

# Analysis and Simulation of Beam Ion Instability in ILC Damping Ring with Multi-gas Species

Lanfa Wang<sup>#</sup> and Mauro Pivi

SLAC National Accelerator Laboratory, Menlo Park, CA

## *Abstract*

Ion induced beam instability is one critical issue for the electron damping ring of the International Linear Collider (ILC) due to its ultra small emittance of 2 pm. The beam ion instability for the latest DTC lattice has been studied both analytically and numerically. Different from previous studies, multi-gas species and exact beam filling patterns have been modeled simultaneously in this study. This feature makes the studies more realistic and accurate. The numerical code has been benchmarked with SPEAR3 experimental data and there is a good agreement between the simulation and observations. It uses the optics from MAD and can handle arbitrary beam filling pattern and vacuum. Our analyses agree well with expensive simulations.

## I. INTRODUCTION

When the beam emittance becomes small, the trapped (or partially trapped) ions can cause beam instability and emittance blow-up as predicted by Raubenheimer and Zimmermann [1]. The beam ion instability in ILC damping rings has been evaluated for various optics designs [2-5]. The DTC lattice has been recently selected as the official baseline lattice for the TDP-II (Technical Design Phase) of International ILC damping ring [6]. The beam ion instability depends on the optics and beam filling pattern. Therefore, it is important to evaluate the beam ion instability with the new lattice design and new beam filling pattern. In the all previous study, single gas species CO is used. It is more accurate to use a realistic vacuum, for instance, the vacuum with multi-gas species from the existing accelerators. Multi-gas species could provide additional damping to the instability as shown late. Single gas-species model overestimates the beam ion instability. Therefore, it is more accurate to use multi-gas species vacuum model.

The beam ion instability is sensitive to the beam filling pattern. Multi-bunch train is very effective on reduction of the ion density near the beam, for instance [2]. The effects of the beam filling pattern depend on both beam density and the mass of ion [7], therefore, the ion species in the vacuum. Hence it is essential to accurately model both the beam filling pattern and the vacuum simultaneously.

The vacuum in the vacuum chamber of different accelerators varies. Table 1 shows the average vacuum in SPEAR3 vacuum chamber. This paper studies the beam ion instability in ILC damping ring baseline lattice DTC02 and DTC04 with a multi-gas model shown in Table 1, which is a good approximation.

The analysis of beam ion instability has been done for single bunch train case [2,4,8,9] and multi-bunch-train case[7]. The wake field model [7] makes it easy to include both the nonlinear effect of space charge and optics. As we will see in this paper, it also is straightforward with wake field to study the instability with multi-gas species vacuum. The effect of optic is first modeled as a frequency spread [9]. Here we extend it to accurately represent the optics effect by integrating the wake along the ring.

Table 2 lists the main parameters and beam filling patterns for three different beam configurations. KCS and DRFS have the same number of bunches and beam current. They are different only in the beam filling pattern: KCS has longer

bunch train and bunch train gap. FP upgrade mode has highest beam current and the longest bunch train. Note that the beam filling pattern period is not exactly, additional bunch train is followed to make the total bunch number as listed.

Table 1: Vacuum in the SPEAR3 vacuum chamber

Gas Species	Mass Number	Percentage in Vacuum
H <sub>2</sub>	2	48%
CH <sub>4</sub>	16	5%
H <sub>2</sub> O	18	16%
CO	28	14%
CO <sub>2</sub>	44	17%

Table 2: Main parameters of ILC DTC damping ring

Parameter	KCS	DRFS	FP upgrade
Energy[GeV]	5.0	5.0	5.0
Circumference[m]	3238.76	3238.76	3238.76
Emittance $\epsilon_x/\epsilon_y$ [pm]	637/2	637/2	637/2
Harmonic number	7022	7022	7022
Number of bunches	1312	1312	2625
Beam current[mA]	389	389	779
Bunch spacing [ $\lambda_{RF}$ ]	4	4	2
Beam Filling period	19	14	29
Fill pattern (1period)			
<i>Train [bunch number]</i>	34	22	44
<i>Gap [in <math>\lambda_{RF}</math>]</i>	45	33	31
<i>Train [bunch number]</i>	34	22	45
<i>Gap [in <math>\lambda_{RF}</math>]</i>	49	33	31
<i>Train [bunch number]</i>		22	
<i>Gap [in <math>\lambda_{RF}</math>]</i>		33	
		23	
		33	
Bunch length[mm]	6	6	6
Energy spread, $\sigma_\delta$	$1 \times 10^{-3}$	$1 \times 10^{-3}$	$1 \times 10^{-3}$
Mom. compaction, $\alpha$	$3.36 \times 10^{-4}$	$3.36 \times 10^{-4}$	$3.36 \times 10^{-4}$
Tunes, $\nu_x / \nu_y / \nu_s$	48.36 / 27.22 / 0.03	48.36 / 27.22 / 0.03	48.36 / 27.22 / 0.03
Damp times $\tau_x / \tau_y / \tau_s$ [ms]	22 / 22 / 11	22 / 22 / 11	22 / 22 / 11
E loss/turn, $U_0$ [MeV]	4.87	4.87	4.87
RF voltage, $V_{RF}$ [MV]	12.83	12.83	12.83

## II. ANALYSIS

The analysis and simulation of the beam ion instability is hard to get good agreements with the observations for a number of reasons. Especially, the analyses may overestimate the instability growth rate if the important factors are not included. First the nonlinear force of ions should be included. The strong nonlinear field causes a low  $Q$  of the wake,

which is about 16 by analysis [10] and 9 by the simulation [7], respectively. Secondly, the optics effect should be included. The ion frequency spread due to the variation of the beam size along the ring provides Landau damping to the beam instability. The optics effect is represented by a frequency spread in the study of single bunch train instability [9].

Here we present a more practical way on how to analytically model both nonlinear space charge force and optics effect in general case where the frequency spread due to the optics can be arbitrary. The wake field of ion-cloud for a constant beam size is given by [7]

$$W_y(s) = \hat{W}_y e^{-\frac{\omega_i s}{2Q_0 c}} \sin\left(\frac{\omega_i s}{c}\right), \quad (1)$$

$$\hat{W}_y = N_i \left( \frac{r_p S_b}{A N_e} \right)^{1/2} \left[ \frac{4}{3} \frac{1}{\sigma_y (\sigma_y + \sigma_x)} \right]^{3/2}. \quad (2)$$

where  $c$  is the speed of light,  $r_p$  is the classical radius of proton,  $A$  is the mass number of the ion,  $N_e$  is electron bunch population,  $S_b$  is bunch spacing and  $\sigma_{x,y}$  is the transverse root mean square (*rms*) beam size of the electron bunches.  $Q_0$  is the  $Q$  value due to the nonlinear space charge field. The coherent ion frequency is

$$f_{i,y} \approx \frac{c}{2\pi} \left( \frac{4N_e r_p}{3AS_b (\sigma_x + \sigma_y) \sigma_y} \right)^{1/2}, \quad (3)$$

In general case the beam size varies along the ring and the vacuum pressure can also be different from section to section along the ring. The total wake of ions can be calculated from the integral along the whole ring

$$W_{ring}(z) = \int_0^c \frac{4}{3} \frac{\omega_{i,y}(s)}{c} \frac{\lambda_i(s)}{\lambda_e} \frac{1}{\sigma_y(s)(\sigma_y(s) + \sigma_x(s))} e^{-\frac{\omega_i(s)z}{2Q_0 c}} \sin\left(\frac{\omega_i(s)z}{c}\right) ds. \quad (4)$$

Where  $\lambda_i$  is the ion line density. The density of ion-cloud linearly increases along the bunch train and exponentially decays during the bunch train gap with a decay time order of the ion oscillation period [7]. Therefore, the above equation can be used to calculate the wake with arbitrary beam filling pattern where the space charge force of ion cloud is negligible.

Figure 1 shows the betatron function of the whole ring in DCT02 design. There is a large variation of betatron function (700%), and hence the beam size, which causes a large variation of the ion frequency as shown in Figure 2. For instance, the frequencies of  $\text{CO}_2^+$ ,  $\text{CO}^+$ ,  $\text{H}_2\text{O}^+$  and  $\text{CH}_4^+$  form a continuous spectrum, which ranges from 30MHz to 120MHz. This large frequency spread provides effective Landau damping to the beam ion instability.

Figure 3 shows the vertical wake of various ions with a total vacuum pressure of 0.5nTorr in KCS configuration. The density of the ions seen by different bunches varies when beam is not uniformly distributed. The wake here is calculated using the average ion density seen by all bunches. The wake couldn't be simple represented by a single resonance model as shown in Eq. (2) due to the large variation of the beam size along the ring. This indicates a low  $Q$  and a large damping to the beam instability. Each wake can be fitted to multiple resonance model according to Eq. (2) and then the fastest growth rate of the beam ion instability can be estimated using the fitted parameters  $\hat{W}_y$ ,  $Q$ , and  $\omega_i$  [7]

$$\frac{1}{\tau} \approx \frac{N_e n_b r_e c}{2\mathcal{Y}_0^2 \omega_\beta} \sum_m \frac{\hat{W}_{y,m}}{\omega_{i,m}} Q_m \quad (5)$$

Here  $n_b$  is the bunch number,  $r_p$  is the classical radius of electron and  $m$  is the number of resonance wakes.

The alternative way, also a robust way, is to calculate the instability modes directly from the wake function shown in Figure 3. This method is straight forward and can be used in general case. In certain situation, it can be difficult and also less accurate to fit the wake according to the resonance model. When the beam is evenly filled along the ring, the exponential growth rate of the coupled bunch instability for mode  $y_j^\mu \propto e^{2\pi\mu j / n_b}$  is given by the imaginary of the coherent frequency shift with mode number  $\mu$  [10]

$$\Omega_\mu - \omega_\beta \approx \frac{r_e c N_0}{4\pi v_\beta \gamma} \sum_{m=0}^{M-1} \left[ \sum_k W_y \left( kC + \frac{m}{M} C \right) e^{2\pi i v_\beta k} \right] e^{2\pi i (\mu + v_\beta) \frac{m}{M}} \quad (6)$$

Here  $v_y$  is the betatron tune,  $M$  is the bunch number and  $C$  is the circumference,  $r_e$  is the classical radius of electron and  $N_0$  is bunch population. Figure 4 shows the estimated growth rate and tune shift due to ions in KCS configuration. A negative growth rate means unstable. There is a minimum growth time of 0.89 ms and maximum tune shift 0.002 for KCS beam. Three unstable regimes are clear shown for  $\text{CO}_2^+$ ,  $\text{CO}^+$ ,  $\text{H}_2\text{O}^+$ . Each regime of the unstable modes is driven by the ions in different sections along the ring which can be roughly seen from Figure 2. However, the distributions of these unstable modes driven by different ions are different due to the difference in their frequencies. The single gas model overestimates the instability with the simple assumption that the mode numbers driven by different ions are the same. It is more interesting that the unstable modes around 900 driven by  $\text{H}_2^+$  are damped, while these modes driven by other ions are unstable. Thence the total growth rate with multi-gas effect is actually reduced. Therefore, it is crucial to model the beam ion instability using multi-gas species vacuum.

Figure 5-6 show the unstable modes for DRFS and FP upgrade configuration, respectively. Benefitting from the shorter bunch train, the DRFS configuration has a longer growth time of 1.2 ms although it as the same beam-current as KCS. The FP upgrade configuration has shortest growth time of 0.7 ms due to its high beam current. The damping effect of  $\text{H}^+$  on the unstable beam is large with FP upgrade beam.

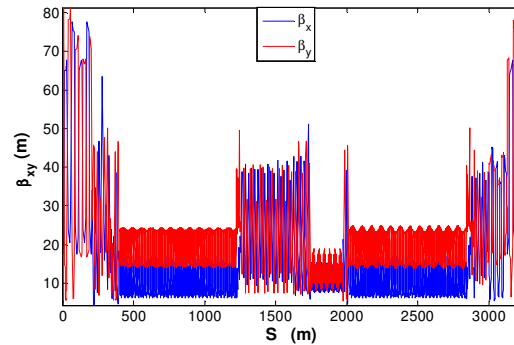


Figure 1. Betatron functions of the DTC02 lattice

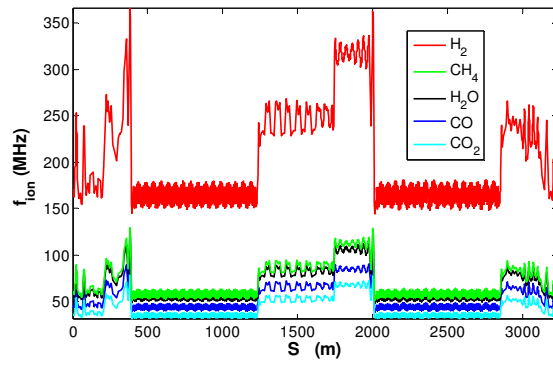


Figure 2. Vertical ion frequency along the ring with KCS beam.

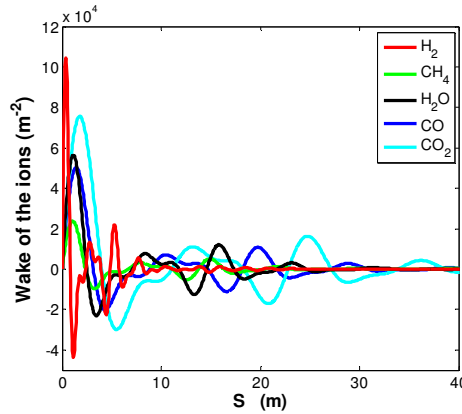


Figure 3. The vertical wake field driven by various ions in the vacuum with KCS beam. The total vacuum pressure is  $0.5 \text{ nTorr}$  with partial gas pressures percentage shown in Table 1.

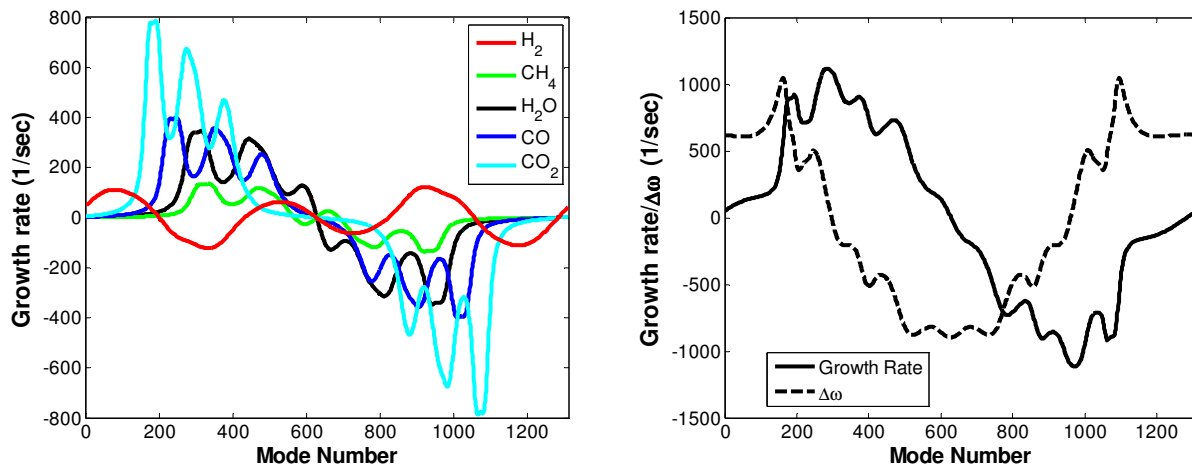


Figure 4. The growth rate of unstable modes driven by various ions (a) and the growth rate and tune shift due to all ions (b) calculated from the wake shown in Figure 3.

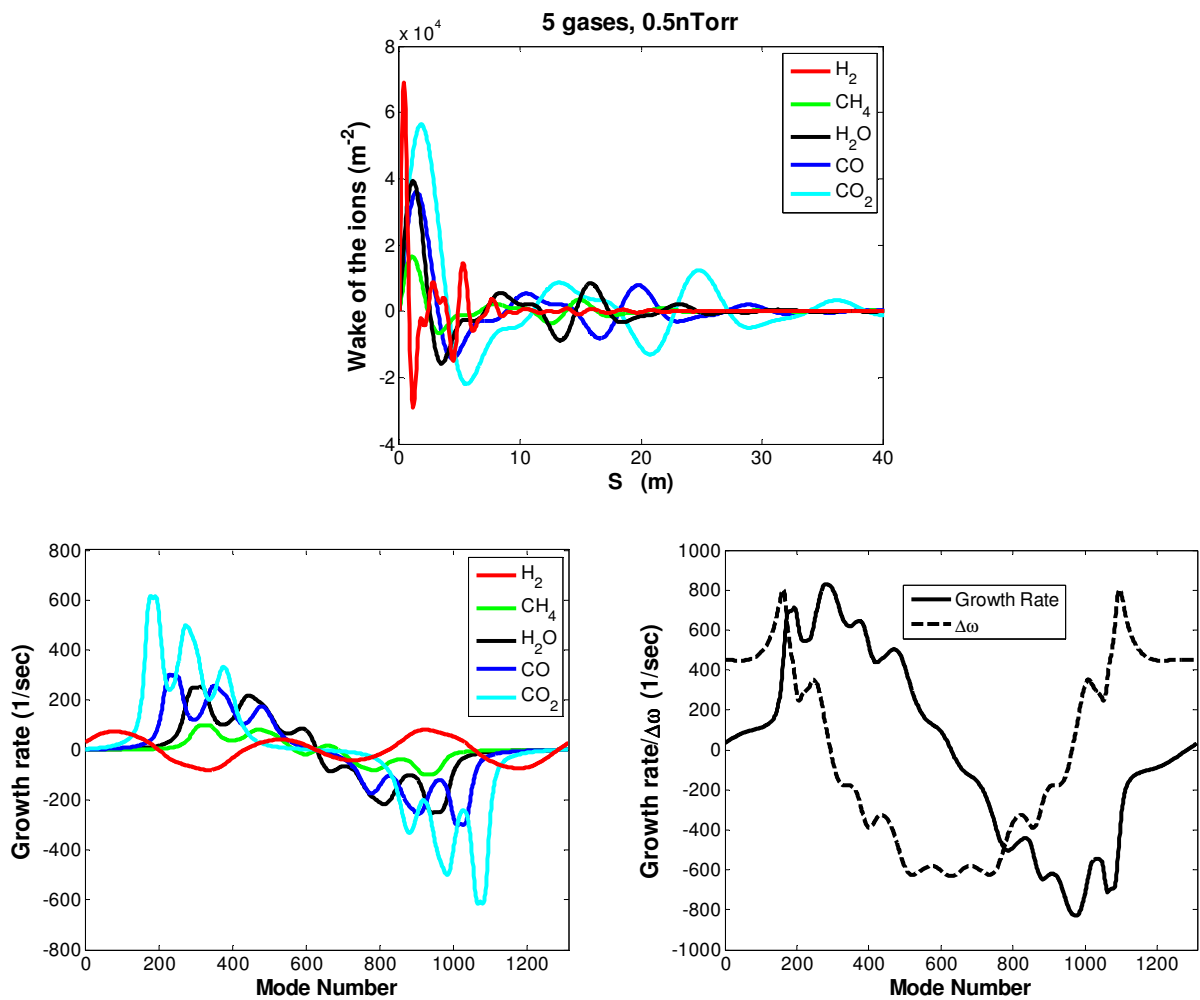
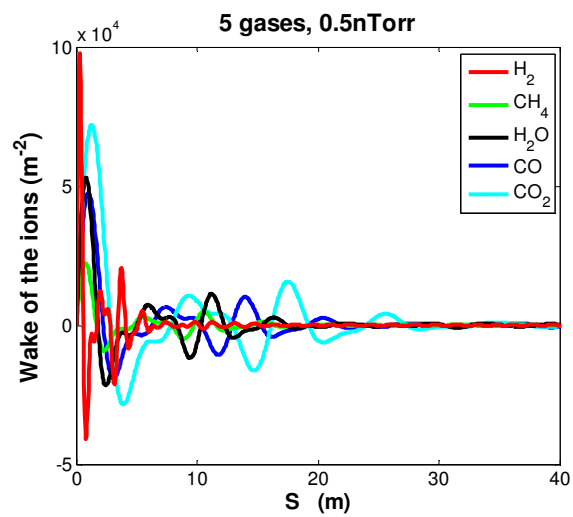


Figure 5. The vertical wake field (a) and unstable modes (b) driven by various ions in the vacuum with DRFS beam. The growth rate and tune shift due to all ions is shown in (c). The total vacuum pressure is 0.5 nTorr with partial gas pressures percentage shown in Table1.



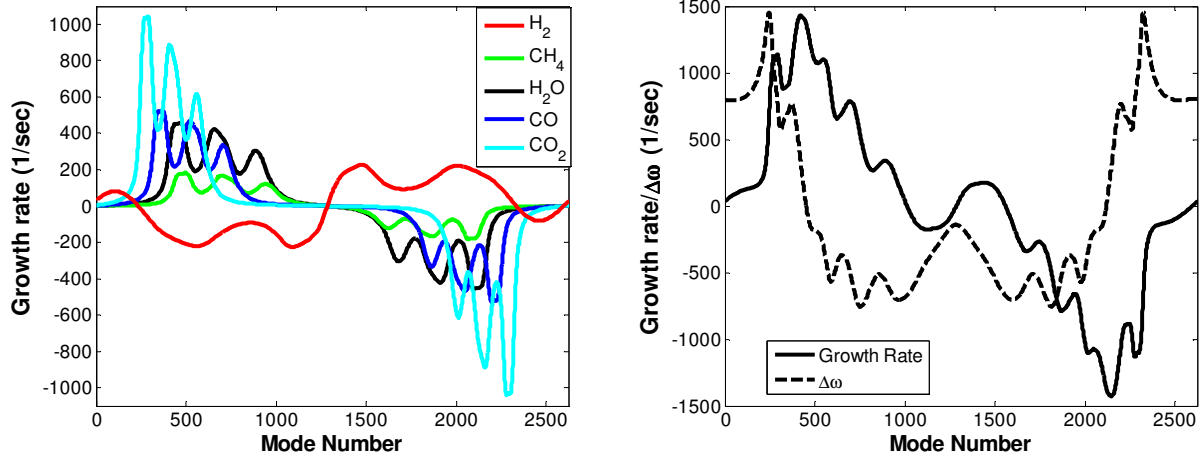


Figure 6. The vertical wake field (a) and unstable modes (b) driven by various ions in the vacuum with FP upgrade beam. The growth rate and tune shift due to all ions is shown in (c). The total vacuum pressure is  $0.5 \text{ nTorr}$  with partial gas pressures percentage shown in Table 1.

### III. SIMULATIONS

Simulation has a number of advantages in the study of the beam-ion instability: the nonlinearity of the ion-cloud is automatically included; the effects of optics and bunch-train gap with arbitrary beam filling pattern can be easily handled; a realistic vacuum model with multi-gas species is straightforward in simulation. A Particle in Cell (PIC) code based on wake-strong model is used here [7]. The code has been benchmarked with SPEAR3 experiment [11] and there is a good agreement.

The SPEAR3 vacuum shown in Table 1 with a total pressure of  $0.5 \text{ nTorr}$  is used in the simulation. A uniform vacuum pressure along the ring is assumed. It is essential to study the beam ion instability using realistic vacuum model with multi-gas species to accurately model the effects of optics, beam filling pattern and possible extra Landau damping due to multi-gas species effect. The exact beam filling patterns shown in Table 2, which are not uniform bunch-train fillings, are used in the simulations.

Figure 7 shows the simulated vertical beam ion instability with KCS beam. There is an exponential growth when the amplitude is smaller than the beam size, and then it grows linearly due to the nonlinearity of space charge force. There is a broad band spectrum and the distribution of the unstable modes agrees well with the analysis shown in Figure 4. Figure 8 shows the instability of DRFS and FP upgrade configuration. The fastest exponential growth times for the three configurations are  $0.61 \text{ ms}$ ,  $0.91 \text{ ms}$  and  $0.40 \text{ ms}$ , respectively. The simulations give slightly shorter growth time than the analyses. Note that the growth time in simulations is given by fitting the maximum vertical amplitude of all bunches.

The optics of DCT is recently updated. Figure 10 shows the beam ion instability with DCT04 optics[6]. There is a smaller vertical emittance of  $1.2 \text{ pm}$  in DCT04 design comparing with  $2 \text{ pm}$  as in DCT02. Therefore the growth time becomes shorter: they are  $0.59 \text{ ms}$ ,  $0.80 \text{ ms}$  and  $0.29 \text{ ms}$  for KCS, DRFS and FP upgrade beam, respectively.

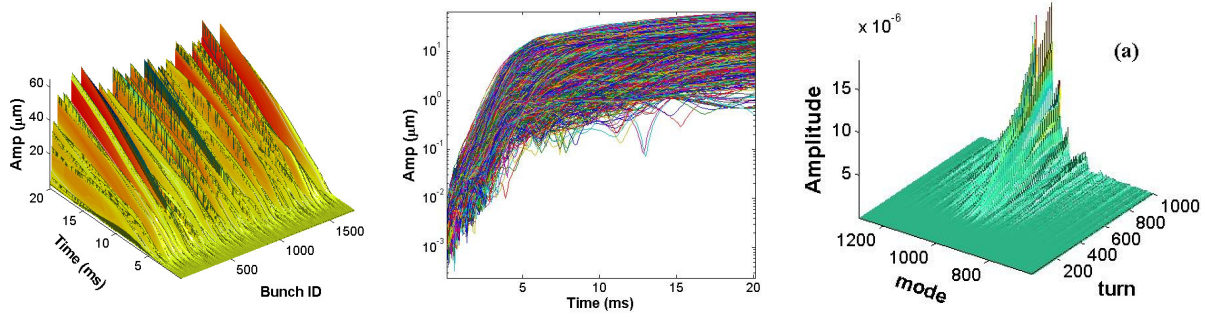


Figure 7. Simulated vertical beam ion instability in KCS configuration: growth of vertical amplitude of all bunches (a) and (b); growth of unstable modes(c). The vertical oscillation amplitude in (a) is in linear scale, while it is in logarithmic scale in (b). The different lines in the plots are for different bunches. There are total 1312 bunches. The vertical instability growth time is  $0.61\text{ ms}$ .

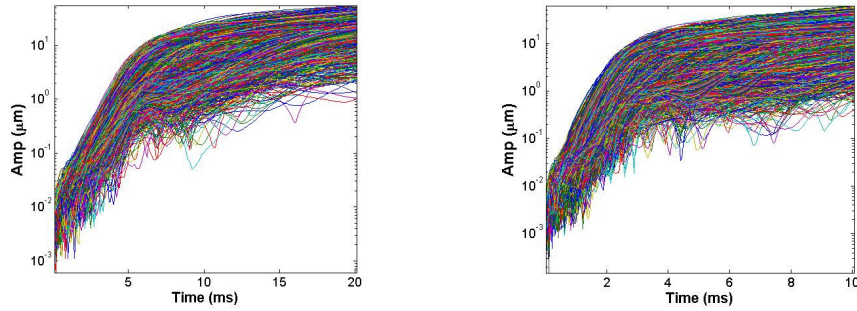


Figure 8. Simulated vertical beam ion instability in DRFS (a) and FP upgrade (b) configuration. There are total 1312 and 2625 bunches, respectively. The vertical instability growth time is  $0.91\text{ ms}$  and  $0.40\text{ ms}$ .

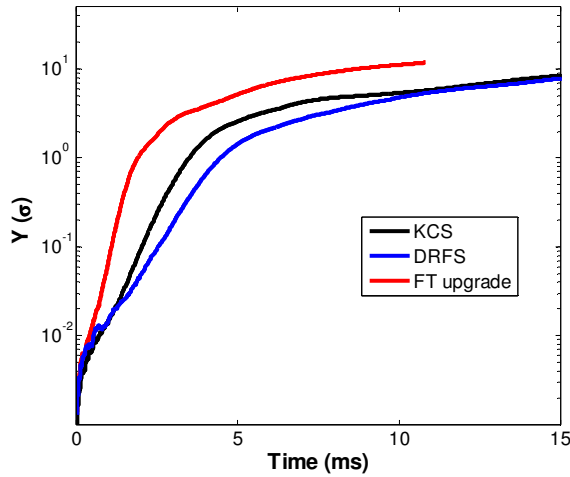


Figure 9. Simulated vertical beam ion instability with DCT04 lattice for various beam configurations

#### IV. ACKNOWLEDGEMENTS

We would like to acknowledge Mark A. Palmer and Susanna Guiducci for their supporting of this work.



## V. SUMMARY AND CONCLUSION

The beam ion instabilities in the new ILC damping ring have been studied using realistic vacuum with multi-gas species, exact beam filling pattern and exact optics. We first time calculate the ion instability using a simple analysis formula, which includes the effect of realistic optics and beam filling pattern. Our analyses agree with the expensive simulation. There is also an excellent agreement in the distributions of unstable modes.

With a conservative total vacuum pressure of  $0.5 \text{ nTorr}$ , the growth times are 0.61 ms, 0.91ms and 0.40 ms for KCS, DRFS and FP upgrade, respectively. The growth time is much shorter the radiation damping time of 11.0 ms. The instability can be mitigated by a large chromaticity in the expense of lifetime and injection efficiency. A bunch-by-bunch feedback can be used to suppress the instability. However, how good a feedback at  $\mu\text{m}$  level is not yet fully confirmed experimentally, for instance, the noise. More R&D is necessary.

## REFERENCES

- [1] T. Raubenheimer and F. Zimmermann, Phys. Rev. E52, No. 5, 5487 (1995).
- [2] L. Wang, T. Raubenheimer and A. Wplski, Proceeding of EPAC06, 2155(2006).
- [3] G. Xia and E. Elsen, Nucl. Inst. Methods, A593, 183 (2008)
- [4] E.S. Kim and K. Ohmi, Japanese Journal of Applied Physics 48, 086501 (2009)
- [5] Guoxing Xia, p1671, Proceedings of 2011 Particle Accelerator Conference, New York, NY, USA (2011)
- [6] <https://wiki.lepp.cornell.edu/ilc/bin/view/Public/DampingRings/WebHome>
- [7] L. Wang, et al., Physical Review Special Topics – Accelerators and Beams, 14, 084401 (2011)
- [8] G.V. Stupakov, T.O. Raubenheimer, F. Zimmermann, Phys. Rev. E52, 5499 (1995)
- [9] G. V. Stupakov, KEK Proceedings 96-6, 243 (1996)
- [10] A. Chao, “Physics of Collective Beam Instabilities in High Energy Accelerators”, Wiley, (1995).
- [11] L. Wang, et al., p814, Proceedings of IPAC2011, San Sebastián, Spain(2011)