

DEVELOPMENT OF A STEADY STATE SIMULATION CODE FOR KLYSTRON AMPLIFIERS

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Abstract

The design of klystrons is based on the intensive utilization of simulation codes, which can evaluate the complete beam-cavities interaction in the case of large signals. In the present work, we present the development of a 2-D steady state simulation code that can self-consistently evaluate the effects of the electromagnetic field on the particles and of the particles back on the field. The algorithm is based on the iterative solution of the power balance equation in the RF structures and it allows determining the amplitude and phase of the electromagnetic field starting from the cavity modes. Some applications of the code to a single cavity and a two cavity klystron are presented and compared with the results obtained from theory and from another code. Effects of space charge forces in the drift tubes are also evaluated.

INTRODUCTION

The tools commonly used to design klystrons can be divided in two main categories: on one side we have the so-called disk codes, which are steady state, very fast (usually 1-D) codes, while on the other side we have the more accurate and general-purpose PIC codes.

In the present work, we present a steady state algorithm that can self-consistently compute the motion of charged particles in the presence of electromagnetic fields: the charged particles contribute to the fields, and the fields act back on the charged particles. The algorithm is valid not only for static fields, as it is in some gun codes [1], but also for RF fields, so that it can be applied to the case of klystron cavities, fed or not. This can be implemented to write a klystron simulation code which finds a steady state solution for the system.

THE SIMULATION ALGORITHM

We derive here an equation that expresses the amplitude and phase of the field inside the cavity as a function of the beam current density and that can be solved iteratively in order to obtain a complete description of the mutual interaction between the electron beam and the klystron resonators.

Cavity Balance Equation

We consider a generic electromagnetic resonator, fed from a RF driver and with an electron beam going through it. Since the natural modes of a resonator form a

complete and orthogonal set, the cavity field can be expanded in terms of these natural modes:

$$\vec{E} = \sum_i \alpha_i \vec{e}_i \quad \vec{H} = \sum_i \beta_i \vec{h}_i \quad (1)$$

With $\beta_i = \frac{\omega_i}{\omega} \alpha_i$. If the cavity is driven at a frequency that is close to the resonant frequency of a particular mode, and the quality factor of that mode is high enough, the amplitude of that mode will be large with respect to all the others. This case is very common for klystron cavities, so it is possible to consider only one mode in the expansions:

$$\vec{E} \cong \alpha \vec{e}_0 \quad \vec{H} \cong \beta \vec{h}_0 \quad (2)$$

The field in the cavity is then equal to the natural field of the design mode of the structure, multiplied by a complex coefficient α , to be determined in amplitude and phase. To get an expression for this coefficient, we apply the Poynting theorem to the volume V of the cavity delimited by the surface S (fig. 1):

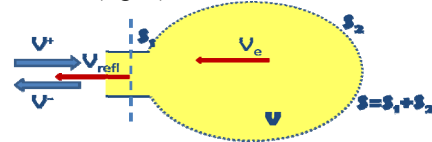


Figure 1: Poynting theorem applied to the cavity (beam pipe not represented).

$$\begin{aligned} & \frac{1}{2} \int_S (\vec{E} \times \vec{H}^*) \cdot \hat{n} dS = \\ & = j\omega \left(\frac{1}{2} \epsilon \int_V (\vec{E} \cdot \vec{E}^*) dV - \frac{1}{2} \mu \int_V (\vec{H} \cdot \vec{H}^*) dV \right) + \\ & \quad - \frac{1}{2} \int_V (\vec{E} \cdot \vec{j}_{beam}^*) dV \end{aligned} \quad (3)$$

The first integral in equation (3) can be separated in two parts: an integral over the waveguide aperture S_1 , that can be expressed by using the transmission line formalism [2], and an integral over the remaining surface S_2 , that can be written by using the Leontovich-Schelkunoff condition for good conductors; we can then assume $\hat{n} \times \vec{H} \cong \beta \vec{h}_0$ on the surface [3], and then use the definition of the quality factor Q_0 of the cavity. With some manipulations [4] the balance equation becomes:

$$-\frac{1}{2} (V^+ + V^-) (I^+ - I^-)^* =$$

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$$= -(1+j)\frac{\omega_0}{Q_0}|\beta|^2u + j\omega|\alpha|^2u\delta + \quad (4)$$

$$-\frac{1}{2}\int_V (\vec{E} \cdot \vec{J}_{beam}^*)dV$$

Where $\delta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$ and $u = \frac{1}{2}\varepsilon \int_V (\vec{e}_0 \cdot \vec{e}_0^*)dV$. The left side of eq. (4) contains the power flowing across the waveguide aperture; the incoming wave can be written in terms of the input power $V^+ = \sqrt{2Z_0P_{IN}}e^{j\phi^+}$, while the outgoing wave can be expressed as $V^- = -V^+ + V^e$, where V^e is the wave emitted from the cavity. Its amplitude depends only on the stored energy, while its phase can be expressed as a function of the phase of the incoming field, V^+ if there is no beam; if there is also the beam, it depends also on the phase of the field inside the cavity (i.e., on the phase of α). We then have the final equation for the complex coefficient α :

$$\alpha^* = \frac{2\sqrt{\frac{\omega_0 u P_{IN}}{Q_{ext}}} e^{j\left(\phi^+ + \arctan\left(\frac{(\frac{\omega_0}{\omega})^2 - Q_0\delta}{Q_0 - Q_{ext}(\frac{\omega_0}{\omega})^2}\right)\right)}}{\omega_0 u \left(\frac{1}{Q_{ext}} + \frac{(\frac{\omega_0}{\omega})^2}{Q_0} + j\left(\frac{(\frac{\omega_0}{\omega})^2}{Q_0} - \delta\right)\right)} + \quad (5)$$

$$-\frac{\frac{1}{2}\int_V (\vec{E} \cdot \vec{J}_{beam}^*)dV}{\omega_0 u \left(\frac{1}{Q_{ext}} + \frac{(\frac{\omega_0}{\omega})^2}{Q_0} + j\left(\frac{(\frac{\omega_0}{\omega})^2}{Q_0} - \delta\right)\right)}$$

The first term in equation (5) is due to the input power coming from the waveguide, and it is obviously different from zero only for the case of the klystron input cavity. The last integral represents the interaction between the cavity field and the beam current density. This current can be written as the sum of the N individual electron currents, so that we get, after some manipulations, the final expression of the coefficient:

$$\alpha^* = \frac{2\sqrt{\frac{\omega_0 u P_{IN}}{Q_{ext}}} e^{j\left(\phi^+ + \arctan\left(\frac{(\frac{\omega_0}{\omega})^2 - Q_0\delta}{Q_0 - Q_{ext}(\frac{\omega_0}{\omega})^2}\right)\right)}}{\omega_0 u \left(\frac{1}{Q_{ext}} + \frac{(\frac{\omega_0}{\omega})^2}{Q_0} + j\left(\frac{(\frac{\omega_0}{\omega})^2}{Q_0} - \delta\right)\right)} + \quad (6)$$

$$\frac{\frac{1}{T}\left(\sum_{i=1}^N q_i \int_0^L \vec{e}_0^*(\rho_i(z), t_i(z), z) \cdot \frac{\vec{v}_i(t_i(z))}{v_{zi}(t_i(z))} dz\right)^*}{\omega_0 u \left(\frac{1}{Q_{ext}} + \frac{(\frac{\omega_0}{\omega})^2}{Q_0} + j\left(\frac{(\frac{\omega_0}{\omega})^2}{Q_0} - \delta\right)\right)}$$

The Simulation Algorithm

In order to get the value of the field coefficient α we need to know the positions and the velocities of all the particles along the cavity. This means that we have to solve the relativistic equations of motion for all the N particles in presence of the electromagnetic field given by (2). The simulation algorithm is the following:

1. Assume an initial value α_1 for the field coefficient;
2. Integrate the equations of motion for each particle through the length of the cavity with the field $\vec{E} = \alpha_1 \vec{e}_0$ and $\vec{H} = \frac{\omega_0}{\omega} \alpha_1 \vec{h}_0$, where \vec{e}_0 and \vec{h}_0 can be obtained from an electromagnetic simulation code as the FEM code developed by S. Tantawi;
3. Calculate the new value α_2 for the field coefficient through eq. (6);
4. If $\alpha_2 = \alpha_1$ (within a certain tolerance) go to the next step; otherwise assume a new value for α_1 and go to step 2.
5. Calculate the steady state solution for the electromagnetic field by eq. (2).

Simulations Results

The routine described in the previous paragraph has been implemented using *Mathematica* [5]. In order to check the self-consistency of the method, the first step has been the simulation of a simple X-band pillbox input cavity (11.424 GHz) with a 100 A-100 KV beam flowing through it. We used, as a first approximation, the analytical field of an ideal cylindrical cavity without beam pipe plus a Brillouin focusing magnetic field, and we neglected the space charge forces between particles.

Fig. 2 shows the outgoing power $P_{oc} = \frac{1}{2}|V_e - V^+|^2$ as a function of the cavity resonant frequency for an input power of 250 W and an external Q of 95, chosen to minimize reflections. We can see that we have a minimum of the outgoing power from the cavity at 11.450 GHz, slightly above the design frequency, as expected in the presence of the electron beam.

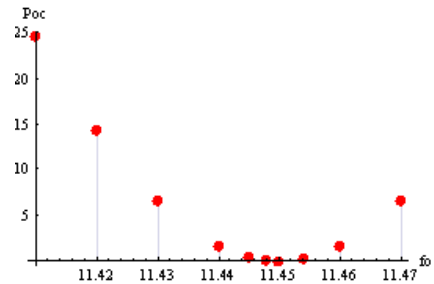


Figure 2: Outcoming power from a pillbox input cavity as a function of resonance frequency (GHz).

The algorithm has then been applied to a cavity with no RF power coming from a driver, like the output cavity of

a two cavity klystron. Space charge forces have been still neglected. In this case the beam itself is responsible of the field excitation and the modulated beam is the one coming from the previous input cavity followed by a 30 cm drift. In this case the cavity resonant frequency and the external Q are optimized to maximize the output power so that we get a maximum efficiency for this simple device of 30% at 11.364 GHz with $Q_{e2}=40$.

The results of the previous simulations have been compared to the ones obtained with klystron kinematic theory and with AJDisk [6], a 1-D simulation code for round and sheet beam klystrons developed at SLAC.

Fig. 3 shows the voltage induced by the beam in the second cavity of a two cavity klystron as a function of the beam current. The new algorithm shows a good agreement with AJDisk in the case of very low current simulations, while the voltage is overestimated in the case of higher beam currents, as expected, since the effect of the space charge forces has not been included yet in the algorithm.

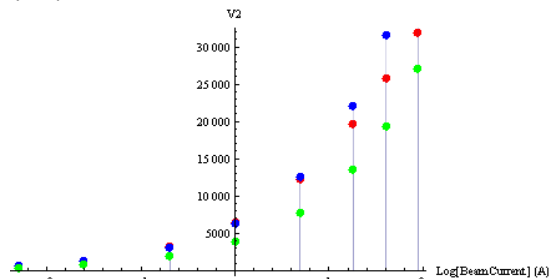


Figure 3: Voltage in the second cavity calculated with the Mathematica algorithm (blue), AJDisk (red) and kinematic theory (green).

The Space Charge Field

An iterative method to evaluate the effects of the space charge fields in the drift tubes between klystron cavities has then been developed and it is being implemented in the code.

We search the steady state solution by applying the method shown in fig.4: the first iteration consists of a simple tracking of the electrons through the drift tube, without considering any kind of space charge field. The result is a complete description of the particles phase space as a function of time inside the pipe between two cavities. After this first tracking we can calculate the space charge potentials generated by the previous particles distribution in every point of the pipe and for every time t . This can be done either analytically (with some approximations), or numerically, by solving the Poisson's equation (in the particles frame and then Lorentz transforming) with an electromagnetic solver. These potentials are then used to integrate the equations of motion during the second iteration. The space charge potentials can then be recalculated and the procedure is iterated until it reaches convergence.

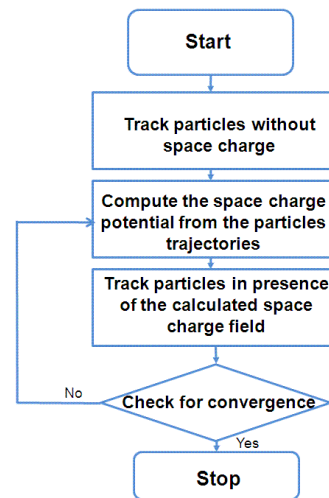


Figure 4: Flow chart for the steady state simulations with space charge.

This routine has been implemented so far only by using the analytical approach and by making some approximations in order to preserve the simplicity and the speed of the code; in particular, free space potentials have been used for the calculation of the space charge fields. Further work has to be done in order to optimize the implementation of the complete space charge algorithm.

CONCLUSIONS

A steady state algorithm which can self-consistently simulate the complete beam-cavity interaction has been presented. The method is based on the iterative solution of a power balance equation, and it determines the amplitude and phase of the electromagnetic field inside the structure starting from the cavity mode field.

The algorithm has been implemented using Mathematica and it has been applied to some simple situations in absence of space charge effects.

The effect of the space charge forces has then been introduced by using an iterative routine and its complete implementation is still in progress.

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