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Status of Monte-Carlo event generators

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Abstract. Recent progress on general-purpose Monte-Carlo event generators is reviewed with emphasis on the simulation of hard QCD processes and subsequent parton cascades.

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INTRODUCTION

Describing full final states of high-energy particle collisions in contemporary experiments is an intricate task. Hundreds of particles are typically produced, and the reactions involve both large and small momentum transfer. The high-dimensional phase space makes an exact solution of the problem impossible. Instead, one typically resorts to regarding events as factorized into different steps, ordered descending in the mass scales or invariant momentum transfers which are involved. In this picture, a hard interaction, described through fixed-order perturbation theory, is followed by multiple Bremsstrahlung emissions off initial- and final-state and, finally, by the hadronization process, which binds QCD partons into color-neutral hadrons. Each of these steps can be treated independently, which is the basic concept inherent to general-purpose event generators. Their development is nowadays often focused on an improved description of radiative corrections to hard processes through perturbative QCD. In this context, the concept of jets is introduced, which allows to relate sprays of hadronic particles in detectors to the partons in perturbation theory [1].

In this talk, we briefly review recent progress on perturbative QCD in event generation. The main focus lies on the general-purpose Monte-Carlo programs HERWIG [2, 3], PYTHIA [4, 5] and SHERPA [6, 7], which will be the workhorses for LHC phenomenology. A detailed description of the physics models included in these generators can be found in [8]. We also discuss matrix-element generators, which provide the parton-level input for general-purpose Monte Carlo.

HARD PROCESSES

Traditionally, event generators implement hard processes at lowest order in the perturbative expansion, i.e. as $2 \to 2$ or at most $2 \to 3$ scatterings. This leads to serious deficiencies in the description of final states with large jet multiplicity, as the production of most jets must in turn be simulated through parton showers. To achieve at least leading-order accuracy for related observables, the computation of tree-level matrix elements with arbitrary final-state multiplicity is required. This task is handled by programs like

ALPGEN [9], AMEGIC [10], COMIX [11], HELAC [12], and MADGRAPH [13]. They are widely used to generate parton-level events, which are processed through event generators for showering and hadronization. As such, although independent programs in principle, matrix-element generators like the above should be regarded as part of the simulation chain in general-purpose Monte Carlo. They can be extended to include new physics models, but implementation and validation are usually cumbersome. This task was alleviated through FEYNRULES [14], a Mathematica package, which allows to automatically derive interaction vertices from virtually arbitrary Lagrangians. Recent tests of the approach using MADGRAPH and AMEGIC have proved very successful [15]. MADGRAPH also allows to automatically construct new routines for evaluating effective operators that are not yet included in the main program [16].

Predictions for observables in multi-jet final states involve high powers of the strong coupling, and thus, they have large associated uncertainties. It is often desirable to improve the description of high-multiplicity events through next-to-leading order calculations. This involves the computation of virtual and real corrections, which can be combined in an automated way using infrared subtraction algorithms [17, 18, 19, 20, 21, 22, 23, 24]. Recently, many tree-level matrix-element generators were therefore extended with a subtraction procedure. AMEGIC was the first generator to automate Catani-Seymour dipole subtraction [25], followed by MADGRAPH [26, 27] and HELAC [28]. A generator-independent program was presented in [29], and a Mathematica package was advertised in [30]. MADGRAPH also provides the FKS subtraction procedure [31]. The computation of $pp \to W$+4 jets [32] and $pp \to Z$+3 jets [33] at next-to-leading order with AMEGIC, $e^+e^- \to$5 jets [34] and a variety of other processes [35] with MADGRAPH, and $pp \to t\bar{t}b\bar{b}$ [36] with HELAC have proved the versatility of the various implementations.

By means of infrared subtraction, almost all parts of next-to-leading order cross sections can nowadays be computed using extended tree-level techniques. The missing piece is the finite remainder of virtual corrections, which often poses the greatest challenge, both because of complexity and numerical stability of the calculation. Tremendous progress was made in this field, leading to new computational algorithms [37, 38, 39, 40, 41, 42, 43] based on generalized unitarity [44, 45]. Fully automated calculations of one-loop corrections have since become available in the BLACKHAT [46], HELACNLO [47], MADLOOP [35], ROCKET [48] and SAMURAI [49] programs, as well as various others [50, 51]. Additionally, more traditional, Feynman-diagram based techniques have been extended [52, 53] and applied for example to the process $pp \to W^+W^-b\bar{b}$ [54]. In this context, tree-level matrix element generators often serve as a framework to carry out the numerical integration over phase-space. A generic interface between the two types of programs was proposed in [55]. Table 1 shows an example of recent next-to-leading order results for W+4 jet production, obtained with BLACKHAT and SHERPA [32]. It exemplifies the possible synergy between general-purpose Monte-Carlo and the so-called one-loop engines.

PARTON SHOWERS

While the production rate of jets in high-energy collisions is often described well by fixed-order matrix elements, jet shapes cannot be reflected in such calculations. On the

TABLE 1. Total cross sections in pb for W+n jet production at the LHC (7 TeV). The NLO result for W+4 jets uses a leading-color approximation. Numerical integration uncertainties are in parentheses, the scale dependence is quoted in super- and subscripts. Table taken from [32].

no. jets	W^- LO	W^- NLO	W^+/W^- LO	W^+/W^- NLO	$W^-n/(n-1)$ LO	$W^-n/(n-1)$ NLO
0	$1614.0(0.5)^{+208.5}_{-235.2}$	$2077(2)^{+40}_{-31}$	1.656(0.001)	1.580(0.004)	—	—
1	$264.4(0.2)^{+22.6}_{-21.4}$	$331(1)^{+15}_{-12}$	1.507(0.002)	1.498(0.009)	$0.1638(0.0001)^{+0.044}_{-0.031}$	0.159(0.001)
2	$73.14(0.09)^{+20.81}_{-14.92}$	$78.1(0.5)^{+1.5}_{-4.1}$	1.596(0.003)	1.57(0.02)	$0.2766(0.0004)^{+0.051}_{-0.037}$	0.236(0.002)
3	$17.22(0.03)^{+8.07}_{-4.95}$	$16.9(0.1)^{+0.2}_{-1.3}$	1.694(0.005)	1.66(0.02)	$0.2354(0.0005)^{+0.034}_{-0.025}$	0.216(0.002)
4	$3.81(0.01)^{+2.44}_{-1.34}$	$3.55(0.04)^{+0.08}_{-0.30}$	1.812(0.001)	1.73(0.03)	$0.2212(0.0004)^{+0.026}_{-0.020}$	0.210(0.003)

other hand, jets do have a finite size in practice, and their substructure has come into the focus of interest in the context of new physics searches [1, 56, 57].

Within the framework of perturbative QCD, the inner structure of jets can be understood in terms of collinear factorization properties of scattering amplitudes. This observation allows to devise parton showers as suitable Monte-Carlo algorithms, which can turn any n-parton event into an $n+1$-parton event by making use of approximate real-radiation cross sections and a unitarity condition. Repeated application of this procedure sums leading and certain subleading logarithmic corrections to the n-parton production process. The difference between existing parton-shower implementations in HERWIG [58, 59], PYTHIA [60] and SHERPA [61, 62] lies in the parametrization of the radiative phase space, the splitting functions which are employed and, in particular, the splitting kinematics: Matrix elements for hard processes involve on-shell partons, but emissions in the parton shower generate a virtuality for the splitter. Hence, four-momentum must be shuffled between partons in some way to be conserved. The collinear approximation does not specify how this should be done.

SHERPA implements a novel dipole-like parton shower [61, 62], which was formally introduced in [63, 64], and which is based on the Catani-Seymour dipole subtraction method in the large-N_c approximation. The advantage compared to traditional parton showers is an improved treatment of soft-collinear regions, where the DGLAP splitting functions are modified to give the correct soft-gluon radiation pattern. This is achieved through a dependence on the momentum of the recoil partner, which is identified as the color partner of the splitter in the large-N_c approximation. Similar ideas have been investigated in HERWIG [65]. Within PYTHIA, recent development focused on improved matching to hard processes at next-to-leading order [66] and on incorporating multiple scattering and rescattering effects into shower simulations [60, 67]. Figure 1 shows a comparison of results from the three different parton-shower algorithms in HERWIG, PYTHIA and SHERPA. It can be seen that, although the programs exhibit large algorithmic differences, their predictions for experimentally observable quantities are very similar.

An alternative way of formulating parton evolution is directly in terms of emission from sets of color-connected partons, called dipoles [71]. This approach was first used in ARIADNE [72], and has recently been revisited by several groups. The first modern event generator to implement a dipole shower was SHERPA [73], while independent programs have been presented in [74, 75]. Spin-dependent splitting functions were recently computed [76, 77]. For most purposes, dipole showers can be considered equivalent to coherence-improved parton showers as discussed in [8]. Some doubt was

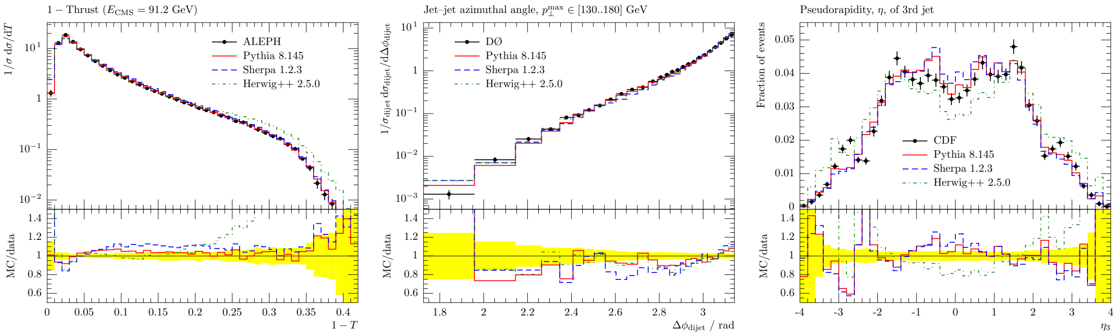

FIGURE 1. Left: Thrust distribution at LEP, measured by ALEPH [68]. Middle: Di-jet azimuthal decorrelation, measured by DØ [69]. Right: Pseudorapidity distribution of the third jet in QCD events as test for color coherence, measured by CDF [70]. Experimental data are compared to predictions from HERWIG, PYTHIA and SHERPA. Figures taken from [8].

cast on the validity of the method by [78], but it is likely that the mismatch with DGLAP evolution found therein is an artifact of the toy model used for the study [79, 80] and is thus not shared by full implementations.

For scattering processes involving partons with very small momentum fraction x compared to the incoming hadrons, logarithms of $1/x$ can be large and should be resummed in the BFKL or CCFM approach. The new parton-level event generator HEJ [81] implements an improved scheme for high-energy evolution [82, 83], based on the BFKL equation in next-to-leading logarithmic approximation. Its predictions often differ significantly from both, next-to-leading order and parton-shower results, see for example [84]. The parton shower programs SMALLX [85] and CASCADE [86, 87] follow similar ideas, but they focus on the CCFM approach. It seems likely that a variety of hard processes at the LHC with momentum fractions below 10^{-4} will be affected by effects described in these Monte-Carlo programs.

MATRIX-ELEMENT PARTON-SHOWER MERGING

Higher-order tree-level calculations and parton showers, as introduced above, are two essentially complementary approaches to simulating perturbative QCD interactions in general-purpose Monte-Carlo. It is desirable to combine both, in order to get an improved description of the event structure. This is best seen with an example, say the production of Drell-Yan lepton pairs plus jets. Describing a particular $l\bar{l}$+n-jet final state with $l\bar{l}$+n-parton tree-level matrix elements gives an estimate of the inclusive production rate and of jet-jet and jet-lepton correlations. However, the inner structure of jets cannot be resolved. Simulating the same final state through matrix elements for $l\bar{l}$ production plus subsequent parton showers gives a description in terms of $l\bar{l}$ plus m partons, where $n \leq m < \infty$. While jets now have substructure, production rates are predicted exclusively and in the collinear approximation. Next-to-leading order calculations of $l\bar{l}$+n-jet production combine parts of both approaches, as they include real-radiation corrections, which can be constrained in the phase space to emulate the Sudakov suppression in parton showers.

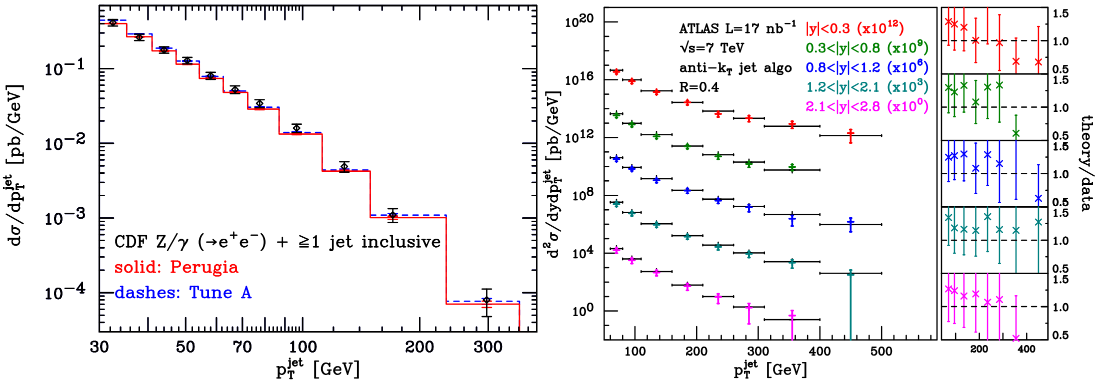

FIGURE 2. Left: Transverse momentum of the first jet in Z+jet events, measured by CDF [88]. Right: Double differential jet cross section, measured by ATLAS [89]. Experimental data are compared to predictions from POWHEG combined with PYTHIA. Figures taken from [90, 91].

Historically, the first generic methods for systematically improving parton showers with higher-order matrix elements were the merging methods pioneered in [92, 93, 94], and further worked out in different varieties at different accuracies and for different parton showers in [95, 96, 97, 98, 99]. In this approach, tree-level matrix elements modified by Sudakov suppression factors are used to describe exclusive n-jet processes with different jet multiplicity. Lately, a new formulation has been proposed which can be proved to preserve the formal accuracy of the parton shower, independent of the process under consideration [100, 101, 102]. It relies on so-called truncated vetoed parton showers [103], which are employed to compute Sudakov suppression factors and to generate soft wide-angle radiation, which can occur before hard emissions. Despite varying degrees of formal accuracy amongst the various merging methods, their respective predictions tend to agree on a level expected from improved leading-order perturbation theory. Corresponding comparisons were presented in [104, 105].

However, the technique still suffers from one major drawback of all tree-level approaches, which is their instability with respect to scale variations. This deficiency necessitates the implementation of NLO virtual corrections. Two universally applicable methods to accomplish this were suggested in the past, which are dubbed MC@NLO [109, 110] and POWHEG [103, 111]. They combine full next-to-leading order predictions for inclusive processes ($l\bar{l}$ production in the above example) with subsequent parton showers, either by defining a suitable subtraction procedure to regularize the real-radiation contribution, or by matrix-element correction of the branching probability in the parton shower combined with suitable local K-factors. Both methods were recently applied to a variety of processes, using the event generation frameworks of HERWIG and PYTHIA [112, 113, 114, 115, 116, 117]. In contrast to MC@NLO, the POWHEG method does not depend on the specific parton-shower algorithm, hence, independent implementations exist [118, 119, 90, 91, 120]. Figure 2 shows some representative results. Within SHERPA, the POWHEG method has been fully automated [121].

Having both tree-level merging and the MC@NLO and POWHEG methods at hand, the question naturally arises, whether those two approaches can be combined. In the

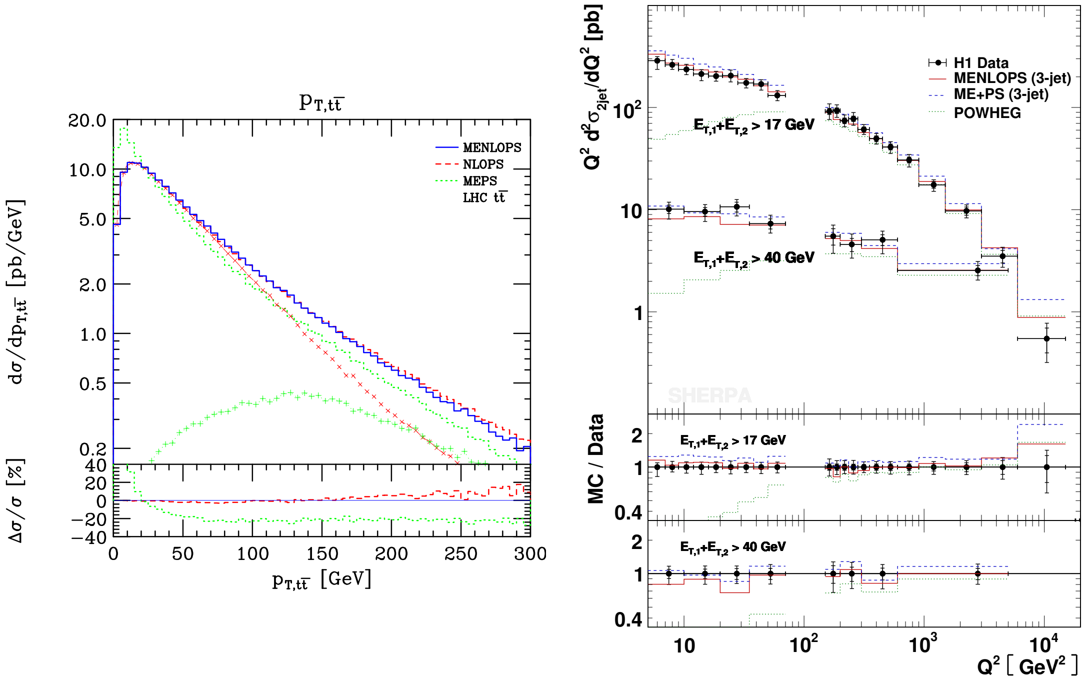

FIGURE 3. Left: Transverse momentum of the $t\bar{t}$-system in $t\bar{t}$+jets production at the LHC (14 TeV). Figure taken from [106]. Right: Di-jet cross section as measured by H1 [107]. Figure taken from [108].

example of Drell-Yan lepton pair plus jets production, this would amount to the simulation of the $l\bar{l}$-production process at next-to-leading order, while $l\bar{l}$+n-jet production with $n > 0$ enters at tree-level. The corresponding proposal was made independently in [106] and in [108]. Figure 3 shows an example for the quality of the corresponding predictions. The problem of including higher-multiplicity next-to-leading order results was investigated in [122] and is currently in the focus of interest.

EVENT GENERATOR VALIDATION AND TUNING

Monte-Carlo event generators have a variety of free parameters, which can be tuned such that predictions better match experimental data. Many of these parameters are connected to fragmentation models and underlying-event simulation, or more general, to models for non-perturbative QCD effects. The resulting parameter space can be quite large, which makes it impossible to find an optimal solution by hand. On the other hand, once a certain set of observables is described satisfactorily, one might want to test the same set of parameters in other analyses.

Recently, two new tools have been developed, which attack these problems using a generator-independent validation and tuning strategy. RIVET [123], as the successor of HZTOOL [124], implements analyses from the LEP, Tevatron and LHC experiments in a common framework and allows simultaneous tests of Monte-Carlo output against all available data. PROFESSOR [125] employs RIVET to semi-automatically find the best point in the parameter space of the event generator. With RIVET, the Tevatron and LHC experiments are given a tool to preserve the full details of their analyses for future exploitation of the data.

SUMMARY

Modern general-purpose event generators are highly sophisticated tools for LHC phenomenology. They often implement perturbative QCD calculations at next-to-leading order in the strong coupling and they provide parton showers to include resummation effects. Their underlying parameters are mostly related to non-perturbative QCD aspects. Many extensions of event generators exist, allowing them to become a platform for testing new physics models and improved descriptions of perturbative QCD in the same framework. The validation and tuning of event generators has been simplified considerably.

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