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Meson Transition Form Factors in Light-Front Holographic QCD

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Abstract

We study the photon-to-meson transition form factors (TFFs) $F_{M\gamma}(Q^2)$ for $\gamma\gamma^* \to M$ using light-front holographic methods. The Chern-Simons action, which is a natural form in 5-dimensional anti-de Sitter (AdS) space, leads directly to an expression for the photon-to-pion TFF for a class of confining models. Remarkably, the predicted pion TFF is identical to the leading order QCD result where the distribution amplitude has asymptotic form. The Chern-Simons form is local in AdS space and is thus somewhat limited in its predictability. It only retains the $q\bar{q}$ component of the pion wavefunction, and further, it projects out only the asymptotic form of the meson distribution amplitude. It is found that in order to describe simultaneously the decay process $\pi^0 \to \gamma\gamma$ and the pion TFF at the asymptotic limit, a probability for the $q\bar{q}$ component of the pion wavefunction $P_{q\bar{q}} = 0.5$ is required; thus giving indication that the contributions from higher Fock states in the pion light-front wavefunction need to be included in the analysis. The probability for the Fock state containing four quarks (anti-quarks) which follows from analyzing the hadron matrix elements, $P_{q\bar{q}q\bar{q}} \sim 10\%$, agrees with the analysis of the pion elastic form factor using light-front holography including higher Fock components in the pion wavefunction. The results for the TFFs for the η and η' mesons are also presented. The rapid growth of the pion TFF exhibited by the *BABAR* data at high Q^2 is not compatible with the models discussed in this article, whereas the theoretical calculations are in agreement with the experimental data for the η and η' TFFs.

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I. INTRODUCTION

The AdS/CFT correspondence between an effective gravity theory on a higher dimensional anti-de Sitter (AdS) space and conformal field theories in physical space-time [1–3] has led to a remarkably accurate semiclassical approximation for strongly-coupled QCD, and it also provides physical insights into its non-perturbative dynamics. Incorporating the AdS/CFT correspondence as a useful guide, light-front holographic methods were originally introduced [4] by matching the electromagnetic (EM) current matrix elements in AdS space [5] to the corresponding Drell-Yan-West (DYW) expression, [6, 7] using light-front (LF) theory in physical space-time. One obtains the identical holographic mapping using the matrix elements of the energy-momentum tensor [8] by perturbing the AdS metric

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right), \tag{1}$$

around its static solution. [9]

A precise gravity dual to QCD is not known, but color confinement can be incorporated in the gauge/gravity correspondence by modifying the AdS geometry in the large infrared (IR) domain $z \sim 1/\Lambda_{\rm QCD}$, which also sets the mass scale of the strong interactions in a class of confining models. The modified theory generates the point-like hard behavior expected from QCD, such as constituent counting rules [10–12] from the ultraviolet (UV) conformal limit at the AdS boundary at $z \to 0$, instead of the soft behavior characteristic of extended objects. [13]

One can also study the gauge/gravity duality starting from the Hamiltonian equation of motion in physical space-time. [14] To a first semiclassical approximation, where quantum loops and quark masses are not included, this leads to a LF Hamiltonian equation which describes the bound-state dynamics of light hadrons in terms of an invariant impact variable ζ which measures the separation of the partons within the hadron at equal light-front time. This allows us to identify the holographic variable z in AdS space with the impact variable ζ. [4, 8, 14]

The pion transition form factor (TFF) between a photon and pion measured in the $e^-e^- \to e^-e^-\pi^0$ process, with one tagged electron, is the simplest bound-state process in

QCD. It can be predicted from first principles in the asymptotic $Q^2 \to \infty$ limit. [11] More generally, the pion TFF at large Q^2 can be calculated at leading twist as a convolution of a perturbative hard scattering amplitude $T_H(\gamma\gamma^* \to q\bar{q})$ and a gauge-invariant meson distribution amplitude (DA) which incorporates the nonperturbative dynamics of the QCD bound-state. [11]

The *BABAR* Collaboration has reported measurements of the transition form factors from $\gamma^*\gamma \to M$ process for the π^0, [15] η, and η' [16, 17] pseudoscalar mesons for a momentum transfer range much larger than previous measurements. [18, 19] Surprisingly, the *BABAR* data for the π^0-γ TFF exhibit a rapid growth for $Q^2 > 15$ GeV^2, which is unexpected from QCD predictions. In contrast, the data for the η-γ and η'-γ TFFs are in agreement with previous experiments and theoretical predictions. Many theoretical studies have been devoted to explaining *BABAR*'s experimental results. [20–33]

Motivated by the conflict of theory with experimental results we have examined in a companion paper [32] existing models and approximations used in the computation of pseudoscalar meson TFFs in QCD, incorporating the evolution of the pion distribution amplitude [11, 34] which controls the meson TFFs at large Q^2. In this article we will study the structure of the meson TFFs which follows from the Chern-Simons (CS) action present in the higher dimensional gravity side. [3, 35] A simple analytical form is found which satisfies both the low energy theorem for the decay $\pi^0 \to \gamma\gamma$ and the QCD predictions at large Q^2, thus allowing us to encompass the perturbative and non-perturbative space-like regimes in a simple model. We choose the soft-wall approach to modify the infrared AdS geometry to include confinement, but the general results are not expected to be sensitive to the specific model chosen to deform AdS space in the IR since the Chern-Simons action is a topological invariant.

After introducing the Chern-Simons structure of the meson transition form factor in AdS space in Section II, the pion transition form factors calculated with the free and dressed currents are presented in Sections III and IV, respectively. The higher Fock state contributions to the pion transition form factor are studied in Section V. The results for the η and η' transition form factors are given in Section VI. Some conclusions are presented in Section VII. Different forms for the pion distribution amplitude from holographic mapping are discussed in Appendix A.

II. THE CHERN-SIMONS STRUCTURE OF THE MESON TRANSITION FORM FACTOR IN ADS SPACE

The pion transition form factor can be studied by exploring the mathematical structure of higher-dimensional forms describing the pion-photon amplitude in AdS space and its holographic mapping to light-front QCD by constructing higher dimensional transition amplitudes. In the case of the $U(1)$ gauge theory the Chern-Simons (CS) action is of the form $\epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$ in the five dimensional Lagrangian [3, 35], where x^M are the five-dimensional coordinates $x^M = (x^\mu, z)$. The Chern-Simons form is the product of three fields at the same point in five-dimensional space corresponding to a local interaction.

In order to compute the transition form factor $F_{\pi\gamma}$ using holographic methods, one needs to relate the five-dimensional CS amplitude to the $\gamma^* \gamma \to \pi^0$ amplitude:

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q \\ \sim (2\pi)^4 \delta^{(4)}\left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma, \quad (2)$$

with the meson field in AdS space identified with the A_z component. [36, 37] In the r.h.s of (2) q and k are the momenta of the virtual and on-shell photons respectively with corresponding polarization vectors $\epsilon_\mu(q)$ and $\epsilon_\mu(k)$. The momentum of the outgoing pion is p_π.

The incoming electromagnetic field propagates in AdS according to $A_\mu(x^\mu, z) = \epsilon_\mu(q) e^{-iq\cdot x} V(q^2, z)$, where $V(q^2, z)$ is the bulk-to-boundary propagator with boundary conditions $V(q^2 = 0, z) = V(q^2, z = 0) = 1$. [5] Since the incoming photon with momentum k is on its mass shell, $k^2 = 0$, its wave function is $A_\mu(x^\mu, z) = \epsilon_\mu(k) e^{ik\cdot x}$.

The propagation of the pion in AdS space is described by a normalizable mode $\Phi_{p_\pi}(x^\mu, z) = e^{-ip_\pi \cdot x} \Phi_\pi(z)$ with invariant mass $p_{\pi\mu} p_\pi{}^\mu = \mathcal{M}_\pi^2$ and plane waves along Minkowski coordinates x^μ. In the chiral limit for massless quarks $\mathcal{M}_\pi = 0$. The normalizable mode $\Phi(z)$ scales as $\Phi(z) \to z^{\tau=2}$ in the limit $z \to 0$, since the leading interpolating operator for the pion has twist two. A simple dimensional analysis implies that $A_z \sim \Phi_\pi(z)/z$, matching the twist scaling dimensions: two for the pion and one for the EM field. Substituting in (2) the expression given above for the the pion and the EM fields propagating in AdS we find ($Q^2 = -q^2 > 0$)

$$F_{\pi\gamma}(Q^2) = \frac{1}{2\pi} \int_0^\infty \frac{dz}{z}\, \Phi_\pi(z) V\big(Q^2, z\big)\,. \tag{3}$$

We have defined our units such that the AdS radius $R = 1$. As will become clear below, the higher dimensional amplitude (2), with the normalization given by (3), can be consistently mapped to physical QCD in the light front, reproducing the asymptotic QCD prediction. Since the pion field is identified as $\Phi_\pi(z) \sim zA(z)$, the CS form $\epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$ is similar in form to an axial current; this correspondence can explain why the resulting pion distribution amplitude has the asymptotic form.

In Ref. [37] the pion TFF was studied in the framework of a CS extended hard-wall AdS/QCD model with $A_z \sim \partial_z \Phi(z)$. The expression for the TFF which follows from (2) then vanishes at $Q^2 = 0$, and has to be corrected by the introduction of a surface term at the IR wall. [37] However, this procedure is only possible for a model with a sharp cutoff. The pion TFF has also been studied using the holographic approach to QCD in Refs. [38, 39].

III. A SIMPLE HOLOGRAPHIC CONFINING MODEL

Conformal invariance can be broken analytically by the introduction of a color-confining dilaton profile $\varphi(z)$ in the action,

$$S = \int d^4x \int dz \sqrt{g} e^{\varphi(z)} \mathcal{L}, \tag{4}$$

thus retaining conformal AdS metrics as well as introducing a smooth IR cutoff. It is not possible in this model to introduce a surface term as in Ref. [37] to match the value of the TFF at $Q^2 = 0$ derived from the decay $\pi^0 \to \gamma\gamma$. Instead, higher Fock components which modify the pion wave function at large distances are required to satisfy this low-energy constraint naturally. Since the higher-twist components have a faster fall-off at small distances, the asymptotic results are not modified.

It is convenient to scale away the dilaton factor in the action by a field redefinition. [40] For example, for a scalar field we shift $\Phi \to e^{-\varphi/2}\Phi$, and the bilinear component in the action is transformed into the equivalent problem of a free kinetic part plus an effective potential $V(\Phi, \varphi)$. The five-dimensional CS action for the redefined pion field is determined by the equation of motion in the presence of the effective potential $V(\Phi, \varphi)$.

A particularly interesting case is a dilaton profile $\exp\left(\pm\kappa^2 z^2\right)$ of either sign, since it leads to linear Regge trajectories consistent with the light-quark hadron spectroscopy. It avoids the ambiguities in the choice of boundary conditions at the infrared wall. [41] In this case the effective potential takes the form of a harmonic oscillator confining potential $\kappa^4 z^2$, and the normalizable solution for a meson of a given twist τ, corresponding to the lowest radial $n = 0$ node, is given by

$$\Phi_\pi^\tau(z) = \sqrt{\frac{2P_\tau}{\Gamma(\tau-1)}}\, \kappa^{\tau-1} z^\tau e^{-\kappa^2 z^2/2}, \tag{5}$$

with normalization

$$\langle \Phi^\tau | \Phi^\tau \rangle = \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi^\tau(z)^2 = P_\tau, \tag{6}$$

where P_τ is the probability for the twist τ mode (5). This agrees with the fact that the field Φ^τ couples to a local hadronic interpolating operator of twist τ defined at the asymptotic boundary of AdS space, and thus the scaling dimension of Φ^τ is τ. The decay constant of the hadron described by the AdS mode (5) is normalized according to (Appendix A)

$$f_\tau = \frac{1}{4\pi} \frac{\partial_z \Phi^\tau(z)}{z}\Big|_{z=0}, \tag{7}$$

and thus the decay constant for the pion ($\tau = 2$)

$$f_\pi = \sqrt{P_{q\bar{q}}}\, \frac{\kappa}{\sqrt{2}\pi}. \tag{8}$$

The QCD asymptotic prediction of the TFF can be computed from first principles by analyzing the local coupling of the free electromagnetic current to the elementary constituents in the interaction representation. [11] To compare with the asymptotic QCD prediction, we first consider a simplified model where the non-normalizable mode $V(q^2, z)$ for the EM current satisfies the "free" AdS equation, dual to the free QCD current, [4] and thus

$$V(Q^2, z) = zqK_1(zQ), \tag{9}$$

with boundary conditions $V(Q^2 = 0, z) = V(Q^2, z = 0) = 1$. Substituting the pion wave function (5) for twist $\tau = 2$ in (3) and using the integral representation for $V(Q^2, z)$

$$zQK_1(zQ) = 2Q^2 \int_0^\infty \frac{tJ_0(zt)}{(t^2 + Q^2)^2} dt, \tag{10}$$

we find upon integration

$$F_{\pi\gamma}(Q^2) = \frac{\sqrt{2P_{q\bar{q}}}\, Q^2}{\pi\kappa} \int_0^\infty \frac{tdt}{(t^2 + Q^2)^2} e^{-t^2/2\kappa^2}. \tag{11}$$

Changing variables as $x = \frac{Q^2}{t^2+Q^2}$ one obtains

$$F_{\pi\gamma}(Q^2) = \frac{P_{q\bar{q}}}{2\pi^2 f_\pi} \int_0^1 dx \exp\left(-\frac{(1-x)P_{q\bar{q}}Q^2}{4\pi^2 f_\pi^2 x}\right). \tag{12}$$

Upon integration by parts, Eq. (12) can also be written as

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - \exp\left(-\frac{(1-x)P_{q\bar{q}}Q^2}{4\pi^2 f_\pi^2 x}\right)\right], \tag{13}$$

where $\phi(x) = \sqrt{3} f_\pi x(1-x)$ is the asymptotic QCD distribution amplitude with f_π given by (8).

Remarkably, the pion transition form factor given by (13) for $P_{q\bar{q}} = 1$ is identical to the results for the pion TFF obtained with an exponential light-front wave function (LFWF) [42]; it also reproduces the leading order QCD result [11] for the TFF at the asymptotic limit, $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2f_\pi$. [43] The leading-twist result (13) does not include non-leading order α_s corrections in the hard scattering amplitude nor gluon exchange in the evolution of the distribution amplitude, since the semiclassical correspondence implied in the gauge/gravity duality does not contain quantum effects such as particle emission and absorption.

The transition form factor at $Q^2 = 0$ can be obtained from Eq. (13),

$$F_{\pi\gamma}(0) = \frac{1}{2\pi^2 f_\pi} P_{q\bar{q}}. \tag{14}$$

The form factor $F_{\pi\gamma}(0)$ is related to the decay width for the $\pi^0 \to \gamma\gamma$ decay,

$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{\alpha^2\pi m_\pi^3}{4} F_{\pi\gamma}^2(0), \tag{15}$$

where $\alpha = 1/137$. The form factor $F_{\pi\gamma}(0)$ is also well described by the Schwinger, Adler, Bell and Jackiw anomaly [44] which gives

$$F_{\pi\gamma}^{\rm SABJ}(0) = \frac{1}{4\pi^2 f_\pi}, \tag{16}$$

in agreement within a few percent of the observed value obtained from the the decay $\pi^0 \to \gamma\gamma$.

Taking $P_{q\bar{q}} = 0.5$ in (14) one obtains a result in agreement with (16). Thus (13) represents a description on the pion TFF which encompasses the low-energy non-perturbative and the high-energy hard domains, but includes only the asymptotic DA of the $q\bar{q}$ component of the pion wave function at all scales. The results from (13) are shown as dotted curves in Figs. 1 and 2 for $Q^2 F_{\pi\gamma}(Q^2)$ and $F_{\pi\gamma}(Q^2)$ respectively. The calculations agree reasonably well with the experimental data at low- and medium-Q^2 regions ($Q^2 < 10$ GeV^2) , but disagree with *BABAR*'s large Q^2 data.

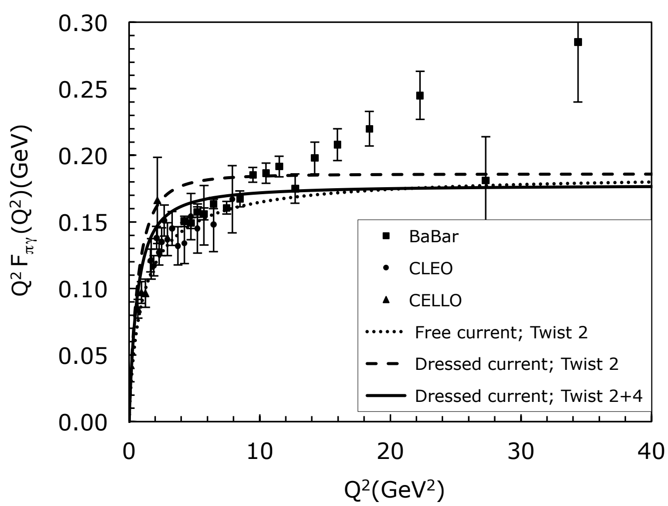

FIG. 1: The $\gamma\gamma^* \to \pi^0$ transition form factor shown as $Q^2 F_{\pi\gamma}(Q^2)$ as a function of $Q^2 = -q^2$. The dotted curve is the asymptotic result predicted by the Chern-Simons form. The dashed and solid curves include the effects of using a confined EM current for twist-two and twist-two plus twist-four respectively. The data are from [15, 18, 19].

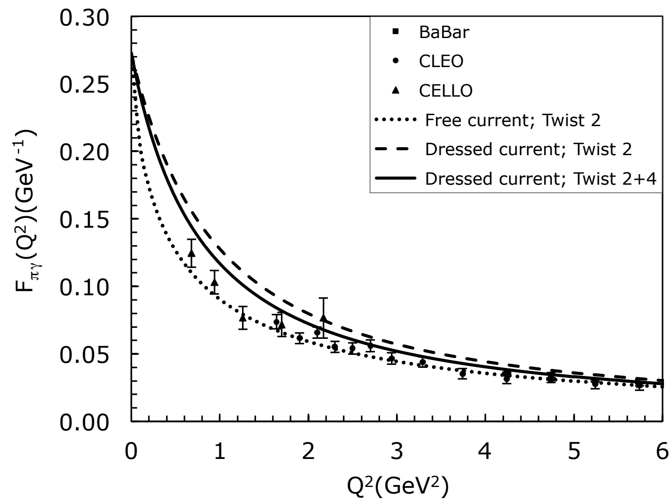

FIG. 2: Same as Fig. 1 for $F_{\pi\gamma}(Q^2)$.

IV. TRANSITION FORM FACTOR WITH THE DRESSED CURRENT

The results described in Section III correspond to a free current propagating on AdS space. It is dual to the electromagnetic point-like current in the Drell-Yan-West light-front formula [6, 7] for the pion form factor.

The DYW formula is an exact expression for the form factor. It is written as a sum of an overlap of LF Fock components with an arbitrary number of constituents. This allows one to map state-by-state to the effective gravity theory in AdS space. However, this mapping has the shortcoming that the multiple pole structure of the time-like form factor cannot be obtained in the time-like region unless an infinite number of Fock states is included. Furthermore, the moments of the form factor at $Q^2 = 0$ diverge term-by-term; for example one obtains an infinite charge radius. [45]

Alternatively, one can use a truncated basis of states in the LF Fock expansion with a limited number of constituents, and the non-perturbative pole structure can be generated with a dressed EM current as in the Heisenberg picture, *i.e.*, the EM current becomes modified as it propagates in a IR deformed AdS space to simulate confinement. The dressed current is dual to a hadronic EM current which includes any number of virtual

$q\bar{q}$ components.

The simple valence $q\bar{q}$ model discussed above should thus be modified at small Q^2 by introducing the dressed current. In the case of soft-wall potential, the EM bulk-to-boundary propagator is

$$V(Q^2, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right), \tag{17}$$

where $U(a, b, c)$ is the Tricomi confluent hypergeometric function. The modified current $V(Q^2, z)$, (17), has the same boundary conditions as the free current (9), and reduces to (9) in the limit $Q^2 \to \infty$. Eq. (17) can be conveniently written in terms of the integral representation [46]

$$V(Q^2, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}. \tag{18}$$

Inserting the pion wave function (5) for twist $\tau = 2$ and the confined EM current (18) in the amplitude (3) one finds

$$F_{\pi\gamma}(Q^2) = \frac{P_{q\bar{q}}}{\pi^2 f_\pi} \int_0^1 \frac{dx}{(1+x)^2}\, x^{Q^2 P_{q\bar{q}}/(8\pi^2 f_\pi^2)}. \tag{19}$$

Eq. (19) gives the same value for $F_{\pi\gamma}(0)$ as (14) which was obtained with the free current. Thus the anomaly result $F_{\pi\gamma}(0) = 1/(4\pi^2 f_\pi)$ is reproduced if $P_{q\bar{q}} = 0.5$ is also taken in (19). Upon integration by parts, Eq. (19) can also be written as

$$Q^2 F_{\pi\gamma}(Q^2) = 8 f_\pi \int_0^1 dx \frac{1-x}{(1+x)^3}\left(1 - x^{Q^2 P_{q\bar{q}}/(8\pi^2 f_\pi^2)}\right). \tag{20}$$

Noticing that the second term in Eq. (20) vanishes at the limit $Q^2 \to \infty$, one recovers Brodsky-Lepage's asymptotic prediction for the pion TFF: $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$. [11]

The results calculated with (19) for $P_{q\bar{q}} = 0.5$ are shown as dashed curves in Figs. 1 and 2. One can see that the calculations with the dressed current are larger as compared with the results computed with the free current and the experimental data at low- and medium-Q^2 regions ($Q^2 < 10$ GeV^2). The new results again disagree with *BABAR*'s data at large Q^2.

V. HIGHER-TWIST COMPONENTS TO THE TRANSITION FORM FACTOR

In a previous light-front QCD analysis of the pion TFF [47] it was argued that the valence Fock state $|q\bar{q}\rangle$ provides only half of the contribution to the pion TFF at $Q^2 = 0$, while the other half comes from diagrams where the virtual photon couples inside the pion (strong interactions occur between the two photon interactions). This leads to a surprisingly small value for the valence Fock state probability $P_{q\bar{q}} = 0.25$. More importantly, this raises the question on the role played by the higher Fock components of the pion LFWF,

$$|\pi\rangle = \psi_2|\bar{q}q\rangle + \psi_3|q\bar{q}g\rangle + \psi_4|q\bar{q}q\bar{q}\rangle + \cdots , \tag{21}$$

in the calculations for the pion TFF.

The contributions to the transition form factor from these higher Fock states are suppressed, compared with the valence Fock state, by the factor $1/(Q^2)^n$ for n extra $q\bar{q}$ pairs in the higher Fock state, since one needs to evaluate an off-diagonal matrix element between the real photon and the multi-quark Fock state. [11] We note that in the case of the elastic form factor the power suppression is $1/(Q^2)^{2n}$ for n extra $q\bar{q}$ pairs in the higher Fock state. These higher Fock state contributions are negligible at high Q^2. On the other hand, it has long been argued that the higher Fock state contributions are necessary to explain the experimental data at the medium Q^2 region for exclusive processes. [48, 49]. The contributions from the twist-3 parts of the two-parton pion distribution amplitude to the pion elastic form factors were evaluated in Ref. [50]. The three-parton contributions to the pion elastic form factor were studied in Ref. [51]. The contributions from diagrams where the virtual photon couples inside the pion to the pion transition form factor were estimated using light-front wavefunctions in Ref. [22, 52]. The higher twist (twist-4 and twist-6) contributions to the pion transition form factor [53] were evaluated using the method of light-cone sum rules in Refs. [30, 33], but opposite claims were made on whether the *BABAR* data could be accommodated by including these higher twist contributions.

It is also not very clear how the higher Fock states contribute to decay processes, such as $\pi^0 \to \gamma\gamma$, [54] due to the long-distance non-perturbative nature of decay processes.

Second order radiative corrections to the triangle anomaly do not change the anomaly results as they contain one internal photon line and two vertices on the triangle loop. Upon regulation no new anomaly contribution occurs. In fact, the result is expected to be valid at all orders in perturbation theory. [55, 56] It is thus generally argued that in the chiral limit of QCD (*i.e.*, $m_q \to 0$), one needs only the $q\bar{q}$ component to explain the anomaly, but as shown below, the higher Fock state components can also contribute to the decay process $\pi^0 \to \gamma\gamma$ in the chiral limit.

As discussed in the last two sections, matching the AdS/QCD results computed with the free and dressed currents for the TFF at $Q^2 = 0$ with the anomaly result requires a probability $P_{q\bar{q}} = 0.5$. Thus it is important to investigate the contributions from the higher Fock states. In AdS/QCD there are no dynamic gluons and confinement is realized via an effective instantaneous interaction in light-front time, analogous to the instantaneous gluon exchange. [57] The effective confining potential also creates quark-antiquark pairs from the amplitude $q \to q\bar{q}q$. Thus in AdS/QCD higher Fock states can have any number of extra $q\bar{q}$ pairs. These higher Fock states lead to higher-twist contributions to the pion transition form factor.

To illustrate this observation consider the two diagrams in Fig. 3. In the leading process, Fig. 3 (a), where both photons couple to the same quark, the valence $|q\bar{q}\rangle$ state has $J^z = S^z = L^z = 0$,

$$|q\bar{q}\rangle = \frac{1}{\sqrt{2}}\Big(\Big| +\frac{1}{2}, -\frac{1}{2}\Big\rangle - \Big| -\frac{1}{2}, +\frac{1}{2}\Big\rangle\Big). \tag{22}$$

Eq. (22) represents a $J^{PC} = 0^{-+}$ state with the quantum numbers of the conventional π meson axial vector interpolating operator $\mathcal{O} = \bar{\psi}\gamma^+\gamma^5\psi$.

In the process involving the four quark state $|q\bar{q}q\bar{q}\rangle$ of the pion, Fig. 3 (b), where each photon couples directly to a $q\bar{q}$ pair, the four quark state also satisfies $J^z = S^z = L^z = 0$ and is represented by

$$\begin{aligned}|q\bar{q}q\bar{q}\rangle = \frac{1}{2}\Big(&\Big| +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}\Big\rangle + \Big| +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}\Big\rangle \\ &- \Big| -\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}\Big\rangle - \Big| -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}\Big\rangle\Big). \end{aligned}\tag{23}$$

The four quark state in Eq. (23) has also quantum numbers $J^{PC} = 0^{-+}$ corresponding

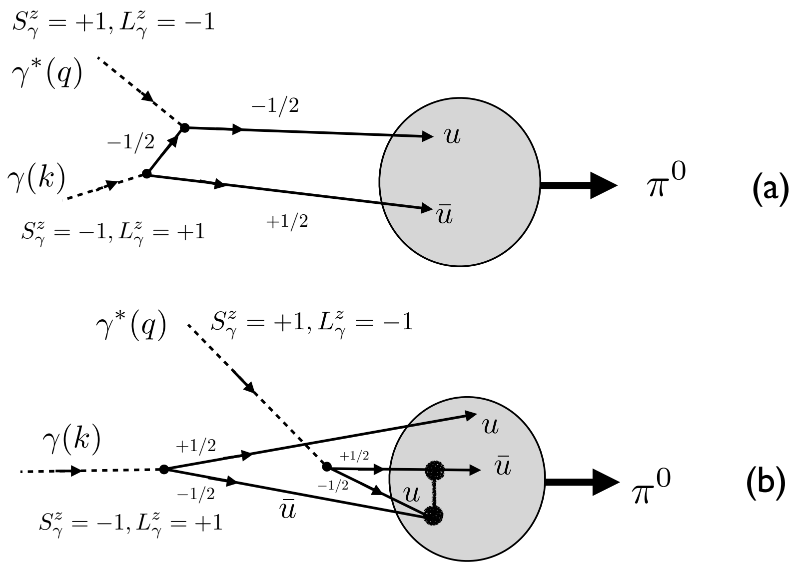

FIG. 3: Leading-twist contribution (a) and twist-four contribution (b) to the process $\gamma\gamma^* \to \pi^0$.

to the quantum numbers of the local interpolating operators $\mathcal{O} = \bar{\psi}\gamma^+\gamma^5\psi\psi\bar{\psi}$ where the scalar interpolating operator $\bar{\psi}\psi$ has quantum numbers $J^{PC} = 0^{++}$.

We note that for the Compton scattering $\gamma H \to \gamma H$ process, similar higher-twist contributions, as illustrated in Fig. 3 (b), are proportional to $\sum_{e_i \neq e_j} e_i e_j$ and are necessary to derive the low energy amplitude for Compton scattering which is proportional to the total charge squared $e_H^2 = (e_i + e_j)^2$ of the target. [58]

Both processes illustrated in Fig (3) make contributions to the two photon process $\gamma^*\gamma \to \pi^0$. Time reversal invariance means that the four quark state $|q\bar{q}q\bar{q}\rangle$ should also contribute to the decay process $\pi^0 \to \gamma\gamma$. In a semiclassical model without dynamic gluons, Fig. 3 (b) represents the only higher twist term which contribute to the $\gamma^*\gamma \to \pi^0$ process. The twist-four contribution vanishes at large Q^2 compared to the leading-twist contribution, thus maintaining the asymptotic predictions while only modifying the large distance behavior of the wave function.

To investigate the contributions from the higher Fock states in the pion LFWF, we write the twist-two and twist-four hadronic AdS components from (5)

$$\Phi_\pi^{\tau=2}(z) \;=\; \frac{\sqrt{2}\kappa z^2}{\sqrt{1+\alpha^2}} e^{-\kappa^2 z^2/2}, \tag{24}$$

$$\Phi_\pi^{\tau=4}(z) \;=\; \frac{\alpha\kappa^3 z^4}{\sqrt{1+\alpha^2}} e^{-\kappa^2 z^2/2}, \tag{25}$$

with normalization

$$\int_0^\infty \frac{dz}{z^3}\left[|\Phi_\pi^{\tau=2}(z)|^2 + |\Phi_\pi^{\tau=4}(z)|^2\right] = 1, \tag{26}$$

and probabilities $P_{q\bar{q}} = 1/(1+|\alpha|^2)$ and $P_{q\bar{q}q\bar{q}} = \alpha^2/(1+|\alpha|^2)$. The pion decay constant follows from the short distance asymptotic behavior of the leading contribution and is given by

$$f_\pi = \frac{1}{\sqrt{1+\alpha^2}}\frac{\kappa}{\sqrt{2}\pi}. \tag{27}$$

Using (24) and (25) together with (18) in equation (3) we find the total contribution from twist-two and twist-four components for the dressed current,

$$F_{\pi\gamma}(Q^2) = \frac{1}{\pi^2 f_\pi}\frac{1}{(1+\alpha^2)^{3/2}}\int_0^1 \frac{dx}{(1+x)^2} x^{Q^2/[8\pi^2 f_\pi^2(1+\alpha^2)]}\left[1 + \frac{4\alpha}{\sqrt{2}}\frac{1-x}{1+x}\right]. \tag{28}$$

The transition from factor at $Q^2 = 0$ is given by

$$F_{\pi\gamma}(0) = \frac{1}{2\pi^2 f_\pi}\frac{1+\sqrt{2}\alpha}{(1+\alpha^2)^{3/2}}. \tag{29}$$

The Brodsky-Lepage's asymptotic prediction for the pion TFF can be recovered from Eq. (28) by noticing that the second term vanishes at $Q^2 \to \infty$ and the similarity between Eq. (20) and the first term in Eq. (28).

Imposing the anomaly result (16) on (29) we find two possible real solutions for α: $\alpha_1 = -0.304$ and $\alpha_2 = 1.568$. [59] The larger value $\alpha_2 = 1.568$ yields $P_{q\bar{q}} = 0.29$, $P_{q\bar{q}q\bar{q}} = 0.71$, and $\kappa = 1.43$ GeV. The resulting value of κ is about 4 times larger than the value obtained from the AdS/QCD analysis of the hadron spectrum and the pion elastic form factor, [60] and thereby should be discarded. The other solution $\alpha_1 = -0.304$ gives $P_{q\bar{q}} = 0.915$, $P_{q\bar{q}q\bar{q}} = 0.085$, and $\kappa = 0.432$ GeV – results that are similar to that found from an analysis of the space and time-like behavior of the pion form factor using LF holographic methods, including higher Fock components in the pion wave function. [60] Semiclassical holographic methods, where dynamical gluons are not presented, are thus compatible with a large probability for the valence state of the order of 90%. On the other hand, QCD analyses including multiple gluons on the pion wave function favor a small probability (25%) for the valence state. [47] Both cases (and examples in between) are examined in Ref. [32].

The results for the transition form factor are shown as solid curves in Figs. 1 and 2. The agreements with the experimental data at low- and medium-Q^2 regions ($Q^2 < 10$ GeV^2) are greatly improved compared with the results obtained with only twist-two component computed with the dressed current. However, the rapid growth of the pion-photon transition form factor exhibited by the *BABAR* data at high Q^2 still cannot be reproduced. So we arrive at a similar conclusion as we did in a QCD analysis of the pion TFF in Ref. [32]: it is difficult to explain the rapid growth of the form factor exhibited by the *BABAR* data at high Q^2 within the current framework of QCD.

VI. TRANSITION FORM FACTORS FOR THE η AND η' MESONS

The η and η' mesons result from the mixing of the neutral states η_8 and η_1 of the SU(3)$_F$ quark model. The transition form factors for the latter have the same expression as the pion transition form factor, except an overall multiplying factor $c_P = 1, \frac{1}{\sqrt{3}}$, and $\frac{2\sqrt{2}}{\sqrt{3}}$ for the π^0, η_8 and η_1, respectively. By multiplying equations (13), (19) and (28) by the appropriate factor c_P, one obtains the corresponding expressions for the transition form factors for the η_8 and η_1.

The transition form factors for the physical states η and η' are a superposition of the transition form factors for the η_8 and η_1

$$\begin{pmatrix} F_{\eta\gamma} \\ F_{\eta'\gamma} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} F_{\eta_8\gamma} \\ F_{\eta_1\gamma} \end{pmatrix}, \tag{30}$$

where θ is the mixing angle for which we adopt $\theta = -14.5^o \pm 2^o$. [61] The results for the η and η' transitions form factors are shown in Figs. 4 and 5 for $Q^2 F_{M\gamma}(Q^2)$, and Figs. 6 and 7 for $F_{M\gamma}(Q^2)$. The calculations agree very well with available experimental data over a large range of Q^2. We note that other mixing schemes were proposed in studying the mixing behavior of the decay constants and states of the η and η' mesons. [62–64] Since the transition from factors are the primary interest in this study it is appropriate to use the conventional single-angle mixing scheme for the states. Furthermore, the predictions for the η and η' transition form factors remain largely unchanged if other mixing schemes are used in the calculation.

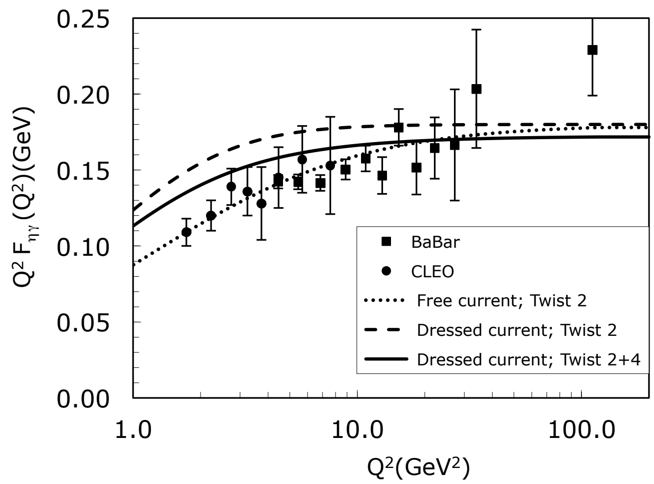

FIG. 4: The $\gamma\gamma^* \to \eta$ transition form factor shown as $Q^2 F_{\eta\gamma}(Q^2)$ as a function of $Q^2 = -q^2$. The dotted curve is the asymptotic result. The dashed and solid curves include the effects of using a confined EM current for twist two and twist two plus twist four respectively. The data are from [15, 18, 19].

VII. CONCLUSIONS

The light-front holographic approach provides a direct mapping between an effective gravity theory defined in a fifth-dimensional warped space-time and a corresponding semiclassical approximation to strongly coupled QCD quantized on the light-front. In addition to predictions for hadron spectroscopy, important outputs are the elastic form factors of hadrons and constraints on their light-front bound-state wavefunctions. The soft wall color-confining AdS/QCD model is particularly successful.

We have studied the photon-to-meson transition form factors $F_{M\gamma}(Q^2)$ for $\gamma^*\gamma \to M$ using light-front holographic methods. The Chern-Simons action, which is a natural

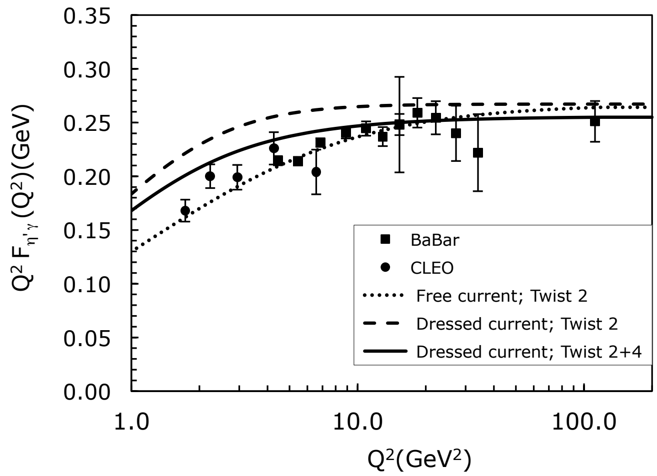

FIG. 5: Same as Fig. 4 for the $\gamma\gamma^* \to \eta'$ transition form factor shown as $Q^2 F_{\eta'\gamma}(Q^2)$.

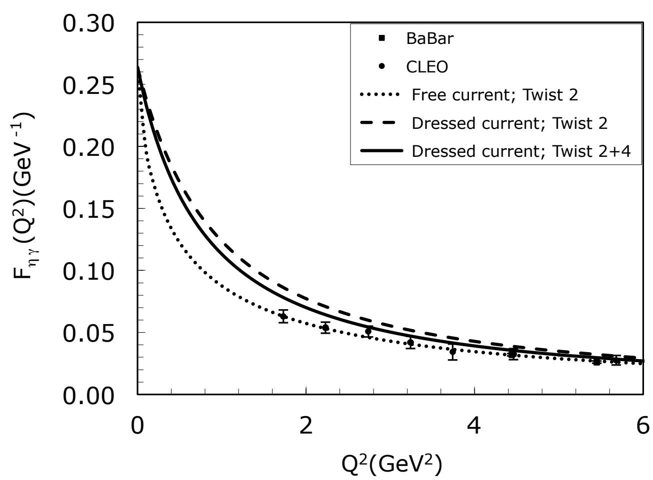

FIG. 6: Same as Fig. 4 for the $\gamma\gamma^* \to \eta$ transition form factor shown as $F_{\eta\gamma}(Q^2)$.

form in 5-dimensional AdS space, leads directly to an expression for the photon-to-pion transition form factor for a class of confining models. Remarkably, the pion transition form factor given by Eq. (13) derived from the CS action is identical to the leading order QCD result where the distribution amplitude has the asymptotic form $\phi(x) \propto x(1-x)$.

The Chern-Simon form is local in AdS space and is thus somewhat limited in its predictability. It only retains the $q\bar{q}$ component of the pion wavefunction, and further, it

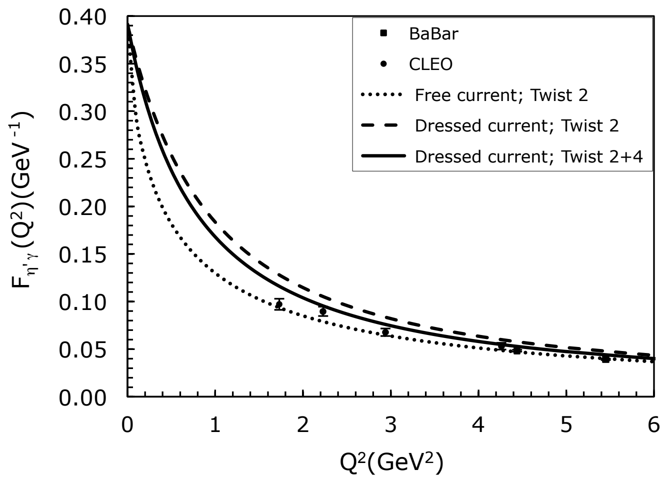

FIG. 7: Same as Fig. 4 for the $\gamma\gamma^* \to \eta'$ transition form factor shown as $F_{\eta'\gamma}(Q^2)$.

projects out only the asymptotic form of the meson distribution amplitude $\phi(x) \propto x(1-x)$. In contrast, the holographic light-front mapping of electromagnetic and gravitational form factors gives the full form of the distribution amplitude $\phi \propto \sqrt{x(1-x)}$ for arbitrary values of Q^2. This apparently contradictory result was first found in Ref. [37] in a hard-wall AdS extended model. This contradiction indicates that the local interaction from the CS action can only represent the point-like asymptotic form. [65] If the QCD evolution for the distribution amplitude is included, the asymptotic DA is recovered. The asymptotic result coincides with the CS amplitude which is only sensitive to short-distance physics.

It is found that in order to describe simultaneously the decay process $\pi^0 \to \gamma\gamma$ and the pion TFF at the asymptotic limit a probability for the $q\bar{q}$ component of the pion wavefunction $P_{q\bar{q}} = 0.5$ is required for the calculations with the free and dressed AdS currents.

We have argued that the contributions from the higher Fock components in the pion light-front wave function also need to be included in the analysis of exclusive processes. In fact, just as in 1+1 QCD, the confining interaction of the LF Hamiltonian in light-front holography leads to Fock states with any number of extra $q\bar{q}$ pairs. These contributions lead to higher-twist contributions to the hadron form factor. We have shown how the effect of the higher Fock states in form factors can be obtained by analyzing the hadron matrix elements of the confined dressed electromagnetic Heisenberg current from the gauge/gravity duality. The probability for the four-quark states obtained in this work, $P_{q\bar{q}q\bar{q}} = 0.085$ is similar to that found from an analysis of the space- and time-like behavior of the pion form factor using LF holographic methods, including higher Fock components in the pion wave function. [60]

The results obtained for the η- and η'-photon transition form factors are consistent with all currently available experimental data. However, the rapid growth of the pion-photon transition form factor exhibited by the *BABAR* data at high Q^2 is not compatible with the models discussed in this article, and in fact is very difficult to explain within the current framework of QCD.

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Appendix A: Distribution Amplitudes From Holographic Mapping

For a two-parton bound state, light-front holographic mapping relates the light-front wavefunction $\psi(x, \zeta, \varphi)$ in physical space-time with a field $\Phi(z)$ which represents a hadronic state in AdS space. The relation is [14]

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}, \tag{A1}$$

where we have factored out the angular dependence φ and the longitudinal, $X(x)$, and transverse mode $\phi(\zeta) = \zeta^{-3/2}\Phi(\zeta)$. The holographic variable z is related to the light-front invariant variable ζ which represents the transverse separation of the quarks within the pion: $z \to \zeta = \sqrt{x(1-x)}|\mathbf{b}_\perp|$. The LF variable x is the longitudinal light-cone momentum fraction $x = k^+/P^+$ and $\mathbf{b}_\perp$ is the impact separation and Fourier conjugate to $\mathbf{k}_\perp$, the relative transverse momentum coordinate.

The LFWF is normalized according to

$$\langle \psi_{q\bar{q}/\pi} | \psi_{q\bar{q}/\pi} \rangle = P_{q\bar{q}}, \tag{A2}$$

where $P_{q\bar{q}}$ is the probability of finding the $q\bar{q}$ component in the pion light-front wavefunction. We choose the normalization of the LF mode $\phi(\zeta) = \langle\zeta|\psi\rangle$ as

$$\langle\phi|\phi\rangle = \int d\zeta\, |\langle\zeta|\phi\rangle|^2 = P_{q\bar{q}}, \tag{A3}$$

and thus the longitudinal mode is normalized as

$$\int_0^1 \frac{X^2(x)}{x(1-x)} = 1. \tag{A4}$$

The factorization of the LFWF given by (A1) is a natural factorization in the light front since the corresponding canonical generators, the longitudinal and transverse generators P^+ and $\mathbf{P}_\perp$ and the z-component of the orbital angular momentum J^z, are kinematical generators which commute with the LF Hamiltonian generator P^-. Using this factorization one can map the elastic electromagnetic and gravitational form factors for arbitrary values of the transverse momentum Q obtaining the specific form $X(x) = \sqrt{x(1-x)}$ [4, 8] for the longitudinal mode. For a harmonic confining potential $U(z) \sim \kappa^4 z^2$ the LFWF is

$$\psi_{q\bar{q}/\pi}(x, \mathbf{b}_\perp) = \frac{\kappa}{\sqrt{\pi}} \sqrt{P_{q\bar{q}}} \sqrt{x(1-x)}\; e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2}. \tag{A5}$$

The pion distribution amplitude in the light-front formalism [11] is the integral of the valence $q\bar{q}$ light-front wavefunction

$$\phi(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \psi_{q\bar{q}/\pi}(x, \mathbf{k}_\perp), \tag{A6}$$

and satisfies the normalization condition which follows from the decay process $\pi \to \mu\nu$ ($N_C = 3$)

$$\int_0^1 \mathrm{d}x\, \phi(x) = \frac{f_\pi}{2\sqrt{3}}, \tag{A7}$$

where $f_\pi = 92.4$ MeV is the pion decay constant. From (A5) we find the distribution amplitude

$$\phi(x) = \frac{4}{\sqrt{3}\pi} \sqrt{x(1-x)}, \tag{A8}$$

and the pion decay constant

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa. \tag{A9}$$

The longitudinal mode $X(x)$ cannot be determined from the mapping of the Hamiltonian equation for bound states as it decouples in the ultra relativistic limit $m_q \to 0$. [14]

As discussed in the article, the CS mapping gives the asymptotic distribution amplitude since the CS maps a point-like pion. The corresponding longitudinal mode in the LFWF is $X(x) = \sqrt{6}x(1-x)$ and

$$\psi_{q\bar{q}/\pi}(x, \mathbf{b}_\perp) = \frac{\kappa}{\sqrt{\pi}} \sqrt{P_{q\bar{q}}} \sqrt{6}\, x(1-x)\; e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2}. \tag{A10}$$

The pion decay constant in this case is

$$f_\pi = \sqrt{P_{q\bar{q}}}\, \frac{\kappa}{\sqrt{2}\pi}. \tag{A11}$$

The evolution of the pion distribution amplitude in $\log Q^2$ is governed by the Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation [11, 34]. It can be expressed in terms of the anomalous dimensions of the Gegenbauer polynomial projection of the DA. If we normalize the full LFWF of the pion by $\langle\psi|\psi\rangle = 1$, we can compute the probability to find the pion in a given component of a Gegenbauer polynomial expansion $X(x) = x(1-x)\sum_n \alpha_n C_n^{(3/2)}(2x-1)$. We find

$$P_n = \frac{(n+2)(n+1)}{4(2n+3)}\alpha_n^2, \tag{A12}$$

where $\sum_n P_n = 1$. For the AdS solution $X(x) = \sqrt{x(1-x)}$ the asymptotic component $\alpha_0 = 3\pi/4$ and the probability to find the pion in its asymptotic state is $P_0 = 3\pi^2/32 \simeq 92.5\,\%$, not too far from the asymptotic result.

The asymptotic form has zero anomalous dimension. The distribution amplitude $\phi(x) \propto \sqrt{x(1-x)}$ derived from LF holographic methods is sensitive to soft physics $1 - x \sim \Lambda_{\rm QCD}/Q^2$, and has Gegenbauer polynomial components with nonzero anomalous dimensions which are driven to zero for large values of Q^2. Expanding the distribution amplitude at any finite scale as $x(1-x)$ times Gegenbauer polynomials, only its projection on the lowest Gegenbauer polynomial with zero anomalous moment survives.

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