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Simple limits on achieving a quasi-linear magnetic compression for an FEL driver

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Abstract

Free electron lasers (FEL) need a very bright electron beam in three dimensions and a high peak charge density. In order to compress an initially longer electron bunch generated from the photoinjector, magnetic bunch compression systems are widely employed. In this paper, first harmonic RF linearization and its associated requirements are reviewed. Meanwhile it is also briefly discussed what is the relation between a proper initial bunch length and main RF frequency, when a harmonic RF linearization is included. Then given a reasonable bunch compression ratio, a proper initial bunch length as a function of the main RF frequency and RF phase is estimated analytically by several approaches, assuming that no harmonic RF section is needed to linearize the energy modulation introduced during main RF acceleration, and at the same time still linearly compress the bunch length. Next the upper limit of the bunch compression ratio in a single stage is evaluated analytically. The analytical relations derived on choosing a proper initial bunch length as a function of main RF frequency are confirmed by numerical simulation. These simple limit provide rough estimations and may be beneficial for choosing bunch compression ratios in different stages of an FEL driver, especially in a first stage bunch compression where there is usually a harmonic RF linearization applied. It may also be useful in evaluating the possibility of low charge operation mode without any harmonic RF linearization, where a shorter initial bunch length can be achieved from the photoinjector.

1 Overview

Soft and hard X-ray free electron lasers (FEL) are being designed, constructed or operated everywhere in the world, since its invention and first demonstration by J. Madey at Stanford University in 1970s [1] [2]. Usually FELs are based on RF acceleration with different RF frequencies and electron bunch compression in stages, such as L-band RF for TTF-FEL [3], FLASH [4] and the XFEL [5], S-band RF for LCLS [6], C-band RF for SPring-8 Compact SASE Source [7] and X-band RF based FELs under study at SLAC. The FEL coherence condition of the electron beam in the undulators requires a high peak current, a small also uniform longitudinally sliced transverse emittance and relative energy spread. The RMS electron bunch length from the photoinjector is normally in the ps scale, with an electron bunch charge in the range of hundreds pC to several nC, which means

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that the initial peak current is usually less than 100 Amp. FEL simulations indicate that a peak current in the kilo Amp range is necessary for electron beam to lase coherently in an undulator and continually accumulate FEL power in a short gain length, as observed from the radiation power gain length (in which FEL lasing power increases by $e = 2.718$ times) formula shown below which is based on 1-D theory [8].

$$L_g = \frac{1}{\sqrt{3}} \left[\frac{2mc}{\mu_0 e} \frac{\gamma^3 \sigma_r^2 \lambda_u}{K^2 I} \right]^{1/3} \tag{1}$$

where γ denotes relativistic beam energy, c speed of light, e electron charge, σ_r transverse electron beam size, λ_u undulator period, K normalized undulator strength and I electron bunch peak current.

Magnetic bunch compression system is then normally adopted during the acceleration process of the electron beam, usually in several (two) stages. The basic idea is to first introduce an energy modulation (chirp) along the electron bunch in its longitudinal direction by an RF acceleration off-crest, then let the electron beam pass by a dispersive region where electrons in the head and tail of the bunch all move relatively towards bunch center. Chicane and wiggler based bunch compressor design were proposed and studied thoroughly in 1990s, mainly for linear collider projects [9] [10] [11]. It is worth to note that for a four dipole chicane (or other shape with only dipoles and drifts), there is a simple relation between second order dispersion T_{566} and first order dispersion R_{56}, which is $T_{566} = -1.5 R_{56}$.

Due to the nonlinear nature of the RF sinusoidal wave and an initially long bunch length, higher order energy chirps can not be neglected. Harmonic RF linearization of the longitudinal phase space of the electron bunch is normally employed to generate a more uniformly compressed bunch in a first stage bunch compression. Without the assistance of harmonic RF, usually one part of the electron bunch is compressed more than the other parts and a single high spike is generated in the current profile, which is not favored by the FEL application. As an example, FLASH managed to generate a final compressed electron bunch with a uniform current profile, after it installed a third harmonic RF section in its first stage bunch compression system. Without this harmonic RF linearization, there is always a single spark on its final bunch current profile. Similarly, LCLS employs an X-band fourth harmonic RF [12] to linearize the electron bunch's longitudinal phase space, again in its first stage bunch compression.

An alternative way to do a linearized bunch compression is proposed and studied [13] [14]. The key point is to eliminate the harmonic RF section, and to apply an optics linearization instead, with a specially designed bunch compressor which includes quadrupole and sextupole magnets and generates a required relation between T_{566} and R_{56}. Similar schemes have been proposed and studied preliminarily before, either in an analytical manner [11] [15] [16], by numerical simulations [11] [16] [17] [18], or in experimental measurements [16].

In this paper, first harmonic RF linearization and its associated requirements are reviewed and discussed. Meanwhile it is also briefly discussed what is the relation between a proper initial bunch length and main RF frequency, when a harmonic RF linearization is included. Then a proper initial bunch length is derived as a function of the main RF frequency and RF phase, given the condition that quasi linear bunch compression is still possible and there is no harmonic RF linearization employed. An upper limit is also derived on the bunch compression ratio in a single stage. Numerical simulations are performed employing different RF frequency from L-band RF to X-band RF, and a good agreement is achieved with the analytical derivations. These simple estimated limit provide a guidance in choosing bunch compression ratio of high charge operation mode (i.e. $250pC$ bunch charge) in its first bunch compression stage, in which there is a harmonic RF section to linearize the energy correlation and cancel the higher order energy chirps from main RF acceleration. In other words, the proper bunch length relation sets an upper limit on the electron bunch length in linac2 and in bunch compressor two, for a high charge operation mode with longer initial bunch length from photoinjector. Hence it also sets a lower limit on the bunch compression ratio in the first stage. These formulae may also be applied to evaluate a proper initial bunch length for a low charge operation mode (i.e. $10pC$ or $20pC$ bunch charge), where no harmonic RF linearization is needed even in the first stage bunch compression process.

One needs to note that these limits only provide rough estimations on the required bunch length and compression ratio, where assumptions have been adopted and no collective effects are included. A detailed bunch compression system design can start with these simple estimations and then be optimized by 3-D simulations with all effects included. As mentioned, except a high peak charge density associated with a short bunch length, FEL lasing also requires a small energy spread. In this paper, only the bunch length compression is discussed in detail. Meantime, the initial un-correlated energy spread (from photoinjector and laser heater), or more generally, the initial longitudinal emittance should be kept small for a more efficient FEL lasing, but large enough to Landau-damp collective effects such as micro-bunching instability.

2 Review of harmonic RF linearization

To linearize the main RF curvature on the energy chirp using another RF system, a harmonic RF section is required which RF frequency is usually several times of the main RF [12] [19]. The original derivations in [12] [19] are reviewed and further discussed in this section. The relative energy offset of any particle after passing by these two RF sections is expressed in the following formula, where the second and third terms on the right side are from the main RF acceleration and harmonic RF deceleration, respectively. The first term on the right side is from the damped initial un-correlated energy offset.

$$\delta(z) = \delta_i \frac{E_{i0}}{E_{f0}} + \frac{eV_0 \cos(\phi + kz_i)}{E_{f0}} + \frac{eV_h \cos(\phi_h + k_h z_i)}{E_{f1}} \tag{2}$$

where V_0 denotes the main RF voltage, δ_i initial relative un-correlated energy offset, E_{i0} initial electron beam energy, E_{f0} central energy after main RF acceleration, e electron charge, E_{f1} central energy after main RF acceleration and harmonic RF deceleration, ϕ main RF phase (with respect to crest), $k = \frac{2\pi f}{c}$ main RF wave number, f main RF frequency, z_i electron's initial longitudinal coordinate, V_h harmonic RF voltage, ϕ_h harmonic RF phase, $k_h = \frac{2\pi f_h}{c}$ the harmonic RF wave number, f_h the harmonic RF frequency and c speed of light.

By using a trigonometric expansion, and Taylor series of the sinusoidal functions 'Sine' and 'Cosine', the z-correlated energy offset $\delta(z)$ could be derived as a polynomial of z, as shown in the formulae below.

$$\delta(z) = a \cdot \delta_i + h_1 z_i + h_2 z_i^2 + h_3 z_i^3 + ... \tag{3}$$

where $a = E_{i0}/E_{f0}$ denotes energy damping ratio, h_1 the first order energy chirp (correlation), h_2 second order energy chirp, and h_3 third order energy chirp.

Up to third order, it is easy to write down the overall energy chirp after passing by these two RF sections, as shown in the formulae below. The overall energy correlation (chirp) is a simple sum of the chirp from main and harmonic RF sections. The expression of these chirp terms are functions of RF frequency, RF voltage and phase, as well as the electron beam energy at different locations.

$$h_1 = h_{1s} + h_{1h} = -\frac{keV_0 \sin\phi}{E_{f0}} - \frac{k_h eV_h \sin\phi_h}{E_{f1}} \tag{4}$$

$$h_2 = h_{2s} + h_{2h} = -\frac{k^2 eV_0 \cos\phi}{2E_{f0}} - \frac{k_h^2 eV_h \cos\phi_h}{2E_{f1}} \tag{5}$$

$$h_3 = h_{3s} + h_{3h} = \frac{k^3 eV_0 \sin\phi}{6E_{f0}} + \frac{k_h^3 eV_h \sin\phi_h}{6E_{f1}} \tag{6}$$

where h_{1s} denotes the first order energy chirp from main RF section, h_{2s} second order energy chirp, and h_{3s} third order energy chirp (the letter 's' reflects that it is assumed to be S-band RF as for LCLS case), h_{1h} the first order harmonic RF energy chirp, h_{2h} second order harmonic RF energy chirp, and h_{3h} third order harmonic RF energy chirp. The electron beam energy relation before and after harmonic RF section is given below.

$$E_{f1} = E_{f0} + V_h \cos\phi_h \tag{7}$$

where E_{f0} denotes the electron beam central energy at the end of the main RF section, E_{f1} the central energy at the end of the harmonic RF section.

One observes that to generate a linearized energy chirp up to second order, the requirement on the harmonic RF could be resolved by letting $h_2 = h_{2s} + h_{2h} = 0$ which is presented in a formula below.

$$V_h \cos\phi_h \left(E_{f0} \cdot k_h^2 + k^2 \cdot V_0 \cos\phi\right) = -E_{f0} k^2 \cdot V_0 \cos\phi \tag{8}$$

where E_{f0} denotes central energy after main RF acceleration.

It is straightforward to observe that one needs a decelerating harmonic RF phase ϕ_h, given an accelerating RF phase ϕ is employed for the main RF system which is the normal case. Similarly solve the zero condition for the third order energy chirp $h_3 = h_{3s} + h_{3h} = 0$, one finds that the third order energy linearization condition reads as shown below.

$$E_{f0} k_h^3 V_h \sin\phi_h + k^3 \cdot V_0 \sin\phi \cdot V_h \cos\phi_h = -E_{f0} k^3 \cdot V_0 \sin\phi \tag{9}$$

In general, one needs two harmonic RF sections to correct both second order chirp h_2 and third order chirp h_3 to zero. A harmonic RF section with higher RF frequency is usually more efficient in linearization.

Taking advantage of the relationship between beam energy before and after the main acceleration RF section, as shown below, one finds a simpler solution for second order linearization requirement.

$$V_0 \cos\phi = E_{f0} - E_{i0} \tag{10}$$

$$V_h \cos\phi_h \left(E_{f0} \cdot k_h^2 + k^2 \cdot (E_{f0} - E_{i0})\right) = -E_{f0} k^2 \cdot (E_{f0} - E_{i0}) \tag{11}$$

As usually the initial electron beam energy from photoinjector is around $5MeV$ which is very small compared with the beam energy at bunch compressor one, $E_{i0} \ll E_{f0}$, one finds that the above formula can then be simplified with $V_0 \cos\phi \approx E_{f0}$, as shown below.

$$V_h \cos\phi_h \left[\left(\frac{k_h}{k}\right)^2 + 1\right] = -E_{f0} \tag{12}$$

Given a fixed design beam energy at bunch compressor one E_{f0}, one finds that a higher harmonic RF wave number k_h requires a smaller harmonic RF voltage V_h for a same decelerating RF phase ϕ_h and fixed main RF wave number k. That means a harmonic RF section with higher frequency is more efficient in correcting the higher order energy chirp. Meantime, one also observes that a harmonic RF phase of $\phi_h = 180°$ gives $\cos\phi_h = -1$ which also minimizes the harmonic RF voltage needed.

Take a third harmonic RF as an example, the required harmonic RF frequency for a main RF section of L-band, S-band, C-band and X-band are 3 GHz, 9 GHz, 20 GHz and 30 GHz, respectively. One then could conclude that while it is easier to employ harmonic RF linearization for L-band main RF and S-band main RF, the limitation from current RF technology makes it difficult to provide a harmonic RF for C-band main RF and X-band main RF, with a required RF frequency above 20 GHz. As already being demonstrated in operation, a 3.9 GHz third harmonic RF (S-band) for FLASH and an 11.4 GHz fourth harmonic RF (X-band) for LCLS both work well in linearizing the energy modulation and in assisting to provide a more uniform final beam current profile.

Choosing a proper initial bunch length with harmonic RF linearization is briefly discussed in this section. Assume a harmonic RF phase of $\phi_h = 180°$, and a harmonic RF voltage V_h which makes the overall second order chirp h_2 equals zero, one finds that the overall energy chirp up to third order can be expressed as shown below. One needs to note that given a harmonic RF phase of $\phi_h = 180°$, the first and third order energy chirp is not affected by harmonic RF deceleration.

$$h_1 = h_{1s} + h_{1h} = -\frac{keV_0 \sin\phi}{E_{f0}} \tag{13}$$

$$h_2 = h_{2s} + h_{2h} = -\frac{k^2 eV_0 \cos\phi}{2E_{f0}} + \frac{k_h^2 eV_h}{2E_{f1}} = 0 \tag{14}$$

$$h_3 = h_{3s} + h_{3h} = \frac{k^3 eV_0 \sin\phi}{6E_{f0}} \tag{15}$$

One then observes that the overall second order energy chirp is zero under these conditions. If the third order energy chirp can be neglected for a given initial bunch length, the longitudinal phase space can be treat as completely linearized.

Using the energy chirp approach which is similar as the one in next section, the overall energy modulation with harmonic linearization can then be rewritten as shown below.

$$\delta(z) = a \cdot \delta_i + (h_1 + h_3 \cdot z^2) z \tag{16}$$

where $h_1 + h_3 \cdot z^2$ denotes the effective first order energy chirp, with the overall second order energy chirp zeroed by harmonic RF section.

One then observes that a proper initial bunch length to make $h_1 + h_3 \cdot z^2 \approx h_1$ should fulfill a condition of $|h_1| \gg |h_3 \cdot \sigma_z^2|$. Using the expressions of first order and third order energy chirp, h_1 and h_3, this condition can be derived as shown below.

$$\frac{6}{k_{RF}^2 \cdot \sigma_z^2} \gg 1 \tag{17}$$

where k_{RF} denotes RF wave number.

One needs to note that in most cases, the initial bunch length from photoinjector is much shorter than the required bunch length to maintain a quasi linear compression discussed above. Hence generally the third order energy chirp from RF acceleration can well be neglected. However, the third order energy chirp from collective effects, such as strong longitudinal wake field in the linac (in particular the second linac) always generates high current spikes in the head and tail of the bunch, thus could not be neglected. This effect is not discussed in this paper.

3 Proper initial bunch length without harmonic RF

In this section, only main RF acceleration section is considered and a proper initial bunch length is derived as a function of RF frequency and RF phase, given that a quasi linear compression is achieved with a reasonable bunch compression ratio adopted for each stage. There are two possible ways to quantify a quasi linear compression. First the contribution to final bunch length from nonlinearities (from RF acceleration and bunch compressors) could be evaluated and limited to a certain percentage. A second way is to compare the final bunch length calculated as the root mean square of the distribution, with the one derived from a numerical fit of the final distribution (Gaussian fit if the initial distribution is Gaussian), and limit the difference between these two to certain amount. It is assumed that no harmonic RF linearization is employed in any stage of the bunch compression. With a low bunch charge such as 10 pC or 20 pC, a much shorter initial bunch length can be achieved from the photoinjector. As mentioned above, the solution of the proper initial bunch length sets an upper limit on the required bunch length at the exit of bunch compressor one, in the high bunch charge operation mode. In other words it sets a lower limit on the bunch compression ratio in this first stage. For the low bunch charge operation mode, the harmonic RF section before bunch compressor one could be switched off, given a short enough initial bunch length is provided by the photoinjector. In this case a quasi linear bunch compression can be achieved if multi stage bunch compression is adopted with a proper compression ratio in each stage.

In the following, a proper initial bunch length is evaluated either with an energy chirp approach, or with a final bunch length approach, assuming that a reasonable bunch compression ratio is adopted in this stage. A proper compression ratio in each stage bunch compression is then discussed and evaluated analytically, in order to maintain a quasi linear compression process.

3.1 Energy chirp approach

First let us recall the energy modulation formula, for any one particle in a bunch, its relative energy offset after passing by an RF acceleration off-crest can be expressed as shown below.

$$\delta(z) = \delta_i \frac{E_{i0}}{E_{f0}} + \frac{eV_0 \cos(\phi + kz_i)}{E_{f0}} = a \cdot \delta_i + \frac{eV_0 \cos(\phi + kz_i)}{E_{f0}} \tag{18}$$

where δ_i denotes the initial un-correlated energy offset, E_{i0} central energy before RF acceleration, E_{f0} central energy after RF acceleration, e electron charge, V_0 the RF voltage, ϕ the RF phase, $k = \frac{2\pi}{\lambda}$ the RF wave number, λ the RF wave length, z_i particle's longitudinal coordinate relative to the bunch center and $a = E_{i0}/E_{f0}$ energy damping ratio.

The first order and second order energy chirp from the main RF acceleration could be described by the RF wave number, RF voltage and RF phase, solved from the same Taylor expansions discussed above, as shown below.

$$h_1 = -\frac{keV_0 \sin\phi}{E_{f0}} \tag{19}$$

$$h_2 = -\frac{k^2 eV_0 \cos\phi}{2E_{f0}} \tag{20}$$

Neglect the energy chirp above third order, the overall energy modulation from this main RF section can then be rewritten as

$$\delta(z) = a \cdot \delta_i + (h_1 + h_2 \cdot z)z \tag{21}$$

where $h_1 + h_2 \cdot z$ denotes the effective first order energy chirp.

One then observes that if z is small enough to make $h_1 + h_2 \cdot z \approx h_1$, the overall energy modulation is almost linear and in that case harmonic RF linearization is not necessary anymore. This condition could be interpreted as $|h_1| \gg |h_2 \cdot \sigma_z|$ and further derived to be the one shown below.

$$\frac{c \cdot \tan\phi}{\pi \cdot f_{RF} \cdot \sigma_z} \gg 1 \tag{22}$$

where c denotes speed of light, ϕ RF phase used when introducing energy chirp, f_{RF} RF frequency, σ_z RMS bunch length.

At this step, a proper initial bunch length can be expressed as a function of the main RF frequency and RF phase, as shown below. Here one assumes that a reasonable bunch compression ratio is adopted, with the detailed requirements discussed in the section below.

$$\sigma_{z,max} = D_0 \frac{2\tan\phi}{k_{RF}} \tag{23}$$

where D_0 denotes an empirical constant which is a small number much less than 1, and it could be fitted from numerical simulation studies employing RF systems with different frequency (such as from L-band RF to X-band RF). As illustrated by the numerical simulation results below, in general D_0 should be less than 0.01 in order to maintain a relatively linear longitudinal phase space and a quasi linear bunch compression without harmonic RF assistance. In general, the required value of

D_0 depends on RF frequency, average RF phase adopted for introducing energy correlation, total compression ratio in all the stages and the number of bunch compression stages. D_0 is inversely proportional to the RF phase and total compression ratio, while it is proportional to the number of bunch compression stages.

To generate a final hard X-ray FEL using normal photoinjector and undulators configuration, usually the electron bunch length needs to be compressed by 30-200 times from its initial value. If more stages of bunch compression is adopted, then one only needs a small compression ratio in each stage. This in turn requires only a small RF phase and small longitudinal dispersion (momentum compaction) of the bunch compressor. In this case, a larger constant D_0 and a longer initial bunch length can be tolerated.

3.2 Final bunch length approach

A proper initial bunch length can also be evaluated by examing the expression of a final bunch length after bunch compression. Under a condition of linear optimal compression $1 + h_1 R_{56} = 0$, a similar required relationship is derived which is $|h_1| \gg |h_2 \sigma_z|$. Assuming the electron bunch preserves a Gaussian distribution after passing by the dispersive region, the RMS bunch length can then be calculated as an integral shown below.

$$\sigma_z{}^2 = \int\int z_f{}^2(z,\delta) \cdot f(z,\delta) dz d\delta \tag{24}$$

where $f(z,\delta)$ denotes a Gaussian distribution in both z and δ, $z_f(z,\delta)$ any particle's final longitudinal coordinate as a function of its initial coordinate and relative energy offset.

Then the expression of $z_f(z,\delta)$ is derived with dispersion terms of the bunch compressor up to third order. After passing by the dispersive region, the longitudinal coordinate (relative to the bunch center) of any particle can be expressed as shown below.

$$z_f(\delta) = z_i + R_{56}\delta + T_{566}\delta^2 + U_{5666}\delta^3 + ... \tag{25}$$

where R_{56} denotes the first order longitudinal dispersion of the bunch compressor, T_{566} the second order dispersion, δ relative energy offset, and U_{5666} the third order dispersion.

Take the correlated relative energy offset $\delta(z)$ in terms of energy chirp up to second order, keep the terms up to second order bunch compressor dispersion, and neglect the initial un-correlated energy offset δ_i in higher order terms above δ^2, the square of $z_f(\delta)$ is then calculated to be a polynomial of z as shown below.

$$\begin{aligned} z_f^2(z,\delta_i) &= a^2 R_{56}^2 \delta_i{}^2 \\ &+ 2R_{56} \cdot a \cdot \delta_i (1 + h_1 R_{56}) \cdot z \\ &+ \left[(1 + h_1 R_{56})^2 + 2R_{56} \cdot a \cdot \delta_i (h_2 R_{56} + h_1^2 \cdot T_{566})\right] \cdot z^2 \\ &+ (\cdots) \cdot z^3 \\ &+ \left[(h_1^2 \cdot T_{566} + h_2 \cdot R_{56})^2 + 4h_1 h_2 T_{566}(1 + h_1 R_{56}) + 2h_2^2 R_{56} T_{566} \cdot a \cdot \delta_i\right] \cdot z^4 \\ &+ (\cdots) \cdot z^5 + (\cdots) \cdot z^6 + (\cdots) \cdot z^7 + (\cdots) \cdot z^8 \end{aligned} \tag{26}$$

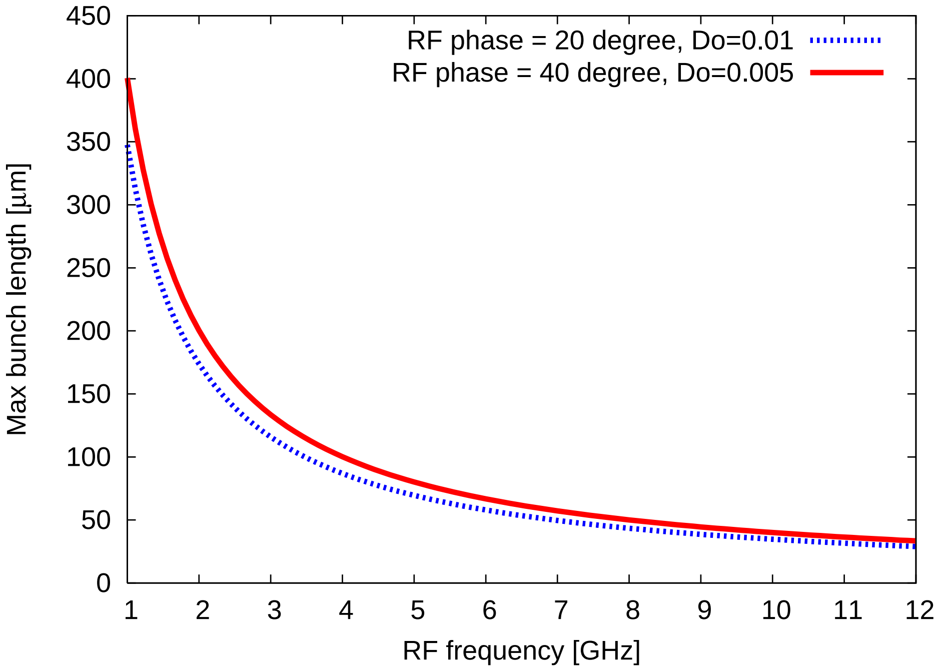

Figure 1: Proper initial bunch length as a funcion of RF frequency, for two given RF phase of 20 and 40 degree, based on formula (23). No harmonic RF linearization is included. A constant of $D_0 = 0.01$ is assumed for 20 degree case, while $D_0 = 0.005$ for 40 degree case.

where δ_i denotes the initial un-correlated energy offset, $z_f(z, \delta)$ the final longitudinal coordinate, and z the initial longitudinal coordinate.

One can then find the expression of the final bunch length by inserting $z_f^2(z, \delta_i)$ into the final bunch length formula and do the integral. Here one needs to note that the integral of all the odd functions is zero, such as $\int\int z \cdot f(z, \delta) dz d\delta = 0$ and $\int\int z^3 \cdot f(z, \delta) dz d\delta = 0$. The expression of final bunch length only has entries from integral of all the even functions. Under a condition of linear optimal compression, $1 + h_1 R_{56} = 0$, one finds that the final bunch length equals

$$\sigma_{z,f} = \sqrt{a^2 R_{56}^2 {\delta_i}^2 + [2R_{56} \cdot a \cdot \delta_i (h_2 R_{56} + h_1^2 \cdot T_{566} + h_2^2 T_{566} \cdot \sigma_z^2) + (h_1^2 \cdot T_{566} + h_2 \cdot R_{56})^2 \cdot \sigma_z^2] \cdot \sigma_z^2} \tag{27}$$

Again assuming a normal four dipole chicane is employed as the bunch compressor, there is a fixed ratio between its first order and second order dispersion terms R_{56} and T_{566}, as $T_{566} = -1.5R_{56}$. From the expression of final bunch length, one observes that the first term $a^2 R_{56}^2 {\delta_i}^2$ is from the initial un-correlated energy spread, and the third term $(h_1^2 \cdot T_{566} + h_2 \cdot R_{56})^2 \cdot \sigma_z^2$ implies that second order chirp h_2 always adds on first order effect given $T_{566} = -1.5R_{56}$. However, a larger RF phase (close to on-crest point) can minimize the contribution from the third term. It is the second term which could be derived to give a similar final conclusion as discussed in the last section, with a required criteria of $|h_2 R_{56} + h_1^2 \cdot T_{566}| \gg |h_2^2 T_{566} \cdot \sigma_z^2|$. To repeat, the required relationship associated with the maximum tolerable bunch length is $|h_1| \gg |h_2 \sigma_z|$.

Now according to formula (23) and the above argument as discussed in this section, one could plot the proper initial bunch length as a function of RF frequency with different RF phase, as shown in Figure 1. One observes that for a lower RF frequency, the proper initial bunch length is longer which is as expected. Another point is that if a higher RF phase can be employed, then the tolerable bunch length can be longer. In the next section, analytical results derived above are checked and confirmed by numerical simulations with multi stage bunch acceleration and compression based on RF systems with different frequency.

3.3 Proper compression ratio in one stage

In this subsection, a proper bunch compression ratio in one stage is discussed and an upper limit is drawn. Limited to first order, a compression ratio is approximated as shown below.

$$C = \frac{1}{1 + h_1 R_{56}} \tag{28}$$

where h_1 denotes a linear energy chirp, R_{56} first order dispersion term of bunch compressor.

From the final bunch length expression as shown in formula (26) one also observes that to keep an effectively linear bunch compression, there should be a limit applied on the bunch compression ratio. That means a bunch compression ratio can not be too large in one stage, otherwise the bunch compression is not linear anymore.

Consider a first order optimal bunch compression case, one has $1 + h_1 R_{56} = 0$ and then could conclude that in this case, the final bunch distribution is only dependent on the nonlinear terms, which obviously breaks the linear compression requirement. So in general, there should be another criteria for a linear bunch compression, which can be derived from formula (26) and shown in the following formula, up to second order. One needs to note that in the coefficient of term z^4, the relatively small un-correlated initial energy offset δ_i and first order associated term $4h_1 h_2 T_{566}(1 + h_1 R_{56})$ are neglected.

$$(1 + h_1 R_{56})^2 \gg (h_1^2 \cdot T_{566} + h_2 \cdot R_{56})^2 \cdot \sigma_z^2 \tag{29}$$

where h_2 denotes a second order energy chirp, T_{566} second order dispersion term of bunch compressor and σ_z RMS bunch length before bunch compression.

The requirement above in turn gives an upper limit on the bunch compression ratio in one stage. As there is a large acceleration in beam energy between subsequent stages, the nonlinear terms in the bunch energy chirp is also damped during acceleration. That damping effect helps to maintain a linear energy correlation. Also as the bunch length goes shorter from one stage to the next one, the new contribution on the nonlinear energy chirp from the following linacs is smaller. Under this kind of configuration with more than two stages bunch compression, a quasi linear bunch profile should be easily achieved at the linac end. The requirement on bunch compression ratio in one stage (especially the first stage) is then approximated as shown below, which is a function of dispersion terms, energy chirp terms and RMS bunch length.

$$C \ll \frac{1}{\sigma_z |h_1^2 \cdot T_{566} + h_2 \cdot R_{56}|} \tag{30}$$

As mentioned above, to generate a final hard X-ray FEL using normal photoinjector and undulators, usually the bunch length needs to be compressed by 30-100 times from its initial value, while achieving a final peak current of 1-3 kA. Combining all the above considerations, a bunch compression ratio between 5 and 10 may be adopted in one stage. Last point is that in general one should choose a weak chicane bunch compressor with small dispersion terms R_{56} and T_{566}, in order to minimize the contribution from higher order terms. On the other hand, that means a larger RF phase (close to on-crest point) is preferable for this purpose. Meanwhile a larger RF phase (close to on-crest point) also makes the system less sensitive to timing jitter effects. However, acceleration at a larger RF phase (close to on-crest point) also introduces a larger correlated energy spread which in turn requires better field quality of the chicane magnets. In next section, two bunch compression systems is built based on the conclusion of these analytical derivations, and tracking simulation is performed on these two FEL drivers. A good agreement is achieved between analytical and numerical simulation results.

4 Confirmation by numerical simulation

Numerical simulation results are presented here, for two FEL driver design based on different RF frequencies, which is L-band RF (1.3 GHz) and X-band RF (12 GHz), respectively. No harmonic RF correction is included here. The initial bunch length of $300\mu m$ and $40\mu m$ are adopted based on formula (23) plus the RF frequency and phase. One could observe Figure 1 and find that $300\mu m$ and $40\mu m$ are the limited bunch length corresponding to RF frequency of 1.3 GHz and 12 GHz, respectively. From the simulation results as shown below, one could conclude that the bunch compression process is quasi-linear, and a quasi-Gaussian bunch density profile is preserved. No spike in density profile is generated. These numerical simulation results confirm the validity of the above analytical derivations.

The accelerator lattice is designed using computer code MAD8 [20], which consists of a first linac section (Linac1) with an electron beam energy from $7MeV$ to $250MeV$, followed by a first stage four-dipole bunch compressor, then a second linac section (Linac2) from $250MeV$ to roughly $2GeV$, and a second stage bunch compressor which is also composed of four dipole magnets. No harmonic RF system is included in either bunch compression stage. The lattice is then translated into another accelerator simulation code Elegant [21], and the tracking simulation is performed therein.

A first example is an L-band RF system (with a frequency of 1.2 GHz) based FEL driver. Initially at a beam energy of $7MeV$, a low charge electron bunch is generated internally in Elegant, with 20 pC bunch charge and an RMS bunch length of $300\mu m$. A constant of $D_0 = 0.005$ is assumed here in choosing the initial bunch length. Its longitudinal phase space and current profile along longitudinal direction is shown in Figure 2 (top), where one observes that initial peak current is roughly 8 Amp. This electron bunch is then accelerated to $250MeV$ in Linac1 under an RF phase of -25 degree, then passing by a four-dipole chicane which has a first-order longitudinal dispersion of $R_{56} = -77mm$. The bunch length is compressed by roughly 10 times in first stage. It is then further accelerated to $2GeV$ in Linac2 under an RF phase of -55 degree, followed by a second chicane bunch compressor again with $R_{56} = -77mm$. During this second bunch compression stage a compression ratio of 20 is achieved, with a final peak current over 3 kA. As discussed before, the nonlinearities in energy chirp from Linac1 RF curvature and higher order dispersions of bunch compressor one is damped during acceleration in Linac2, a relatively linear bunch profile is achieved

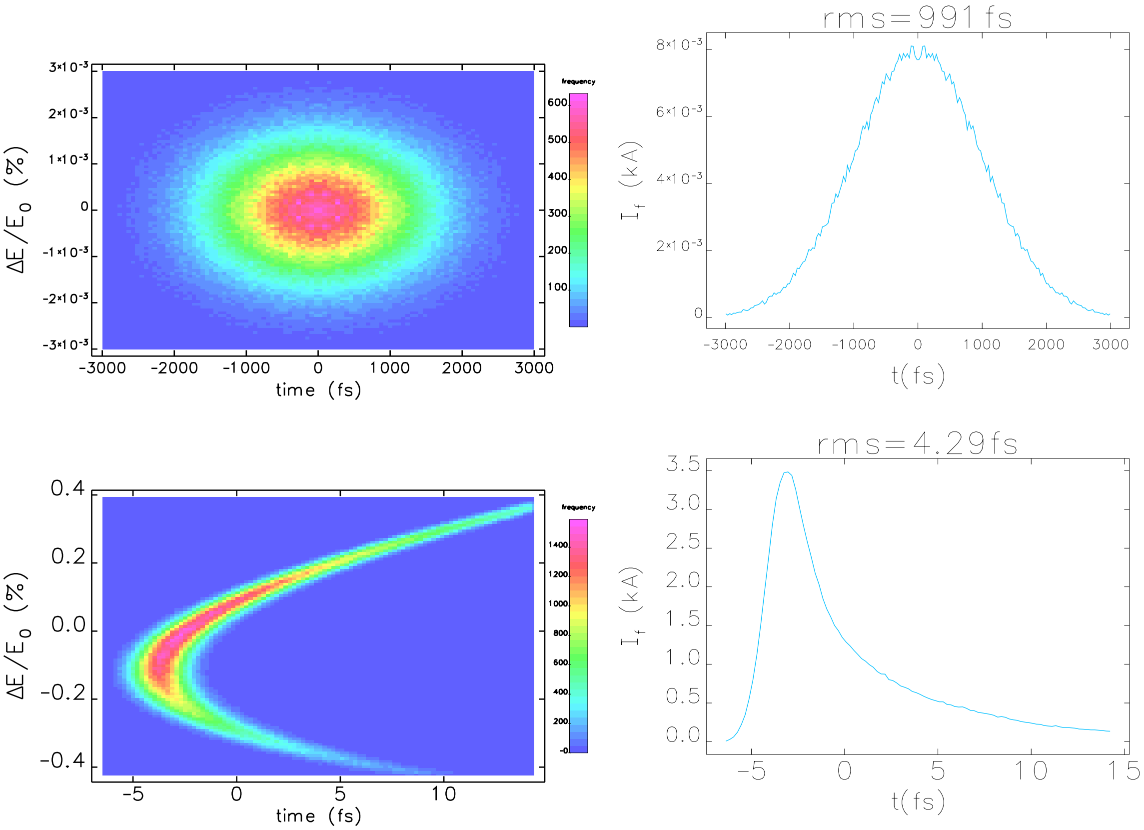

Figure 2: Two stage bunch compression employing L-band RF cavities, without harmonic RF linearization. A compression ratio of 200 is achieved. An average RF phase of -45 degree is chosen to compress the electron bunch length linearly. Space charge, wake fields and other collective effects are not included here. Top left: initial longitudinal phase space at the exit of photoinjector; top right: initial current profile, with an RMS bunch length of $300\mu m$; bottom left: final longitudinal phase space at the exit of bunch compressor two; bottom right: final current profile, with an RMS bunch length of $1.5\mu m$. A constant of $D_0 = 0.005$ is adopted here. Good agreement is achieved between these simulation results and the simple theoretical estimation as shown in Figure 1.

at the end, as shown in Figure 2 (bottom). To simplify the optimization of different parameters during tracking simulation, here no collective effects such as wake fields, space charge, incoherent synchrotron radiation (ISR) or coherent synchrotron radiation (CSR) are included. One observes that the electron bunch is successfully compressed in a linear manner and a final peak current of $3kA$ is achieved. The simulation results agree with the analytical derivations as shown in Figure 1.

Here one needs to note that if more stages of bunch compression are adopted for this case, such as three stages or four stages, a larger constant D_0 and a longer initial bunch length can be tolerated. For the two stage bunch compression case discussed above, another point to adopt a shorter initial bunch length is that one needs to keep the correlated energy spread relatively smaller, given a larger RF phase (close to on-crest point) employed such as 25 degree.

A second example is a similar FEL driver design which is based on all X-band (11.4 GHz) RF acceleration [22]. ASTRA simulations show that with a low bunch charge of 10 pC, the RMS bunch length can be tuned to around $40\mu m$, by fine tuning of the laser system in the X-band photoinjector [22]. That translates into a constant of $D_0 = 0.01$ in choosing the proper initial bunch length. The normalized transverse emittance can be preserved at a smaller value of $0.14\mu m \cdot rad$. The optics design and two stage bunch compression configuration are similar with the L-band FEL driver case discussed above, and is not presented in details here. Due to its high frequency and short wavelength, X-band RF is more efficient in introducing an energy chirp (correlation). Also for such a reason, relatively smaller RF phase is employed for this X-band accelerator, with an average value of -15 degree. A first order longitudinal dispersion R_{56} in a range between $-7mm$ and $-10mm$ is selected for either of two chicane compressors which is composed of four dipole magnets. In this example, all the collective effects like ISR, CSR, space charge and wake fields are included, and the compression ratio, RF phase and chicane design parameters are optimized accordingly, to generate a more linear final current profile [22]. A third X-band linac section is employed to accelerate the electron bunch to a beam energy of $6GeV$. In this process the energy correlation established in the previous linac sections is also removed mainly using the longitudinal wake field effects. An initial and a final longitudinal phase space and bunch current profile is shown in Figure 3. Again one observes that a quasi linear bunch compression is achieved, plus that the residual energy correlation is successfully removed in Linac3. A final peak current over $3kA$ with a pulse length of $2fs$ is generated. Here again the simulation results agree well with the analytical derivations as shown in Figure 1.

It is noted that if one extends these two stage bunch compression systems into three stage or more, the final bunch current profile could be more uniform, due to the reasons discussed above. In details, the bunch compression ratio can be smaller in each single stage, and the dispersion terms R_{56} and T_{566} are also smaller in the bunch compressors. The relative damping effect of the nonlinear energy chirp is also stronger given that more stages bunch compression is adopted.

5 Conclusion and discussion

In this paper, analytical formulae are derived to estimate an initial proper bunch length which is suitable for a linear bunch compression in multi stages, with or without the assistance of harmonic RF linearization. This proper initial bunch length is found to be a function of RF frequency and RF phase, for the case without harmonic RF. In order to achieve a quasi linear bunch compression,

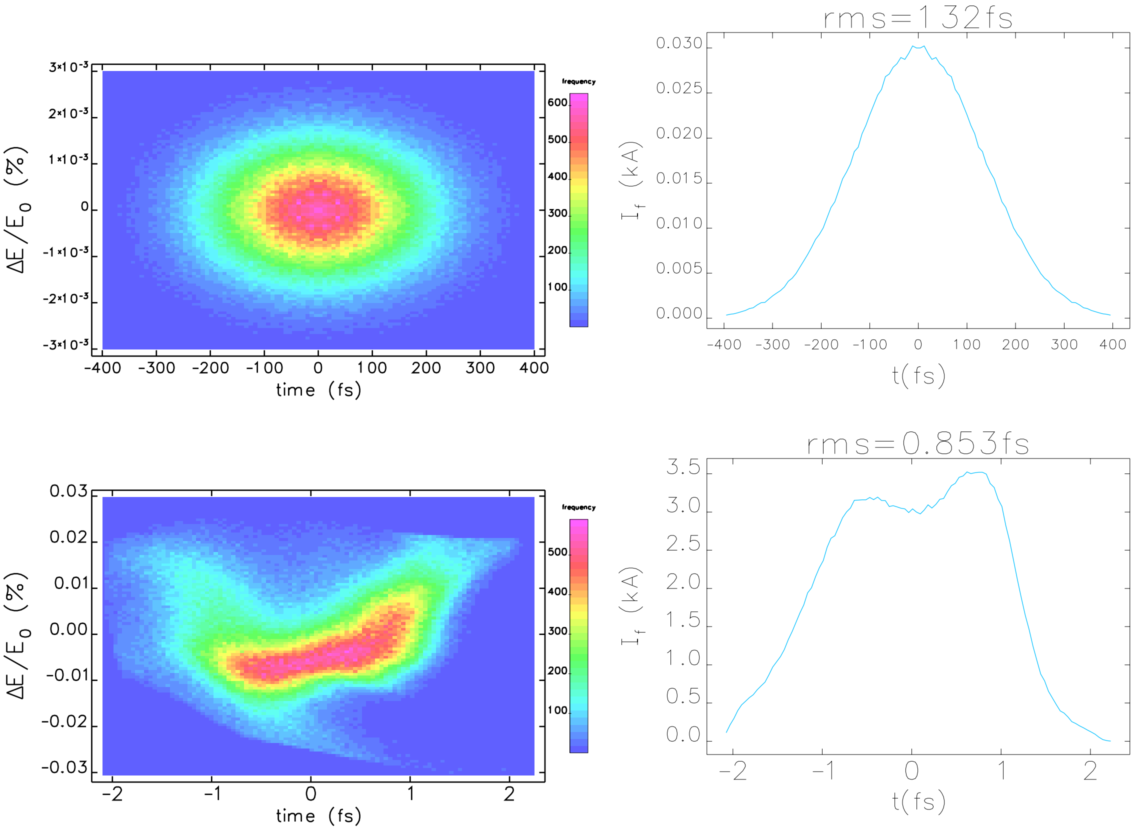

Figure 3: Two stage bunch compression employing X-band RF cavities, without harmonic RF linearization. A compression ratio of 150 in all is achieved. An average RF phase of -15 degree is chosen to compress linearly. Space charge, wake fields and other collective effects like ISR and CSR are all included in the tracking simulation. Top left: initial longitudinal phase space; top right: initial current profile, with an RMS bunch length of $40\mu m$; bottom left: final longitudinal phase space; bottom right: final current profile, with an RMS bunch length of less than $0.3\mu m$. A constant of $D_0 = 0.01$ is adopted for this case. Good agreement is achieved between these simulation results and the simple theoretical estimation as shown in Figure 1.

the initial bunch length should be shorter than this proper initial tolerable bunch length. Besides bunch compression ratio should not be too large in one stage, in order to minimize the impact of RF nonlinear curvature and higher order dispersions from chicane bunch compressor. Two numerical simulation examples of hard x-ray FEL driver design are given where these derivations are partially checked and confirmed. The RF frequency of the main acceleration part are chosen to be 1.2 GHz and 11.4 GHz, respectively. These derivations may be benificial for an FEL driver design which operation mode is mainly low bunch charge and which could achieve an initially short bunch length from photoinjector. On the other hand, these analytical derivations can also be applied to evaluate the lower limit of bunch compression ratio in a first stage with harmonic RF linearization, for high bunch charge operation mode such as $250pC$. In other words, it sets an upper limit on the maximum bunch length in linac2 and at the entrance of bunch compressor two. It is noted here that these limits only provide rough estimations on the required bunch length and compression ratio, where assumptions have been made and no collective effects are included. A detailed bunch compression system design can start with these simple formulae and then be optimized by 3-D simulations with all effects included.

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