

## Measurement of the CKM phase angle $\alpha$

---

**J. William Gary\***

*Department of Physics and Astronomy*

*University of California*

*Riverside, CA, 92521 USA*

*E-mail: bill.gary@ucr.edu*

A summary is given of direct experimental measurements of the CKM phase angle  $\alpha$ . The direct measurements are based on branching fractions and CP asymmetries in  $B$  meson decays to  $\pi\pi$ ,  $\rho\rho$  and  $\rho\pi$  final states, Decays to the  $a_1(1260)\pi$  final state are also discussed.

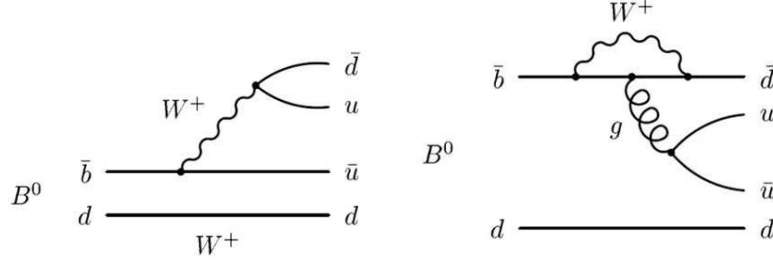
*Flavor Physics and CP Violation: FPCP 2009*

*May 27 - June 1 2009*

*Lake Placid, NY, USA*

---

\*Speaker.



**Figure 1:** Tree (left) and penguin (right) diagrams for  $b \rightarrow u\bar{u}d$  processes,

## 1. Introduction

Over-constraint of the so-called Unitarity Triangle (UT) [1] is a principal goal of the  $B$ -factory experiments, Babar at SLAC and Belle at KEK, allowing non-trivial tests of the Cabibbo-Kobayashi-Maskawa (CKM) mechanism for quark-flavor mixing and violation of the combined charge-parity (CP) symmetry, as well as searches for the effects of physics beyond the Standard Model (SM). Here, a summary is given of the current status of direct measurements of the UT angle  $\alpha$ . The angle  $\alpha$  is sometimes referred to as  $\phi_2$ . For simplicity, we use the common notation in which the three UT angles are denoted  $\alpha$ ,  $\beta$ , and  $\gamma$ . In terms of the CKM matrix elements  $V_{ij}$ , where  $i = u, c, t$  and  $j = d, s, b$  are quark indices,  $\alpha$  is defined by  $\alpha = \arg(-V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)$ .

Direct measurements of  $\alpha$  are provided by  $b \rightarrow u\bar{u}d$  quark-level processes, corresponding to such two-body  $B$  meson decays as  $B \rightarrow \pi\pi$ ,  $\rho\rho$ ,  $\rho\pi$ , or  $a_1(1260)\pi$ . The tree-level amplitudes of these decays (Fig. 1 left) contain the CKM matrix element  $V_{ub}$ . In the Wolfenstein parametrization [1],  $V_{ub}$  contains the phase term  $\exp(-i\gamma)$ . Mixing of neutral  $B$  mesons ( $B^0 \rightarrow \bar{B}^0$  transitions) introduces a phase term  $\exp(-2i\beta)$  [1]. Restricting attention to mixing-induced CP asymmetries, namely CP asymmetries that arise due to interference between direct decays of  $B^0$  mesons to a certain final state and decays to that same final state after the  $B^0$  mixes to form a  $\bar{B}^0$ , one therefore obtains a relative phase of  $-2\beta$  from the mixing and  $-2\gamma$  from the difference between the  $b \rightarrow u$  (from the  $B^0$ ) and  $\bar{b} \rightarrow \bar{u}$  (from the  $\bar{B}^0$ ) transitions. Since  $\alpha + \beta + \gamma = \pi$ , this makes measurements of CP asymmetries in tree-level  $b \rightarrow u\bar{u}d$  processes directly sensitive to  $\alpha$ , i.e.,  $-2\beta - 2\gamma \sim -2\alpha$ .

The well-known formalism for the time-dependent CP asymmetry  $a_{CP}(t)$  in decays of neutral  $B$  mesons to a CP eigenstate  $f_{CP}$ , such as  $\pi^+\pi^-$ ,  $\rho^+\rho^-$ , or  $\rho^0\rho^0$ , is given in [1]. To briefly summarize,

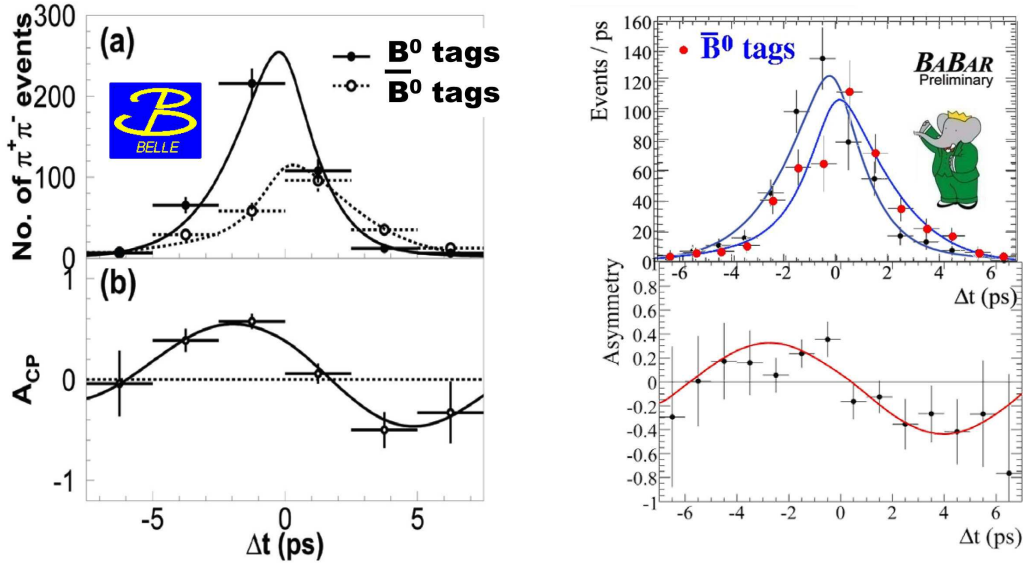
$$a_{CP}(t) \equiv \frac{\Gamma[B^0(t) \rightarrow f_{CP}] - \Gamma[\bar{B}^0(t) \rightarrow f_{CP}]}{\Gamma[B^0(t) \rightarrow f_{CP}] + \Gamma[\bar{B}^0(t) \rightarrow f_{CP}]} \quad (1.1)$$

$$= S \sin(\Delta mt) - C \cos(\Delta mt) \quad (1.2)$$

where  $S$  and  $C$  are CP violating (CPV) coefficients that depend on a parameter denoted  $\lambda$ , with  $\lambda$  defined by the ratio of the  $\bar{B}^0$  and  $B^0$  decay amplitudes to  $f_{CP}$ :

$$S = -\frac{2\text{Im}\lambda}{1 + |\lambda|^2}; \quad C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}; \quad \lambda = e^{-2i\beta} \frac{\bar{A}}{A} \quad (1.3)$$

$$A = \langle f_{CP} | H | B^0 \rangle \quad \bar{A} = \langle f_{CP} | H | \bar{B}^0 \rangle. \quad (1.4)$$



**Figure 2:** The time-dependent CP asymmetry  $a_{CP}(t)$  from  $B^0 \rightarrow \pi^+\pi^-$  decays measured by Belle [2] (left) and Babar [3] (right).

The phase of  $2\beta$  in  $\lambda$  comes from the mixing. If a single diagram dominates the decays, then  $|A| = |\bar{A}|$  so that  $|\lambda| = 1$  and  $C = 0$ . If this single diagram is the tree diagram, then  $\bar{A}/A = \exp(-2i\gamma)$  as discussed above, leading to  $S = \sin 2\alpha$ . Thus, measurement of  $a_{CP}$  in  $b \rightarrow \bar{u}ud$  processes directly determines  $\alpha$ , assuming that tree-level diagrams dominate. On the other hand, if the  $C$  coefficient is found to be *non-zero*, then  $|A| \neq |\bar{A}|$ , implying that other diagrams are present, such as loop (penguin) diagrams (Fig. 1 right). These penguin amplitudes are not directly proportional to  $V_{ub}$  and thus spoil the determination of  $\alpha$ .

The principal channels that have been used for the direct determination of  $\alpha$  are  $B \rightarrow \pi\pi$ ,  $\rho\rho$ , and  $\rho\pi$ . In addition, the  $B \rightarrow a_1(1260)\pi$  channel should soon provide independent information on  $\alpha$ . The  $\rho$  and  $a_1$  mesons are reconstructed through their decays to  $\pi\pi$  and  $\pi\pi\pi$ , respectively. Therefore all the channels used to measure  $\alpha$  are reconstructed through all-pionic final states. Belle and Babar use Cherenkov radiation and specific energy loss to identify charged pions. The typical detection efficiencies are better than 95% and the mis-identification rates around 10%. Neutral pions are reconstructed primarily through their decays to two photons, with efficiencies and purities around 50% and 70%, respectively. Key variables in the reconstruction of  $B$  mesons are the energy difference  $\Delta E = E_B^* - E_{beam}^*$  and beam-energy-substituted (or beam-energy-constrained) mass  $m_{ES}$  (or  $m_{bc}$ ) =  $\sqrt{(E_{beam}^*)^2 - (p_B^*)^2}$ , where  $E_B$  ( $p_B$ ) is the energy (3-momentum) of the  $B$  meson candidate,  $E_{beam}$  is the beam energy, and the asterisk denotes the center-of-mass frame. For correctly reconstructed  $B$  mesons,  $\Delta E$  peaks at zero and  $m_{ES}$  at the  $B$  meson mass.

## 2. $B \rightarrow \pi\pi$

The time-dependent CP asymmetry  $a_{CP}(t)$  measured in  $B^0 \rightarrow \pi^+\pi^-$  decays is shown in Fig. 2. The left-hand side shows the results from Belle [2] and the right-hand side from Babar [3]. These

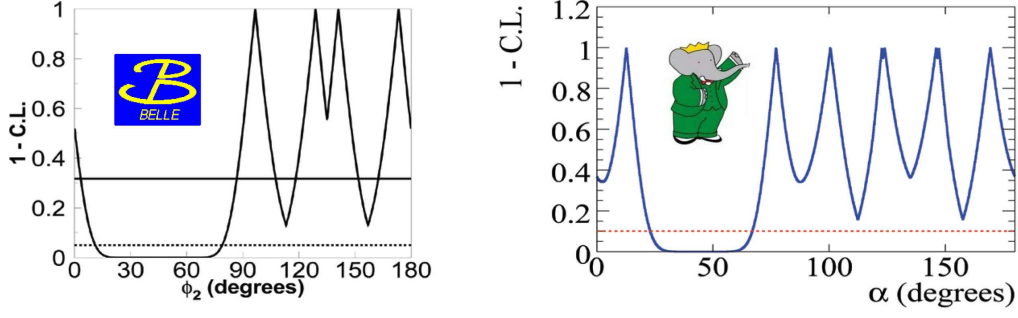
results are based on event samples of  $535 \times 10^6$  and  $467 \times 10^6 e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$  events, respectively. This Babar sample is their complete and final one. The CPV parameters are determined to be  $S = -0.61 \pm 0.10 \pm 0.04$  ( $-0.68 \pm 0.10 \pm 0.03$ ) and  $C = -0.55 \pm 0.08 \pm 0.05$  ( $-0.25 \pm 0.08 \pm 0.02$ ) for Belle (Babar). There is a difference of 1.9 standard deviations between the Belle and Babar results for  $C$  [4]. The more important point is that both experiments find  $C$  to be non-zero, implying that penguin diagrams are present and that  $S$  in the  $\pi^+\pi^-$  channel does not directly measure  $\alpha$ .

In the presence of penguin terms, one can still define  $a_{CP}(t) = S \sin(\Delta mt) - C \cos(\Delta mt)$  as in eq. (1.2), but now the CPV parameter  $S$  determines an *effective* angle  $\alpha_{eff}$  given by  $S \equiv \sqrt{1 - C^2} \sin 2\alpha_{eff}$ . A famous paper by Gronau and London [5] shows how to apply isospin symmetry to the family of  $B \rightarrow \pi\pi$  decays to determine the difference  $\delta\alpha = \alpha - \alpha_{eff}$ . Combining the result for  $\alpha_{eff}$  (from  $a_{CP}(t)$  in  $B^0 \rightarrow \pi^+\pi^-$  decays) with the result for  $\delta\alpha$  (from the isospin analysis) then leads to a measurement of  $\alpha$ .

The basic idea of the isospin analysis is that in  $B^+ \rightarrow \pi^+\pi^0$  decays, the two pions must have total isospin  $I = 1$  or  $2$ , since  $I_3 = 1$ . For the penguin terms (Fig. 1 right), only  $I = 0$  or  $1$  is allowed since the gluon carries  $I = 0$  and isospin is conserved in strong interactions. However,  $I = 1$  is forbidden by Bose-Einstein statistics. Thus  $B^+ \rightarrow \pi^+\pi^0$  decays must proceed uniquely through the tree diagram. By combining the information from  $B^+ \rightarrow \pi^+\pi^0$  decays with that from  $B^0 \rightarrow \pi^+\pi^-$  and  $\pi^0\pi^0$  decays assuming isospin symmetry, one can therefore disentangle the contributions of the tree and penguin terms and use the tree terms to determine  $\alpha$ . There are three  $B \rightarrow \pi\pi$  charge states:  $\pi^+\pi^-$ ,  $\pi^+\pi^0$  and  $\pi^0\pi^0$ . The relationship between the three complex amplitudes yields a triangle, and similarly for the three  $\bar{B}$  amplitudes. The difference between the  $B$  and  $\bar{B}$  triangles determines  $\delta\alpha$  [5]. There are four possible relative orientations of the two triangles. Furthermore, there is a two-fold ambiguity in the solution to  $\alpha_{eff} \sim \arcsin(S)$  (i.e., either  $\alpha_{eff}$  or  $90^\circ - \alpha_{eff}$ ). The Gronau-London method is therefore characterized by an eight-fold discrete ambiguity in the determination of  $\alpha$ .

The required measurements to determine  $\alpha$  using the Gronau-London isospin method are the branching fractions for all three charge modes,  $B \rightarrow \pi^+\pi^-$ ,  $\pi^+\pi^0$  and  $\pi^0\pi^0$ , the time integrated CP asymmetry for  $B^0 \rightarrow \pi^0\pi^0$ , and the  $S$  and  $C$  results for  $B^0 \rightarrow \pi^+\pi^-$ . All these measurements have been made based on data samples of at least  $227 \times 10^6 B\bar{B}$ , and typically many more, events [3, 6]. Although not necessary for the isospin analysis, the CP asymmetry in  $B^+ \rightarrow \pi^+\pi^0$  decays has also been measured [7] and found to be consistent with zero, i.e., the rates of  $B^+ \rightarrow \pi^+\pi^0$  and  $B^- \rightarrow \pi^-\pi^0$  decays are the same. This is a necessary consistency check to verify that  $B^+ \rightarrow \pi^+\pi^0$  decays are dominated by a single (i.e., tree) diagram and that isospin-violating terms such as electroweak penguin diagrams can be neglected.

The results of the isospin analysis are shown in Fig. 3, on the left for Belle [2] and on the right for Babar [3]. These plots are so-called  $\alpha$ -scans, produced by finding the minimum  $\chi^2$  value in a fit of the isospin triangle to the data, for an assumed value of  $\alpha$ . The results are converted to a confidence level (C.L.) using frequentist statistical techniques. The value assumed for  $\alpha$  is then changed and the process repeated. The multiple spikes seen in the plots arise from the discrete ambiguities discussed above. Babar excludes the region  $23 < \alpha < 67^\circ$  at 90% C.L. Belle excludes a wider region:  $11 < \alpha < 79^\circ$  at 95% C.L.



**Figure 3:**  $\alpha$  scans from  $B \rightarrow \pi\pi$  decays from Belle [2] (left) and Babar [3] (right).

### 3. $B \rightarrow \rho\rho$

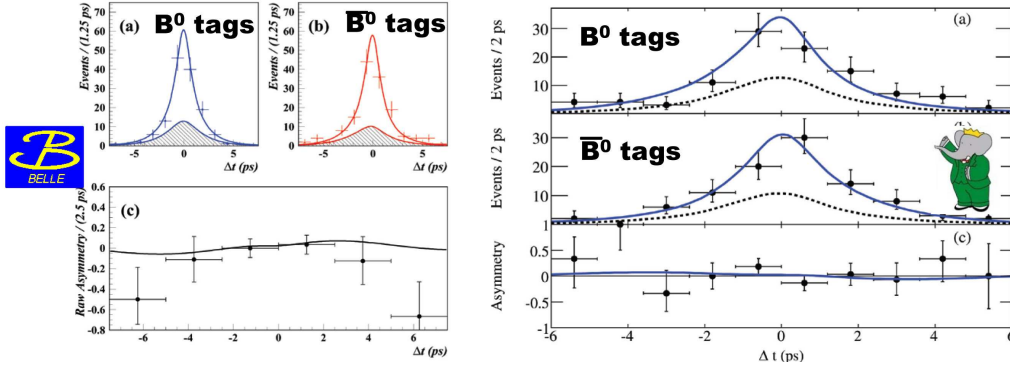
$B \rightarrow \rho\rho$  decays are more complex than  $B \rightarrow \pi\pi$  decays since  $\rho$  mesons have non-zero spin. Because the  $B$  meson has spin 0, the two spin 1  $\rho$  mesons must have relative orbital angular momentum  $L = 0, 1$  or  $2$ . The CP eigenvalue of the  $\rho\rho$  system is  $(-1)^L$  and thus depends on  $L$ . The formalism described in the Introduction to determine  $\alpha$  requires definite CP eigenstates  $f_{CP}$ .

To identify a group of  $B \rightarrow \rho\rho$  events with a definite CP eigenvalue, it is convenient to describe the decays as a superposition of three polarization amplitudes: one longitudinal (helicity = 0) amplitude with  $L = 0$  or  $2$  (thus CP = +1) and two transverse (helicity  $\pm 1$ ) amplitudes with  $L = 0, 1$ , or  $2$  (thus CP is mixed). The contributions of the longitudinal and transverse amplitudes can be separated by considering the distribution of the two  $\rho$  helicity angles  $\theta_1$  and  $\theta_2$ , after integrating over the azimuthal angle between the two  $\rho$  decay planes:

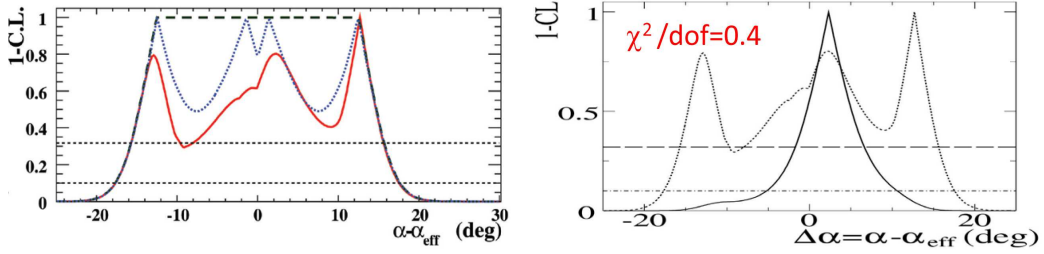
$$\frac{1}{\Gamma} \frac{d^2\Gamma}{d\cos\theta_1 d\cos\theta_2} = \frac{9}{16} [4f_L \cos^2\theta_1 \cos^2\theta_2 + (1 - f_L) \sin^2\theta_1 \sin^2\theta_2], \quad (3.1)$$

where  $\theta_1$  is the angle between the direction of flight of the charged pion from the  $\rho$  decay and the boost from the  $B$  rest frame, evaluated in the rest frame of one of the  $\rho$  mesons, and  $\theta_2$  is the analogous angle with respect to the other  $\rho$  meson. The longitudinal polarization fraction  $f_L$  is determined by fitting Eq. (3.1) to data (this is done simultaneously with the fit to determine the branching fractions). Since the longitudinally-polarized events represent a CP eigenstate, they may be used to determine the CPV parameters  $S$  and  $C$  from  $a_{CP}(t)$  and for the Gronau-London isospin analysis, exactly as described for  $B \rightarrow \pi\pi$  decays. It turns out that the longitudinal component dominates  $B \rightarrow \rho\rho$  decays (see below) and it is not necessary to further consider the transverse terms.

The results for  $a_{CP}(t)$  for longitudinally-polarized  $B^0 \rightarrow \rho^+\rho^-$  decays are shown in Fig. 4. The results from Belle [8] (Babar [9]) are based on  $535 \times 10^6$  ( $384 \times 10^6$ )  $B\bar{B}$  events. The CPV parameters are found to be  $S = 0.19 \pm 0.30 \pm 0.08$  ( $-0.17 \pm 0.20 \pm 0.06$ ) and  $C = -0.16 \pm 0.21 \pm 0.08$  ( $-0.01 \pm 0.15 \pm 0.06$ ) for Belle (Babar). Babar determines  $f_L = 0.992 \pm 0.024^{+0.026}_{-0.013}$  while Belle assumes  $f_L = 1.0$ . Thus, in contrast to the observation in  $B^0 \rightarrow \pi^+\pi^-$  decays, the  $C$  coefficient in  $B^0 \rightarrow \rho^+\rho^-$  decays is consistent with zero, implying that penguin contributions are small. Therefore the value of  $\alpha_{eff}$  extracted from the  $S$  coefficient should be close to the true value  $\alpha$ . It is



**Figure 4:** The time-dependent CP asymmetry  $a_{CP}(t)$  from  $B^0 \rightarrow \rho^+\rho^-$  decays measured by Belle [8] (left) and Babar [9] (right).



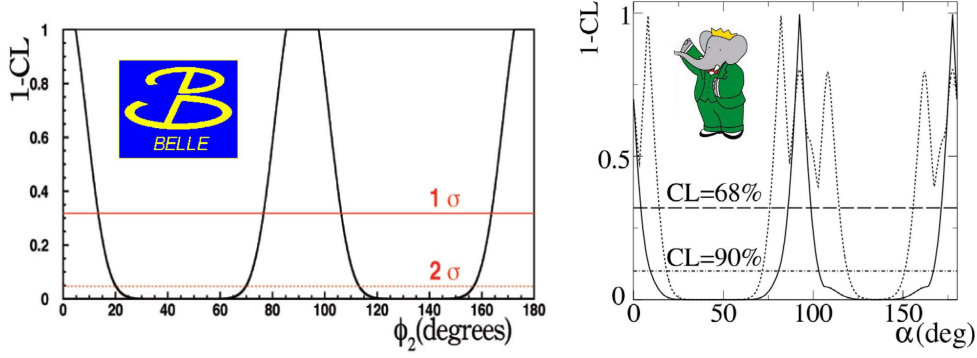
**Figure 5:** Babar results for the constraint  $\delta\alpha = \alpha - \alpha_{eff}$ , obtained by applying the Gronau-London isospin analysis, incorporating updated results for  $B^0 \rightarrow \rho^0\rho^0$  [10] (left) and  $B^+ \rightarrow \rho^+\rho^0$  [11] (right) events.

further observed that the  $S$  coefficient is consistent with zero (no CPV), meaning that  $\alpha_{eff}$  (or  $\alpha$ ) is either around  $0^\circ$  or  $90^\circ$ .

The  $\rho\rho$  system also allows measurement of  $S$  and  $C$  coefficients in  $B^0 \rightarrow \rho^0\rho^0$  decays (the analogous possibility in  $B^0 \rightarrow \pi^0\pi^0$  decays is not practical since the two photons from  $\pi^0$  decay cannot be used to construct a decay vertex). Using their final data sample of about  $465 \times 10^6 B\bar{B}$  events, Babar observes  $B^0 \rightarrow \rho^0\rho^0$  decays with a significance of 3.3 standard deviations [10], measures  $f_L = 0.75^{+0.11}_{-0.14} \pm 0.05$ , and determines  $S = 0.3 \pm 0.7 \pm 0.2$  and  $C = 0.2 \pm 0.8 \pm 0.3$  for the longitudinally-polarized component.

For the Gronau-London isospin analysis, one also needs the branching fraction of the  $B^+ \rightarrow \rho^+\rho^0$  channel. The  $B^+ \rightarrow \rho^+\rho^0$  analysis is difficult because of large backgrounds and correlations. Babar has recently updated this study [11] using their final data sample. They find a result for the branching fraction,  $(23.7 \pm 1.4 \pm 1.4) \times 10^{-6}$ , that is significantly larger than their previous result  $(16.8 \pm 2.2 \pm 2.3) \times 10^{-6}$  [12]. The reason for the increase is an improved method to account for correlations. The CP asymmetry in  $B^+ \rightarrow \rho^+\rho^0$  decays is found to be consistent with zero, implying that contributions from non-isospin-conserving amplitudes are small, analogous to the result mentioned above for  $B^+ \rightarrow \pi^+\pi^0$  decays.

Babar results on the constraint  $\delta\alpha = \alpha - \alpha_{eff}$ , obtained by applying the isospin analysis, are shown in Fig. 5. In this case there are 10 inputs: the three branching fractions, the three  $f_L$  values,



**Figure 6:**  $\alpha$  scans from  $B \rightarrow \rho\rho$  decays from Belle [13] (left) and Babar [11] (right).

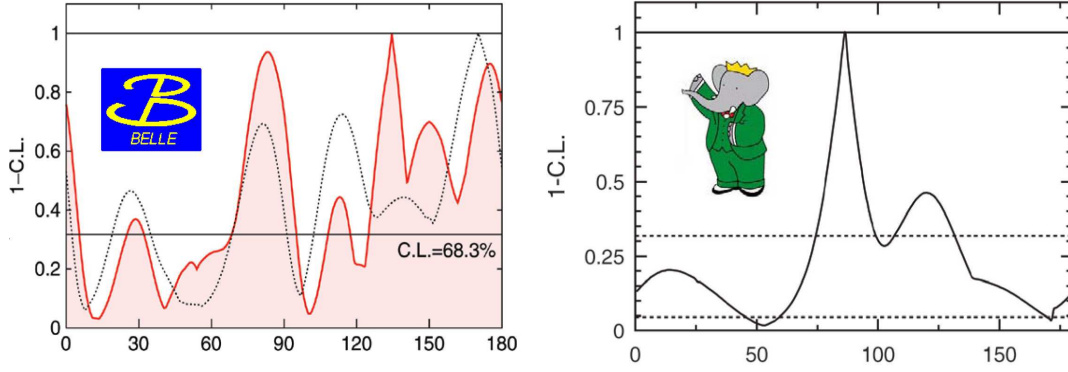
and the two  $S$  and two  $C$  coefficients. The left plot illustrates the impact of  $S$  and  $C$  from  $\rho^0\rho^0$  events. The horizontal dashed line shows the constraint without  $S$  or  $C$ . The dotted curve shows the result if  $C$  is used but not  $S$ . The solid line shows the constraint if both  $S$  and  $C$  are used. It is seen that using the  $S$  and  $C$  values measured in  $B^0 \rightarrow \rho^0\rho^0$  decays helps to resolve the discrete ambiguities. The right plot shows the effect of the updated  $B^+ \rightarrow \rho^+\rho^0$  branching fraction. The dotted curve shows the constraint using the previous [12] measurement (this is the same as the solid curve in the left plot). The solid curve shows the constraint using the new (larger) branching fraction [11] measurement. The larger branching fraction found in the new study flattens both the  $B$  and  $\bar{B}$  isospin triangles, effectively removing the four-fold discrete ambiguity associated with the unknown relative orientation of the two triangles.

The  $\alpha$  scans from the  $B \rightarrow \rho\rho$  isospin analyses are shown in Fig. 6, for Belle [13] (left) and Babar [11] (right). Neglecting the SM-disfavored solution near  $\alpha = 0$  or  $180^\circ$ , Belle obtains  $\alpha = (91.7 \pm 14.9)^\circ$  and Babar  $(92.4^{+6.0}_{-6.5})^\circ$ . The more stringent result for Babar is due primarily to the updated branching fraction result for  $B^+ \rightarrow \rho^+\rho^0$ . It is seen that the results for  $\alpha$  are much more precise than those obtained from  $B \rightarrow \pi\pi$  (Fig. 3).

#### 4. $B \rightarrow \rho\pi$

$\rho\pi$  is not a CP eigenstate, so to extract  $\alpha$  from this channel the formalism discussed in the Introduction needs to be generalized to simultaneously consider  $B^0(\bar{B}^0) \rightarrow \rho^+\pi^-$  and  $B^0(\bar{B}^0) \rightarrow \rho^-\pi^+$  decays.

Two methods have been proposed to eliminate penguin contributions and extract  $\alpha$  from the  $\rho\pi$  channel. The first method [14] is an isospin analysis based on the three neutral ( $B^0 \rightarrow \rho^+\pi^-$ ,  $\rho^-\pi^+$ ,  $\rho^0\pi^0$ ) and two charged ( $B^+ \rightarrow \rho^+\pi^-$ ,  $\rho^0\pi^+$ ) modes, and is effectively an extension of the Gronau-London isospin triangle method used for  $\pi\pi$  and  $\rho\rho$  events. Since there are five amplitudes, the method leads to  $B$  and  $\bar{B}$  pentagons. There are discrete ambiguities in the determination of  $\alpha$ , as in the  $\pi\pi$  or  $\rho\rho$  studies. The precision of this method is expected to be small. The second method [15] is a time-dependent Dalitz plot analysis of the three neutral  $(\rho\pi)^0$  modes. One determines the relative phases and moduli of the six amplitudes in which either the  $B^0$  or  $\bar{B}^0$  decays to



**Figure 7:**  $\alpha$  scans from  $B \rightarrow \rho\pi$  decays from Belle [16] (left) and Babar [17] (right).

one of the three neutral  $\rho\pi$  states, through their interference in the  $B^0(\bar{B}^0) \rightarrow \pi^+\pi^-\pi^0$  Dalitz plot. Isospin relations allow the six amplitudes to be combined to extract  $\alpha$  without discrete ambiguities.

The  $\pi^+\pi^-\pi^0$  Dalitz plot analysis has been performed by both Belle [16] and Babar [17]. An amplitude  $A_{3\pi}$  is written as the sum of the amplitudes for a  $B^0$  to decay to the three neutral  $\rho\pi$  states. A similar amplitude  $\bar{A}_{3\pi}$  is the corresponding sum of the three  $\bar{B}^0 \rightarrow (\rho\pi)^0$  amplitudes. The expression for the time-dependent and Dalitz plot-position-dependent  $B^0 \rightarrow \pi\pi\pi$  decay rate depends on  $A_{3\pi}$ ,  $\bar{A}_{3\pi}$ , and sine and cosine terms analogous to those in Eq. (1.2) [15]. Inserting the expressions for  $A_{3\pi}$  and  $\bar{A}_{3\pi}$  into the expression for the time-dependent decay rate leads to an expression with 26 real-valued coefficients denoted  $U$  and  $I$  that are determined in a fit to the time-sliced Dalitz plot. The Belle and Babar results for the  $U$  and  $I$  coefficients are in good agreement with each other (see, e.g., [4]). Results for  $\alpha$  are then derived from the measured  $U$  and  $I$  terms. The  $\alpha$  scans are shown in Fig. 7. The Belle (left) results are shown both with (solid) and without (dotted) information from the charged  $B^+ \rightarrow (\rho\pi)^+$  modes. The information from the charged modes is incorporated using the pentagon relations mentioned above.

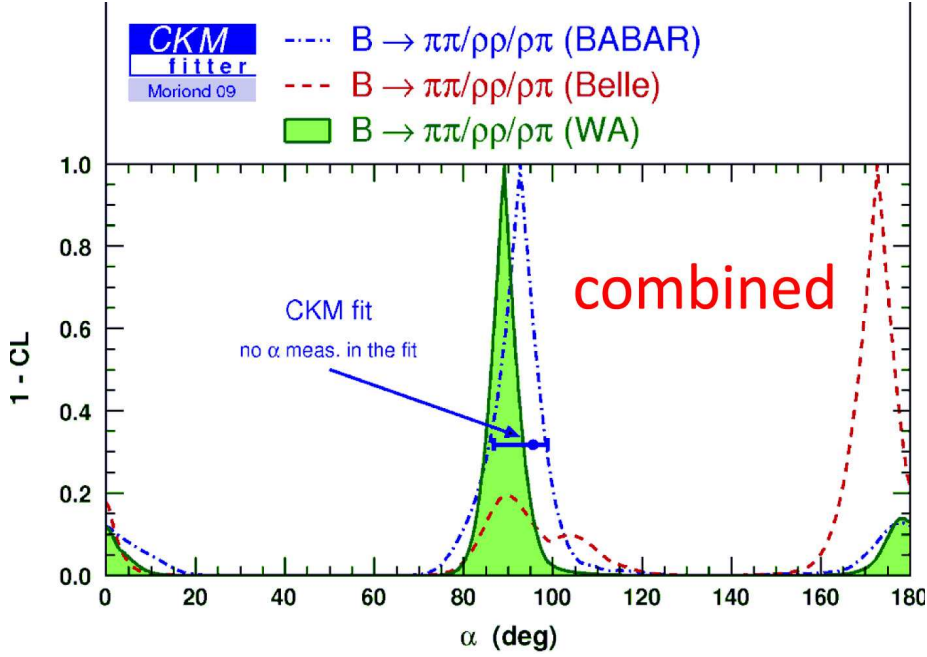
The Babar (right) results exhibit the resolution of discrete ambiguities that is expected, namely a single preferred solution around  $\alpha \approx 90^\circ$ . The result  $\alpha = (87_{-13}^{+45})^\circ$  obtained from  $\rho\pi$  decays is less precise than the result from  $\rho\rho$  decays but does not contain the ambiguity around 0 or  $180^\circ$  seen in Fig. 6.

## 5. Combined $\pi\pi$ , $\rho\rho$ and $\rho\pi$ results

The shaded (green) curve in Fig. 8 shows the results on  $\alpha$  from the CKMfitter group [18], obtained by combining the Belle and Babar results from the  $B \rightarrow \pi\pi$ ,  $\rho\rho$  and  $\rho\pi$  channels, i.e., by combining the results shown in Figs. 3, 6, and 7. This scan is constructed using frequentist techniques similar to those used by the experiments. Analogous combinations of data have been presented by the UTfit group [19]. However, these latter results, unlike those of Fig. 8, do not at the time of this writing incorporate the final Babar measurement of the  $B^+ \rightarrow \rho^+\rho^0$  branching fraction.

The combined result for  $\alpha$  is  $\alpha = (89 \pm 4.3)^\circ$ , a precision of 4.8%. This can be compared with a precision of 6.2% in the summer of 2008. The increase in precision is mostly due to the updated measurement of the  $B^+ \rightarrow \rho^+\rho^0$  branching fraction by Babar.





**Figure 8:** Combined  $\alpha$  results from  $B \rightarrow \pi\pi$ ,  $B \rightarrow \rho\rho$ , and  $B \rightarrow \rho\pi$  decays [18].

Most of the precision in the measurement of  $\alpha$  comes from the  $\rho\rho$  data, as discussed above, while the  $\rho\pi$  data are important to resolve the discrete ambiguity of the  $\rho\rho$  analysis at  $\alpha \approx 0$  and  $180^\circ$ . The data point with horizontal error bars in Fig. 8 shows the result for  $\alpha$  obtained from indirect SM constraints such as measurements of  $|V_{us}|$ ,  $|V_{ud}|$ ,  $|V_{ub}|$ ,  $|V_{cb}|$  and CPV in kaon decays. The indirect result is seen to agree well with the direct result, establishing an important non-trivial test of the SM description of CPV.

## 6. $B \rightarrow a_1(1260)\pi$

Finally, a brief mention will be made of the  $B \rightarrow a_1(1260)\pi$  channel, with  $a_1 \rightarrow \pi\pi\pi$ . A Dalitz plot analysis, such as that performed for the  $B \rightarrow \rho\pi$  channel, is considered to be too complex to undertake and so a quasi-two-body approach [20] is taken instead. This method contains an inherent four-fold discrete ambiguity. Combining the results from  $B^0 \rightarrow a_1^+\pi^-$  and  $a_1^-\pi^+$  decays with the corresponding  $\bar{B}^0$  decays, and selecting the solution that is consistent with the SM, Babar obtains an effective result  $\alpha_{eff} = (78.6 \pm 7.3)^\circ$  [21]. To evaluate the contributions of penguin terms and determine the correction  $\delta\alpha = \alpha - \alpha_{eff}$ , SU(3) flavor symmetry is invoked [20]. The SU(3) relations to constrain  $\delta\alpha$  require measurement of the branching fractions of the SU(3)-related channels  $B \rightarrow a_1K$  and  $B \rightarrow K_{1A}(1270)\pi$ . The  $a_1K$  branching fraction has been measured by Babar [22]. The  $K_{1A}(1270)\pi$  analysis is complicated since the  $K_{1A}(1270)$  is a mixture of the  $K_1(1270)$  and  $K_1(1400)$  states and their interference must be considered. Preliminary results [23] from Babar on the  $K_{1A}(1270)\pi$  channel are being finalized and are expected to constrain  $\delta\alpha$  with accuracy  $16^\circ$  or smaller.

## 7. Summary

The precision on the determination of  $\alpha$  has improved considerably since the summer of 2008, mostly because of a new measurement of the  $B^+ \rightarrow \rho^+ \rho^0$  branching fraction by Babar.  $\alpha$  is now known to better than 5% accuracy, making its precision roughly comparable to that of the CKM phase angle  $\beta$ . Although the main constraints on  $\alpha$  come from  $B \rightarrow \rho\rho$  and  $\rho\pi$  decays, independent results from  $B \rightarrow a_1(1260)\pi$  decays are expected soon, which may further improve the measurement of  $\alpha$ .

## References

- [1] See C. Amsler *et al.* (Particle Data Group), Phys. Lett. B667 (2008) 1, section on CP violation and references therein.
- [2] Belle Collab., Phys. Rev. Lett. 98 (2007) 211801.
- [3] Babar Collab., arXiv:0807.4226 (2008).
- [4] Heavy Flavor Averaging Group, <http://www.slac.stanford.edu/xorg/hfag>, Unitarity Triangle Parameters.
- [5] M. Gronau and D. London, Phys. Lett. B65 (1990), 3381.
- [6] Belle Collab., Phys. Rev. Lett. 99 (2007) 121601; hep-ex/0610065; Babar Collab., Phys. Rev. D75 (2007) 012008; Phys. Rev. D76 (2007) 091102; CDF Collab., hep-ex/0612018.
- [7] Belle Collab., Nature 452 (2008) 332; Babar Collab., Phys. Rev. D76 (2007) 091102.
- [8] Belle Collab., Phys. Rev. D76 (2007) 011104.
- [9] Babar Collab., Phys. Rev. D76 (2007) 052007.
- [10] Babar Collab., Phys. Rev. D78 (2008) 071104.
- [11] Babar Collab., Phys. Rev. Lett. 102 (2009) 141802.
- [12] Babar Collab., Phys. Rev. Lett. 97 (2006) 261801.
- [13] Belle Collab., Phys. Rev. D78 (2008) 111102.
- [14] H.J. Lipkin *et al.*, Phys. Rev. D44 (1991) 1454; M. Gronau, Phys. Lett. B265 (1991) 389.
- [15] A.E. Snyder and H.R. Quinn, Phys. Rev. D48 (1993) 2139; H.R. Quinn and J.P. Silva, Phys. Rev. D62 (2000) 054002.
- [16] Belle Collab., Phys. Rev. Lett. 98 (2007) 221602.
- [17] Babar Collab., Phys. Rev. D76 (2007) 012004.
- [18] CKMfitter Group (J. Charles *et al.*), Eur. Phys. J. C41, 1-131 (2005), <http://ckmfitter.in2p3.fr>.
- [19] M. Bona *et al.* (UTfit Collaboration), JHEP 0603, 080 (2006), <http://www.utfit.org>.
- [20] M. Gronau and J. Zupan, Phys. Rev. D73 (2006) 057502.
- [21] Babar Collab., Phys. Rev. Lett. 98 (2007) 181803.
- [22] Babar Collab., Phys. Rev. Lett. 100 (2008) 051803.
- [23] Babar Collab., arXiv:0807.4760 (2008).