# Study of the $K^{+} K^{-}$invariant－mass dependence of $C P$ asymmetry in $\boldsymbol{B}^{+} \rightarrow \boldsymbol{K}^{+} \boldsymbol{K}^{-} \boldsymbol{K}^{+}$decays 

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As a followup to the latest BABAR amplitude analysis of the decay $B^{+} \rightarrow K^{+} K^{-} K^{+}$, we investigate the $K^{+} K^{-}$invariant-mass dependence of the $C P$ asymmetry and compare it to that obtained by the LHCb collaboration. The results are based on a data sample of approximately $470 \times 10^{6} B \bar{B}$ decays, collected with the BABAR detector at the PEP-II asymmetric-energy $B$ factory at the SLAC National Accelerator Laboratory.

A study of $C P$ violation in a Dalitz-plot analysis of $B^{+} \rightarrow K^{+} K^{-} K^{+}$decays was performed by the BABAR collaboration [1]. Based on this existing analysis, we exploit the ${ }_{s} \mathcal{P} l o t$ technique [2] to investigate the $K^{+} K^{-}$invariant-mass dependence of the $C P$ asymmetry, $A_{C P}=\frac{\Gamma\left(B^{-}\right)-\Gamma\left(B^{+}\right)}{\Gamma\left(B^{-}\right)+\Gamma\left(B^{+}\right)}$. The dependence of the $C P$ asymmetry on $K^{+} K^{-}$invariant mass is compared to a recent preliminary result from the LHCb collaboration [3], where the direct $C P$ asymmetry in $B^{+} \rightarrow K^{+} K^{-} K^{+}$over the entire phase space excluding charm decays was measured to be

$$
\begin{equation*}
A_{C P}\left(B^{+} \rightarrow K^{+} K^{-} K^{+}\right)=-0.046 \pm 0.009(\text { stat. }) \pm 0.005(\text { syst. }) \pm 0.007\left(J / \psi K^{ \pm}\right) \tag{1}
\end{equation*}
$$

The first quoted uncertainty is statistical, the second is systematic, and the third is due to the uncertainty on the measured value of the $C P$ asymmetry in $B \rightarrow J / \psi K^{ \pm}$decays (see below). This result has a significance of $3.7 \sigma$ to be non-zero and is claimed to be the first evidence of $C P$ violation observed in inclusive charmless $B$ decays. The corresponding measurement from $B A B A R$ is

$$
\begin{equation*}
A_{C P}\left(B^{+} \rightarrow K^{+} K^{-} K^{+}\right)=-0.017_{-0.014}^{+0.019}(\text { stat. }) \pm 0.014(\text { syst. }) \tag{2}
\end{equation*}
$$

where no significant $C P$ violation is observed, although it is not inconsistent with the result from LHCb .
The analysis method used to extract $A_{C P}$ is rather different between the experiments. BABAR performs an amplitude analysis, based on a maximum-likelihood fit to the Dalitz plot as well as the output of a neural network based on event shape variables and the kinematic variables $m_{\mathrm{ES}}$ and $\Delta E[1]$. The energy-substituted mass is defined as $m_{\mathrm{ES}} \equiv \sqrt{\left(s / 2+\mathbf{p}_{i} \cdot \mathbf{p}_{B}\right)^{2} / E_{i}^{2}-p_{B}^{2}}$ and the energy difference $\Delta E \equiv E_{B}^{*}-\frac{1}{2} \sqrt{s}$, where $\left(E_{B}, \mathbf{p}_{B}\right)$ and $\left(E_{i}, \mathbf{p}_{i}\right)$ are the

[^0]four-vectors of the $B$ candidate and the initial electron-positron system measured in the laboratory frame, respectively. The asterisk denotes the $e^{+} e^{-} \mathrm{CM}$ frame, and $s$ is the invariant mass squared of the electron-positron system. Signal events peak at the $B$ mass $\left(\approx 5.279 \mathrm{GeV} / c^{2}\right)$ for $m_{\mathrm{ES}}$, and at zero for $\Delta E$. The inclusive $A_{C P}$ is calculated by separately integrating over the Dalitz plane the efficiency-corrected charmless isobar amplitudes for $B^{+}$and $B^{-}$. The LHCb result is obtained by fitting the $K^{+} K^{-} K^{+}$and $K^{-} K^{+} K^{-}$invariant mass distributions, integrated over the Dalitz plot without any efficiency correction, and calculating $A_{C P}^{R A W}=\frac{N^{-}-N^{+}}{N^{-}+N^{+}}$. This raw asymmetry is corrected by their observed $J / \psi K^{ \pm}$asymmetry of $-0.014 \pm 0.007$ to subtract residual charge asymmetries in production and detection. This correction uses the world-average measured asymmetry of $0.001 \pm 0.007$ [4] for $B^{ \pm} \rightarrow J / \psi K^{ \pm}$. In the LHCb analysis, to remove contributions from the charm decays $B^{ \pm} \rightarrow \bar{D}^{0}\left(D^{0}\right) h^{ \pm}$(where $h$ stands for $K$ or $\pi$ ) with $\bar{D}^{0}\left(D^{0}\right) \rightarrow h^{+} h^{-}$, a $m_{K^{+} K^{-}}$veto was applied at $\pm 30 \mathrm{MeV} / c^{2}$ around the $D^{0}$-mass value. The inclusive $A_{C P}$ extracted by LHCb is the integral over all the observed events in the $K^{+} K^{-} K^{+}$Dalitz plane. Unlike BABAR, LHCb does not include a correction for varying efficiency across the phase space, but evaluates a systematic uncertainty of $0.15 \%$ due to this effect.

LHCb also obtained the raw asymmetry as a function of the squared $K^{+} K^{-}$invariant mass. They observe a broad structure in the asymmetry at $m_{K^{+} K^{-}}^{2} \approx 1.6 \mathrm{GeV}^{2} / c^{4}$. peaking at $A_{C P} \approx-0.2$. The $B A B A R$ publication did not directly include this study, although Fig. 8 in the $B A B A R$ paper shows the $m_{K^{+} K^{-}}$distributions for $B^{+}$and $B^{-}$ separately. In this note, we have reproduced the binning and Dalitz plot cuts of the LHCb study in order to directly compare the mass dependence of $A_{C P}$ between the two experiments. The $B A B A R A_{C P}$ distributions were produced with the ${ }_{s} \mathcal{P}$ lot technique, using the $m_{\mathrm{ES}}$ and $\Delta E$ variables, which are not correlated to each other or to the $K^{+} K^{-}$ invariant mass. In Fig 1 we show the extracted charge asymmetry as a function of the lower of the two $K^{+} K^{-}$ masses, $m_{K^{+} K^{-} \text {,low }}$.



FIG. 1: Left: $A_{C P}$ as a function of $m_{K^{+} K^{-}, \text {low }}^{2}$ in $B^{+} \rightarrow K^{+} K^{-} K^{+}$from LHCb (solid dots) and BABAR (open dots). The LHCb distribution is $A_{C P}^{R A W}$. The distribution from $B A B A R$ is obtained by the ${ }_{s} \mathcal{P}$ lot technique. For both experiments the error bars are statistical only. The systematic effects for BABAR are estimated to be approximately 0.01 . The BABAR data points on the plot are shifted to the right by $0.1 \mathrm{GeV}^{2} / c^{4}$ for clarity. Right: The difference between the $B A B A R$ and LHCb asymmetries, $A_{C P}(B A B A R)-A_{C P}^{R A W}(L H C b)$. Also shown is the average shift of $0.045 \pm 0.021$.

Although the errors on the $B A B A R$ data are approximately 2 times larger than those of LHCb , the pattern of the $C P$ asymmetry as a function of $m_{K^{+} K^{-} \text {,low }}^{2}$ agrees very well. The $\chi^{2}$ between the data is 16.1 for 16 bins. There does appear to be, however, a clear overall shift between the measured LHCb and $B A B A R$ asymmetries, as shown in the right hand plot of Fig. 11 The average difference between the binned $A_{C P}$ measurements is $0.045 \pm 0.021$ and appears to be flat across the spectrum. To obtain this average, we weighted the binned $A_{C P}$ values by their respective errors.

The $K^{+} K^{-}$invariant-mass spectrum in the region $1.3-1.7 \mathrm{GeV} / c^{2}$ includes contributions from at least the $f_{0}(1500)$, $f_{2}^{\prime}(1525)$, and $f_{0}(1710)$, as well as a broad non-resonant contribution [1]. Considering the many varying strong phases involved, as well as the differing quark content of the different resonances, it is not surprising to see significant direct $C P$ violation in this region of phase space.

For completeness, we also include similar plots the higher of the two $K^{+} K^{-}$masses, $m_{K^{+} K^{-} \text {,high, in Fig. 2, Here, }}$, the average shift is $0.053 \pm 0.021$. The average shifts in asymmetry observed in $m_{K^{+} K^{-}, \text {low }}$ and $m_{K^{+} K^{-} \text {,high }}$ are similar but not identical. This behavior is expected due to the fact that we calculate the average of binned $A_{C P}$ values weighted by the error and not by the number of signal events in each bin. The errors are influenced by the background distributions, which are different in the two variables.

In summary, we performed a study of the $K^{+} K^{-}$invariant-mass dependence of the $C P$ asymmetry in $B^{+} \rightarrow$ $K^{+} K^{-} K^{+}$decays, based on a published BABAR Dalitz-plot analysis [1]. The BABAR data support the variation of the $C P$ asymmetry over the Dalitz plot seen by LHCb. Nevertheless, a difference exists between the $C P$ asymmetries


FIG. 2: Left: $A_{C P}$ as a function of $m_{K^{+} K^{-}, \text {high }}^{2}$ in $B^{+} \rightarrow K^{+} K^{-} K^{+}$from LHCb (solid dots) and BABAR (open dots). The LHCb distribution is $A_{C P}^{R A W}$. The distribution from BABAR is obtained by the ${ }_{s} \mathcal{P}$ lot technique. For both experiments the error bars are statistical only. The systematic effects for BABAR are estimated to be approximately 0.01 . The BABAR data points on the plot are shifted to the right by $0.1 \mathrm{GeV}^{2} / c^{4}$ for clarity. Right: The difference between the $B A B A R$ and LHCb asymmetries, $A_{C P}(B A B A R)-A_{C P}^{R A W}(L H C b)$. Also shown is the average shift of $0.053 \pm 0.021$.
measured by $B A B A R$ and $L H C b$. This difference appears to be consistent with being uniform across the phase space and is found to be $0.045 \pm 0.021$ between the $B A B A R A_{C P}$ distribution as a function of $m_{K^{+} K^{-}}$,low and that obtained by LHCb. A compatible difference is observed in $m_{K^{+} K^{-}, \text {high. }}$. These values are consistent with the difference between the inclusive $A_{C P}$ obtained by the two experiments. The shift, while consistent with zero within 2 standard deviation, explains the different conclusions between the two experiments concerning effects in specific regions of the phase space: the hint of direct $C P$ asymmetry in $B^{+} \rightarrow \phi(1020) K^{+}$that was seen by BABAR but not confirmed by LHCb, and the fact that $B A B A R$ finds a negative asymmetry with a smaller magnitude than LHCb around $m_{K^{+} K^{-}}^{2} \approx 1.6 \mathrm{GeV}^{2} / c^{4}$. Further experimental investigation is needed to draw definitive conclusions on the source of $C P$ violation in $B^{+} \rightarrow K^{+} K^{-} K^{+}$ decays.

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