18] of quintessence to demonstrate this method, which can be summarized as follows. For each dark energy model, we look for a *charac-*

teristic, $Q(z)$, which in general can vary with the redshift but is equivalent to a constant parameter within the domain of the model. We further define the *measure of consistency*, $\mathcal{M}(z)$, as the derivative of $Q(z)$ with respect to the redshift z. The observational data should allow a null value for $\mathcal{M}(z)$ if the corresponding dark energy model is consistent with them. If, however, the $\mathcal{M}(z) = 0$ line lies outside certain confidence region, then that dark energy model is ruled out by the observational data at the corresponding confidence level. To obtain the constraint on the measure of consistency $\mathcal{M}(z)$ from the observational data, a parametrization of the relevant physical quantity, such as the equation of state or the luminosity distance, is required. We have invoked a broadly used form of parametrization of the equation of state,[35, 36],[2]

$$w(z) = p_{\rm DE}(z)/\rho_{\rm DE}(z) = w_0 + w_a(1-a) = w_0 + w_a z/(1+z)\,, \tag{1}$$

where $p_{\rm DE}(z)$ and $\rho_{\rm DE}(z)$ are the pressure and the energy density of dark energy, respectively. The two parameters in Eq. (1) and the normalized matter density at present, Ω_m, define the parameter space, (w_0, w_a, Ω_m), through which the information from the observational data can be extracted. A recent work that is close in spirit to ours is that of Zunckel and Clarkson,[10] who proposed consistency test of the cosmological constant via a direct parametrization of the luminosity distance. In our terminology, they use $Q(z) = 1 - \rho_{\rm DE}(z)/\rho_c$, which is equivalent to the constant Ω_m in the domain of the cosmological constant, where ρ_c is the critical density at present. Sahni et al.[9] also proposed null test of the cosmological constant via the diagnostic $Om(z) = (H^2(z)/H_0^2 - 1)/[(1+z)^3 - 1]$, which is equivalent to the constant Ω_m in the domain of the cosmological constant, where $H(z)$ is the Hubble expansion rate and H_0 is the Hubble constant.

In this paper, we take the more recent data set and apply our method of consistency test to five dark energy models, including the cosmological constant, the generalized Chaplygin gas as the dark energy component,[31, 32] and three quintessence models: exponential, power-law and inverse-exponential potentials[17, 18]. The data set we use includes a recently compiled "Constitution set" of SN Ia data,[37]–[43] the cosmic microwave background (CMB) measurement from the five-year Wilkinson Microwave Anisotropy Probe (WMAP) observation,[11] and the baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey (SDSS)[44] and the 2dF Galaxy Redshift Survey (2dFGRS)[45].

A conventional way to determine how well a dark energy model can fit the observational data is the *model-based approach*, in which one optimizes the parameters of each dark energy model based on the observational data and then statistically assesses the goodness of fit (see Ref. 34, for example). In such approach one has to obtain the best fit for each set of parameters specific to the particular dark energy model, which could be tedious. In particular, in order to optimize the parameters of a quintessence model one has to solve the field equation numerically for each point in the parameter space, which can be computationally intensive and time consuming.[4] In contrast, in our approach we first constrain the parameters of the chosen parametrization through the observational data, and then test consistency of each dark energy model based on this set of parameters. This can be more efficient than the model-based approach when one deals with a large number of dark energy models. It is also more direct and therefore much faster to constrain the parameter space (w_0, w_a, Ω_m) than to optimize the parameters of a quintessence model in the model-based approach. The potential downside of our method, however, would be that as long as one invokes a specific form of parametrization, one might have simultaneously imposed a prior, or bias, against certain dark energy models. This issue requires a separate investigation and we are currently pursuing that.[46] We note that the two methods are different in spirit. The goodness of fit describes how well a model can fit the observations. The consistency test, on the other hand, examines whether the condition necessary for a model is excluded by the observations. With in mind the pros and cons mentioned above, we believe that the two methods, that is, the model-based and ours, should be complimentary to each other in the pursuit of revealing the nature of dark energy.

II. CONSISTENCY TEST OF DARK ENERGY MODELS

A. Formalism

In this paper we consider a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe and assume that it is dominated by pressureless matter and dark energy in the present epoch. The Hubble expansion rate, $H \equiv \dot{a}/a$, is given by the Friedmann equations as

$$\begin{aligned} H^2(z) &= \frac{8\pi G_N}{3}\left[\rho_m(z) + \rho_{\rm DE}(z)\right] \\ &= H_0^2\left[\Omega_m(1+z)^3 + (1-\Omega_m)\exp\left(3\int_0^z \left[1+w(z')\right]\frac{dz'}{1+z'}\right)\right], \end{aligned} \tag{2}$$

where the dark energy density

$$\rho_{\rm DE}(z) = \rho_c(1-\Omega_m)\exp\left(3\int_0^z [1+w(z')]\,\frac{dz'}{1+z'}\right), \tag{3}$$

and

$$\rho_c \equiv \frac{3H_0^2}{8\pi G_N}\,. \tag{4}$$

For quintessence as a dark energy model, the quintessence field and the potential are related to the equation of state, the Hubble expansion rate, and the dark energy density as follows [8, 47].

$$\phi(z)-\phi_0 = \pm\int_0^z \frac{\sqrt{[1+w(z')]\,\rho_{\rm DE}(z')}}{H(z')}\frac{dz'}{1+z'}\,, \tag{5}$$

$$V(z) = [1-w(z)]\,\rho_{\rm DE}(z)/2\,. \tag{6}$$

We perform the consistency test of five dark energy models including the cosmological constant, the exponential potential, the power-law potential, the inverse-exponential potential, and the generalized Chaplygin gas as the dark energy component.

For the cosmological constant, the energy density ρ_Λ is a constant. We define the characteristic $Q_\Lambda(z)$ as the dark energy density $\rho_{\rm DE}(z)$, which in general would evolve with the redshift but is equivalent to the constant parameter ρ_Λ within the cosmological constant domain,

$$Q_\Lambda(z) \;\equiv\; \rho_{\rm DE}(z) \tag{7}$$

$$\;=\; \rho_\Lambda \quad \text{for the cosmological constant.} \tag{8}$$

In the same spirit, for the exponential potential,

$$V_{\rm exp}(\phi) = V_1\exp\left[-\phi/M_1\right]\,, \tag{9}$$

we identify M_1 as the characteristic constant parameter and accordingly define the characteristic $Q_{\rm exp}(z)$,

$$Q_{\rm exp}(z) \;\equiv\; -V(z)\left(\frac{dV}{d\phi}\right)^{-1}(z) \tag{10}$$

$$\;=\; M_1 \quad \text{for the exponential potential.} \tag{11}$$

For the power-law potential,

$$V_{\rm power}(\phi) = m^{4-n}\phi^n\,, \tag{12}$$

we define the following characteristic corresponding to the index n,

$$Q_{\rm power}(z) \equiv \left[1 - V(z)\left(\frac{dV}{d\phi}(z)\right)^{-2}\frac{d^2V}{d\phi^2}(z)\right]^{-1} \tag{13}$$

$$= n \quad \text{for the power-law potential.} \tag{14}$$

For the inverse-exponential potential,

$$V_{\text{inverse-exp}}(\phi) = V_2 \exp\left[M_2/\phi\right] \,, \tag{15}$$

the characteristic is defined as

$$Q_{\text{inverse-exp}}(z) \equiv -\frac{4}{V(z)}\left(\frac{dV}{d\phi}(z)\right)^3\left[\frac{d^2V}{d\phi^2}(z) - \frac{1}{V(z)}\left(\frac{dV}{d\phi}(z)\right)^2\right]^{-2} \tag{16}$$

$$= M_2 \quad \text{for the inverse-exponential potential.} \tag{17}$$

As the dark energy component, the generalized Chaplygin gas has an equation of state govern by

$$p_{\rm DE}(z) = -A/\left[\rho_{\rm DE}(z)\right]^{\alpha} \,, \tag{18}$$

where $\alpha \neq -1$ and $A > 0$. The corresponding characteristic is defined as

$$Q_{\rm Chaplygin}(z) \equiv -\frac{\rho_{\rm DE}(z)}{w(z)}\frac{dw}{dz}(z)\left(\frac{d\rho_{\rm DE}}{dz}(z)\right)^{-1} - 1 \tag{19}$$

$$= \alpha \quad \text{for the generalized Chaplygin gas.} \tag{20}$$

We then define the measure of consistency $\mathcal{M}_i(z)$ as the derivative of the characteristic $Q_i(z)$ with respect to the redshift for each dark energy model,

$$\mathcal{M}_i(z) \equiv \frac{dQ_i}{dz}(z) \tag{21}$$

$$= 0 \quad \text{for the corresponding dark energy model,} \tag{22}$$

where i denotes "Λ", "exp", "power", "inverse–exp" and "Chaplygin", respectively. $\mathcal{M}_i(z)$ can in general evolve with the redshift but should be constant zero in the domain of the corresponding dark energy model.

B. Observational data and constraint

We use the combined data set from three types of observations including the SN Ia observation, the CMB measurement, and the BAO measurement.

We use the Constitution set of SN Ia data compiled by Hicken et al.,[43],[37]–[42] which provides the information of the luminosity distance and the redshift. The luminosity distance-redshift relation is given by

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}. \tag{23}$$

We use the CMB shift parameter measured by the five-year WMAP observation,[11]

$$R = \sqrt{\Omega_m H_0^2}\int_0^{1090.04} \frac{dz}{H(z)} = 1.710 \pm 0.019. \tag{24}$$

We use the BAO measurement from the joint analysis of the SDSS and 2dFGRS data,[44, 45] which gives

$$D_V(0.35)/D_V(0.2) = 1.812 \pm 0.060, \tag{25}$$

where

$$D_V(z_{\rm BAO}) = \left[(1+z_{\rm BAO})^2 D_A^2(z_{\rm BAO})\frac{z_{\rm BAO}}{H(z_{\rm BAO})}\right]^{1/3}, \tag{26}$$

and $D_A(z)$ is the angular diameter distance,

$$D_A(z) = \frac{1}{1+z}\int_0^z \frac{dz'}{H(z')}. \tag{27}$$

The constraint of the parameter space (w_0, w_a, Ω_m) is obtained by fitting the three parameters to this combined data set. The best fit of the parameters are found to be

$$w_0 = -0.89^{+0.12}_{-0.14}, \quad w_a = -0.18^{+0.71}_{-0.74}, \quad \Omega_m = 0.25^{+0.03}_{-0.02}.$$

The two-dimensional constraint of (w_0, w_a) is shown in Fig. 1.

C. Results of the consistency test

For the consistency test of each dark energy model, we reconstruct $\mathcal{M}_i(z)$ via the constraint of (w_0, w_a, Ω_m), with the use of the equations in Sec. I and Sec. II A. We perform the test in the redshift region $0 < z < 1.55$, where the influence of dark energy on the expansion of the universe is most significant. This region is covered by the current SN Ia observations, which is the most sensitive type of observations to probe the behavior of dark energy. If the $\mathcal{M}_i(z) = 0$ line lies outside certain confidence region, the corresponding dark energy model is ruled out at that confidence level. Adopting the constraint obtained in Sec. II B, we find that the $\mathcal{M}_{\rm exp}(z) = 0$ line lies outside the 95.4% confidence region while the null lines of the other four models lie inside the 68.3% confidence region. The results are shown in Fig. 2.

III. SUMMARY

We have preformed consistency test of five dark energy models, including the cosmological constant, the generalized Chaplyngin gas, and three quintessence models: exponential, power-law, and inverse-exponential potentials. This test gives a simple signature if a dark energy model is ruled out by the observational data. It can be done efficiently via the constraint of a single set of parameters deduced from the observational data, and can test quintessence models without solving the field equation.

Through our approach and invoking the broadly used parametrization of the equation of state, the exponential potential is found to be ruled out at the 95.4% confidence level based on the current observational data. The other four dark energy models remain consistent with the current observations down to the 68.3% confidence level. It is worth noticing that in our previous work the power-law potential was ruled out at the 68.3% confidence level based on a different data set [8]. One noticeable change in the new data set is that the cosmological constant is contained in the 68.3% confidence region, which was not so in the previous one. Whether our method can discriminate between the power-law potential and the cosmological constant can in principle be studied with the Monte Carlo test [46]. The flat cosmological constant model was examined by Davis et al [34]. via the model-based approach, in which they used the same data set except the three-year WMAP data instead. They found the goodness of fit for the flat cosmological constant model to be 43.7% while we find the model at least consistent with the data at the 68.3% confidence level. The two results are not in conflict with each other.

Our method of the consistency test can in principle be applied not only to other dark energy models but also to other models explaining the acceleration of the expansion, as long as one looks for the characteristic $Q(z)$ corresponding to a constant parameter of each model. One can also choose a different parametrization for better discriminating power between the models in regard. The discriminating power of the method with different forms of parametrization and the possible bias imposed by the chosen parametrization should be studied via the Monte Carlo test [46].

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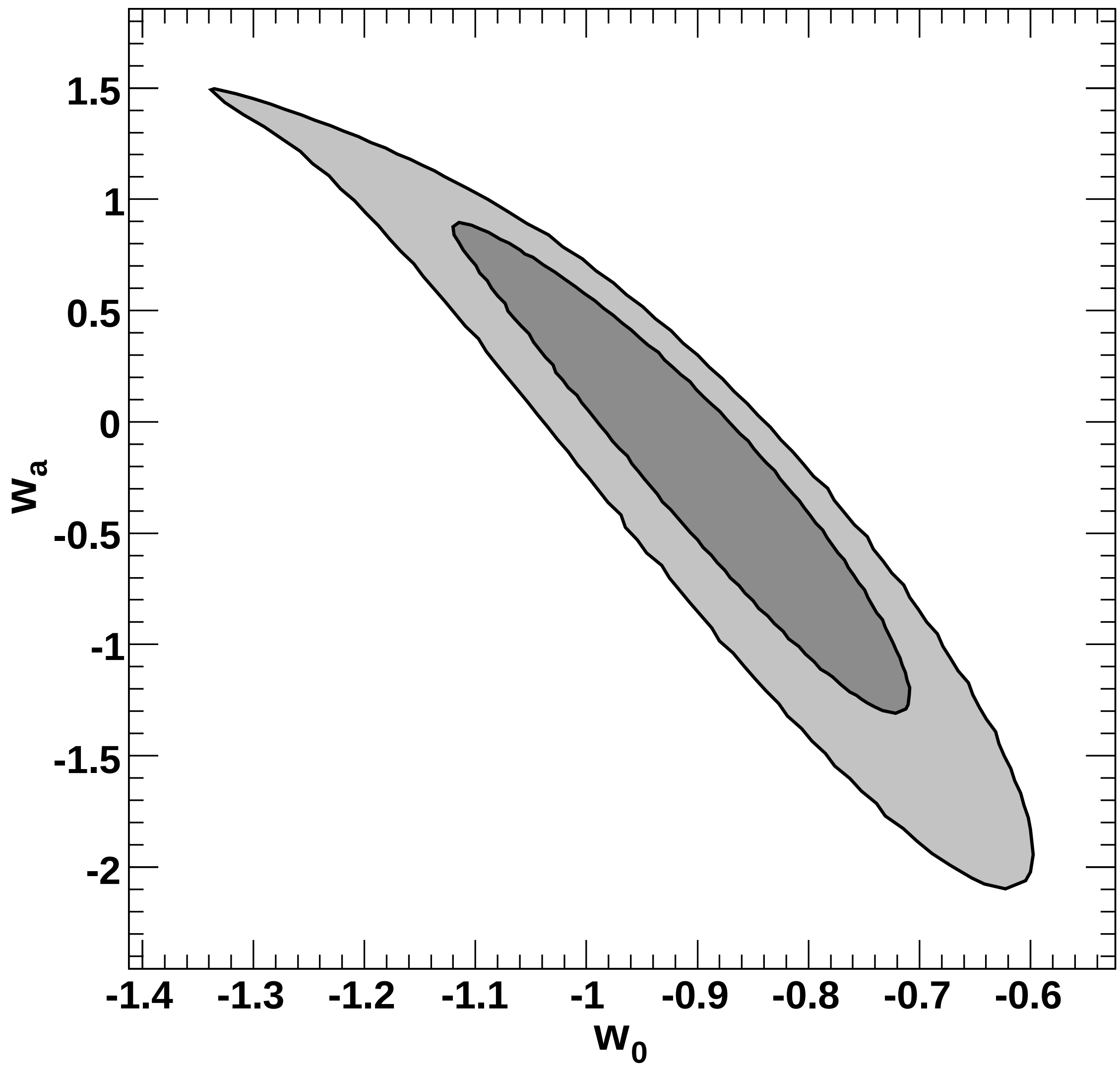

FIG. 1: The joint two-dimensional constraint of (w_0, w_a) based on a combined data set including the Constitution set of SN Ia data, the CMB measurement from the five-year WMAP, and the BAO measurement from the SDSS and 2dFGRS. The dark and light gray areas correspond to the 68.3% and 95.4% confidence regions, respectively.

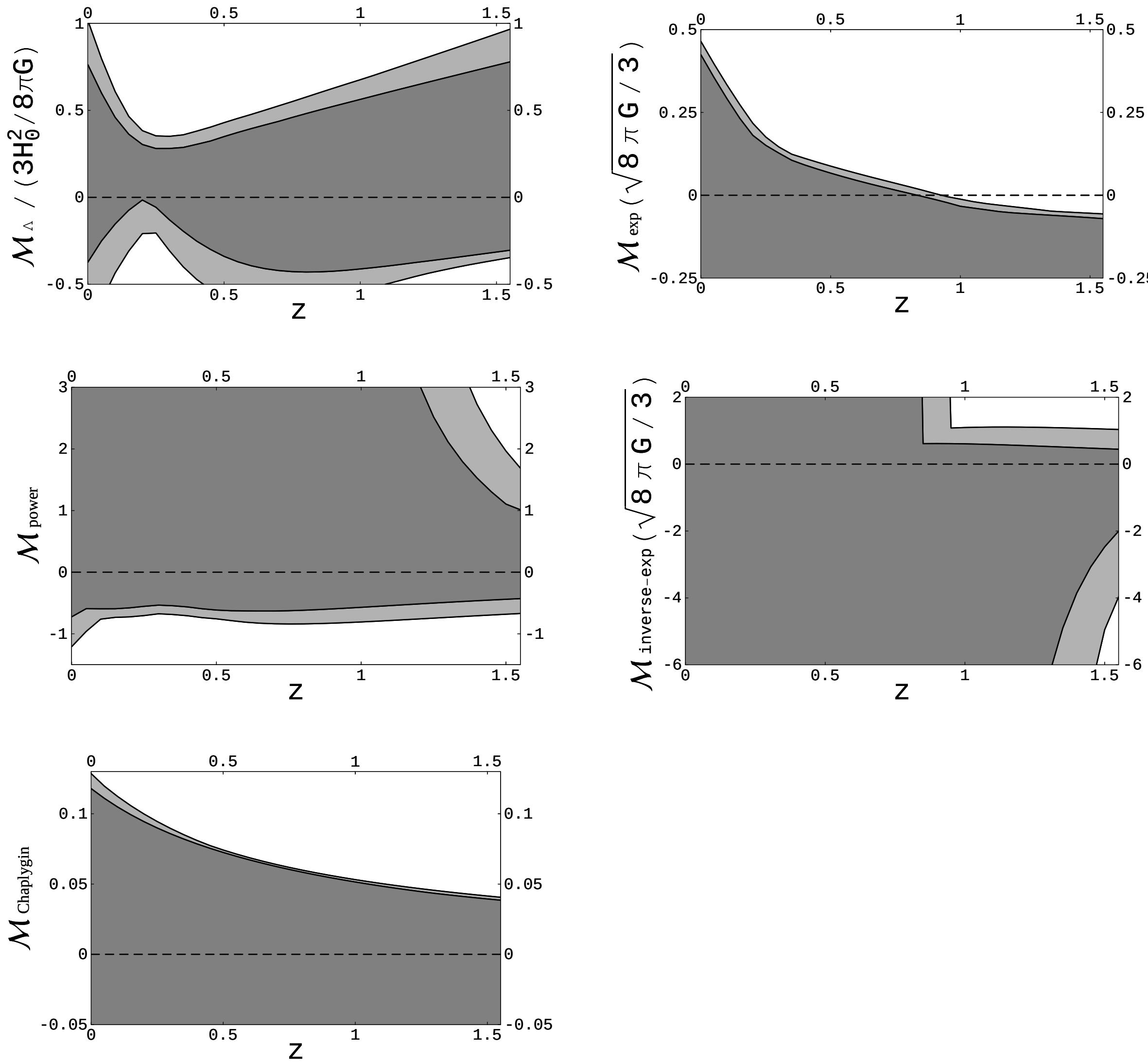

FIG. 2: The measure of consistency of the five dark energy models. The dark and light gray areas correspond to the 68.3% and 95.4% confidence regions, respectively. The $\mathcal{M}_{\rm exp}(z) = 0$ line lies outside the 95.4% confidence region for $0.95 < z < 1.55$. The null lines of the measure for the other four models lie inside the 68.3% confidence regions for $0 < z < 1.55$. This indicates that the exponential potential is ruled out at the 95.4% confidence level while the other four dark energy models are still consistent with the current observational constraints down to the 68.3% confidence level.