SLAC-PUB-15293

BASIS LIGHT-FRONT QUANTIZATION: A NEW APPROACH TO NON-PERTURBATIVE SCATTERING AND TIME-DEPENDENT PRODUCTION PROCESSES*

James P. Vary[a], Xingbo Zhao[a], Anton Ilderton[b]
Heli Honkanen[c], Pieter Maris[a], Stanley J. Brodsky[d]

[a]Dept. of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA
[b]Physics Dept., Umeå University, 901-87 Umeå Sweden
[c]Dept. of Physics, Penn State University, University Park, PA 16802, USA
[d]SLAC National Accelerator Laboratory, Stanford University, CA 94309, USA

(Received December 14, 2012)

Hamiltonian light-front quantum field theory constitutes a framework for deriving invariant masses, correlated parton amplitudes of self-bound systems and time-dependent scattering amplitudes. By choosing the light-front gauge and adopting an orthonormal basis function representation, we obtain a large, sparse, Hamiltonian matrix for mass eigenstates that is solvable by adapting *ab initio* no-core methods of nuclear many-body theory. In the continuum limit, the infinite matrix limit, we recover full covariance. There is considerable freedom in the choice of the orthonormal and complete set of basis functions with key considerations being convenience and convergence properties. We adopt a two-dimensional harmonic oscillator basis for transverse modes that corresponds with eigensolutions of the soft-wall anti-de Sitter/quantum chromodynamics (AdS/QCD) model obtained from light-front holography. We outline our approach and present preliminary results for non-linear Compton scattering, evaluated non-perturbatively, where a strong (possibly time-dependent) laser field excites an electron that emits a photon.

DOI:10.5506/APhysPolBSupp.6.257
PACS numbers: 11.10.Ef, 11.15.Tk, 12.20.Ds

1. Introduction

Recent intense interest in strong-field dynamics, ranging from (a) the observed anomalous enhancement of lepton production in ultrarelativistic nuclear collisions at RHIC [1], (b) a prediction for photon yield depletion at the LHC [2], and (c) novel proposals for producing supercritical fields

* Presented at the Light Cone 2012 Conference, Kraków, Poland, July 8–13, 2012.

Work supported in part by US Department of Energy contract DE-AC02-76SF00515.

using next-generation lasers [3, 4], all point to the importance of developing new methods for solving QED in its non-perturbative domain. Ultimately, the goal is to address time-dependent strong scattering problems in QCD as well. An ideal tool for such problems is Hamiltonian light-front formalism (see, *e.g.*, Ref. [5]), in which the gauge theory is quantized on the light front and the physical states are expanded in a Fock-space basis. The Hamiltonian is represented as an operator acting on these Fock states. Since time is set along the light front, the ground state of the free theory is also a ground state of the full interacting theory and the formalism is Lorentz frame independent. Since we employ a basis rather different from the traditional plane wave basis, we refer to our method as Basis Light-Front Quantization (BLFQ).

2. Outline of BLFQ

We define our light-front coordinates as $x^{\pm} = x^0 \pm x^3$, $x^{\perp} = (x^1, x^2)$, where the variable x^+ is light-front time and x^- is the longitudinal coordinate. We adopt the "null plane" $x^+ = 0$ for our quantization surface. Here we adopt basis states for each constituent that consist of transverse 2D harmonic oscillator (HO) states combined with discretized longitudinal plane waves. This basis function approach follows [6–8] and is buttressed by successful anti-de Sitter-QCD models [9].

The HO states are characterized by a principal quantum number n, orbital quantum number m, and HO energy Ω. Working in momentum space, it is convenient to write the 2D oscillator as a function of the dimensionless variable $\rho = |p^{\perp}|/\sqrt{M_0 \Omega}$, and M_0 has units of mass. The orthonormalized HO wave functions in polar coordinates (ρ, φ) are then given in terms of the generalized Laguerre polynomials, $L_n^{|m|}(\rho^2)$, by

$$\Phi_{nm}(\rho, \varphi) = \sqrt{\frac{2\pi}{M_0 \Omega}} \sqrt{\frac{2n!}{(|m|+n)!}} e^{im\varphi} \rho^{|m|} e^{-\rho^2/2} L_n^{|m|}\left(\rho^2\right) \tag{1}$$

with eigenvalues $E_{n,m} = (2n + |m| + 1)\Omega$.

Our longitudinal modes, ψ_k, are defined for $-L \leq x^- \leq L$ with periodic (antiperiodic) boundary conditions for the photon (electron), *i.e.*,

$$\psi_k(x^-) \; = \; \frac{1}{\sqrt{2L}} \, e^{i \, \frac{\pi}{L} k \, x^-} \, , \tag{2}$$

where $k \in \{1, 2, 3, \ldots\}$ ($k \in \{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots\}$). We neglect the photon zero mode. Then the full 3D single-particle basis state is

$$\Psi_{k,n,m}(x^-, \rho, \varphi) \; = \; \psi_k(x^-) \Phi_{n,m}(\rho, \varphi) \, . \tag{3}$$

Following Ref. [5], we introduce the total invariant mass-squared M^2 for the low-lying physical states in terms of a Hamiltonian H times a dimensionless number for the conserved total light-front momentum K through

$$M^2 = P^+P^- - P_\perp P_\perp = KH - P_\perp P_\perp \,, \tag{4}$$

where the correction for total transverse center of mass kinetic energy depends on the HO scale $1/\sqrt{M_0\Omega}$ and is easily removed from each solution.

We have solved for the electron's anomalous magnetic moment (a_e) when confined to an external trap and we have taken the limit where the trap vanishes [10]. Here we remove the trap altogether. We adopt a Fock space of electron and electron–photon configurations consistent with a total angular momentum projection of 1/2 and total HO quanta (sum over constituents' $2n + |m|$) limited by $N_{\max}$. The transverse infrared properties are still regulated by the finite HO basis and, as demonstrated in Fig. 1, extrapolation to the infinite basis reproduces the expected Schwinger result.

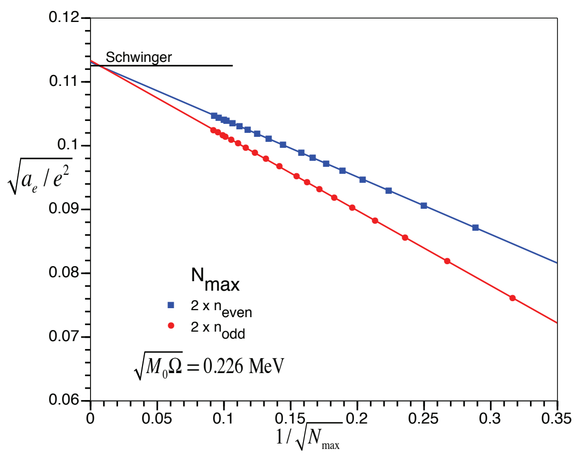

Fig. 1. (Color online) Square root of the electron anomalous magnetic moment (a_e), normalized by e^2, *versus* the inverse square root of the HO basis cutoff $N_{\max}$ at $K = 80.5$. Symbols are BLFQ results: squares (circles) for even (odd) $N_{\max}/2$, spanning $N_{\max} = 10 \ldots 118$. Lines are linear extrapolations based on all points shown and agree favorably with the Schwinger result $\sqrt{1/8\pi^2} = 0.11254$ [11].

3. Extension to include time-dependent processes

We have recently extended the BLFQ to include the non-perturbative time-evolution of light-front systems [12] (tBLFQ). Light-front time evolution of quantum states $|\psi; x^+\rangle$ is governed by the Schrödinger equation,

$$i\frac{\partial}{\partial x^+}|\psi; x^+\rangle = \tfrac{1}{2}P^-\left(x^+\right)|\psi; x^+\rangle \,. \tag{5}$$

In our QED demonstration case [12], the Hamiltonian $P^-(x^+)$ contains two parts; P^-_{QED} is the full Hamiltonian of QED, and $V(x^+)$ is the time-dependent interaction term introduced by the background field, so

$$P^-\left(x^+\right) = P^-_{\mathrm{QED}} + V\left(x^+\right) . \tag{6}$$

It is natural to adopt the interaction picture, where states are defined by

$$|\psi; x^+\rangle_{\mathrm{I}} = e^{\frac{i}{2}P^-_{\mathrm{QED}}x^+}|\psi; x^+\rangle . \tag{7}$$

These states obey the equation

$$i\frac{\partial}{\partial x^+}|\psi; x^+\rangle_{\mathrm{I}} = \tfrac{1}{2}V_{\mathrm{I}}(x^+)|\psi; x^+\rangle_{\mathrm{I}} , \tag{8}$$

in which V_{I}, "the interaction Hamiltonian in the interaction picture", is

$$V_{\mathrm{I}}(x^+) = e^{\frac{i}{2}P^-_{\mathrm{QED}}x^+}V(x^+)e^{-\frac{i}{2}P^-_{\mathrm{QED}}x^+} . \tag{9}$$

The formal solution to (8) is

$$|\psi; x^+\rangle_{\mathrm{I}} = \mathcal{T}_+ \exp\left(-\frac{i}{2}\int_0^{x^+} V_{\mathrm{I}}(x'^+)dx'^+\right)|\psi; 0\rangle_{\mathrm{I}} . \tag{10}$$

In tBLFQ, we solve Eq. (10) non-perturbatively. Some approximation must still be made in order to yield a (numerically) tractable system, but instead of resorting to perturbation theory, we use a Fock-space truncation (in the following example, we use the simplest nontrivial truncation, namely restricting to electron plus electron–photon states, along with $N_{\mathrm{max}} = 8$).

We first use BLFQ to solve for a set of QED mass eigenstates sufficient to include those that will be excited by the laser for its given strength and time duration; this set includes, for example, states with a range of longitudinal motions spanning several "K-segments", since the laser injects longitudinal momentum into the system. We then shine the laser (taken here as the classical source $e\mathcal{A}^-(x^-) = 2ma_0\cos(\omega x^-)$ with $m = 0.511$ MeV, $a_0 = 0.5$, $\omega = 1.0$ MeV, chosen so that it makes transitions between states with longitudinal momenta differing by $\Delta K = 2$) on the target (lowest mass eigenstate in lowest K-segment of longitudinal motion) and time-evolve the states in the basis of the original mass eigenstates according to Eq. (10). The basis uses $\sqrt{M_0\Omega} = 0.511$ MeV along with three K-segments with $K = 1.5$, 3.5, and 5.5 which allows the system to accelerate without (or with) excitation.

With the chosen laser field and basis space we present results for the time evolution of the electron system in Fig. 2. The tBLFQ evolution of the system is both coherent and non-perturbative — yielding the invariant mass distribution for electron + photon final states up to the (arbitrary) time the laser is switched off. Note that by the time at $x^+ = 14\,\mathrm{MeV}^{-1}$, the initial $K = 1.5$ ground state has nearly vanished.

This example demonstrates: (a) tBLFQ is able to generate first-principles-based, non-perturbative results for processes in quantum field theory; and, (b) the full quantum configuration (wavefunction) of the system is available for analyzing dynamical processes at any intermediate time.

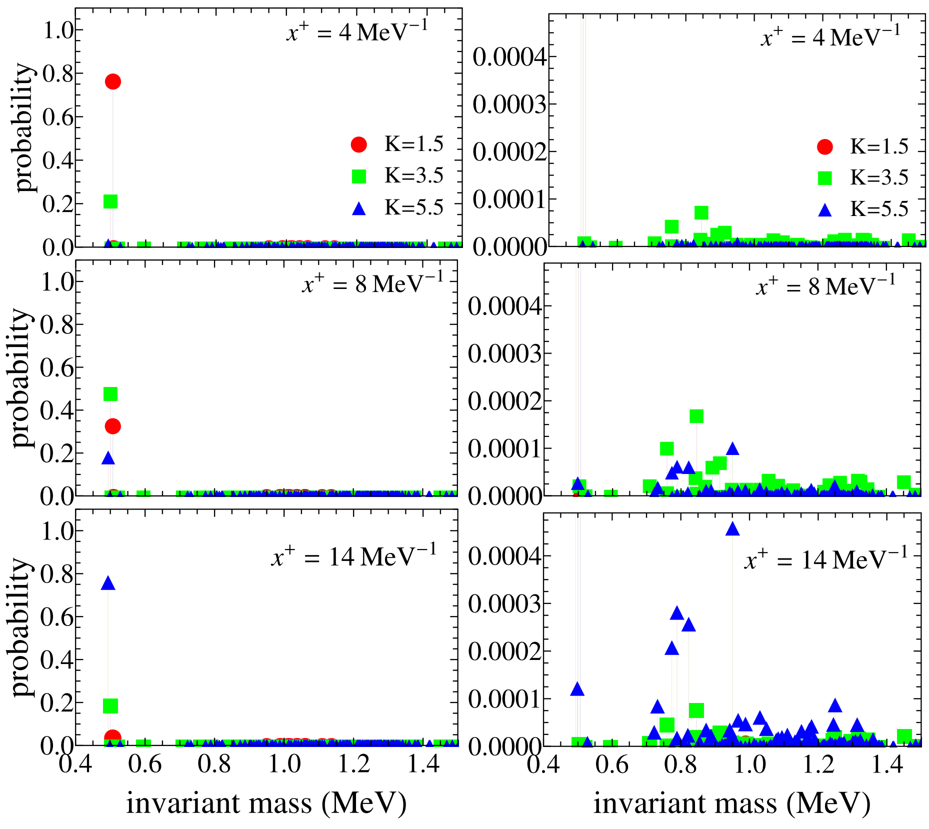

Fig. 2. (Color online) Non-perturbative time evolution of the single electron system in a laser field switched on at $x^+ = 0$ using tBLFQ. From top to bottom, the panels in each row successively correspond to light-front-time $x^+ = 4.0,\ 8.0,\ 14.0\,\mathrm{MeV}^{-1}$. Each dot represents an eigenstate of P^-_{QED}. The y-axis is the probability of finding this state, and the x-axis is the state's invariant mass. The left-hand panels show the evolution of the three ground states in $K = 1.5,\ 3.5,\ 5.5$ segments respectively and the right panels (with the y-axis expanded) show details of the excited state evolutions.

We acknowledge fruitful discussions with K. Tuchin and G.F. de Teramond. This work, SLAC-PUB-15293, was supported in part by the DOE Grant Nos. DE-FG02-87ER40371, DE-FC02-09ER41582 (SciDAC-2/UNEDF), DESC0008485 (SciDAC-3/NUCLEI), DE-FG02-93ER40771, and by the DOE Contract No. DE-AC02-76SF00515 and by US NSF grant 0904782.

REFERENCES

[1] A. Adare *et al.* [PHENIX Collaboration], *Phys. Rev.* **C81**, 034911 (2010).

[2] K. Tuchin, *Phys. Rev.* **C83**, 017901 (2011) [`arXiv:1008.1604v2 [nucl-th]`].

[3] T. Heinzl, A. Ilderton, *Eur. Phys. J.* **D55**, 359 (2009) [`arXiv:0811.1960 [hep-ph]`].

[4] G.V. Dunne, *Eur. Phys. J.* **D55**, 327 (2009) [`arXiv:0812.3163 [hep-th]`].

[5] S.J. Brodsky, H.C. Pauli, S.S. Pinsky, *Phys. Rep.* **301**, 299 (1998).

[6] J.P. Vary *et al.*, *Phys. Rev.* **C81**, 035205 (2010).

[7] J.P. Vary *et al.*, *Nucl. Phys. Proc. Suppl.* **199**, 64 (2010).

[8] P. Maris *et al.*, *Acta Phys. Pol. B Proc. Suppl.* **6**, 321 (2013), this issue.

[9] G.F. de Teramond, S.J. Brodsky, *Phys. Rev. Lett.* **102**, 081601 (2009).

[10] H. Honkanen, P. Maris, J.P. Vary, S.J. Brodsky, *Phys. Rev. Lett.* **106**, 061603 (2011).

[11] J.S. Schwinger, *Phys. Rev.* **73**, 416 (1948).

[12] X. Zhao, A. Ilderton, P. Maris, J.P. Vary, in preparation.