# NUMERICAL VERIFICATION OF THE POWER TRANSFER AND WAKEFIELD COUPLING IN THE CLIC TWO-BEAM ACCELERATOR *

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## Abstract

The Compact Linear Collider (CLIC) provides a path to a multi-TeV accelerator to explore the energy frontier of High Energy Physics. Its two-beam accelerator (TBA) concept envisions complex 3D structures, which must be modeled to high accuracy so that simulation results can be directly used to prepare CAD drawings for machining. The required simulations include not only the fundamental mode properties of the accelerating structures but also the Power Extraction and Transfer Structure (PETS), as well as the coupling between the two systems. Time-domain simulations will be performed to understand pulse formation, wakefield damping, fundamental power transfer and wakefield coupling in these structures. Applying SLAC's parallel finite element code suite, these large-scale problems will be solved on some of the largest supercomputers available. The results will help to identify potential issues and provide new insights on the design, leading to further improvements on the novel two-beam accelerator scheme.

## INTRODUCTION

The Office of Science in the U. S. DOE is promoting the use of High Performance Computing (HPC) in projects relevant to its mission via the 'Scientific Discovery through Advanced Computing' (SciDAC) program which began in 2001. Since 1996, SLAC has been developing a parallel accelerator modeling capability, first under the DOE Grand Challenge and now under SciDAC, for use on HPC platforms to enable the large-scale electromagnetic simulations needed for improving existing facilities and optimizing the design of future machines.

T3P is the time-domain module within SLAC's parallel electromagnetic code suite ACE3P, and is used for simulations of wakefields and transient effects. T3P solves Maxwell's equations via the inhomogeneous vector wave equation for the time integral of the electric field $\mathbf{E}$:

$$\left(\varepsilon\frac{\partial^2}{\partial t^2} + \sigma_{\text{eff}}\frac{\partial}{\partial t} + \nabla\times\mu^{-1}\nabla\times\right)\int^{\mathbf{t}}\mathbf{E}\,\mathrm{d}\tau = -\mathbf{J}, \quad (1)$$

with permittivity $\varepsilon=\varepsilon_0\varepsilon_r$ and permeability $\mu=\mu_0\mu_r$. For simplicity in the computations, a constant value of the effective conductivity $\sigma_{\text{eff}}=\tan\delta\cdot 2\pi\mathrm{f}\cdot\varepsilon$ is assumed for dielectric materials by fixing a frequency $f$, and the losses are

specified by the loss tangent $\tan\delta$. The computational domain is discretized into an unstructured mesh of quadratic (10-point, curved) tetrahedral elements and $\int^t\mathbf{E}\,\mathrm{d}\tau$ in Eq. (1) is expanded into a set of hierarchical Whitney (tangentially continuous) vector basis functions $\mathbf{N_i}(\mathbf{x})$ up to order $p$ within each element:

$$\int^t\mathbf{E}(\mathbf{x},\tau)\,\mathrm{d}\tau = \sum_{i=1}^{N_p}e_i(t)\cdot\mathbf{N_i}(\mathbf{x}). \quad (2)$$

For illustration, $N_1$=8, $N_2$=20 and $N_6$=216. Time integration is performed via the unconditionally stable implicit Newmark-Beta scheme. It requires the solution of a sparse linear system at every time step, typically performed iteratively with a conjugate gradient method (and a diagonal or incomplete Cholesky preconditioner) with an upper bound for the relative residual error of $10^{-12}$.

As is common for wakefield computations of rigid beams, the electric current source density $\mathbf{J} = \rho\mathbf{v}$ in Eq. (1) is given by a Gaussian line charge density $\rho = Q\delta(x - x0)\delta(y - y0)G(z - z0)$, moving at the speed of light along the beam line (here: along $z$-direction). For calculations of transverse wakefields, excitation by a dipole current is realized with two co-moving charges $+Q/2$ and $-Q/2$ separated by $2d$, twice the transverse offset. For structures with two-fold (or four-fold) symmetry, electric boundary conditions in the dipole symmetry plane can be used to restrict the simulation to one half (or a quarter) of the geometry.

More detailed information about the methods used in T3P and their parallel scalability has been published earlier [1].

## RESULTS

In the following, numerical verification of transverse wakefield damping in the CLIC Two-Beam Accelerator (TBA) is presented - for the PETS decelerator structure, the TD24 accelerator structure and a combined system of the basic unit of the TBA, consisting of one PETS, two TD26 accelerator structures and a connecting waveguide network. More information about the structures can be found at [2], [3].

### Single Structure Calculations and Code Benchmarking

For the PETS, a beam with bunch length $\sigma_z$=2 mm is driven along the beam pipe axis with a transverse horizontal offset of 2.5 mm, in a quarter structure in conjunction
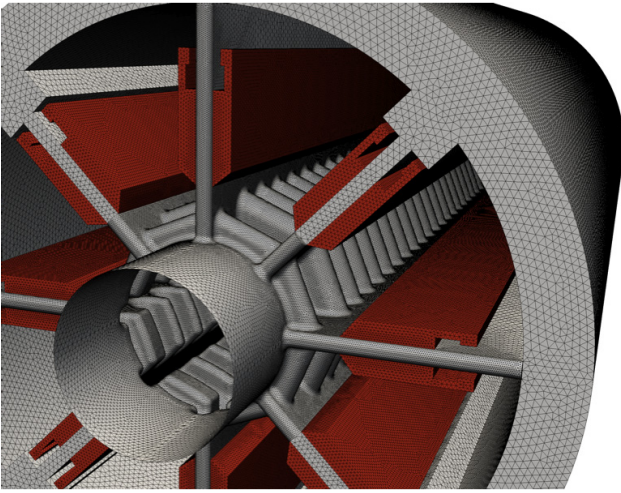
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Figure 1: Unstructured conformal mesh model used for T3P simulations of the PETS. A quarter model consists of about 10 million tetrahedral elements, many of which are curved. The mesh for the dielectric loads (highlighted in red) is refined to resolve the smaller local wavelengths.



Figure 3: Transverse wake potential of the PETS computed with T3P as a function of the order $p$ of the vector basis functions. For $p$=2, the time step is halved (to 0.25 ps) for optimal accuracy. Comparison to GdfidL results shows good agreement.

with electric boundary conditions in the vertical symmetry plane and magnetic boundary conditions in the horizontal symmetry plane. The effective conductivity $\sigma_{\mathrm{eff}}$ for the lossy dielectrics is calculated with parameters $f$=12 GHz, $\varepsilon_r$=13 and $\tan\delta$=0.2. Figures 1 and 2 show the used mesh model and a snapshot of the excited wakefields in the PETS as calculated with T3P. Figure 3 shows the transverse wake potential of the PETS computed with T3P and comparison to GdfidL results (first-order accuracy in fields and geometry via the finite difference cut-cell method).

For the TD24 accelerating structure, a single bunch of length $\sigma_z$=2 mm is driven along the beam pipe axis at a horizontal transverse offset of 1 mm. Figure 4 show the

used mesh model to T3P simulations. By using the broadband waveguide boundary conditions, the waveguides can be shortened without sacrificing simulation accuracy.

Figure 5 shows a snapshot of the transverse wakefields in the TD24 as calculated with T3P. For numerical truncation of the model at the waveguide ports, T3P uses broadband waveguide boundary conditions based on mode expansion into 2D port modes. This leads to better numerical absorption of excited modes at or near the waveguide cutoff than with the conventional first-order "ABC" absorbing boundary conditions. Figure 6 shows the transverse wake potential of the TD24 accelerating structure as calculated with T3P and GdfidL. Good agreement is found.
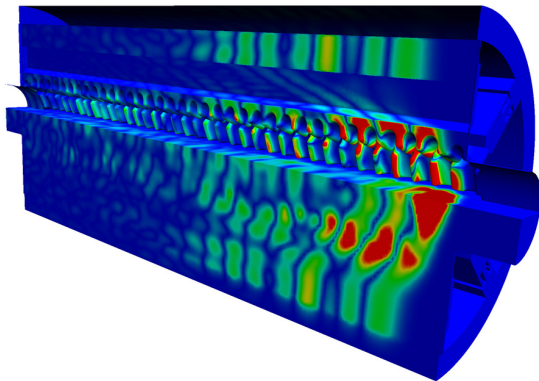


Figure 2: Snapshot of electric field magnitude in the PETS as computed with T3P for a beam transiting at an offset to excite transverse wakefields (via electric/magnetic boundary conditions). Strong damping in the lossy dielectric loads is observed.
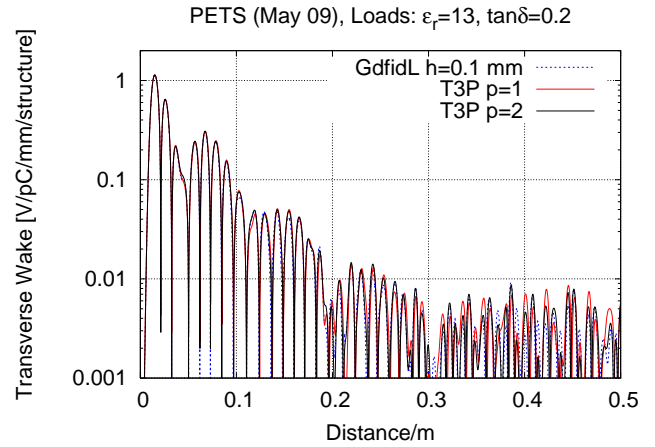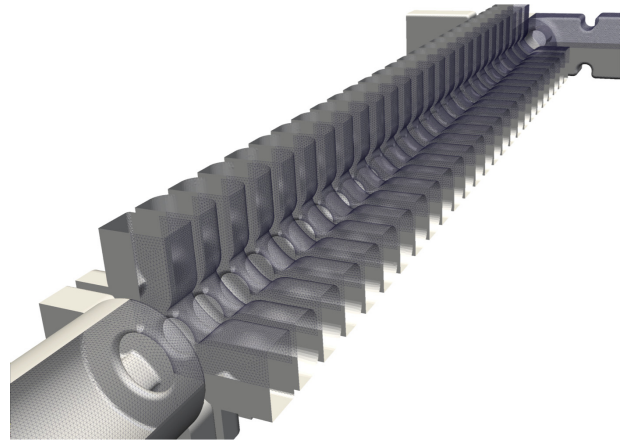


Figure 4: Unstructured conformal mesh model used for T3P simulations of the TD24 accelerating structure. A quarter model consists of about 4 million tetrahedral elements, many of which are curved.
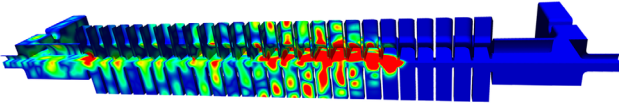
Figure 5: Snapshot of electric field magnitude in the TD24 accelerating structure as computed with T3P for a beam transiting at an offset to excite transverse wakefields (monopoles excluded via proper electric/magnetic boundary conditions).
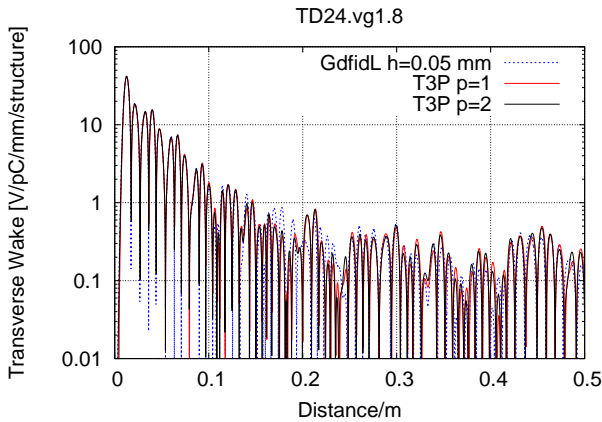


Figure 6: Transverse wake potential of the TD24 accelerating structure computed with T3P and GdfidL.

## Coupled Structure Calculations

The large-scale modeling capability of T3P is applied to full-scale 3D electromagnetic time-domain simulations of a basic unit of the CLIC Two-Beam Accelerator (TBA). Figures 7 and 8 show the tetrahedral mesh model used for T3P calculations of power transfer and wakefield coupling. The average element edge length is 0.9 mm, with an rms of only 0.25 mm. This is indicative of the significant effort spent on producing a high quality mesh model by compositing small surfaces from the CAD model into larger surfaces before generating the actual mesh, and thereby limiting the number of overall elements to only 28 million. Note that this mesh is intended for wakefield calculations with an rms bunch length $\sigma_z$=3 mm. Accurate simulations of a more realistic bunch length (e.g., $\sigma_z$=1 mm, the current design parameter) would require creating a mesh consisting of roughly one billion elements to obtain sufficient accuracy in modeling the higher frequency components. While this is currently not impossible, the required computational resources would be prohibitively large for routine simulations.

**Coupled Structure T3P Simulation Parameters.** The following time-domain calculations of coupling effects in the basic unit of the CLIC TBA were performed with T3P, using the coupled structure mesh model shown in figure 7. The main simulation parameters are a PETS bunch length of $\sigma_z$=3 mm, a discretization time step of $dt$=2 ps,
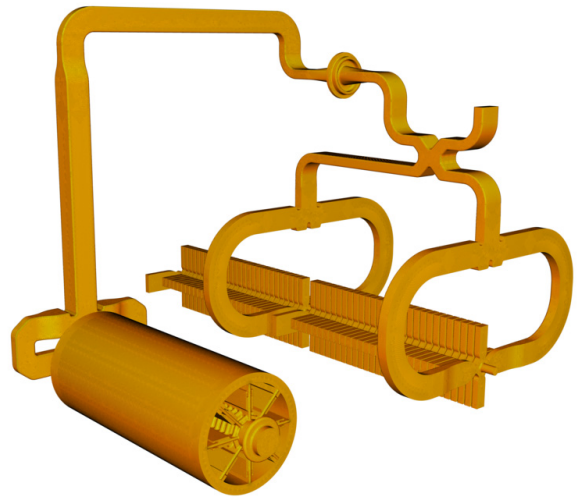


Figure 7: Unstructured tetrahedral mesh model used for T3P simulations of the complete basic unit of the CLIC Two-Beam Accelerator consisting of one power extraction and transfer structure (PETS), two TD26 accelerating structures (AS), and a connecting waveguide network. A detailed view of the mesh model is shown in figure 8.
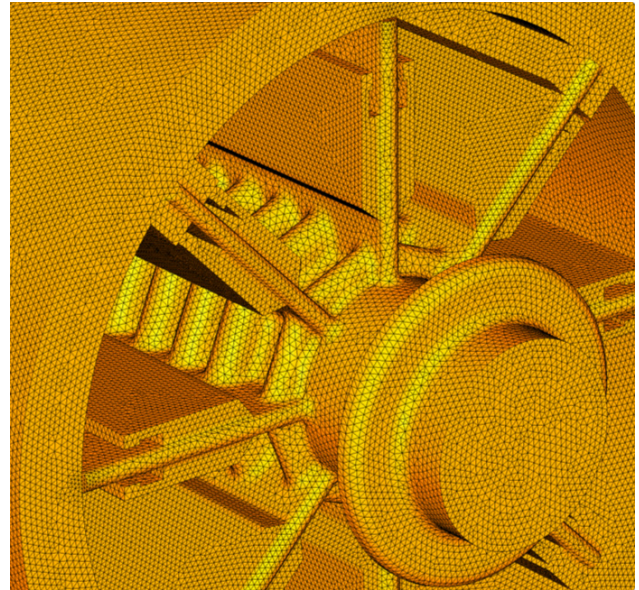


Figure 8: Detailed view of the mesh model shown in Fig. 7.

a total simulation time of 100 ns, and the use of first-order basis functions ($p$=1) in quadratic elements (i.e., curved, where needed). All waveguides are terminated with first-order absorbing boundary conditions. Ohmic wall losses are not included. Calculations were performed on *Hopper2*, NERSC's latest supercomputer, a Cray XE6 with 150k compute cores. A typical run time for such a simulation is about 20 hours on 1200 cores, and about an additional 10 hours for creating 1900 volume monitor field snapshots for later post-processing into animations or still frames (about 3 GB per volume snapshot).

**Three Different PETS Bunch Parameters.** In the following, results from three T3P simulations with different PETS bunch parameters are presented:

- *"single on-axis"*: one on-axis PETS bunch (charge Q)

- *"multiple on-axis"*: multiple on-axis PETS bunches with a repetition rate of 11.994 GHz (AS design fundamental frequency)

- *"single dipole X"*: one dipole PETS bunch realized by charge +Q/2 at X-offset 5 mm and charge -Q/2 at X-offset -5 mm

**Power Transfer - "single on-axis" Case.** Figure 9 shows a temporal snapshot of the *"single on-axis"* case, and figure 10 shows longitudinal energy transfer results.
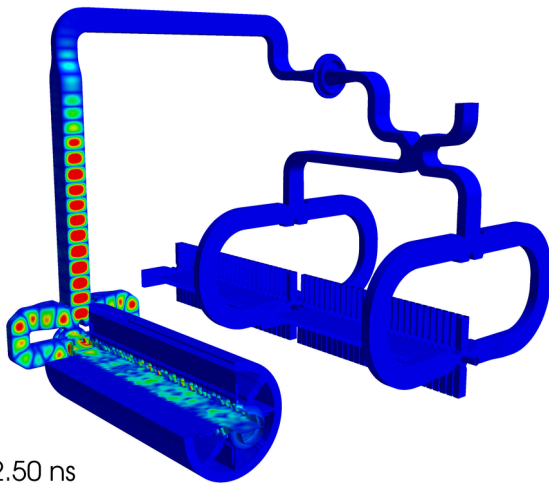


t=02.50 ns

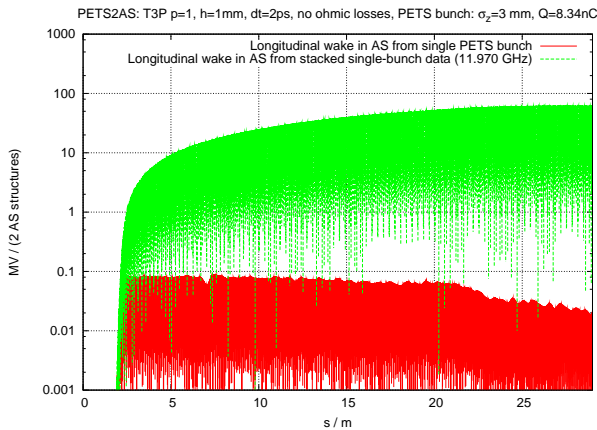Figure 9: Snapshot of transient electric fields for the *"single on-axis"* case simulated with T3P.



Figure 10: In red: Longitudinal wakefield (energy gain) calculated from the *"single on-axis"* case, as experienced by a main beam bunch transiting two TD26 AS. In green: Longitudinal wakefield obtained by stacking the single bunch data.

Stacking the single-bunch longitudinal wakefield at a repetition rate of 11.97 GHz resulted in the highest achievable energy gain of 62 MV in the two AS. While this is in good agreement with the design specifications, it also means that the *numerical* synchronous frequency differs by 24 MHz from the design frequency of 11.994 GHz. This difference can be attributed to the numerical inaccuracies stemming from using low-order $p=1$ basis functions, relatively large average element edge sizes of 1 mm, and a relatively large time step of 2 ps. Further, simulating with a 3 mm PETS bunch length instead of the design bunch length of 1 mm should result in a reduction of the electric fields by a form factor of approx. 0.777, and not including ohmic wall losses in the simulations increases the fields towards the downstream ends of the AS by a significant amount. Clearly, quantitative conclusions about the power transfer efficiency cannot be drawn from the current results, but spectral analysis can still help identify higher-order mode effects. More work is needed before a detailed numerical study of the power transfer mechanism is possible.

**Power Transfer - "multiple on-axis" Case.** Figure 11 shows a temporal snapshot of the *"multiple on-axis"* case. Based on the observations about the numerical approximations for the *"single on-axis"* case, these results are mainly used for illustration.
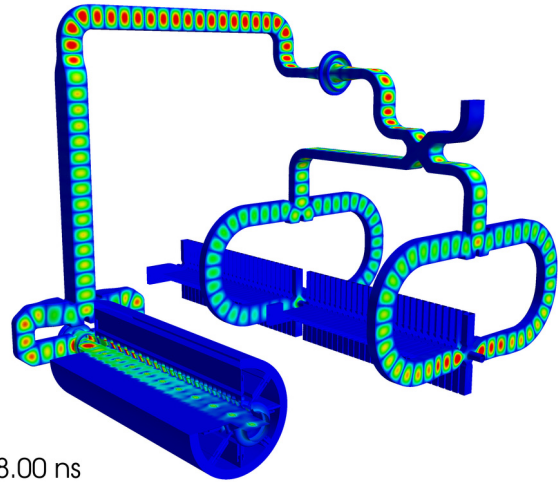


t=08.00 ns

Figure 11: Snapshot of transient electric fields for the *"multiple on-axis"* case simulated with T3P.

**Wakefield Coupling - "single dipole X" Case.** For numerical investigation of potentially dangerous transverse wakefield coupling from the PETS to the AS, a T3P simulation of wakefields excited by a dipole bunch in the PETS is performed (c.f. section "Introduction"). Figure 12 shows a temporal snapshot of the *"single dipole X"* case, and figure 13 shows the coupled longitudinal and transverse dipole wake potentials in the two AS.

Some mode conversion can be seen in the waveguide network bends, and the resulting longitudinal wakefield in
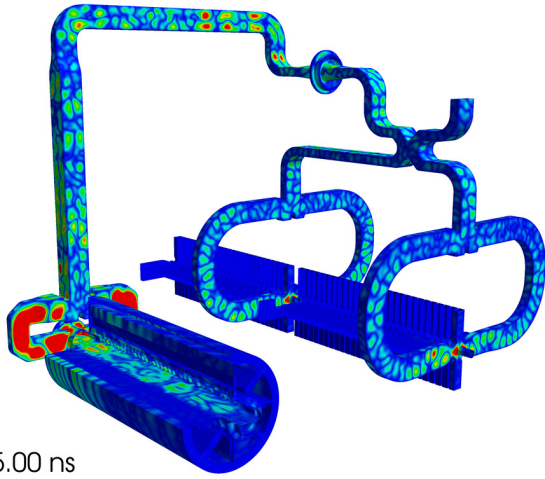
t=15.00 ns

Figure 12: Snapshot of transient electric fields for the *"single dipole X"* case simulated with T3P.

the AS created by a PETS bunch offset can lead to energy spread in the main beam. A significant amount of higher-order trapped modes are visible in the PETS output coupler. First investigations indicate that this might be sextupole modes at around 17 GHz, which are overly excited by the large transverse PETS bunch offset of 5 mm. Proper determination of the source of those transverse wakes is important since the PETS bunch has a $17\times$ larger charge than the main beam and also has more relaxed offset tolerances. Note that the simulation was performed with $p=1$ and with absorbing boundary conditions at the waveguide ports and beampipe ends. To confirm these results, more studies and further T3P simulations with higher accuracy are planned. Similar to the single structure simulations shown above, using broadband waveguide boundary conditions and higher-order basis functions will improve field representation and reduce numerical dispersion errors. Including ohmic wall losses, these high-fidelity T3P simulations are expected to deliver quantitative answers about the wakefield coupling in the CLIC TBA design.
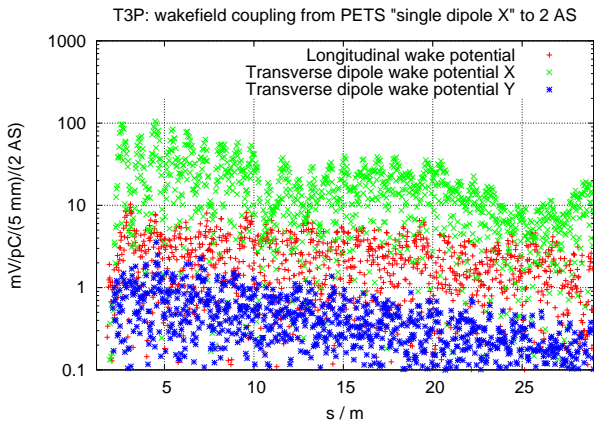


Figure 13: T3P results for wakefield coupling from the PETS to the AS for the *"single dipole X"* case.

## SUMMARY

SLAC's parallel electromagnetic code suite ACE3P employs state-of-the-art parallel 3D Finite Element methods on curved conformal unstructured tetrahedral meshes with higher-order field representation. Operating on DOE supercomputing facilities, the scalable electromagnetic time-domain code T3P allows large-scale modeling of transient effects and wakefields in complex accelerator structures with unprecedented accuracy.

In this study, T3P is used to calculate wakefield effects in the CLIC two-beam accelerator (TBA) concept. Single structure simulations of the PETS and the TD24 accelerating structure (AS) are benchmarked against GdfidL results, and good agreement is found.

For the first time, power transfer and dipole wakefield coupling effects are computed for the fully 3D combined system of one PETS, two TD26 AS and a connecting waveguide network. These first results demonstrate that T3P is ideally suited to calculate broadband impedances of such a complex system, and to allow a wide array of numerical diagnostics with fast-turnaround times. While coupled longitudinal and transverse wakefield effects have been calculated, the results indicate that more work and additional higher-accuracy T3P simulations are needed before definite conclusions can be drawn.

High-fidelity electromagnetic modeling of the TBA concept with ACE3P codes has the potential to significantly benefit the CLIC rf design. It also paves the way to large-scale modeling of challenging future designs.

## ACKNOWLEDGMENTS

## REFERENCES

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