# Accessing the distribution of linearly polarized gluons in unpolarized hadrons 

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#### Abstract

Gluons inside unpolarized hadrons can be linearly polarized provided they have a nonzero transverse momentum. The simplest and theoretically safest way to probe this distribution of linearly polarized gluons is through $\cos 2 \phi$ asymmetries in heavy quark pair or dijet production in electron-hadron collisions. Future Electron-Ion Collider (EIC) or Large Hadron electron Collider ( LHeC ) experiments are ideally suited for this purpose. Here we estimate the maximum asymmetries for EIC kinematics.


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## INTRODUCTION

Linearly polarized gluons in an unpolarized hadron, carrying a light-cone momentum fraction $x$ and transverse momentum $\boldsymbol{p}_{T}$ w.r.t. to the parent's momentum, are described by the transverse momentum dependent distribution (TMD) $h_{1}^{\perp g}\left(x, \boldsymbol{p}_{T}^{2}\right)[1,2,3]$. Unlike the quark TMD $h_{1}^{\perp q}$ of transversely polarized quarks inside an unpolarized hadron (also frequently referred to as Boer-Mulders function) [4], $h_{1}^{\perp g}$ is chiral-even and $T$ even. This means it does not require initial or final state interactions (ISI/FSI) to be nonzero. Nevertheless, as any TMD, $h_{1}^{\perp g}$ can receive contributions from ISI or FSI and therefore can be process dependent, in other words, non-universal, and its extraction can be hampered in nonfactorizing cases.

Thus far no experimental studies of $h_{1}^{\perp g}$ have been performed. As recently pointed out, it is possible to obtain an extraction of $h_{1}^{\perp g}$ in a simple and theoretically safe manner, since unlike $h_{1}^{\perp q}$ it does not need to appear in pairs [3]. Here we will discuss observables that involve only a single $h_{1}^{\perp g}$ in semi-inclusive DIS to two heavy quarks or to two jets, which allow for TMD factorization and hence a safe extraction. The corresponding

[^0]hadroproduction processes run into the problem of factorization breaking [3, 5].

## AZIMUTHAL ASYMMETRIES

We first consider heavy quark (HQ) production, $e(\ell)+h(P) \rightarrow e\left(\ell^{\prime}\right)+Q\left(K_{1}\right)+\bar{Q}\left(K_{2}\right)+X$, where the four-momenta of the particles are given within brackets, and the heavy quarkantiquark pair in the final state is almost back-to-back in the plane perpendicular to the direction of the exchanged photon and hadron. We look at the heavy quarks created in the photon-gluon fusion process, which can be distinguished kinematically from intrinsic charm production; e.g., from the $Q \bar{Q}$ invariant mass distribution. The calculation proceeds along the lines explained in Refs. [2, 6]. We obtain for the cross section integrated over the angular distribution of the back-scattered electron $e\left(\ell^{\prime}\right)$ :

$$
\begin{equation*}
\frac{d \sigma}{d y_{1} d y_{2} d y d x_{B} d^{2} \boldsymbol{q}_{T} d^{2} \boldsymbol{K}_{\perp}}=\frac{\alpha^{2} \alpha_{s}}{\pi s M_{\perp}^{2}} \frac{\left(1+y x_{B}\right)}{y^{5} x_{B}}\left(A+B \boldsymbol{q}_{T}^{2} \cos 2 \phi\right) \delta\left(1-z_{1}-z_{2}\right) . \tag{1}
\end{equation*}
$$

This expression involves the standard DIS variables: $Q^{2}=-q^{2}$, where $q$ is the momentum of the virtual photon, $x_{B}=Q^{2} / 2 P \cdot q, y=P \cdot q / P \cdot \ell$ and $s=(\ell+P)^{2}=2 \ell \cdot P=$ $2 P \cdot q / y=Q^{2} / x_{B} y$. Furthermore, we have for the HQ transverse momenta $K_{i \perp}^{2}=-\boldsymbol{K}_{i \perp}^{2}$ and introduced the rapidities $y_{i}$ for the HQ momenta (along photon-target direction). We denote the proton mass with $M$ and the heavy (anti)quark mass with $M_{Q}$. For the partonic subprocess we have $p+q=K_{1}+K_{2}$, implying $z_{1}+z_{2}=1$, where $z_{i}=P \cdot K_{i} / P \cdot q$. We introduced the sum and difference of the HQ transverse momenta, $K_{\perp}=\left(K_{1 \perp}-K_{2 \perp}\right) / 2$ and $q_{T}=K_{1 \perp}+K_{2 \perp}$, considering $\left|q_{T}\right| \ll\left|K_{\perp}\right|$. In that situation, we can use the approximate HQ transverse momenta $K_{1 \perp} \approx K_{\perp}$ and $K_{2 \perp} \approx-K_{\perp}$ denoting $M_{i \perp}^{2} \approx M_{\perp}^{2}=$ $M_{Q}^{2}+\boldsymbol{K}_{\perp}^{2}$. The azimuthal angles of $\boldsymbol{q}_{T}$ and $\boldsymbol{K}_{\perp}$ are denoted by $\phi_{T}$ and $\phi_{\perp}$ respectively, and $\phi \equiv \phi_{T}-\phi_{\perp}$. The functions $A$ and $B$ depend on $y, z\left(\equiv z_{2}\right), Q^{2} / M_{\perp}^{2}, M_{Q}^{2} / M_{\perp}^{2}$, and $\boldsymbol{q}_{T}^{2}$.

The angular independent part $A$ is non negative and involves only the unpolarized TMD gluon distribution $f_{1}^{g}, A \equiv e_{Q}^{2} f_{1}^{g}\left(x, \boldsymbol{q}_{T}^{2}\right) \mathscr{A}^{e g \rightarrow e Q \bar{Q}} \geq 0$. We focus on the magnitude $B$ of the $\cos 2 \phi$ asymmetry, which is determined by $h_{1}^{\perp g}$. Namely,

$$
\begin{equation*}
B=\frac{1}{M^{2}} e_{Q}^{2} h_{1}^{\perp g}\left(x, \boldsymbol{q}_{T}^{2}\right) \mathscr{B}^{e g \rightarrow e Q \bar{Q}} \tag{2}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathscr{B}^{e g \rightarrow e Q \bar{Q}}=\frac{1}{2} \frac{z(1-z)}{D^{3}}\left(1-\frac{M_{Q}^{2}}{M_{\perp}^{2}}\right) a(y)\left\{[2 z(1-z) b(y)-1] \frac{Q^{2}}{M_{\perp}^{2}}+2 \frac{M_{Q}^{2}}{M_{\perp}^{2}}\right\} \tag{3}
\end{equation*}
$$

$D \equiv D\left(z, Q^{2} / M_{\perp}^{2}\right)=1+z(1-z) Q^{2} / M_{\perp}^{2}, a(y)=2-y(2-y), b(y)=[6-y(6-y)] / a(y)$.
Since $h_{1}^{\perp g}$ is completely unknown, we estimate the maximum asymmetry that is allowed by the bound

$$
\begin{equation*}
\left|h_{1}^{\perp g(1)}(x)\right| \leq f_{1}^{g}(x), \tag{4}
\end{equation*}
$$



FIGURE 1. Upper bound of $\left|\left\langle\cos 2\left(\phi_{T}-\phi_{\perp}\right)\right\rangle\right|$ defined in Eq. (5) as a function of $\left|\boldsymbol{K}_{\perp}\right|$ at different values of $Q^{2}$, with $y=0.01$ and $z=0.5$.
where the superscript (1) denotes the $n=1$ transverse moment (defined as $f^{(n)}(x) \equiv$ $\left.\int d^{2} \boldsymbol{p}_{T}\left(\boldsymbol{p}_{T}^{2} / 2 M^{2}\right)^{n} f\left(x, \boldsymbol{p}_{T}^{2}\right)\right)$. The function $R$, defined as the upper bound of the absolute value of $\left\langle\cos 2\left(\phi_{T}-\phi_{\perp}\right)\right\rangle$,

$$
\begin{equation*}
\left|\left\langle\cos 2\left(\phi_{T}-\phi_{\perp}\right)\right\rangle\right| \equiv\left|\frac{\int d^{2} \boldsymbol{q}_{T} \cos 2\left(\phi_{T}-\phi_{\perp}\right) d \sigma}{\int d^{2} \boldsymbol{q}_{T} d \sigma}\right|=\frac{\int d \boldsymbol{q}_{T}^{2} \boldsymbol{q}_{T}^{2}|B|}{2 \int d \boldsymbol{q}_{T}^{2} A} \leq \frac{\left|\mathscr{B}^{e g \rightarrow e Q} \bar{Q}\right|}{\mathscr{A}^{e g \rightarrow e Q \bar{Q}} \equiv R, ~, ~} \tag{5}
\end{equation*}
$$

is depicted in Fig. 1 as a function of $\left|\boldsymbol{K}_{\perp}\right|(>1 \mathrm{GeV})$ at different values of $Q^{2}$ for charm (left panel) and bottom (right panel) production. We have selected $y=0.01, z=0.5$, and taken $M_{c}^{2}=2 \mathrm{GeV}^{2}, M_{b}^{2}=25 \mathrm{GeV}^{2}$. Such large asymmetries would probably allow an extraction of $h_{1}^{\perp g}$ at EIC (or LHeC).

If one keeps the lepton plane angle $\phi_{\ell}$, there are other azimuthal dependences, such as a $\cos 2\left(\phi_{\ell}-\phi_{T}\right)$. The bound on $\left|\left\langle\cos 2\left(\phi_{\ell}-\phi_{T}\right)\right\rangle\right|$, denoted as $R^{\prime}$, is shown in Fig. 2 in the same kinematic region as in Fig. 1. One can see that $R^{\prime}$ can be larger than $R$, but only at smaller $\left|\boldsymbol{K}_{\perp}\right| . R^{\prime}$ falls off more rapidly at larger values of $\left|\boldsymbol{K}_{\perp}\right|$ than $R$. We note that it is essential that the individual transverse momenta $K_{i \perp}$ are reconstructed with an accuracy $\delta K_{\perp}$ better than the magnitude of the sum of the transverse momenta $K_{1 \perp}+K_{2 \perp}=q_{T}$. This means one has to satisfy $\delta K_{\perp} \ll\left|q_{T}\right| \ll\left|K_{\perp}\right|$, which will require a minimum $\left|K_{\perp}\right|$.

The cross section for the process $e h \rightarrow e^{\prime}$ jetjet $X$ can be calculated in a similar way and is analogous to Eq. (1). In particular, the explicit expression for $B$ can be obtained from the one for HQ production taking $M_{Q}=0$, while $A$ now depends also on $x_{B}$ and receives a contribution from the subprocess $\gamma^{*} q \rightarrow g q$ as well, not just from $\gamma^{*} g \rightarrow q \bar{q}$. Therefore, the maximal asymmetries (not shown) are smaller than for HQ pair production.


FIGURE 2. Same as in Fig. 1, but for the upper bound $R^{\prime}$ of $\left|\left\langle\cos 2\left(\phi_{\ell}-\phi_{T}\right)\right\rangle\right|$.

## CONCLUSIONS

Studies of the azimuthal asymmetry of jet or heavy quark pair production in ep collisions can directly probe $h_{1}^{\perp g}$, the distribution of linearly polarized gluons inside unpolarized hadrons. Breaking of TMD factorization is expected in $p p$ or $p \bar{p}$ collisions, hence a comparison between extractions from these two types of processes would clearly signal the dependence on ISI/FSI. The contribution of $h_{1}^{\perp g}$ to diphoton production has also been studied [7]. Since the proposed measurements are relatively simple (polarized beams are not required), we believe that the experimental determination of $h_{1}^{\perp g}$ and the analysis of its potential process dependence will be feasible in the future.

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